IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

SNAPSHOT, A LONG TERM ECONOMY-WIDE MODEL OF AUSTRALIA : PRELIMINARY OUTLINE

by

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1. INTRODUCTION

In this paper we outline a long term economy-wide model which we propose to implement with Australian data. Our theoretical framework is based on the snapshot idea of Manne, Bruno, Evans and others,[1] i.e., we attempt a detailed description of the economy for a particular year well into the future. The snapshot approach has the advantage of allowing us to analyse the effects of demographic and technological developments while abstracting both from short-run cyclical phenomena and from the dynamics of adjustment paths. In order to restrict further the range of issues with which the model must deal, we treat both the demographic structure and the (after tax) distribution of income in the snapshot year as exogenous. Potential users of the model may find these simplifications acceptable in that the long-run personal distribution of disposable real income tends to be politically

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1. Alan S. Manne, "Key Sectors of the Mexican Economy 1960-70," Chapter 16 in Alan S. Manne and H. M. Markowitz (eds) Studies in Process Analysis (New York : John Wiley and Sons, 1963), pp. 379-400.

   Michael Bruno, "A Programming Model for Israel," Chapter 12 in Irma Adelman and Eric Thorbecke (eds), The Theory and Design of Economic Development (Baltimore : Johns Hopkins, 1966), pp. 327-352.

   H. David Evans, A General Equilibrium Analysis of Protection : The Effects of Protection in Australia (Amsterdam : North-Holland, 1972), pp. xv + 216.

and socially determined, whilst a major effort has already been expended in developing demographic scenarios for Australia.[1]

Our aim is to exploit the strengths of the snapshot approach in dealing with long-run questions. In particular, we propose to use our model to examine the likely effects of demographic and technological changes on

(a) the industrial composition of the economy ;

(b) the appropriate skill composition of the workforce and likely manpower requirements ; and

(c) the Australian standard of living.

Several modifications of the Manne/Bruno/Evans framework will be attempted. The two most important are the following:[2]

(i) The demand effects of changing demography will be taken into account by specifying several "representative consumers," one for each identifiable socio-economic/demographic group.

(ii) Investment behaviour will be modelled in a way which ensures its consistency both with endogenously determined growth rates and with exogenously specified relative rates of return.

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1. Commonwealth of Australia, National Population Enquiry (W. D. Borrie, Chairman), Population and Australia - A Demographic Analysis and Projection (Canberra : Australian Government Publishing Service, 1975), Vols. I & II, pp. xxxiv + 760 (hereafter "the Borrie Report").

2. Computational algorithms for handling both these modifications are described in Peter B. Dixon, The Theory of Joint Maximization (Amsterdam : North-Holland, 1975), and Peter B. Dixon and Matthew W. Butlin, "The Evans Model of Protection : An Interpretation and Review," Paper presented to the Fifth Conference of Economists, Brisbane, 1975.

On the other hand, in this paper we will follow Manne, Bruno and Evans in assuming constant returns to scale. Also, our treatment of international trade will be rudimentary. Models in which there are constant returns to scale, perfectly elastic supply curves for imports and demand curves for exports, and infinite substitution elasticities between domestic and imported commodities,[1] tend to exhibit extreme levels of international specialization.[2] Hence, to prevent our model solutions from being dominated by unrealistic trade patterns, we will set the shares of domestic markets captured by imports exogenously (with base year shares as points of reference), whilst levels of exports will be specified exogenously on the basis of long term projections made by other workers.[3] The solution of SNAPSHOT will, as a result, give

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1. This assumption has been relaxed in the International Monetary Fund's exchange rate model: see J. R. Artus and R. R. Rhomberg, "A Multilateral Exchange Rate Model," I.M.F. Staff Papers, 20, November, 1973. Work along similar lines is also under way at the I.A.C.: see Peter B. Dixon, "The Theoretical Structure of the ORANI Module," Impact of Demographic Change on Industry Structure in Australia, Working Paper 0-01, Industries Assistance Commission, Melbourne, October, 1975. Some of that work may eventually be used to modify SNAPSHOT.

2. In Manne's (op. cit.) early Mexican model exports are set exogenously whilst the solution of the model is generated by the minimization of the foreign exchange cost of loans required to finance imports capable of satisfying exogenously specified demands. For Bruno's (op. cit.) model of Israel, exogenous limits were specified within which exports and imports are free to vary. In H. David Evans' (op. cit.) study of the costs of protection of Australian industries, an attempt was made to specify international trade endogenously. Unfortunately, this was not successful, and in the empirical part of the work the use of a large number of exogenous bounds on growth rates of particular industries led to trade being very nearly exogenous.

3. E.g., F. H. Gruen and others, Long Term Projections of Agricultural Supply and Demand : Australia, 1965 to 1980, Monash University, Department of Economics: Report of a project commissioned by the United States Department of Agriculture, 1967, 2 Volumes, pp. xix + 650 (also published in the U.S. - Washington, U.S.D.A., May, 1968, pp. ix + 480).

the long-run impact of demographic and technological developments in the economy under the assumption of protection policies which would maintain the domestic market shares of local industries. In particular, the labor force demand patterns and industry structure implied by such policies will be seen in the light of recently made demographic projections.[1]

2. MODEL SPECIFICATION

The model can be divided into twelve parts; consumption, investment, capital stock, rates of return, international trade, the price system, the balance of trade, commodity cost structure, product market clearing, labor, wages and GNP. Each will be examined in turn. It should be kept in mind that all of the relationships specified in this section are intended to describe variables of the snapshot year.

2.1 Consumption[2]

The households are divided into m groups. The groups will be chosen to reflect different consumption patterns, with socio-economic characteristics such as age and household size as the basis of definition. The consumption preferences of the different groups will be represented by different utility functions and demand behaviour will be derived from solving standard constrained maximization problems.

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1. Commonwealth of Australia, National Population Enquiry, op. cit.
2. The notation used in this paper is listed in the Appendix.

A variety of functional forms for the utility function $U_i(C_i)$ of consumer group i will be considered in operational versions of the model. Irrespective of the final functional form chosen -- in which regard, the Klein-Rubin function[1] must be considered a strong candidate -- the utility maximizing framework implies demand functions of the form

$$C_i = f_i(p, Z_i) \tag{1.1}$$

where C_i is the consumption vector for consumer group i in the snapshot year, p is the relevant vector of commodity prices and Z_i is total expenditure by group i, which is defined by

$$Z_i = (1 - s_i)\alpha_i(\text{GNP}) \quad , \tag{1.2}$$

where α_i is the share of GNP which is disposable income for group i and s_i is group i's average propensity to save. (Notice that $\sum_i \alpha_i \leq 1$ is the total household disposable income as a fraction of GNP.)[2]

The disposable income shares, α_i, of each group are assumed to be independent of the before-tax earnings of that group, the latter being endogenous in SNAPSHOT. That is, the real after-tax income distribution is set exogenously in the model, with an endogenous set of tax and transfer policies implicit, but not modelled, in the background. To put it another way, the tax and transfer policies are assumed to be whatever is necessary to reconcile the exogenous after-tax income distribution with the endogenous before-tax distribution generated by the solution of the model.

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1. L. R. Klein and H. Rubin, "A Constant-Utility Index of the Cost of Living," Review of Economic Studies, Vol. XV, 1948-49, pp. 84-87.

2. Normally Σa_i would not exceed 1. However, household disposable income theoretically could exceed GNP if foreign capital inflow were large enough.

This is of major importance to an understanding of SNAPSHOT, since it breaks the link between the consumption expenditures of individuals and their before-tax earnings from wages and assets. Of course, the exogenously specified distribution of after-tax real incomes may be varied, and the sensitivity of key variables in the model to this distribution ascertained.

The alienation of the distribution of real disposable personal income from the economic to the social and political arena is consistent with widely accepted interpretations of observed long term patterns in the distribution of personal incomes in western countries. It may be, however, that the continuance of observed trends of income distribution imply that liberal policies of public investment in education continue to be followed.[1]

The s_i may be treated either exogenously or endogenously. For example, s_i may be allowed to depend on relative prices, as in the extended linear expenditure system, ELES.[2] For simiplicity, in this paper the s_i will be taken as exogenous (although extensions to encompass endogenous s_i are contemplated in later work).

2.2 Investment

The capital stock, $\overline{K(0)}$, in each industry at the base year is known and exogenously given to the model. The capital stock available in the snapshot year in industry q is obtained by multiplying the base year

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1. Jan Tinbergen, Income Distribution : Analysis and Policies, (Amsterdam : North-Holland, 1975).

2. Constantino Lluch, "The Extended Linear Expenditure System," European Economic Review, Vol. 4, No. 1 (April, 1973), pp. 21-32.

capital stock by the growth factor $(1 + h_q)^t$ where the endogenous variable h_q is the average rate of growth of capital over the t-year snapshot period. Thus, in matrix notation,

$$(2.1) \qquad K(t) = (I + \hat{h})^t (\overline{K(0)}) ,$$

where K(t) is the n-vector of levels of capital stock by industries in the snapshot year and $(\hat{\ })$ throughout this paper indicates the operation of converting a vector ( ) into the corresponding diagonal matrix.

It is assumed that investment by each industry in the snapshot year is sufficient to maintain the snapshot period rates of growth h_q. Hence capital stocks available in the post snapshot year (t + 1) are

$$(2.2) \qquad K(t + 1) = (I + \hat{h}) (K(t))$$

and gross investments in the snapshot year are

$$(2.3) \qquad J = K(t + 1) - (I - \hat{\eta}) (K(t)) ,$$

where J is the vector of gross investments by using industries, and $\hat{\eta}$ is the diagonal matrix of industry-specific depreciation rates applicable to the capital stocks, K(t), over the t^{th} year.[1]

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1. Depreciation here should be interpreted liberally to include retirement (viz., obsolescence) of the capital stock due to changes in relative prices and technology, as well as physical deterioration of plant. Hence in determining the depreciation rates in particular industries, care will be needed to ensure that account is taken of accelerated obsolescence of plants in industries where major technical improvements are projected.

The capital accumulation process for industry j is illustrated in Figure 1.

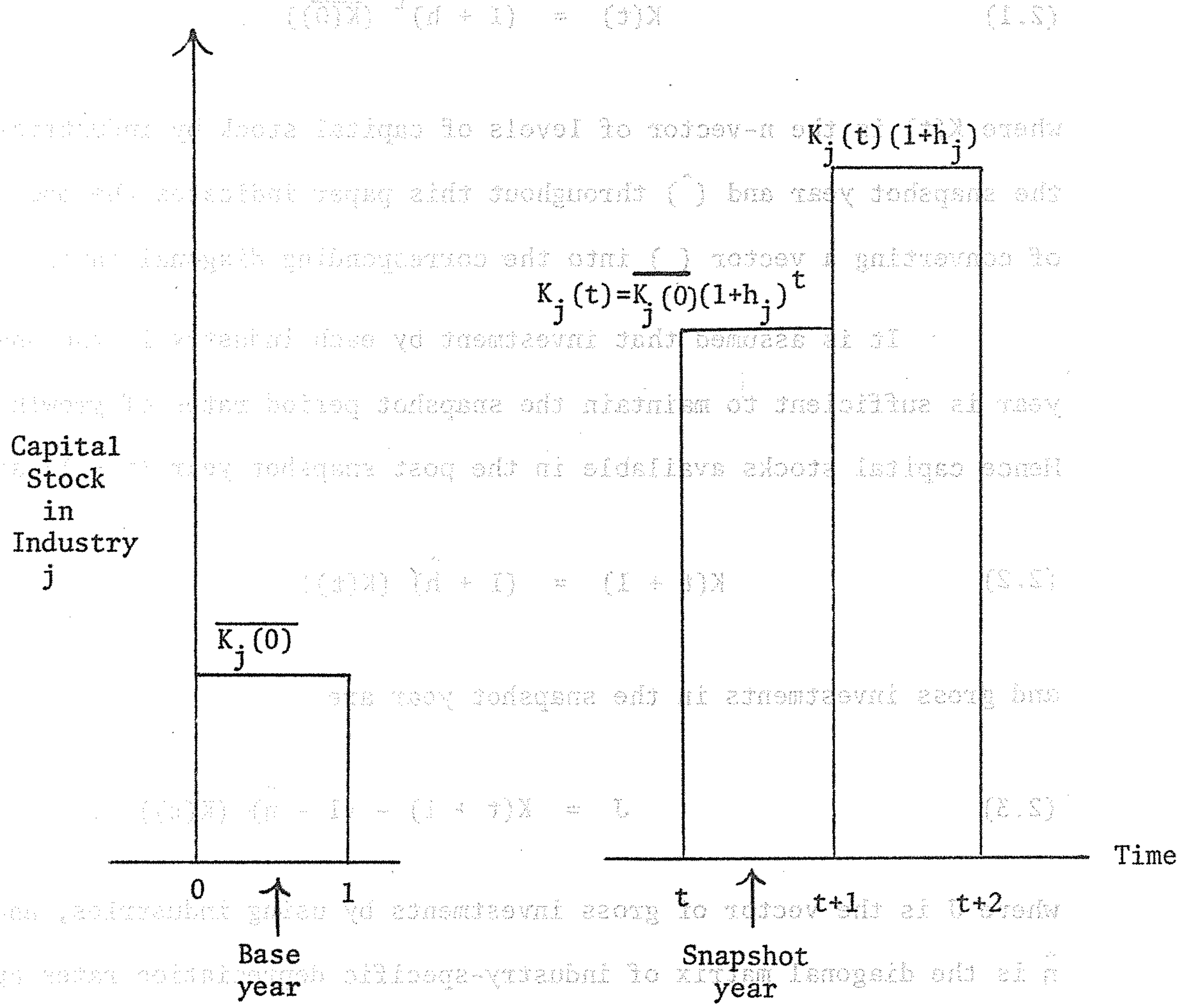

Figure 1

The capital stock, $K_j(t)$, is assumed to be available at the beginning of the snapshot year and is then the capital stock level throughout the snapshot year. On the first day of the year following the snapshot year, the industry's capital stock rises to $K_j(t + 1)$.

2.3 Capital Stock

The vector of industry outputs, X, in the snapshot year is constrained by the availability of capital in each industry. Hence,

(3.1) $$X \leq K(t) \quad ,$$

and

(3.2) $$\hat{\Pi} \, (X - K(t)) = 0 \quad ,$$

(where capital is measured in units of capacity output and Π is the vector of rental prices on capital by industries).

Equation (3.2) means that if X_j were less than $K_j(t)$, then because there would be excess capital stock in industry j, the annual rentals payable on units of capital in industry j would be zero (i.e., $\Pi_j = 0$) .

2.4 Rates of Return

Actual rates of return are defined as annual net rental values of capital per unit cost of constructing capital net of depreciation, i.e.,

(4.1) $$(\Pi_j / p'k_{.j}) - \eta_j$$

for $j = 1, \ldots, n$, where $k_{.j}$ is the j^{th} column of K, K itself being a square matrix of capital coefficients[1];

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1. Notice that K is a square matrix whose $(i, j)^{th}$ element is the quantity of good i required to produce one unit of capacity expansion in industry j. K should not be confused with the vectors of capital stocks K(0), K(t) and K(t + 1) in the base, snapshot and post-snapshot years respectively.

or in matrix notation, the vector of rates of return is

(4.2) $$\widehat{(p'K)}^{-1} \Pi - \eta \quad .$$

The relative rates of return on capital required to induce investment in each industry are exogenously set for the model, i.e., such diverse factors as risk and degree of industry concentration are implicitly taken into account in the model by allowing differences in nominal rates of return between industries (which do not necessarily represent real differences in certainty-equivalent terms). Thus, those factors which cause differences in the base year among the minimum rates of return acceptable to investors in different industries, are assumed to persist and to result in the same relative rates of return being acceptable in the snapshot period. The absolute rate of return, however, is endogenous. Thus,

(4.3) $$r = \beta \begin{pmatrix} \bar{r}_1 \\ \cdot \\ \cdot \\ \cdot \\ \bar{r}_n \end{pmatrix} = \beta \bar{r} \quad ,$$

where r is the vector of minimum acceptable rates of return (by industry), β is an endogenous variable reflecting the absolute rate of return demanded on new capital formation for Australian industry, and $\bar{r}$ is the exogenous vector specifying relativities.

This formulation of the rates of return does allow industries to be affected by changes in profitability caused by demographic and technological change and by changes in protection policy. The effects of these changes, however, are on the size (capital, output, employment) of the industry and not on its rate of return to capital, except to the extent that any such change affects the overall absolute rate of return on Australian capital. Thus, reduced profitability in an industry may cause some firms to leave the industry, reducing investment, output and employment in the industry, but probably not significantly affecting the rate of return to capital for that industry in the long run.

Some support for our adoption of exogenous relative rates of return is provided by the fact that there appears to be no relationship between the rates of return to capital of an industry and its level of tariff protection.[1] The effect of an increase in tariff protection, it seems, is to increase the size of an industry, rather than to raise the rate of return to its capital. Moreover, it is at least of some comfort to note that Evans found that his major results were insensitive to changes in his choice of relative values for the components of r.[2]

It will be recalled that the yield on newly purchased capacity is $(\Pi_j/p'k_{.j}) - \eta_j$. This rate cannot exceed the minimum rate r_j needed to induce investment. (If it were to do so, additional new investment would drive the rental rate down to equality with the rate of return necessary to induce investment.) If the yield on newly purchased capacity in industry j is less than the minimum rate needed to induce investment, then there will be no investment in industry j (i.e., $J_j = 0$) .

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1. Industries Assistance Commission, Annual Report 1973/74, Table 3.4.1 and Table 4.2.3.

2. Evans, op. cit.. The apparent reason for this lack of sensitivity is the relatively small share of the costs of gross output represented by returns to capital.

To summarize,

(4.4) $$r_j \geq (\Pi_j/p'k_{.j}) - \eta_j \qquad (j = 1, \ldots, n) \ ,$$

and

(4.5) $$J_j(r_j - (\Pi_j/p'k_{.j}) + \eta_j) = 0 \qquad (j = 1, \ldots, n) \ .$$

In matrix notation, (4.4) and (4.5) become, respectively:

(4.6) $$r \geq \widehat{(p'K)}^{-1} \Pi - \eta \ ,$$

and

(4.7) $$\hat{J}[r - \widehat{(p'K)}^{-1} \Pi + \eta] = 0 \ .$$

An alternative formulation might allow the rate of return on capital for each industry to be determined by the model. This could result, however, in the model specifying unrealistic relativities between industries. But in the context of long-run analysis, too much faith should not be placed in the preservation of historically observed patterns of yields on capital in different industries. This being said, the base period relativities remain the starting point for our analysis.

2.5 International Trade

As noted in section 1, a major problem in traditional textbook models of international trade is the tendency toward solutions showing excessive international specialization in production. In the present paper we obviate this problem at the expense of making international trade largely exogenous. In fact, for exports E, we postulate an exogenous vector $\bar{E}$ and force

$$(5.1) \qquad E = \bar{E} ,$$

where $\bar{E}$ is a projection based on considerations of trends in world markets and long-run comparative advantage. In the case of import-competing industries, on the other hand, we postulate exogenous import shares of the domestic market, i.e.,

$$(5.2) \qquad M \lesseqgtr \hat{\gamma} X ,$$

where $\hat{\gamma}$ is a diagonal matrix with typical element γ_j being the base year import share in industry j (or, alternatively, some other exogenously given hypothetical set of import shares), M is the (quantity) vector of imports and X (as before) is the vector of domestic outputs.

Some distortions in the price system may be necessary to attain (5.1) and (5.2). One possibility is that there are licences to ration participation in international trade. In the next section, however, we will assume that there is a system of tariffs and export taxes and subsidies which induce (5.1) and (5.2).

2.6 The Price System

We adopt the small country assumption, i.e., foreign currency prices of exports and imports are exogenous. If $\overline{p^e}$ is the f.o.b. vector of foreign currency export prices and $\overline{p^m}$ is the c.i.f. vector of foreign currency import prices, then

$$(6.1) \qquad \overline{p^m} > \overline{p^e} .$$

The gap between $\overline{p^m}$ and $\overline{p^e}$ is caused by transport, insurance and freight costs. "Non-traded" goods (e.g., construction) are characterized by very large gaps between import and export prices (or $\overline{p_j^m}$ close to infinity and $\overline{p_j^e}$ negative).

The domestic price vector p will satisfy two sets of conditions. First,

$$p = \Theta\overline{p^e} + \xi \quad , \tag{6.2}$$

where Θ is the exchange rate ($A per unit of foreign currency) and ξ the vector (endogenously determined) of export taxes (ξ_j positive) or subsidies (ξ_j negative) consistent with the achievement of (5.1). (In the case of a clearly non-exportable commodity (say, commodity j) $\overline{p_j^e}$ would be small, zero or even negative, and the export projection, $\overline{E}_j$, would be zero. Equation (6.2) remains valid, if rather uninformative.) The second set of conditions are

$$p \leqslant \Theta(I + \hat{\bar{\tau}})\, \overline{p^m} + \phi \tag{6.3}$$

$$\hat{M}\, [p - \Theta(I + \hat{\bar{\tau}})\, \overline{p^m} - \phi] = 0 \tag{6.4}$$

$$\hat{\phi}\, [M - \hat{\gamma}X] = 0 \quad , \tag{6.5}$$

where $\hat{\bar{\tau}}$ is an exogenously given diagonal matrix of ad valorem tariff rates, and ϕ is an endogenously determined vector of excess tariff revenues per unit of imports. The imposition of excess tariffs may be necessary to limit imports to be consistent with (5.2). If $M_j < \gamma_j X_j$, then (6.5) implies that $\phi_j = 0$.

For "non-traded" good, j, $\overline{p_j^m}$ will be set very large. Then (6.3) will hold as a strict inequality and (6.4) implies that the import level is zero.

2.7 The Balance of Trade

The balance of trade deficit, expressed in foreign currency is

$$(7.1) \qquad \overline{B} = \left(\overline{p^m}\right)' M - \left(\overline{p^e}\right)' E \ .$$

$\overline{B}$ will be set exogenously at a figure reflecting our projection of average annual long term capital inflow.

2.8 Commodity Cost Structure

The price of a commodity will be less than or equal to the sum of the cost of inputs, wage costs and the rental price of capital to make the good (where the rental price component may contain an element of "pure" profit due to an imperfectly competitive market structure). Should price be less than the costs of production then this implies that the commodity would not be produced. That is,

$$(8.1) \qquad p'(I - A) - w'\ell - \Pi' \leq 0$$

and

$$(8.2) \qquad [p'(I - A) - w'\ell - \Pi'] \hat{X} = 0 \quad ,$$

where A is the input-output coefficient matrix, w is the H-vector of wage rates by occupation, and ℓ is the H × n matrix of labor requirements by occupation and industry per unit of output in the snapshot year.

2.9 Product Market Clearing

Domestic production plus imports of each good must be at least sufficient to satisfy demands for consumption, investment, government purchases, exports, and intermediate usage. That is,

(9.1) $$X + M \geq \sum_{i=1}^{m} C_i + KJ + \overline{G} + E + AX \quad ,$$

where $\overline{G}$ is the (exogenous) vector of government commodity purchases. If supply for any good exceeds demand, then the relevant price is zero. Hence,

(9.2) $$\hat{p}\left[X + M - \sum_{i=1}^{m} C_i - KJ - \overline{G} - E - AX\right] = 0 \quad .$$

2.10 Labor

The total number of people in the workforce ($\overline{N}$) in the snapshot year is specified exogenously, and hence if L (= $(L_1, \ldots, L_H)'$) is the vector of labor of different occupational types demanded by the economy in the snapshot year, the assumption of full employment (which we make for the total workforce) implies

(10.1) $$\overline{N} = \underline{1}'L$$

(where $\underline{1}$ is a vector of H units).

One of the aims of SNAPSHOT is to indicate likely long term manpower needs. Hence the percentage of the workforce needed in different occupations o (o = 1, ..., H) in the snapshot year are endogenous; that is, the ratios (L_o/N; o = 1, ..., H) are endogenous.

On the technological side, the matrix ℓ of labor requirements by occupation per unit of output in each industry in the snapshot year is specified exogenously. The vector of labor demands induced by the (endogenously determined) vector of gross outputs X is ℓX; i.e.,

(10.2) $$L = \ell X .$$

The exogenous total labor supply (10.1) therefore implies

(10.3) $$\underline{1}'\ell X = \overline{N}$$

as a constraint on total output, X .

2.11 Wages

In seeking to explain movements in Australian wage relativities, Hancock and Moore[1] concluded that the lesson to be learnt from the observed compression in wage relativities in the early 1960's and a widening in relativities to 1972 was not that statements

> "about secular stability should be substituted for earlier claims about secular compression. ... Both inferences involve the fallacy of supposing that some arbitrarily selected historial wage structure shares with the existing wage structure the property of lying on a long-run equilibrium path."

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1. K. Hancock and K. Moore, "The Occupational Wage Structure in Australia since 1914," British Journal of Industrial Relations, Vol. X, 1972, pp. 107-122.

They conclude by saying that

> "the factors shaping Australia's occupational wage structure (insofar as it can be described in terms of institutionally prescribed rates) have included a tendency to allow the skilled worker's rate to bear the brunt of economic adjustments; a tendency for differentials to expand at times of falling prices and to contract in inflation; and a strongly held belief in a social minimum. None of these, however, can be regarded as a constant force : the intensity of each has varied with the personnel and attitudes of the arbitration tribunals. In 1971 the unknown in the situation is the possibility that the social minimum - in the form of the minimum wage - may prove to be highly dynamic. If this occurs, the wage structure is likely to become more compressed. It should be said, however, that over-award payments are currently growing rapidly. As market rates diverge further from prescribed rates, the significance of the latter is steadily diminished."

From this work and similar conclusions from other work[1] in the field, it seems that a reasonable assumption to make about wage relativities over the long run is that they will remain constant. There simply is no

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1. B. Hughes, "The Wages of the Strong and the Weak," The Journal of Industrial Relations, Vol. 15, 1973, pp. 1-24. Hughes comments that "action by strong unions to establish a relative wage advance will tend to be frustrated by matching wage increases elsewhere. Secondly, it may be doubted that social forces will allow the exploitation of large, relative wage gains by strong unions." In K. Hancock and B. Hughes, "Relative Wages, Institutions and Australian Labor Markets," The Flinders University of South Australia, Institute of Labor Studies, Working Paper Series, No. 1, May, 1973, the conclusion is reached "that wage differences are not related to power variables in any simple and general fashion."

documented basis for making any assumption about factors shaping Australia's occupational wage structure to any particular widening or narrowing of wage relativities. In the absence of such information the assumption of fixed wage relativities appears no worse than any other assumption. As well, the exogenous treatment of wage relativities means that alternative scenarios on relative wages are easily handled, provided they are not heavily dependent on variables endogenous to SNAPSHOT. If δ is the (endogenous) scalar determining the absolute level of wages, and $\overline{w}$ is the vector of (exogenously given) wage relativities, then the vector w showing wages for the H occupational groups in the snapshot year is:

$$w = \delta(\overline{w}) \,. \tag{11.1}$$

2.12 Gross National Product

The GNP for the snapshot year is the sum of wages, rents on capital and tariff revenue less export subsidies, i.e.,

$$GNP = w'L + \Pi'K(t) + \left[\Theta(\overline{p^m})'\hat{\overline{\tau}} + \phi'\right] M - \xi'E \,. \tag{12.1}$$

It is reassuring to check that our model satisfies the national income identity, i.e., GNP plus the deficit on the balance of trade equals aggregate expenditure (by households, for investment, and by the government).

From (9.2) we have

$$p'(I - A)X + p'M - p'E = p' \sum_{i=1}^{m} C_i + p'KJ + p'\overline{G} \,. \tag{12.2}$$

Then using (8.2), (10.2), (3.2), (6.2), (6.4) and (12.1) we can rewrite (12.2) as

$$\text{(12.3)} \qquad GNP + \Theta\left[(\bar{p}^m)'M - (\bar{p}^e)'E\right] = p'\sum_{i=1}^{m} C_i + p'KJ + p'\bar{G} \ .$$

3. SUMMARY AND CONCLUSION

The equations of the model are summarized in Table 1.

TABLE 1 : SUMMARY OF EQUATIONS SPECIFYING THE SNAPSHOT MODEL

Equation No.	Equation	No. of Equation Equiva-lents	Description
(1.1)	$C_i = f_i(p, Z_i)$, $i=1,\dots,m$	nm	Consumer demand functions
(1.2)	$Z_i = (1 - s_i)\alpha_i(GNP)$	m	Level of total private expenditure
(2.1)	$K(t) = (I + \hat{h})^t\ (\overline{K(0)})$	n	Capital stocks in snapshot year
(2.2)	$K(t + 1) = (I + \hat{h})\ (K(t))$	n	Post-snapshot year capital stocks
(2.3)	$J = K(t + 1) - (I - \hat{\eta})(K(t))$	n	Gross investments
(3.1)	$X \leq K(t)$, and	n	Capacity constraint and complementary slack condition
(3.2)	$\hat{\Pi}(X - K(t)) = 0$		
(4.3)	$r = \beta\bar{r}$	n	Absolute rate of return on capital

continued ...

Table 1 continued ...

Equation No.	Equation	No. of Equation Equiva-lents	Description
(4.6)	$r \geqslant \widehat{(p'K)}^{-1}\,\Pi - \eta$	n	Rate of return on capital and complementary slack condition
(4.7)	$\hat{J}\,(r - \widehat{(p'K)}^{-1}\,\Pi + \eta) = 0$		
(5.1)	$E = \overline{E}$	n	Level of exports
(5.2)	$M \leqslant \hat{\gamma}X$, and	n	Import restraint and complementary slack condition
(6.5)	$\hat{\phi}\,(M - \hat{\gamma}X) = 0$		
(6.2)	$p = \Theta\overline{p^e} + \xi$	n	Export price equation
(6.3)	$p \leqslant \Theta(I + \hat{\overline{\tau}})\overline{p^m} + \phi$, and	n	Import price equation and complementary slack condition
(6.4)	$\hat{M}\left[p - \Theta(I + \hat{\overline{\tau}})\overline{p^m} - \phi\right] = 0$		
(7.1)	$\overline{B} = (\overline{p^m})'M - (\overline{p^e})'E$	1	Balance of trade definition
(8.1)	$p'(I - A) - w'\ell - \Pi' \leqslant 0$	n	Commodity cost structure and complementary slack condition
(8.2)	$\left[p'(I - A) - w'\ell - \Pi'\right]\hat{X} = 0$		
(9.1)	$X + M \geqslant \sum_{i=1}^{m} C_i + KJ + \overline{G} + E + AX$	n	Product market clearing and complementary slack condition
(9.2)	$\hat{p}\left[X + M - \sum_{i=1}^{m} C_i - KJ - \overline{G} - E - AX\right] = 0$		
(10.1)	$\overline{N} = 1'L$	1	Full employment of total labor force
(10.2)	$L = \ell X$	H	Production labor requirements
(11.1)	$w = \delta(\overline{w})$	H	Sets wage relativities
(12.1)	$GNP = w'L + \Pi'K(t) + \left[\Theta(\overline{p^m})'\hat{\overline{\tau}} + \phi'\right]M - \xi'E$	1	Gross national product

Total Number of Equations $(12 + m)n + m + 2H + 3$

The endogenous variables are:

C_i, p, Z_i, K(t), h, GNP, K(t+1), J, X, Π, r, β, E, M,

ϕ, Θ, ξ, w, L, δ, numbering nm + n + m + n + n + 1 + n

n + n + n + n + 1 + n + n + n + 1 + n + H + H + 1

= (12 + m)n + m + 2H + 4 in all.

The exogenous variables, behavioural and technological parameters are $\overline{N}$, $\overline{r}$, $\overline{p^e}$, $\overline{p^m}$, $\overline{\tau}$, $\overline{B}$, $\overline{G}$, $\overline{w}$, $\overline{E}$, $\overline{K(0)}$, t, α_i, s_i, η, K, γ, ℓ, and A .

The number of equations can be increased by one via homegeneity. If the variables comprising a solution for the model adopt values (C_i^*, p^*, Z_i^*, $K(t)^*$, h^*, GNP^*, $K(t+1)^*$, J^*, X^*, Π^*, r^*, β^*, E^*, M^*, ϕ^*, Θ^*, ξ^*, w^*, L^*, δ^*), then (C_i, λp^*, λZ_i^*, $K(t)^*$, h^*, λGNP^*, $K(t+1)^*$, J^*, X^*, $\lambda\Pi^*$, r^*, β^*, E^*, M^*, $\lambda\phi^*$, $\lambda\Theta^*$, $\lambda\xi^*$, λw^*, L^*, $\lambda\delta^*$) is also a solution, where λ is any positive scalar. Our model says nothing about the absolute price level. Thus one variable, say δ, can be assigned an arbitrary value (say unity). Hence we can add the equation

$$\delta = 1 .$$

This increases the number of equations to (12 + m)n + m + 2H + 4, equal to the number of endogenous variables.

Whilst we recognize that equality of the number of endogenous variables and the number of equations is neither necessary nor sufficient for the existence of a solution, it is nevertheless true that in the majority of well behaved and interpretable economic models, this condition is met.

In this paper we have said nothing about a solution procedure for the model, concentrating rather on its behavioural, technological and accounting specification. This procedure is in marked contrast to the approach in the development programming models[1] where it is conventional to specify an objective function for optimization as part of the economic design of the model. While such objective functions will doubtless prove important in the design of an algorithm to obtain a solution to SNAPSHOT, we should emphasize once again that SNAPSHOT includes more than one economic agent and that separate objective functions are employed at least in the case of the m representative consumers distinguished.

The algorithm which we plan to use is Dixon's jointmax.[2] Details of how that algorithmic approach will be used to solve SNAPSHOT constitute the subject matter of a separate forthcoming paper.[3]

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1. E.g., Manne, op. cit., Bruno, op. cit., and Evans, op. cit..

2. Peter B. Dixon, The Theory of Joint Maximization, op. cit..

3. Peter B. Dixon, "A Jointmax Algorithm for the Solution of SNAPSHOT," (forthcoming).

APPENDIX

DEFINITION OF NOTATION

Endogenous Variables in the Snapshot Year

		Number of Variables
C_i	consumption of commodities by consumer group i	(nm)
p	commodity prices	(n)
Z_i	total expenditure of consumer group i	(m)
GNP	gross national product	(1)
h	average rate of growth of capital in each industry over the t-year snapshot period	(n)
K(t)	industry levels of capital stock in the snapshot year	(n)
K(t+1)	industry levels of capital stock in the year after the snapshot year	(n)
J	gross investments by using industries	(n)
X	outputs of commodities	(n)
Π	rental prices on capital by industries	(n)
r	minimum acceptable rates of return by industry	(n)
β	variable reflecting the absolute rate of return demanded on new capital formation for Australian industry	(1)
E	exports of commodities (quantity)	(n)
M	imports of commodities (quantity)	(n)
Θ	exchange rate ($A per unit of foreign currency)	(1)

continued ...

		Number of Variables
ϕ	excess tariff revenue per unit of imports	(n)
ξ	export tax (ξ_j positive) or subsidy (ξ_j negative)	(n)
w	wage rates by occupation before taxes	(H)
L	the number of labor units in each occupational group in the snapshot year	(H)
δ	variable reflecting the absolute level of wages before taxes for the Australian labor force	(1)

Exogenous Variables in the Snapshot Year

		Number of Variables
s_i	consumer group i's average propensity to save out of disposable income	(m)
α_i	share of GNP which is disposable income for group i	(m)
$\overline{K(0)}$	industry levels of capital stock in the base year	(n)
t	number of years of the snapshot period	(1)
η	industry specific depreciation rates applicable to the the industry capital stocks, K(t), over the t^{th} year	(n)
K	capital matrix in the snapshot year, K_{ij} is the input of good i required to create a unit of capital stock for industry j	(n × n)
$\bar{r}$	relative rates of return on capital required to induce investment in each industry	(n)
$\bar{E}$	exports of commodities	(n)
γ	import shares of the domestic markets	(n)
$\bar{p}^e$	export prices (f.o.b.) in foreign currency	(n)
$\bar{p}^m$	import prices (c.i.f.) in foreign currency	(n)

continued ...

		Number of Variables
$\bar{\tau}$	ad valorem tariff rates	(n)
$\bar{B}$	balance of trade deficit in foreign currency	(1)
A	input-output coefficients matrix	(n × n)
ℓ	labor requirements by occupation and industry per unit of output in the snapshot year	(H × n)
$\bar{G}$	government purchases of commodities	(n)
$\bar{N}$	total number of people in the workforce in the snapshot year	(1)
$\bar{w}$	relative wage rates, before taxes, for the various occupational groups	(H)

In addition to the above list of exogenous variables, U_i, the utility function for the i^{th} consumer group, will be exogenously specified.