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SHORT-RUN MACROECONOMIC CLOSURE OF ORANI : AN ALTERNATIVE TO THE IMPACT PARADIGM

by

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ABSTRACT

In this paper, a small stylized CGE model solved using Johansen's method sequentially is presented. This model, called MMO, illustrates an alternative to the so-called Impact paradigm for dealing with ORANI's lack of short-run macroeconomic closure. The approach embodied in MMO does not rely upon the assumption of micro/macro separability. Instead, it achieves the desired objective by incorporating financial assets directly into the decision-making processes of individual agents and by having market clearing relationships for financial stocks as well as for monetary and real commodity flows.

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Philip D. Adams*

1 Introduction

The ORANI module of the IMPACT project is a very large example of the Johansen (1960) class of computable general equilibrium (CGE) model.[1] Like all models built in the Walrasian tradition, ORANI concentrates upon the representation of real commodity flows and abstracts from nearly all financial considerations when projecting the consequences of any exogenous shock. This neglect of monetary phenomena is one of the reasons why there are no mechanisms in the model suitable for determining:

'(I) the extent to which induced changes in the real exchange rate will be realised as changes in the domestic inflation rate relative to the foreign rate or as changes in the nominal exchange rate;

(II) the extent to which induced changes in the buoyancy of the labour market will be realised as changes in real wages or as changes in employment;

[and]

(III) the extent to which changes in national income will be realised as changes in aggregate absorption or as changes in the balance of trade' (Cooper, McLaren and Powell, 1985, pp. 415-6).

Since there is no financial sector and because the user must routinely set exogenous many of the important macroeconomic variables listed in (I) through (III) above, ORANI is said to lack a short-run macroeconomic closure. Similarly, the model also lacks a long-run macroeconomic closure. This means that even if, from the long-run perspective, a dichotomy exists between the real sectors modelled by ORANI and the financial sectors presently not modelled, there are still insufficient mechanisms for endogenizing every macroeconomic variable simultaneously with the large set of real structural variables upon which the model concentrates.

Without a short-run macroeconomic closure, ORANI cannot take care of the necessary national income accounting for the large aggregates of the economy in a way which recognises the existence of a business cycle. This apparent shortcoming is by design and two separate methods have been developed to deal with it. The first, documented in a series of recent articles by Russel Cooper, Keith McLaren and Alan Powell, assumes separability of the macro from the microeconomy and maintains that in the short-run

> '... financial and money markets, as well as fiscal actions, are only important for individual industries insofar as they exert a real effect on the big components of national income: namely, private consumption, private capital formation and government spending' (Powell 1981, p. 242).

On the basis of these assumptions the authors have developed a two-level treatment known as the 'Impact paradigm'.[2] At the level of the macroeconomy, a continuous-time macrodynamic model endogenises important business cycle aggregates, which, in turn, are used as control totals for a conditional general equilibrium disaggregation of the economy by

ORANI.[3] Although this approach has been implemented successfully, there remain one or two inherent problems that limit its general acceptance. The most important of these is inconsistency in aggregation (i.e., there are macrorelations in the macro module which cannot be derived as explicit aggregates of microrelations in ORANI-- see Cooper, McLaren and Powell, 1985, pp. 417-8).

The second approach, developed by Meagher and Parmenter (1985), is less ambitious than the first. The aim of these authors is to enhance the ORANI structural form with a representation of the Australian fiscal system without having to introduce additional financial variables into the model. This enables them to keep the number of distinctively macroeconomic elements to a select minimum; whereas in the Impact paradigm the macro model used to close ORANI is accepted as a fully specified unit. In the Meagher/Parmenter or national and government accounts (NAGA) closure of ORANI it is common to exogenize the public sector borrowing requirement rather than aggregate absorption. This accords better with the realities of the exercise of macroeconomic policy.

The long-run neutrality of money is a maintained position by the ORANI model builders.[4] This reduces the number of issues to be addressed in the model's long-run macroeconomic closure (the development of which has recently been completed).[5] Most effort has been directed towards providing the model with the ability to determine what proportion of a shock-induced change in the aggregate capital stock is financed by overseas capital inflow as opposed to domestic saving. ORANI is augmented with equations allowing the endogenisation of aggregate consumption and hence of aggregate investment. These equations include expressions explaining the domestic share in the ownership of capital, the level of domestic savings, and the rate of growth in real domestic saving over the period between the base year and the long-run solution year.[6]

The purpose of this paper is to present a small stylised CGE model which illustrates an alternative approach for providing ORANI with a short-run macroeconomic closure (and conceptually even a long-run macroeconomic closure). This approach, which Cooper, McLaren and Powell (1985) call the 'extended Walrasian paradigm', introduces financial flows and stocks directly into the decision-making processes of individual agents and enlarges the number of markets typically considered in a CGE model to include those for financial assets.[7] The model, called MMO, is based wherever possible upon 'skeletal ORANI' -- the teaching device described fully in Dixon, Parmenter, Sutton and Vincent (1982, ch.2).

The 'extended Walrasian paradigm' is the cleanest possible method for integrating real and financial sectors within a CGE framework and for providing such models with a macroeconomic closure. Its theoretical foundations can be traced back to Walras himself who, faced with the problem of explaining why individuals hold cash balances within a general equilibrium system, treated money as if it were a consumption good by adding it to the utility function (see Walras, 1900, p. 297). The theory has been subsequently developed by Patinkin (1965), Friedman (1956), Tobin (1969 and 1980), Niehans (1978) and Grandmont (1983), along with a host of others. Perhaps most important among the contentious issues emerging from these studies has been the rationales used to explain why individuals choose to hold a diversity of financial assets including low-yield obligations of the government as well as (potentially) high-yield securities issued by the private sector. In MMO we have incorporated uncertainty of future asset yields to explain this aspect of real-world behaviour. Thus as in the Tobin (1958) and Markowitz (1959) mean-variance model of portfolio behaviour, investors in MMO are always faced with the choice between a greater proportion of low-risk assets offering low expected yields and a greater proportion of relatively high-risk assets offering correspondingly high expected yields.

The remainder of this paper is organised as follows. In section 2, the solution procedure proposed for MMO is explained. In section 3, the theoretical structure of the model is presented; while section 4 contains concluding remarks and some suggestions for future research. Certain coefficients of the model are defined in Appendix A, and a model summary and variable list are provided in Appendices B and C, respectively.

2 The Sequential Johansen Solution Procedure

The most common form of ORANI solution has been described as a 'contemporaneous differential comparative static' (or CDCS solution, for short -- see Cooper, McLaren and Powell, 1985, p. 417). As with the macrodynamic method, attention is focussed upon deviations from 'control'. More specifically, an ORANI CDCS solution has the following form: given a shock in the exogenous variables introduced at some predetermined base year, endogenous variable i will differ in the short-run (i.e., after about two years) by x per cent from the value it would have had in the absence of the shock; while in a more distant future year (the long-run) it will differ by z per cent from the value it would have taken in that year. For ORANI, the most commonly reported short-run results are generally of the neo-classical type; although results with a more 'neo-Keynesian' flavour can be obtained (see, for example, Wright and Cowan, 1980). The short-run solutions favoured by the Impact team allow for a rigid labour market, but are otherwise neo-classical, reflecting the economy's adjustment to the initial disturbance within an environment where industry-specific capital stocks are exogenous and industry rates of return are allowed to vary endogenously. The long-run results (again, neo-classical) are achieved by setting an environment that allows industry capital stocks to vary and eventually to eliminate the initial disturbances in rates of return.

Another form of ORANI solution, a 'forecast', has been developed recently by Dixon, Parmenter and Horridge (1986). The difference between it and a CDCS solution is illustrated in Figure 1.1. Period 0 represents the base year and period t, the solution year. The line AB depicts the adjustment path through time of an endogenous

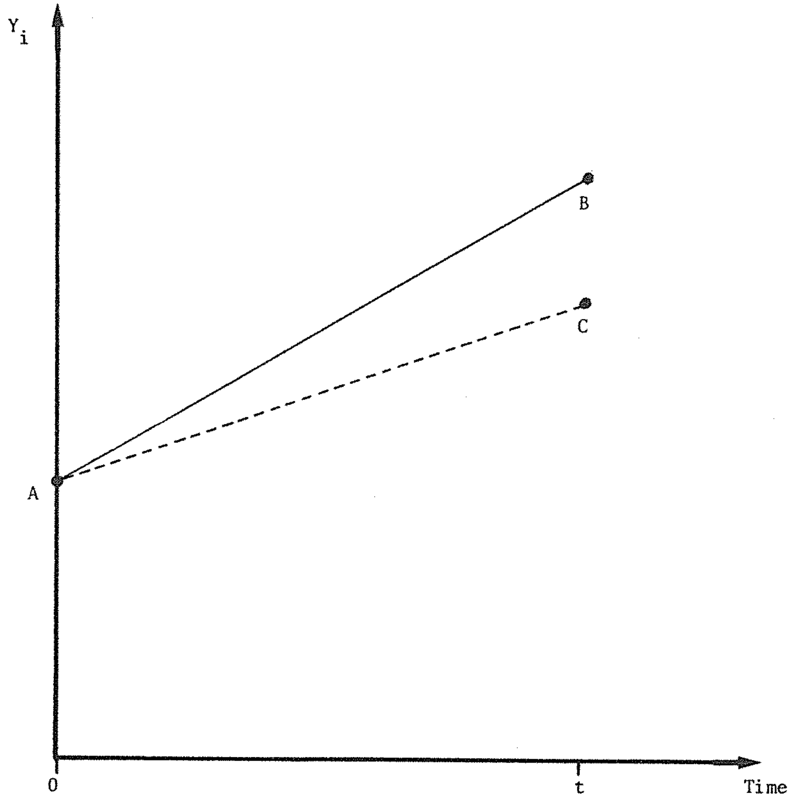

Figure 1.1 : The shock and control paths of endogenous variable Y_i

variable Y_i following an exogenous shock injected in the base year. If the shock had not occurred, then Y_i would have followed its control path depicted by the line AC. To compute a CDCS solution for Y_i we need only measure the gap CB at year t. Clearly such a solution need say nothing about the difference (C-A). On the other hand, the forecasting solution for Y_i requires us to estimate what happens between year 0 and year t (i.e.,between A and B in the presence of the shock, or between A and C in its absence).

The remainder of this paper deals exclusively with CDCS solutions. This type of solution is the most appropriate model-based input to policy analysis for which the model developed in a later section of this paper is designed.

We will see shortly that MMO is a dynamic model, containing a number of variables the current values of which depend on past values for themselves and/or other variables. For instance, there are equations explaining expectations of the future as functions of events in the recent past, and equations enforcing stock/flow consistency through time. To use the model for anything other than short-run CDCS analysis in the presence of so many dynamic equations requires considerable information about the effects of the shock in periods prior to the one being considered. Ideally, this information should be consistent with the predictions of the model for those periods. One way to ensure a measure of such consistency is to solve the model using a sequential form of Johansen's (1960) solution algorithm.[8] This method generates a sequence of linked annual CDCS solutions which will have imposed upon it consistency over time. Whether this sequence of solutions can be interpreted as a path along which the model is moving towards a global state of long-run equilibrium (i.e., a path upon which each successive solution represents a longer and longer run outcome, in the sense that more and more variables are free to adjust) is questionable. If such an interpretation was possible, then MMO would not only achieve a short-run macroeconomic closure, but as well, a long-run macroeconomic closure.

To describe the sequential Johansen solution procedure we must first establish some of the basic features of the economy being modelled by MMO. We begin by dividing the space of time into infinitely many discrete periods, each of identical length (not necessarily equivalent to a calendar year). In this space there exists an economy with an incomplete set of futures markets: wherein at each period, all agents make decisions in light of current events conditional upon their perception of what the future will look like. These expectations cover a specific (to each agent) subset of system variables and are not necessarily rational. For example, they may be formed adaptively (as in Cagan, 1956 or Nerlove, 1958), so that the expectation made at t of a price in t+1, $E_t(P(t+1))$, is generated by:

$$E_t(P(t+1)) = E_{t-1}(P(t)) + \gamma\,[P(t) - E_{t-1}(P(t))] \quad ,$$

where:

γ is a positive parameter ($0 < \gamma < 1$);

and

$P(t)$ is the actual price in period t.

This economy is said to have attained a state of <u>temporary equilibrium</u> in every period, whenever all markets existing in that period are cleared regardless of whether expectations held for that period have, or have not, been realised.[9] A temporary equilibrium therefore can be likened to a 'snapshot' at a given point of time of a fully dynamic economy evolving through time. It differs from the equilibrium of a static system for two reasons:

(i) 'each "period" is not self contained; it is constrained by past history and by the state of expectations about the future;

[and]

(ii) it is generally not a position of rest but is subject to certain dynamic forces; if x_t denotes the position of a temporary equilibrium, it has, within it, certain systematic forces $\dot{x}_t$ so that a temporary equilibrium can be properly understood only in terms of the pair (x_t, $\dot{x}_t$)' (Nagatani 1981, p. 34).

MMO represents this economy in its state of temporary equilibrium at some period t, by a M vector of non-linear implicit behavioural functions, F_t, such that:

$$F_t(U(t-1),\ U(t)) = 0 \qquad (2.1),$$

where:

$U(\tau)$ is a vector of current variables (if $\tau=t$) or of variables lagged one period (if $\tau=t-1$).[10]

The model (2.1) consists of M mathematically independent equations in 2N variables -- N current and N lagged. The lagged variables are arguments of the dynamic equations in the model which consist mainly of expectation functions that combine information from the past (in this case (t-1)) and present (t) to predict the future. Other equations in the model impose conditions at period t, such as: equality between (flow and stock) demand and supply on all current markets; prices reflect costs; capital investment is financed from specified sources outside the investing industries; and the behaviour of consumers and producers is determined from the maximization of objective functions subject to relevant intertemporal constraints.

To find a solution of (2.1) we follow the approach of Johansen (1960).[11] First, we approximate the non-linear structural form of the model at period t by expressing all variables as proportional changes; thereby yielding the following linear system of M equations in 2N variables:

$$A_t(t-1)\ u(t-1) + A_t(t)\ u(t) \qquad (2.2),$$

where:

$A_t(\tau)$ is a (M×N) matrix of coefficients each of which is a function (specific to period τ) of the initial values of the variables in period τ $\{U(\tau),\ U(\tau-1)\}$;

and

$u(\tau)$ is a N vector of current variables (if τ=t) or of variables lagged one period (if τ=t-1), expressed as proportional changes.

Together, the matrices $A_t(t-1)$ and $A_t(t)$ comprise the model's coefficient tableau for t. Elements of this tableau consist typically of shares (such as cost and sales shares at t) derived directly from the period t data base, and of miscellaneous parameters (for example CES substitution elasticities) which have, where possible, been estimated econometrically. The model's period t data base consists of input/output data pertaining to both t-1 and t. These data represent the 'control' (or ceteris paribus) input/output position of the economy at t (and at t-1).[12] As the model moves through time these data will require frequent updating. In contrast, the econometrically estimated parameters remain unchanged throughout a simulation.

If $M < N$, that is, if the number of mathematically independent equations in the model is less than the number of current variables, then $G = N - M$ current variables must be chosen as exogenous to the system (the N lagged variables at t are treated as predetermined). The remaining current variables are called endogenous variables at t.[13] The user does not necessarily have to make a once-and-for-all choice of which current variables shall be exogenous. Each such choice of course, corresponds to a different thought experiment by him/her. A great advantage of the Johansen solution method in general, is that it facilitates moving from one closure of the model to another; that is, it makes it easier operationally for the user to alter his/her choice of the endogenous:exogenous split.

An economically meaningful choice of current variables to be made exogenous, in the sense that an arbitrary assignment of values to the exogenous variables (current or lagged) at t cannot violate any equation of the model, permits (2.2) to be rewritten, as:

$$C_1(t)y(t) + C_2(t)x(t) = 0 \quad , \qquad (2.3),$$

where:

$y(t)$ is the M vector of proportional-change variables chosen from the set $u(t)$ to be endogenous to the system at t;

$C_1(t)$ is a $(M \times M)$ matrix of coefficients chosen from the columns of $A_t(t)$ which correspond to the M current endogenous variables;

$x(t)$ is the $(G+N)$ $(=(2N - M))$ length vector of proportional-change variables which are predetermined at t;

and

$C_2(t)$ is a $(M \times (G+N))$ matrix of coefficients consisting of all N columns from the matrix $A_t(t-1)$, and the G columns of $A_t(t)$ corresponding to the exogenous variables at t.

Rearrangement of (2.3) yields the solution for $y(t)$:

$$y(t) = C_1(t)^{-1}C_2(t)x(t) \qquad (2.4),$$

where the $(M \times (G+N))$ matrix $C(t) = [C_1(t)^{-1}C_2(t)]$ is a matrix of elasticities. The (i,j)'th element of $C(t)$ is the elasticity of endogenous variable i at t with respect to predetermined variable j.[14]

Before the user of MMO can conduct a CDCS thought experiment using (2.4), he/she must first define a <u>base period</u>, choose a <u>suitable closure</u> of the model, formulate a <u>shock</u>, and then assemble a set of <u>coefficient tableaux</u> to allow a solution to be computed.

The base period, say period s, is here defined as the date of reference separating the economy's known past from its unknown future.

For most practical purposes, we can define s to be the most recent period in the economy's life for which a well-documented data base for the model -- the base period data base -- is available.

When the base period has been identified, the user can then choose a suitable closure. The closure of the model will also set the length of run, r; that is, the number of periods separating the end of the base period from the beginning of the solution period. For instance, take a closure for MMO that replicates wherever possible the ORANI neo-classical short-run closure (achieved in part by fixing industry capital stocks and allowing industry rates of return to vary endogenously). Then, the length of run plus the solution period would be just long enough to allow one to ignore the impact of the shock on industry capital stocks in use. For ORANI this length of run corresponds to a calendar time of about two years -- see Cooper, McLaren and Powell (1985).

Once the base period and suitable closure have been selected, the user then can formulate a shock consistent with the particular CDCS thought experiment he/she wishes to conduct. The shock can be conveniently represented as a matrix, say Z(s). Each column of Z(s) corresponds to a period in the economy's 'present or future' (i.e., s, s+1, s+2, ...), and each row corresponds to an exogenous variable (there are G rows). The elements of Z(s) are interpreted as proportional deviations from control: the (m,n)'th element is the proportional deviation in the exogenous variable m from its control value at the end of period n (i.e., its value at the end of n predicted under *ceteris paribus* assumptions for the evolution of the economy up to and including period n).

The final ingredient necessary to compute a solution of MMO using 2.4 is the coefficient tableau for the solution period. This tableau is made up of miscellaneous parameters and shares derived from the model's data base for that period. As we have seen, these data are functions of the variables of the system along a control path which, in general, cannot be known ex ante. However, in some applications of MMO, it may be sufficient to assume that if the length of run is short enough, any differences between the well-documented base period data base and the subsequent solution period data base can be ignored.[15] For the present we assume that such an r is implied by any closure of MMO that holds industry capital stocks fixed. Thus, given s, Z(s) and such a 'short-run closure', we can assemble a solution tableau for s+r+1 and compute a solution of the model for that period, as:

$$\begin{bmatrix} y_1(s+r+1) \\ k(s+r+2) \end{bmatrix} = C(s+r+1) \begin{bmatrix} x_1(s+1+r) \\ x_1(s+r) \\ y_1(s+r) \\ k(s+r+1) \\ k(s+r) \end{bmatrix} \qquad (2.5),$$

where:

$\begin{bmatrix} y_1(s+r+1) \\ k(s+r+2) \end{bmatrix}$ is the vector of current endogenous variables previously denoted y(s+r+1);

k(s+r+2) is a vector of proportional changes in capital stocks at s+r+2 (notice that we are treating the period s+r+2 capital stocks as endogenous at s+r+1 on the assumption that the flow of investment in s+r+1 is endogenous and has no effect on capital stocks until the beginning of the next period);

$x_1(s+r+1)$, $x_1(s+r)$ and $y_1(s+r)$, respectively, are the components of $x(s+r+1)$ corresponding to current and lagged exogenous variables (excluding capital stocks at s+r+1 and s+r) and lagged endogenous variables;

and

$k(s+r+1)$ and $k(s+r)$, respectively, are vectors of proportional changes in capital stocks at s+r+1 and s+r.

The vector $x_1(s+r+1)$ is the (r+1)'th column of Z(s) excluding the element corresponding to k(s+r+1). Note that in simulations, $x_1(s+r)$ and $y_1(s+r)$ will normally be set to zero, as will k(s+1+r) and k(s+r) in short-run simulations.

Included in the response, $y_1(s+r+1)$, is a change in industry investment plans. These changes have no impact on capital stocks until the beginning of period (s+r+2). At that instant the vector of capital stocks differs by k(s+r+2) per cent from the level it would have reached if the shock had not taken place. The response, $y_1(s+r+1)$ also captures the effects of the shock on the input/output structure of the economy in period (s+r+1).

Under the assumptions made above, solving MMO for t = s+r+1 is just like obtaining a neo-classical short-run solution with ORANI. However, as we will see, to solve MMO for t > s+r+1 requires that we use Johansen's method sequentially so as to update the values of all stock variables.

The remainder of this section draws upon Dixon, Parmenter and Sutton (1978, pp. 8-11). Their method is based upon the idea that, whatever procedure is followed with respect to the control solution, it will be necessary to update the coefficients $\{C_1(\tau),\ C_2(\tau);\ \tau = s+r+1, s+r+2, \ldots\}$ to take account of the projected effects over time of the shock under consideration. For $t = s+r+2$, then, we need to consider both the effect of the change $k(s+r+2)$ in capital stocks and the impact of the change in the input/output structure of the economy obtained from (2.5). This is done by computing:

$$\begin{bmatrix} y_1(s+r+2) \\ k(s+r+3) \end{bmatrix} = C^*(s+r+2) \begin{bmatrix} x_1(s+r+2) \\ x_1(s+r+1) \\ y_1(s+r+1) \\ k(s+r+2) \\ k(s+r+1) \end{bmatrix} \qquad (2.6),$$

where:

$x_1(s+r+2)$ and $x_1(s+r+1)$, respectively, are drawn from the $(r+2)$'th and $(r+1)$th columns of $Z(s)$ excluding the elements corresponding to the capital stocks;

and

$C^*(s+r+2)$ is computed from a coefficient tableau formed, in part, from a synthetic data base for $(s+r+2)$ which is a simple update transformation, derived using $y_1(s+r+1)$, of the previous period's data base.

In general, we have:

$$\begin{bmatrix} y_1(s+r+\tau) \\ k(s+r+\tau+1) \end{bmatrix} = C^*(s+r+\tau) \begin{bmatrix} x_1(s+r+\tau) \\ x_1(s+r+\tau-1) \\ y_1(s+r+\tau-1) \\ k(s+r+\tau) \\ k(s+r+\tau-1) \end{bmatrix} \quad (\tau = 2,3,\ldots) \qquad (2.7),$$

where:

$x_1(s+r+\tau)$ and $x_1(s+r+\tau-1)$, respectively, are drawn from the $(r+\tau)$'th and $(r+\tau-1)$'th columns of $Z(s)$ excluding the capital stock elements;

and

$C^*(s+r+\tau)$ is computed as a simple update transformation, using $y(s+r+\tau+1)$, of the previous period's data base.

In the theoretical literature, the concept of long-run equilibrium is often associated with notions of stationarity (where there is no growth in any economic magnitudes) or of steady growth under a 'golden rule'. With respect to the requirements of applied economists, both definitions have little applicability. In ORANI, for example, long-run equilibrium is defined as the state of the economy after an interval of time (the long-run) that is of sufficient length to allow the size and distribution of capital stocks across industries to adjust fully to eliminate the initial impact on relative rates of return induced by the shock.

> 'Notice that apart from this plausibility requirement -- effectively that sufficient time be allowed for capital to be mobile between industries -- the long-run is an arbitrary number of calendar years' (Powell, Cooper and McLaren 1983, p.11 -- my emphasis).

A similar definition could be used in the context of MMO. If so, we would refer to both $y_1(s+r+h)$ and $k(s+r+h)$, where h is large and sufficient for the above plausibility requirement to be met, as our indicators of the long-run effects of the shock, $Z(s)$. Another possibility would be to focus on a length of run sufficient for the effects of a monetary shock to be neutral in the sense of having negligible impact on real magnitudes.

3 The Theoretical Specification of MMO

3.1 Overview

MMO is a small stylised model designed to illustrate the 'extended Walrasian paradigm' within a CGE framework. The main features of the model are:

i) multi-product industries and multi-industry products;

ii) substitution between domestically produced commodities and imported commodities with the same name (the Armington, 1969, 1970 assumption);

iii) a financial sector;

iv) a full intertemporal choice-theoretic framework to explain consumer demand for goods and financial assets;

v) international capital flows (more precisely, flows of claims to ownership in overseas and domestic industries);

and

vi) an endogenous explanation of how total domestic demand is split between consumption and investment.

Features i), and ii) are shared with 'skeletal ORANI' (hereinafter MO), while features iii), iv) and vi) are specific to MMO.

There are five agents recognized in MMO: domestic industries 1 and 2, the representative household (hereinafter the consumer), the central government, and the 'overseas entity' (i.e., the rest of the world). With respect to commodities, there are two goods (both non-storable) and two primary factors. The latter are industry-specific capital and a single type of labour.

In the model's financial sector, there are four broad classes of assets distinguished, namely, money, government bonds, equities and 'overseas financial assets'. A brief description of each now follows.

i) Units of money are created by the central government. They are broadly defined and thus may yield a non-zero market rate of return. In keeping with the assumption commonly made for macroeconomic models of exchange rate determination (see, for example, Mussa, 1982), money is not internationally tradeable. Thus private domestic (overseas) residents are not allowed to hold foreign (domestic) currency.

ii) Bonds are variably priced perpetuities which act as instruments of government debt. In the model no distinction is made between short and long bonds or between safe and risky bonds.

iii) Equities are 'ordinary shares' representing claims to the ownership of real capital in each domestic industry. Equity is issued fully paid-up at the current market price to finance the net investment plans of each industry. For convenience we abstract from the effects of equity changes, such as stock splits, that alter the number of shares held by a stockholder but do not affect his/her claims on the industry's assets.

(iv) 'Overseas financial assets' are aggregates of overseas bonds and claims to ownership of overseas real capital.

In MMO, investments offer the prospect of capital gain only, with no income in the form of interest or dividends accruing to investors. Following Merton (1973a, pp. 868-9), it is assumed that markets for each financial asset are structured so that: there are no transactions costs or problems with indivisibility of assets; and there are a sufficient number of investors with incomparable wealth levels to make each investor believe that he/she can buy and sell as much of an asset as he/she wants at the market price.

A diagramatic overview of the structure of MMO is given in Figures 3.1 and 3.2. Both diagrams are designed to give the reader an overall perspective of the model's general structure without requiring him/her to have prior knowledge of its precise structural form.

Figure 3.1 illustrates the directions of real material and primary factor flows in MMO. As shown, the output of each industry consists of goods 1 and 2 and of industry - specific (non-shiftable) fixed capital. Production of goods 1 and 2 is achieved using a multi-product output technology that is separate from the technology used to produce fixed capital. The latter commodity, created in the current period, is assumed to remain unproductive until the start of the next period.

Domestically produced and imported goods with identical names are treated as distinct in the model. This is achieved using nested utility and production functions which take explicit account of the substitution possibilities existing between goods from both sources. The idea of using nested functions to model the demands for imports at the

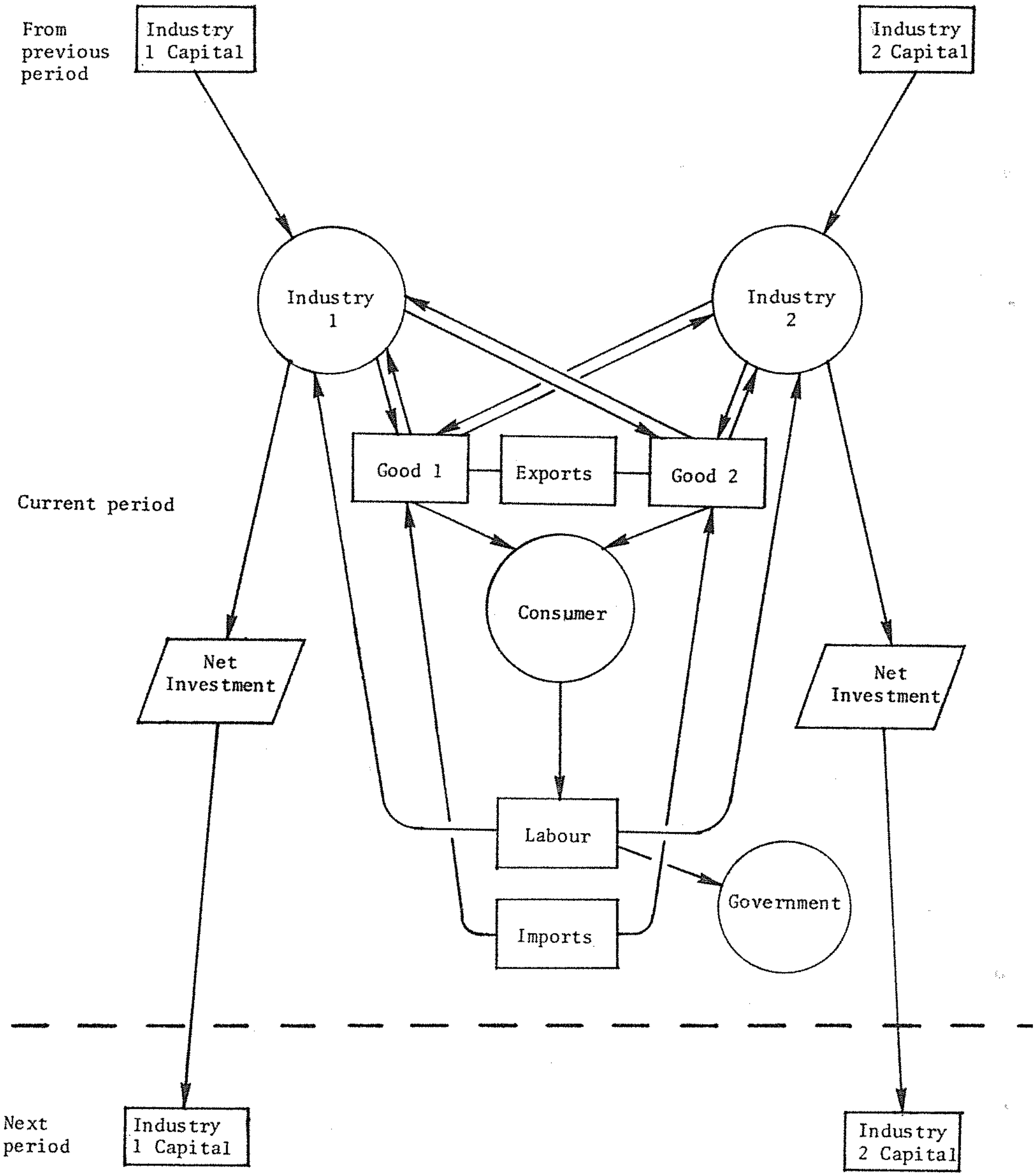

Figure 3.1 : Real material and primary factor flows in MMO

microeconomic level was first put forward in Armington (1969 and 1970) and is used extensively in ORANI.

The government is assumed to produce a single public good, the flows of which are not illustrated, using only units of labour. This good does not enter any utility or production function in the private sector.

Figure 3.2 illustrates the agents who hold and who supply each of the financial assets recognised in MMO. As depicted, the only agents in the model who are permitted to hold financial portfolios are the consumer and the overseas entity. On the supply side, domestic industries 1 and 2 finance their net investment by issuing new equity; while the government finances its budget deficit either by monetarisation or by selling bonds to the private sector in the domestic or overseas economies. Consistent with the domestic economy being small with respect to world financial markets, the supply on the domestic market of 'overseas financial assets' is assumed to be infinitely elastic at the going world price.

Before presenting the full theoretical specification of MMO the reader should be aware of the notational conventions used from now on (notationally, the remainder of this section is self-contained).

i) The term 'commodity' is used to refer to goods, financial assets and primary factors.

ii) Upper-case characters are used to indicate the initial absolute value of a variable; whereas the proportional-change in that variable

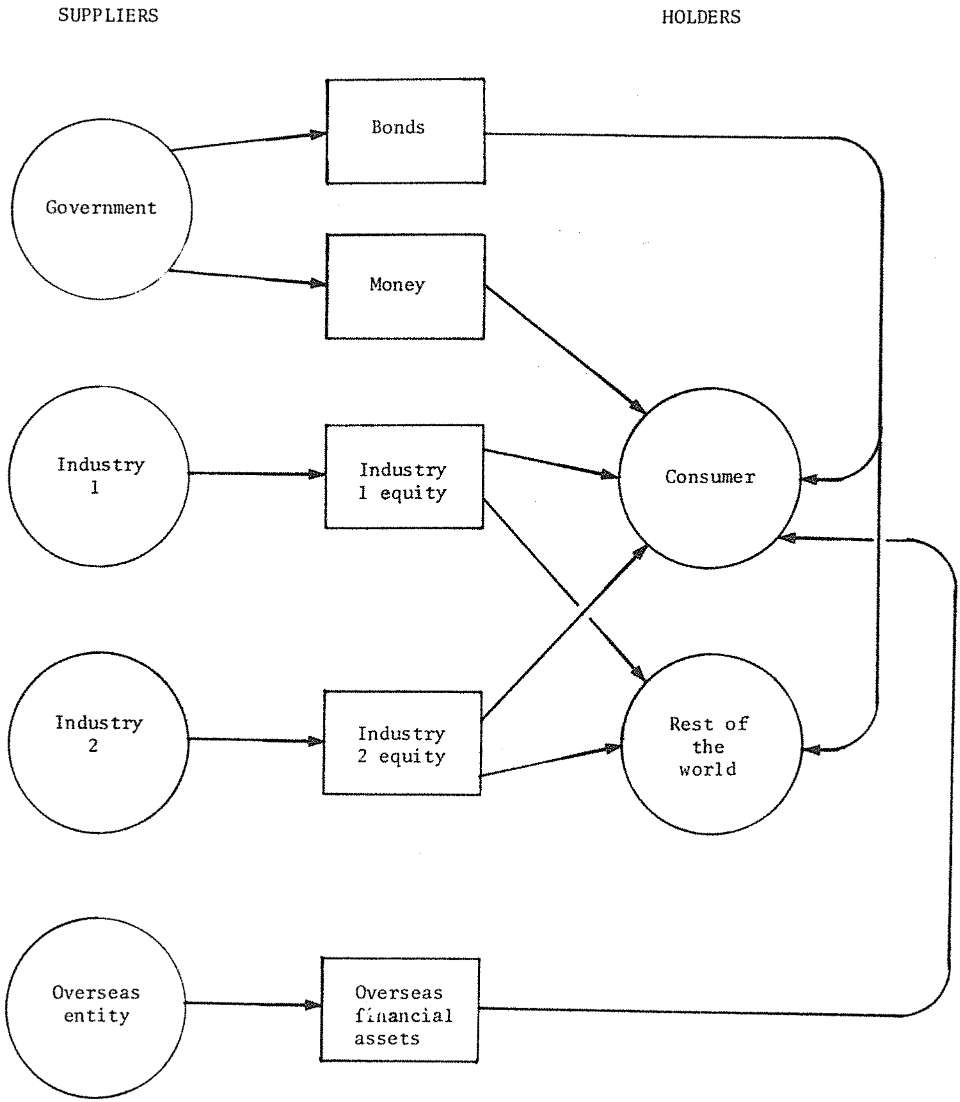

Figure 3.2 : Movements of financial assets among agents in MMO

is indicated by the equivalent lower-case lettering. For example, the proportional-change in a variable whose initial absolute value is denoted by B, is written as b (where b = dB/B). Note though that changes in any variable which may pass through zero are expressed as absolute changes, not as proportional changes.

iii) Several of the more commonly used symbols and their interpretations are listed below (the remainder are fully described in the text):

$X^{(t)}_{(iz)j}$ = actual demand in period t by user j for commodity i of type z;

$Y^{(t)}_{(iz)j}$ = actual supply in period t by producer j of commodity i of type z;

$V^{(t)}_{(iz)j}$ = actual holdings at <u>end</u> of period t by agent j of commodity i of type z;

and

$P^{(n)(t)}_{(iz)j}$ = the actual price in period t of item i of type z.

The superscript (n) on the price variable, $P^{(n)(t)}_{(iz)j}$, allows us to distinguish between different types of good prices. In particular, n = 0 identifies basic prices (i.e., the prices received by producers excluding all sales taxes). Purchasers' prices of inputs to current production are identified by n = 1; purchasers' prices of inputs to capital creation have n = 2; superscript n = 3 identifies prices paid by the household. Finally, the superscript n = * identifies prices in foreign currency. The possible values for j are 1 through 5: domestic industry 1 ($j = 1$); domestic industry 2($j = 2$); the consumer ($j = 3$); the central government ($j = 4$); and the overseas entity ($j = 5$). The subscript i can take the values 1, 2, 3 or 4. When i is 1 and 2 we refer to the goods 1 and 2, respectively. If i is 1 or 2, then z can either be 1 to indicate that i is produced domestically, or 2 to show that i is an imported good. Primary factors are identified by $i = 3$: labour ($i=3$, $z=1$); and fixed capital ($i=3$, $z=2$). When i is 4 we refer to financial assets: money ($i=4$, $z=0$); equities issued by industry 1 ($i=4$, $z=1$); equities issued by industry 2 ($i=4$, $z=2$); government bonds ($i=4$, $z=3$); and overseas financial assets ($i=4$, $z=4$).

The equations of MMO can be classified under seven headings. These are, in order of presentation:

i) equations describing consumer and other final demands for goods, and equations explaining consumer and overseas demands for financial assets;

ii) equations describing industry demands for primary factors and intermediate commodity inputs for current production, and equations describing the composition of industry outputs;

iii) equations determining levels of industry investment and the supply of equities;

iv) pricing equations setting pure profits from all activities to zero;

v) equations explaining the governments' budget deficit and how it is financed;

vi) market clearing equations for primary factors, goods and financial assets;

and

vii) macroeconomic equations.

3.2 Final demand equations and portfolio rules

3.2.1 Consumer demand for goods and financial assets

To model consumer behaviour in MMO we require a methodology which will recognise that the individual is a portfolio manager as well as a consumer. One such model which manages to handle both aspects of behaviour simultaneously is the Extended Linear Expenditure System with Assets (hereinafter ELESA), a full description of which is given in Adams (1986).[16] The ELESA generalises Lluch's (1973) Extended Linear Expenditure System (ELES), using the continuous-time model of individual behaviour developed in Merton (1969, 1971 and 1973b). Uncertainty plays an important role in this framework, and is introduced by assuming that returns on all financial assets are, from the perspective of the consumer, stochastic processes. In this world of perfect capital markets and (hence) zero transactions costs, it is the uncertainty of future asset yields which induces the consumer to hold a diversified portfolio.

In the original formulation of the ELESA, income flowing from labour and capital gains was recognised, but no allowance was made for the effects of income taxation. Similarly, utility was made a function of consumption alone, but the possibility of substitution in consumption between imported and domestically produced items with the same name, was ignored. Taxation and substitution between imported and domestic goods play important roles in MMO and must be explicitly accounted for in any theory of consumer behaviour used therein.

In what follows, we reformulate the ELESA to accommodate both factors which were previously ignored in its construction. This is done at first within the original continuous-time framework, the notational implications of which require that we adopt the following convention. All variables expressed in continuous-time are represented by symbols which are identical to those of their discrete-time analogues except that the former have two (instead of one) time superscripts. The first superscript indicates the current instant of historical time, t; while the second refers to an instant in consumer planning time (extending from t into the future). For example, in continuous-time notation, $X_{(11)}^{(t,h)}$, is the rate at which the domestically produced good is expected, from the viewpoint of the consumer at t, to be consumed at the notional instant $h (h \geq t)$ (note that the user subscript $j = 3$ has for the moment been omitted for notational convenience).

Consider, then, the consumer whose income is generated by capital gains and the yield on an exogenously given endowment of human wealth. Following Merton (1971), the expectation held for the price of financial asset z at h, $P_{(4z)}^{(t,h)}$, is assumed to be generated by the following stationary (in planning time) log-normal process:

$$\frac{dP_{(4z)}^{(t,h)}}{P_{(4z)}^{(t,h)}} = A_{(z)}\, dh + \omega_{(z)}\, dZ_{(z)}^{(t,h)} \qquad (z = 0,\ldots, 4) \quad (3.2.1),$$

where $A_{(z)}$ is the expected instantaneous 'permanent' proportional change in the price of financial asset z, per unit time (i.e., the expected long-run, or average, change per unit time); $(\omega_z)^2$ is the instantaneous variance per unit time (assumed to be a parameter of the model rather than a choice-variable); and $dZ_{(z)}^{(t,h)}$ is a normally distributed variable with mean zero and variance dh. The latter, called a Wiener process, is the continuous-time analogue of the difference equation defining a simple random-walk -- see Cox and Miller (1965). The processes defined in (3.2.1) are also known as Ito processes -- see Ito (1951).

To derive the relevant budget constraint faced by the consumer, we must do so initially in discrete-time, with time increments of Δh, and then take limits to obtain the true continuous-time specification. Following the schema laid down in Merton (1971, pp. 377-79), we assume that the consumer enters an arbitrary period of planning time, h, with planned non-human wealth:

$$W^{(t,h-\Delta h)} = \sum_{z=0}^{4} V_{(4z)}^{(t,h-\Delta h)} \; P_{(4z)}^{(t,h)} \qquad (3.2.2),$$

where $V_{(4z)}^{(t,h-\Delta h)}$ is the quantity of financial asset z expected to be held at the end of $h-\Delta h$ (a liability is represented by a negative value for $V_{(4z)}$). (Hereinafter $W^{(t,h)}$ is referred to as just wealth, although strictly speaking this symbol denotes the value of wealth planned for the end of h from the viewpoint of t.) In (3.2.2) $V_{(4z)}^{(t,h-\Delta h)}$ is the correct term because this is the quantity of financial asset z expected to be purchased for the portfolio in $h-\Delta h$; and it is $P^{(t,h)}_{(4z)}$ because that is the expected price of the asset at h (see Merton, 1971, p. 378).

Expected consumption and income levels and the new portfolio are then chosen simultaneously to obey:

$$(1 - T^{(t,h)})(P^{(t,h)}_{(31)}Y^{(t,h)}_{(31)})\Delta h - \sum_{i=1}^{2} P^{(t,h)}_{(i\cdot)}X^{(t,h)}_{(i\cdot)}\Delta h = \sum_{z=0}^{4} (V^{(t,h)}_{(4z)} - V^{(t,h-\Delta h)}_{(4z)})\, P^{(t,h)}_{(4z)} \qquad (3.2.3),$$

in which $T^{(t,h)}$ is the expected average tax rate on the flow of consumer wage income at h; $P^{(t,h)}_{(31)}Y^{(t,h)}_{(31)}$ is the expected (before-tax) flow of wage income during h, per unit time; and $P^{(t,h)}_{(i\cdot)}X^{(t,h)}_{(i\cdot)}$ is expected expenditure during h on 'effective units' of good i, per unit time. An effective unit of good i is defined as a CES aggregation of domestically produced good i and imported good i. (For operational purposes, it will be necessary at a later stage to define the unobservable $P^{(t,h)}_{(i\cdot)}$ in terms of the observable component prices $P^{(t,h)}_{(i1)}$ and $P^{(t,h)}_{(i2)}$.)

Incrementing (3.2.3) by Δh gives:

$$(1-T^{(t,h+\Delta h)})(P_{(31)}^{(t,h+\Delta h)}Y_{(31)}^{(t,h+\Delta h)})\Delta h - \sum_{i=1}^{2} P_{(i\cdot)}^{(t,h+\Delta h)}X_{(i\cdot)}^{(t,h+\Delta h)}\Delta h$$

$$= \sum_{z=0}^{4} (V_{(4z)}^{(t,h+\Delta h)} - V_{(4z)}^{(t,h)})\, P_{(4z)}^{(t,h+\Delta h)}$$

$$= \sum_{z=0}^{4} (V_{(4z)}^{(t,h+\Delta h)} - V_{(4z)}^{(t,h)})(P_{(4z)}^{(t,h+\Delta h)} - P_{(4z)}^{(t,h)})$$

$$+ \sum_{z=0}^{4} (V_{(4z)}^{(t,h+\Delta h)} - V_{(4z)}^{(t,h)})\, P_{(4z)}^{(t,h)} \qquad (3.2.4).$$

Letting Δh approach zero produces the continuous-time versions of (3.2.2) and (3.2.4), respectively:

$$W^{(t,h)} = \sum_{z=0}^{4} V_{(4z)}^{(t,h)}P_{(4z)}^{(t,h)} \qquad (3.2.5),$$

and

$$(1-T^{(t,h)})(P_{(31)}^{(t,h)}Y_{(31)}^{(t,h)})\, dh - \sum_{i=1}^{2} P_{(i\cdot)}^{(t,h)}X_{(i\cdot)}^{(t,h)}dh$$

$$= \sum_{z=0}^{4} dV_{(4z)}^{(t,h)}dP_{(4z)}^{(t,h)} + \sum_{z=0}^{4} dV_{(4z)}^{(t,h)}P_{(4z)}^{(t,h)} \qquad (3.2.6).$$

The penultimate step in the derivation of the continuous-time budget constraint is total differentiation of (3.2.5). Notice though, that because expected asset prices are generated by stochastic processes, (3.2.5) is not differentiable in the usual sense; instead, a stochastic differential calculus as described in Merton (1982) must be used. This done we obtain:

$$dW^{(t,h)} = \sum_{z=0}^{4} V_{(4z)}^{(t,h)} \, dP_{(4z)}^{(t,h)} + \sum_{z=0}^{4} dV_{(4z)}^{(t,h)} \, dP_{(4z)}^{(t,h)}$$

$$+ \sum_{z=0}^{4} dV_{(4z)}^{(t,h)} \, P_{(4z)}^{(t,h)} \qquad (3.2.7).$$

Substitution of (3.2.6) into (3.2.7) yields the required expression for the instantaneous change in wealth:

$$dW^{(t,h)} = \sum_{z=0}^{4} V_{(4z)}^{(t,h)} dP_{(4z)}^{(t,h)} + (1 - T^{(t,h)})(P_{(31)}^{(t,h)} Y_{(31)}^{(t,h)}) \, dh$$

$$- \sum_{i=1}^{2} P_{(i\cdot)}^{(t,h)} X_{(i\cdot)}^{(t,h)} \, dh \qquad (3.2.8).$$

The first term on the right hand side of (3.2.8) is the expected instantaneous flow of income from capital gains at h. The remaining terms represent the net addition to wealth from sources other than capital gains. Unlike income from labour, income from capital gains accrues to the consumer tax-free.

A common device for simplifying the analysis in models of portfolio behaviour is to assume that one financial asset, say money (z=0), is risk-free (i.e., $\omega_{(0)} = 0$). This assumed, we can rewrite (3.2.8) after substitution of (3.2.1), as:

$$dW^{(t,h)} = \sum_{z=1}^{4} [(A_{(z)} - A_{(0)}) Q_{(4z)}^{(t,h)} W^{(t,h)} dh + \omega_{(z)} Q_{(4z)}^{(t,h)} W^{(t,h)} dZ_{(z)}^{(t,h)}]$$

$$+ \{A_{(0)} W^{(t,h)} + (1 - T^{(t,h)})(P_{(31)}^{(t,h)} Y_{(31)}^{(t,h)})\} \, dh$$

$$- \sum_{i=1}^{2} P_{(i\cdot)}^{(t,h)} X_{(i\cdot)}^{(t,h)} \, dh \qquad (3.2.9),$$

where $Q^{(t,h)}_{(4z)} = P^{(t,h)}_{(4z)}V^{(t,h)}_{(4z)}/W^{(t,h)}$ is the proportion of wealth $W^{(t,h)}$ expected to be invested in financial asset z at h ($\sum_{j=0}^{4} Q^{(t,h)}_{(4j)} = 1$). The reader should note that, in this context, the assumption of risklessness does not imply perfect foresight. It does, however, infer static expectations in a world apparently subject to stochastic variability.

Having established (3.2.9) we now turn our attention to the description of the consumer's choice problem. Let U( ) be the instantaneous intertemporal utility function which has planning time and the time rate of consumption of effective units of goods 1 and 2 as its only arguments. At t, the consumer seeks to choose a bundle of goods , $X^{(t,t)}_{(is)}$ (i,s = 1,2), and a portfolio of financial assets, $Q^{(t,t)}_{(4z)}$(z = 0,...,4), which will maximise the criterion functional

$$E \int_t^\infty U(X^{(t,h)}_{(1\cdot)}, X^{(t,h)}_{(2\cdot)}, h)\, dh \quad ,$$

subject to:

$$X^{(t,h)}_{(i\cdot)} = CES(X^{(t,h)}_{(i1)}, X^{(t,h)}_{(i2)}) \qquad (i = 1,2);$$

exogenously given expected time paths for labour earnings $\{P^{(t,h)}_{(31)}Y^{(t,h)}_{(31)}\}$, commodity prices $\{P^{(t,h)}_{(ij)}$; i,j = 1,2} and the tax rate $\{T^{(t,h)}\}$, all of which are assumed to be stationary over the plan and set equal to their actual values at t [$P^{(t,h)}_{(31)}Y^{(t,h)}_{(31)} = P^{(t,t)}_{(31)}Y^{(t,t)}_{(31)}$, $P^{(t,h)}_{(ij)} = P^{(t,t)}_{(ij)}$, $T^{(t,h)} = T^{(t,t)}$; $h \in [t,\infty)$];

an initial endowment of non-human wealth $W^{(t,t)}$;

and

the accumulation relationships, equations (3.2.1) and (3.2.9).

E_t is the conditional expectation operator, conditional upon knowing actual values for total consumer wealth and asset prices at t.

As in Lluch (1973), the instantaneous utility function U( ) is assumed to have the following Klein-Rubin (or Stone-Geary) form:

$$U(X_{(1\cdot)}^{(t,h)}, X_{(2\cdot)}^{(t,h)}, h) = \exp(-\delta(h-t))\, V(X_{(1\cdot)}^{(t,h)}, X_{(2\cdot)}^{(t,h)}) \qquad (3.2.10),$$

in which

$$V(X_{(1\cdot)}^{(t,h)}, X_{(2\cdot)}^{(t,h)}) = \sum_{i=1}^{2} \beta_i \log(X_{(i\cdot)}^{(t,h)} - \bar{X}_{(i\cdot)}) \qquad (3.2.11),$$

where δ is the constant time-preference discount rate; and the β's and $\bar{X}_{(i\cdot)}$'s are parameters with the β's constrained to be positive and summing to unity. $\bar{X}_{(i\cdot)}$ is often called the subsistence level for $X_{(i\cdot)}^{(t,h)}$ and is invariant to changes in both historical and planning time.

With our focus of attention on the initial instant of planning time t, a solution to the above problem can be obtained in two steps. Step 1 uses results already established in Adams (1986) to obtain 'optimal rules' for $X_{(i\cdot)}^{(t,t)} (i = 1,2)$ and $Q_{(4z)}^{(t,t)}$ $(z = 0, \ldots, 4)$. Step 2 takes as control the values for $X_{(i\cdot)}^{(t,t)} (i = 1,2)$ obtained in step 1 and allocates these effective demands among corresponding goods from domestic and imported sources.

Step 1

If the wage variable used in Adams (1986) is redefined as after-tax labour income, then (with a given stationary average tax rate) the problem considered there is identical to the stage 1 problem being considered here. Having made this substitution, we find by construction that the optimal consumption and portfolio rules corresponding to equations (9) and (10) of Adams (1986) are:

$$X^{(t,t)}_{(i.)}P^{(t,t)}_{(i.)} = \bar{X}_{(i.)}P^{(t,t)}_{(i.)} + \beta_i\mu\,(\zeta^{(t,t)})$$

$$- \beta_i\mu\,(\sum_{j=1}^{2}\bar{X}_{(j.)}\,P^{(t,t)}_{(j.)}) \qquad (i = 1,2) \quad (3.2.12),$$

and

$$Q^{(t,t)}_{(40)} = [1 - \sum_{i=1}^{4} Q^{(t,t)}_{(4i)}] \qquad ,$$

$$Q^{(t,t)}_{(4z)}W^{(t,t)} = \frac{\sum_{i=1}^{4}\omega^{(iz)}(A_{(i)} - A_{(0)})}{A_{(0)}}\,(\zeta^{(t,t)} - \sum_{j=1}^{2}\bar{X}_{(j\cdot)}P^{(t,t)}_{(j\cdot)})$$

$$(z = 1,\ldots, 4) \quad (3.2.13),$$

in which, by definition:

$$\mu = \delta\,/\,A_{(0)},$$

$$\zeta^{(t,t)} = A_{(0)}W^{(t,t)} + (1 - T^{(t,t)})(P^{(t,t)}_{(31)}Y^{(t,t)}_{(31)});$$

and where $\omega^{(ij)}$ is the (i,j)'th element of the inverse of the (4 x 4) covariance matrix of financial asset returns. Hereinafter $\zeta^{(t)}$ is referred to as the expected instantaneous flow of 'safe disposable' income at t (i.e., the expected instantaneous flow of after-tax income from human and non-human sources, where the latter is imputed at the 'safe' rate of return $A_{(0)}$).

Equation (3.2.12) is a modified form of Lluch's ELES; while (3.2.13) explains the consumer's portfolio behaviour as a system of linear functions in safe disposable income. The latter expression is consistent with the more traditional mean-variance model of portfolio behaviour. It implies that the demand for risky asset k is an increasing function of a weighted sum of rates of return on each risky asset relative to the safe rate of return (the weight $\omega^{(iz)}$ being a measure indicative of the relative security endowed from investing in asset i rather than asset z).

Step 2

With $X^{(t,t)}_{(i\cdot)}$ given by (3.2.11), $X^{(t,t)}_{(i1)}$ and $X^{(t,t)}_{(i2)}$ will be chosen to minimize the total cost of purchasing that quantity of effective inputs,

$$P^{(t,t)}_{(i\cdot)} X^{(t,t)}_{(i\cdot)} = \sum_{s=1}^{2} P^{(t,t)}_{(is)} X^{(t,t)}_{(is)} \quad ,$$

subject to the CES aggregation

$$X^{(t,t)}_{(i\cdot)} = B \left[\sum_{s=1}^{2} \alpha_s X^{(t,t)^{-\rho(i)}}_{(1s)} \right]^{-1/\rho(i)} \qquad (i = 1,2) \quad (3.2.14),$$

where B and the α_s 's are positive parameters, and $\rho(i) > -1$, but does not equal zero. (It is conventionally assumed that the value of B is defined so that the sum of the α's is unity).

The first-order conditions for solving this problem are:

$$P^{(t,t)}_{(is)} = \lambda B \left[\sum_{j=1}^{2} \alpha_j X^{(t,t)^{-\rho(i)}}_{(ij)} \right]^{-((1+\rho_{(i)})/\rho_{(i)})} \alpha_s X^{(t,t)^{-(1+\rho_{(i)})}}_{(is)} \qquad (i,s = 1,2) \quad (3.2.15),$$

and

$$X^{(t,t)}_{(i\cdot)} = B\left[\sum_{s=1}^{2} \alpha_s X^{(t,t)^{-\rho(i)}}_{(is)}\right]^{-1/\rho_{(i)}} \qquad (i = 1,2) \quad (3.2.16),$$

where λ is the Lagrangian multiplier. From these it can be shown that the demand functions have the following form:

$$X^{(t,t)}_{(is)} = X^{(t,t)}_{(i\cdot)}\frac{1}{B}\left[\sum_{j=1}^{2} \alpha_j \left[\frac{P^{(t,t)}_{(ij)}\alpha_s}{P^{(t,t)}_{(is)}\alpha_j}\right]^{(\rho_{(i)}/(1+\rho_{(i)}))}\right]^{-1/\rho_{(i)}}$$

$$(i,s = 1,2) \quad (3.2.17)$$

(see Dixon, Bowles and Kendrick, 1980, pp. 295-7).

Equations (3.2.8), (3.2.12), (3.2.13) and (3.2.17) form the core of MMO's sub-model of consumer behaviour. By averaging with respect to planning time over the historical period t (which has unit length) we can obtain the discrete-time analogues of these continuous-time equations and hence derive their final computational (proportional-change) forms. In so doing we assume that the consumer continually replans and so realisations of every variable in historical time are seen as a succession of instants h = t in planning time.

After substitution for $Q^{(t,t)}_{(4z)}$, the discrete-time version of (3.2.8) can be written as:

$$W^{(t)}_3 = W^{(t-1)}_3 + \sum_{z=0}^{4} Q^{(t)}_{(4z)3} W^{(t)}_3 (P^{(t)}_{(4z)} - P^{(t-1)}_{(4z)})/P^{(t)}_{(4z)}$$

$$- \sum_{i=1}^{2} P^{(3)(t)}_{(i\cdot)} X^{(t)}_{(i\cdot)3} + (1 - T^{(t)}_3)(P^{(0)(t)}_{(31)} Y^{(t)}_{(31)3})$$

$$(3.2.18),$$

where $W_3^{(t)} = \sum_{z=0}^{4} P_{(4z)}^{(t)} V_{(4z)3}^{(t)}$, is the nominal value of wealth held by the consumer at the end of period t (similarly, $V_{(4z)3}^{(t)}$ is the quantity of asset z held by the consumer at the end of t, and $Q_{(4z)3}^{(t)} = P_{(4z)}^{(t)} V_{(4z)3}^{(t)} / W_3^{(t)}$). Note that we have reintroduced our full notation including all user subscripts and price identifying superscripts (e.g., the superscript (3) on $P_{(i\cdot)}^{(3)(t)}$ indicates that this is the purchaser's price paid by the consumer). The discrete-time form of equation (3.2.12) is:

$$X_{(i\cdot)3}^{(t)} P_{(i\cdot)}^{(3)(t)} = \bar{X}_{(i\cdot)} P_{(i\cdot)}^{(3)(t)} + \beta_i \mu^{(t)} (\zeta^{(t)})$$

$$- \beta_i \mu^{(t)} \left(\sum_{j=1}^{2} P_{(j\cdot)}^{(3)(t)} \bar{X}_{(j\cdot)} \right) \quad (i = 1,2) \quad (3.2.19),$$

in which, by definition:

$$\mu^{(t)} = \delta / A_{(0)3}^{(t)},$$

$$\zeta^{(t)} = A_{(0)3}^{(t)} W_3^{(t)} + (1 - T_3^{(t)})(P_{(31)}^{(0)(t)} Y_{(31)3}^{(t)});$$

and where the superscript (t) on the A variables indicates, under the assumption of continual replanning, that even though the consumer plans as if he expects $A^{(t)}$ to prevail forever, at t+1 he may substitute a new set $\{A^{(t+1)}\}$ if realisations in the financial sector during t were not as expected. Finally, the discrete-time forms of (3.2.13) and (3.2.17) are:

$$Q_{(40)3}^{(t)} = [1 - \sum_{i=1}^{4} Q_{(4i)3}^{(t)}] \quad ,$$

$$Q_{(4z)3}^{(t)} W_3^{(t)} = \frac{\sum_{i=1}^{4} \omega_3^{(iz)} (A_{(i)3}^{(t)} - A_{(0)3}^{(t)})}{A_{(0)3}^{(t)}} \left(\zeta^{(t)} - \sum_{j=1}^{2} P_{(j\cdot)}^{(3)(t)} \bar{X}_{(j\cdot)}\right)$$

$$(z = 1, \ldots, 4) \quad (3.2.20),$$

and

$$X^{(t)}_{(is)3} = X^{(t)}_{(i\cdot)3} \frac{1}{B} \left[\sum_{j=1}^{2} \alpha_j \left[\frac{P^{(3)(t)}_{(ij)} \alpha_s}{P^{(3)(t)}_{(is)} \alpha_j} \right]^{(\rho_{(i)}/(1+\rho_{(i)}))} \right]^{-1/\rho_{(i)}}$$

$$(i,s = 1,2) \quad (3.2.21).$$

The simplest option to explain the process by which the consumer generates values for the $A^{(t)}_{(z)3}$ variables is linear extrapolation. This assumed, we can immediately write:

$$A^{(t)}_{(z)3} = \frac{P^{(t-1)}_{(4z)} - P^{(t-2)}_{(4z)}}{P^{(t-2)}_{(4z)}} + \Upsilon_{(z)3} \left[\frac{P^{(t-1)}_{(4z)} - P^{(t-2)}_{(4z)}}{P^{(t-2)}_{(4z)}} - \frac{P^{(t-2)}_{(4z)} - P^{(t-3)}_{(4z)}}{P^{(t-3)}_{(4z)}} \right]$$

$$(z = 0,\ldots, 4) \quad (3.2.22).$$

where $\Upsilon_{(z)3}$ $(z = 0,\ldots, 4)$ are parameters. Equation (4.2.22) says that expected permanent changes in the price of financial asset z, from the viewpoint of the consumer at t, equals the immediate past rate of change in price of z plus a correction which allows for any trend exhibited over the previous period. If $\Upsilon_{(z)3}$ is positive, then the consumer extrapolates this trend; while if $\Upsilon_{(z)3}$ is negative, then the trend is expected to be reversed.

Our next task is to convert equations (3.2.18) through (3.2.22) into the form required by the Johansen solution method. Each relation must be expressed as a function of proportional-change variables (where the change is between two outcomes occurring at the same notional period of time, t, rather than between two outcomes occurring at different periods of time). Accordingly, we obtain (with all lower-case symbols denoting proportional changes in the variables represented by the equivalent upper-case characters):

$$w_3^{(t)} = w_3^{(t-1)}B^{(t)1} + \sum_{z=0}^{4}\left[dQ^{(t)}_{(4z)3}\, B^{(t)2}_{(z)} + (p^{(t)}_{(4z)} - p^{(t-1)}_{(4z)})B^{(t)3}_{(z)}\right]$$

$$- \sum_{i=1}^{2} (p^{(3)(t)}_{(i\cdot)} + x^{(t)}_{(i\cdot)3})B^{(t)4}_{(i\cdot)} - dT^{(t)}_3\, B^{(t)5}$$

$$+ (p^{(0)(t)}_{(31)} + y^{(t)}_{(31)3})B^{(t)6} \qquad (3.2.23),$$

$$x^{(t)}_{(i\cdot)3} = d\mu^{(t)}B^{(t)7}_{(i\cdot)} + d\zeta^{(t)}B^{(t)8}_{(i\cdot)} - p^{(3)(t)}_{(i\cdot)}B^{(t)9}_{(ij)}$$

$$- p^{(3)(t)}_{(j\cdot)}B^{(t)10}_{(ij)} \qquad (i = 1,2;\ j \neq i) \quad (3.2.24),$$

$$dQ^{(t)}_{(40)3} = - \sum_{i=1}^{4} dQ^{(t)}_{(4i)3} \qquad (3.2.25),$$

$$dQ^{(t)}_{(4z)3} + w_3^{(t)}Q^{(t)}_{(4z)3} = \sum_{i=1}^{4} (dA^{(t)}_{(i)3}B^{(t)11}_{(iz)} - dA^{(t)}_{(0)3}B^{(t)12}_{(iz)})$$

$$+ d\zeta^{(t)}B^{(t)13}_{(z)} - \sum_{j=1}^{2} p^{(3)(t)}_{(j\cdot)}B^{(t)14}_{(jz)}$$

$$(z = 1,\ldots, 4) \quad (3.2.26),$$

$$x^{(t)}_{(is)3} = x^{(t)}_{(i\cdot)3} - \sigma^{(3)}_{(i)} (p^{(3)(t)}_{(is)} - \sum_{j=1}^{2} p^{(3)(t)}_{(ij)} S^{(t)}_{(ij)3})$$

$$(i,s = 1,2) \quad (3.2.27),$$

and

$$dA^{(t)}_{(z)3} = p^{(t-1)}_{(4z)}B^{(t-1)15}_{(z)} - p^{(t-2)}_{(4z)}B^{(t-2)16}_{(z)} + p^{(t-3)}_{(4z)}B^{(t-3)17}_{(z)}$$

$$(z = 0,\ldots, 4) \quad (3.2.28),$$

in which:

$$w_3^{(t)} = \sum_{z=0}^{4} (p_{(4z)}^{(t)} B_{(z)}^{(t)18} + dV_{(4z)3}^{(t)} B_{(z)}^{(t)18} / V_{(4z)3}^{(t)}) \qquad (3.2.29),$$

$$d\mu^{(t)} = -dA_{(0)3}^{(t)} B^{(t)19} \qquad (3.2.30),$$

$$d\zeta^{(t)} = dA_{(0)3}^{(t)} W_3^{(t)} + w_3^{(t)} B^{(t)20} - dT_3^{(t)} B^{(t)21}$$

$$+ (p_{(31)}^{(0)(t)} + y_{(31)3}^{(t)}) B^{(t)22} \qquad (3.2.31);$$

and where $\sigma_{(i)}^{(3)} = 1/(1+\rho_{(i)})$, is the elasticity of substitution in consumption between good i from domestic and overseas sources; $S_{(ij)3}^{(t)}$ is the share of total consumer spending on good i at t which is devoted to good i from source j; and the B's are explained in Appendix A. To avoid division by zero, all variables which may pass through zero (e.g., the A and Q variables) are expressed in absolute rather than proportional change form.

To interpret (3.2.23), (3.2.24) and (3.2.26), we need an equation defining the $p_{(i\cdot)}$ in terms of the proportional changes $p_{(i1)}$ and $p_{(i2)}$ in the exogenous component prices. By analogy with the way equations (12.18) and (12.19) were derived from (12.7) and (12.8) in Dixon, Parmenter, Sutton and Vincent (1982, pp. 78-81), from (3.2.15) and (3.2.16) we find in proportional changes that:

$$p_{(i\cdot)}^{(3)(t)} = \sum_{s=1}^{2} S_{(is)3}^{(t)} p_{(is)}^{(3)(t)} \qquad (i = 1,2) \qquad (3.2.32).$$

This says that the change in the composite price $P_{(i\cdot)}^{(3)(t)}$ is a share weighted sum of changes in each of the component prices.

We conclude this sub-section with a summary of the proportional-change form of MMO's sub-model of consumer behaviour. Equation (3.2.23) explains the change in the consumer's holding of non-human wealth at the intersection of two solution periods as a function of changes in financial asset prices and goods prices, and of changes in income from capital gains and labour. The second system of equations (3.2.24) relates changes in effective demands for goods 1 and 2 to changes in the 'marginal propensity to consume' out of safe disposable income (μ), to changes in safe disposable income and to changes in the prices of effective inputs of goods 1 and 2. The third equation (3.2.25) relates the change in the share of consumer wealth held as money to changes in each of the risky asset shares. Equation (3.2.26) explains the change in the value of risky asset z held at t as a function of changes in 'risky' rates of return relative to the safe rate of return, of the change in safe disposable income and of changes in the prices of effective units of goods 1 and 2. The fifth system of equations (3.2.27) relates the change in demand for good i from source s to the change in effective demand for good i and to changes in the prices of imported good i and domestic good i. The final set of equations (3.2.28) explains the change in the expected permanent price of financial asset z as a function of changes in the price of z during each of the previous three periods.

3.2.2 Overseas demand for domestic goods and financial assets

The export demand functions in MMO replicate the corresponding relationships in MO (see Dixon, Parmenter, Sutton and Vincent, 1982, p. 22 for further details). If $X^{(t)}_{(i1)5}$ is the volume of exports of good i, then:

$$P^{(*)(t)}_{(i1)} = h_i(X^{(t)}_{(i1)5})\ F_{(i1)5} \qquad (i = 1,2) \qquad (3.2.33),$$

where $P^{(*)(t)}_{(i1)}$ is the foreign currency price per unit of export of good i (i.e. the f.o.b. price in foreign currency), h_i is a nondecreasing function of $X^{(t)}_{(i1)5}$; and $F_{(i1)5}$ is a shift variable used to simulate changes in the foreign demand curve (i.e., if there is an increase (decrease) in foreign demand for i, then $F_{(i1)5}$ will increase (decrease)).

The linear proportional-change form of (3.2.33) is:

$$p^{(*)(t)}_{(i1)} = -\gamma_{(i1)}x^{(t)}_{(i1)5} + f_{(i1)5} \qquad (i = 1,2) \quad (3.2.34),$$

in which $\gamma_{(i1)}$, a positive parameter, is the reciprocal of the foreign elasticity of demand for domestic good i, i.e.:

$$-\gamma_{(i1)} = \frac{\partial h_i}{\partial X^{(t)}_{(i1)5}}\,\frac{X^{(t)}_{(i1)5}}{h_i} \qquad (i = 1,2).$$

The reader should note that (3.2.34) does not explain $x^{(t)}_{(i1)5}$; it merely posits in price/quantity space an equation the shape of which is determined by the value given γ.

With overseas demand for goods explained by (3.2.34), our attention now turns to the overseas demand for financial assets. As for the domestic consumer, we model the portfolio behaviour of the overseas entity using a modified form of the ELESA. However, in the absence of a full treatment of the overseas economy, only a limited version of the system used in sub-section 3.2.1 is needed to explain the foreigner's portfolio behaviour. This version, which accommodates all characteristics peculiar to the overseas entity in MMO, is derived below (note that wherever the reformulation of this sub-section overlaps that of the previous sub-section, the duplicated material is ignored and the reader's attention is drawn to the former for any missing details). Again we work initially in continuous-time, then in discrete-time with the transition to proportional changes left until the end.

As with the consumer, we assume that the expectation held by the overseas entity for the price of (internationally tradeable) financial asset z at planning point h, $P^{(*)(t,h)}_{(4z)5}$, is generated by an Ito process, i.e.:

$$\frac{dP^{(*)(t,h)}_{(4z)5}}{P^{(*)(t,h)}_{(4z)5}} = A^{(*)}_{(z)5}\, dh + \omega_{(z)5}\, dz^{(t,h)}_{(z)5} \quad (z = 1,\dots, 4) \qquad (3.2.35),$$

in which we have reverted to our previous continuous-time notation with the addition of a superscript (*) and a subscript (5). The former is used to distinguish variables expressed in overseas currency from variables expressed in domestic currency, while the latter is used to distinguish expected realisations from the viewpoint of the overseas entity from those of the consumer. An example of this notation is $A^{(*)}_{(z)5}$ which denotes the expected instantaneous permanent change in the overseas price of financial asset z as seen from the viewpoint of the foreigner at t ($A_{(z)}$ on the other hand is the expected permanent change in the domestic price of asset z as seen by the consumer).

Given (3.2.35), the overseas entity's accumulation relationship for wealth (the planned value of which for h is denoted $W^{(*)(t,h)}_{5}$) must be expressed as the stochastic differential equation:

$$dW^{(*)(t,h)}_{5} = \sum_{z=1}^{4} V^{(t,h)}_{(4z)5}\, dP^{(*)(t,h)}_{(4z)5} + C^{(*)(t,h)}_{5}\, dh \qquad (3.2.36),$$

where $C_5^{(*)(t,h)}$ is the expected instantaneous net addition to overseas wealth at h from sources other than capital gains, per unit time.

For convenience, let the overseas financial assets aggregate be, from the perspective of the overseas entity, risk-free (i.e., $\omega_{(4)5} = 0$). Then, upon substitution of (3.2.35), (3.2.36) becomes:

$$dW_5^{(*)(t,h)} = \sum_{z=1}^{3} [(A_{(z)5}^{(*)} - A_{(4)5}^{(*)})\, Q_{(4z)5}^{(*)(t,h)}\, W_5^{(*)(t,h)}\, dh]$$

$$+ \sum_{z=1}^{3} \omega_{(z)5}\, Q_{(4z)5}^{(*)(t,h)}\, W_5^{(*)(t,h)}\, dZ_{(z)5}^{(t,h)}$$

$$+ A_{(4)5}^{(*)}\, W_5^{(*)(t,h)}\, dh + C^{(*)(t,h)}\, dh \qquad (3.2.37),$$

where, analogous to before, $Q_{(4z)5}^{(*)(t,h)} = P_{(4z)5}^{(*)(t,h)}\, V_{(4z)5}^{(t,h)} / W_5^{(*)(t,h)}$.

At t, the intertemporal choice problem for the overseas entity can be expressed as follows. Choose a bundle of goods and a portfolio of financial assets to maximise, over an infinite planning horizon, the expected present value of a utility stream having the same Klein-Rubin form as that specified in (3.2.10) and (3.2.11), subject to:

> exogenously given expected time paths for wage earnings and goods prices all of which are assumed to be stationary over the (foreigner's) plan and set equal to their actual values at t;
>
> an initial endowment of non-human wealth $W_5^{(*)(t,t)}$;

and

> the accumulation relationships, equations (3.2.36) and (3.2.37).

With our attention focussed on the initial instant of planning time h = t, and by analogy with the arguments by which a solution was found to the problem being considered in Adams (1986), we can assert that the portfolio component of the solution to the foreigner's choice problem is:

$$\frac{Q^{(*)(t,t)}_{(4z)5}\ W^{(*)(t,t)}_{5}}{F_{(4z)5}} = \frac{\sum_{i=1}^{3} \omega_5^{(iz)}\ (A^{(*)}_{(i)5} - A^{(*)}_{(4)5})}{A^{(*)}_{(4)5}}\ A^{(*)}_{(4)5} W^{(*)(t,t)}_{5}$$

$$+ \frac{\sum_{i=1}^{3} \omega_5^{(iz)}\ (A^{(*)}_{(i)5} - A^{(*)}_{(4)5})}{A^{(*)}_{(4)5}}\ \eta^{(*)(t,t)}$$

$$(z = 1, \ldots, 3),$$

$$Q^{(*)(t,t)}_{(44)5} = [1 - \sum_{z=1}^{3} Q^{(*)(t,t)}_{(4z)5}]\ F_{(44)5} \qquad (3.2.38),$$

where $\omega_5^{(iz)}$ is the foreign counterpart of the domestic $\omega^{(iz)}$; $F_{(4z)5}$ is a shift variable allowing an exogenous treatment of overseas demand for financial asset z; and $\eta^{(*)(t,t)}$ is the instantaneous flow of foreign wage income net of total 'subsistence expenditure' (i.e., the overseas equivalent of $P^{(t,t)}_{(31)}\ Y^{(t,t)}_{(31)} - \sum_{i=1}^{2} P^{(t,t)}_{(i\cdot)}\ \bar{X}_{(i\cdot)}$).

The discrete-time analogue of (3.2.38) is:

$$\frac{Q_{(4z)5}^{(*)(t)}\, W_5^{(*)(t)}}{F_{(4z)5}} = \frac{\sum_{i=1}^{3} \omega_5^{(iz)} (A_{(i)5}^{(*)(t)} - A_{(4)5}^{(*)(t)})}{A_{(4)5}^{(*)(t)}}\, A_{(4)5}^{(*)(t)} W_5^{(*)(t)}$$

$$+ \frac{\sum_{i=1}^{3} \omega_5^{(iz)} (A_{(i)5}^{(*)(t)} - A_{(4)5}^{(*)(t)})}{A_{(4)5}^{(*)(t)}}\, \eta^{(*)(t)}$$

$$(z = 1, \ldots, 3),$$

$$Q_{(44)5}^{(*)(t)} = [1 - \sum_{z=1}^{3} Q_{(4z)5}^{(*)(t)}]\; F_{(44)5} \qquad (3.2.39),$$

in which:

$$W_5^{(*)(t)} = \sum_{z=1}^{4} P_{(4z)}^{(*)(t)}\; V_{(4z)5}^{(t)} \quad ,$$

and where values for the $A^{(*)}$ variables are generated by the linear extrapolative hypothesis, i.e., as in (3.2.22)

$$A_{(z)5}^{(*)(t)} = \frac{P_{(4z)}^{(*)(t-1)} - P_{(4z)}^{(*)(t-2)}}{P_{(4z)}^{(*)(t-2)}} + \gamma_{(z)5} \left[\frac{P_{(4z)}^{(*)(t-1)} - P_{(4z)}^{(*)(t-2)}}{P_{(4z)}^{(*)(t-2)}} \right]$$

$$- \gamma_{(z)5} \left[\frac{P_{(4z)}^{(*)(t-2)} - P_{(4z)}^{(*)(t-3)}}{P_{(4z)}^{(*)(t-3)}} \right]$$

$$(z = 1, \ldots, 4) \qquad (3.2.40),$$

with $\gamma_{(z)5}$ a parameter. Note that the user subscripts j = 5 on the financial asset price variables are now redundant.

Finally, the linear proportional-change forms of (3.2.39) and (3.2.40) are, respectively:

$$dQ^{(*)(t)}_{(4z)5} + w^{(*)(t)}_{5}B^{(t)23}_{(z)} = dF_{(4z)5}\,B^{(t)24}_{(z)} + \sum_{i=1}^{3} dA^{(*)(t)}_{(i)5}B^{(t)25}_{(iz)}$$

$$- \sum_{i=1}^{3} dA^{(*)(t)}_{(4)5}B^{(t)26}_{(iz)} + d\eta^{(*)(t)}B^{(t)27}_{(z)}$$

$$(z = 1,\ldots, 3),$$

$$dQ^{(*)(t)}_{(44)5} = - \sum_{z=1}^{3} dQ^{(*)(t)}_{(4z)5}F_{(44)5} + dF_{(44)5}[1 - \sum_{i=1}^{3} Q^{(*)(t)}_{(4i)5}]$$

$$(3.2.41)$$

and

$$dA^{(*)(t)}_{(z)5} = p^{(*)(t-1)}_{(4z)}B^{(t-1)28}_{(z)} - p^{(*)(t-2)}_{(4z)}B^{(t-2)29}_{(z)} + p^{(*)(t-3)}_{(4z)}B^{(t-3)30}_{(z)}$$

$$(z = 1,\ldots, 4) \quad (3.2.42),$$

in which

$$w^{(*)(t)}_{5} = \sum_{z=1}^{4} (p^{(*)(t)}_{(4z)}\,B^{(t)31}_{(z)} + dV^{(t)}_{(4z)5}\,B^{(t)31}_{(z)}/\,V^{(t)}_{(4z)5}) \quad (3.2.43),$$

and where the B's contained in (3.2.41), (3.2.42) and (3.2.43) are defined in Appendix A.

3.2.3 Final government demand for goods

The government in MMO produces a single public good using units of labour as the only inputs to production. Accordingly there is no final government demand for goods.

3.2.4 Industry demand for inputs to the production of fixed capital

Following Dixon, Parmenter and Rimmer (1984, pp. 500-1) we describe the final demand equations for inputs to the production of fixed capital in industry j, as solutions to the following problem. For industry j (j = 1,2) at t, choose the inputs

$$X^{(2)(t)}_{(iz)j} \qquad (i,z = 1,2)$$

to minimize the total cost of producing units of fixed capital

$$\sum_{i=1}^{2} \sum_{z=1}^{2} P^{(2)(t)}_{(iz)} X^{(2)(t)}_{(iz)j} \qquad (3.2.44),$$

given the number of industry-specific fixed capital units created by industry j at t, $I^{(t)}_{(32)j}$, and subject to the nested production function:

$$I^{(t)}_{(32)j} = \mathrm{Min}\left[\frac{X^{(2)(t)}_{(1\cdot)j}}{D^{(2)(t)}_{(1\cdot)j}} , \frac{X^{(2)(t)}_{(2\cdot)j}}{D^{(2)(t)}_{(2\cdot)j}}\right] \qquad (3.2.45),$$

$$X^{(2)(t)}_{(i\cdot)j} = \mathrm{CES}\left[X^{(2)(t)}_{(i1)j}, X^{(2)(t)}_{(i2)j}\right] \quad (i = 1,2) \qquad (3.2.46),$$

where $X^{(2)(t)}_{(iz)j}$ is the demand for good i of type z used for the creation of units of fixed capital in industry j at t (note that the superscript (2) identifies inputs to the production of fixed capital; whereas the superscript (1) identifies inputs to current production); $X^{(2)(t)}_{(1\cdot)j}$ is the demand for an effective unit of good i used to create a unit of fixed capital in industry j; and the $D^{(2)(t)}_{(i\cdot)j}$ are positive parameters. An 'effective unit' is here defined as a CES aggregate of units of the imported and domestically produced good with the same name. Thus imported and domestic supplies of good i are allowed to be imperfect substitutes in the production of fixed capital.

The production technology encapsulated in (3.2.45) and (3.2.46) does not involve the use of primary factors. This is in keeping with the convention adopted in ORANI (and many other CGE models) where it is assumed that the use of primary factors in capital creation is accounted for via inputs from the construction industries.

The solution to this cost minimization problem has the following proportional-change form:

$$x^{(2)(t)}_{(iz)j} = i^{(t)}_{(32)j} - \sigma^{(2)}_{(i)j} \left(p^{(2)(t)}_{(iz)} - \sum_{q=1}^{2} S^{(2)(t)}_{(iq)j}\, p^{(2)(t)}_{(iq)}\right) \qquad (i,z,j = 1,2) \qquad (3.2.47),$$

where: $\sigma^{(2)}_{(i)j}$ is the elasticity of substitution between intermediate inputs of good i from domestic and overseas sources used for the creation of fixed capital in industry j; and $p^{(2)(t)}_{(iz)}$ is the proportional-change in the price of good (iz) used by industry j at t for the creation of capital (the superscript (2) allows us to recognize that the flow of good (iz) for input to capital creation may attract a different tax than the flow of the same good to current production).

3.3 Industry inputs and outputs for current production

The current production technology available to each domestic industry in MMO is based upon the same assumption of 'input-output separability' as used in MO.[17] Under this assumption, an industry's input and output decisions can be treated separately thereby yielding relatively simple supply response and input demand equations of the forms:

$$Y_{(i1)j}^{(t)} = Y_{(i1)j}(P_{(i1)}^{(0)(t)}, P_{(i2)}^{(0)(t)}, Z_j^{(t)}) \quad (i,j = 1,2) \quad (3.3.1a),$$

$$X_{(iz)j}^{(1)(t)} = X_{(iz)j}^{(1)}(P_{(11)}^{(1)(t)}, P_{(12)}^{(1)(t)}, P_{(21)}^{(1)(t)}, P_{(22)}^{(1)(t)}, Z_j^{(t)})$$

$$(i,z,j = 1,2) \quad (3.3.1b),$$

and

$$X_{(3z)j}^{(t)} = X_{(32)j}(P_{(31)}^{(1)(t)}, P_{(32)j}^{(t)}, Z_j^{(t)}) \quad (z,j = 1,2) \quad (3.3.1c),$$

in which $Y_{(i1)j}^{(t)}$ is the production of good i by industry j at t; $X_{(iz)j}^{(1)(t)}$ is the demand at t by industry j for good i from source z for purpose (1) (i.e., for current production); and $X_{(3z)j}^{(t)}$ is the demand for primary factor z by industry j at t. According to (3.3.1a), $Y_{(i1)j}^{(t)}$ is a function of the basic prices of goods produced by industry j at t, $P_{(i1)}^{(o)(t)}$ (i = 1,2), and of industry j's activity level at t, $Z_j^{(t)}$ (where $Z_j^{(t)}$ is a scalar).[18] The demand for good i from source z (equation (3.3.1b)) is a function of input goods prices, $P_{(iz)j}^{(1)(t)}$(i,z = 1,2), and of $Z_j^{(t)}$; while the demand for primary factor z (equation (3.3.1c)) depends upon the price of labour inputs purchased for current production, $P_{(31)}^{(1)(t)}$, the rental rate on a unit of capital employed in industry j, $P_{(32)j}^{(t)}$, and again j's activity level.

3.3.1 Industry supplies

In MMO, the equation (3.3.1a) is determined as the solution to the problem of maximizing industry j's revenue at t subject to given product prices, and the CET (constant elasticity of transformation) transformation frontier:

$$Z_j^{(t)} = \underset{i=1,2}{\mathrm{CET}} \left(Y_{(i1)j}^{(t)} \right) \qquad (j = 1,2) \qquad (3.3.2).$$

The supply relations which solve this problem have the following proportional-change forms:

$$y_{(i1)j}^{(t)} = z_j^{(t)} - \beta_j \left[p_{(i1)}^{(0)(t)} - \sum_{q=1}^{2} R_{(q1)j}^{(t)} \, p_{(q1)}^{(0)(t)} \right] \qquad (i,j = 1,2) \qquad (3.3.3),$$

where $R_{(q1)j}^{(t)}$ is the share accounted for by sales of good q in industry j's revenue at t and β_j $(-\infty < \beta_j < 0)$ is the elasticity of transformation between goods one and two produced by industry j.[19] According to equation (3.3.3), the output of good i by industry j at t will increase, in the absence of any price changes, with the activity level of j, Z_j. However, if the price of good 1 increases relative to the price of good 2, then industry j will transform its output mix in favour of good 1.

3.3.2 Input demands

On the input side, equations (3.3.1b) and (3.3.1c) are found as solutions to a cost-minimization problem subject to the following nested input technology:

$$\mathrm{Min} \left[\frac{X_{(1\cdot)j}^{(1)(t)}}{D_{(1\cdot)j}^{(1)(t)}}, \; \frac{X_{(2\cdot)j}^{(1)(t)}}{D_{(2\cdot)j}^{(1)(t)}}, \; \frac{X_{(3\cdot)j}^{(1)(t)}}{D_{(3\cdot)j}^{(1)(t)}} \right] = Z_j^{(t)} \qquad (j = 1,2) \qquad (3.3.4a),$$

$$X_{(i\cdot)j}^{(1)(t)} = \mathrm{CES} \left(X_{(i1)j}^{(1)(t)}, X_{(i2)j}^{(1)(t)} \right) \qquad (i,j = 1,2) \qquad (3.3.4b),$$

$$X_{(3\cdot)j}^{(t)} = \mathrm{CES} \left(X_{(31)j}^{(t)}, X_{(32)j}^{(t)} \right) \qquad (j = 1,2) \qquad (3.3.4c),$$

where $D^{(1)(t)}_{(2\cdot)j}$ (i,j = 1,2) are positive parameters. According to (3.3.4), industry j uses 'effective' units of goods and 'effective' units of primary factors in fixed proportions to produce a unit of activity at t, $z^{(t)}_j$. Thus no substitution is permitted between intermediate inputs and primary factors, nor between intermediate inputs of goods 1 and 2. However, from (3.3.4b and c), capital can be substituted for labour and imported inputs can be substituted for each domestic good.

The input demand functions which solve the cost minimization problem for given input prices, have the following proportional-change forms:

$$x^{(1)(t)}_{(iz)j} = z^{(t)}_j - \sigma^{(1)}_{(i)j}\left(p^{(1)(t)}_{(i1)} - \sum_{q=1}^{2} S^{(1)(t)}_{(iq)j}\, p^{(1)(t)}_{(iq)}\right)$$

(i,z,j = 1,2) (3.3.5),

$$x^{(t)}_{(31)j} = z^{(t)}_j - \sigma^{(1)}_{(3)j}\, S^{(t)}_{(32)j}\left(p^{(1)(t)}_{(31)} - p^{(t)}_{(32)j}\right) \quad (j = 1,2) \quad (3.3.6),$$

and

$$x^{(t)}_{(32)j} = z^{(t)}_j - \sigma^{(1)}_{(3)j} S^{(t)}_{(31)j}\left(p^{(t)}_{(32)j} - p^{(1)(t)}_{(31)}\right) \quad (j = 1,2) \quad (3.3.7),$$

where:

$\sigma^{(1)}_{(i)j}$ is either the elasticity of substitution in industry j between intermediate inputs of good i from domestic and overseas sources (if i = 1 or 2), or the elasticity of substitution between fixed capital and labour in industry j (if i = 3)(note that the superscript (1) indicates that these elasticities pertain to the current production process of industry j);

$S^{(1)(t)}_{(iq)j}$ is the share in the total cost of good i used as input to the current production of industry j, represented by good i from source q;

and

$S^{(t,t)}_{(3z)j}$ is the share in total primary factor costs of industry j at t represented by primary factor z.

3.4 The creation of industry-specific fixed capital and the supply of equities

In sub-section (3.2.4) we described the technology for creating fixed capital in industry j, but left unanswered the question of how much gross investment was undertaken. It is now time to consider this problem (i.e., we describe the MMO theory for the $I^{(t)}_{(32)j}$ $(j = 1,2)$).

MMO's theory of investment is based on the so-called 'q' theory suggested by Tobin (1969).[20] The latter can be summarised as follows.

> 'The neoclassical theory of corporate investment is based on the assumption that the management seeks to maximise the present net worth of the company, the market value of the outstanding shares. An investment project should be undertaken if and only if it increased the value of the shares. The securities markets appraise the project, its expected contributions to the future earnings of the company and its risks. If the value of the project as appraised by investors exceeds the cost, then the company's shares will appreciate to the benefit of existing stockholders. That is, the market will value the project more than the cash used to pay for it.' (Tobin and Brainard, 1977, p. 242).

This suggests that the rate of investment is an increasing function of 'marginal q', the ratio of the market value of new additional investment goods to their replacement cost.

Marginal q is of course unobservable. However, we can observe 'average q', the ratio of the market value of existing capital to its replacement cost and to a good approximation the two are equal.[21] With this assumed we can write:

$$I^{(t)}_{(32)j} = I_{(32)j}(P^{(t)}_{(4j)}/\Pi^{(t)}_{j}) + d_j V^{(t-1)}_{(32)j} \qquad (j = 1,2) \quad (3.4.1),$$

where $I_{(32)j}$ is a non-decreasing function of 'average q' for industry j; $P^{(t)}_{(4j)}$, the price at period t of industry j's equity, is the market's valuation of a unit of industry j's capital existing at t; $\Pi^{(t)}_{j}$ is the unit price of new capital equipment created in that industry at t; and d_j is the rate at which j's stock of physical capital $(V^{(t-1)}_{(32)j})$ depreciates during t. Dee (1983), p. 18) notes that the price $P^{(t)}_{(4j)}$ is not necessarily equal to the actual production cost of capital equipment in that industry (i.e., $\Pi^{(t)}_{j}$). Certainly in the long-run, when capital stocks are free to adjust, equality can be assumed. In that situation 'economic logic indicates that a normal equilibrium value for q is 1 for reproducable assets which are in fact being reproduced' (Tobin and Brainard, 1977, p. 238). However, in the short-run, when capital stocks are held fixed, $P^{(t)}_{(4j)}$ will be whatever is required to ensure that the existing capital stock is willingly held . This price may or may not equal the replacement price $\Pi^{(t)}_{j}$.

For the purposes of this paper, the function $I_{(32)j}$ in (3.4.1) is determined such that:

$$I^{(t)}_{(32)j} = F_{(32)j}(P^{(t)}_{(4j)}/\Pi^{(t)}_j)^{\lambda^{(t)}_j} + d_j V^{(t-1)}_{(32)j} \quad (j = 1,2) \quad (3.4.2),$$

where $F_{(32)j}$ is a shift variable and $\lambda^{(t)}_j$, a positive parameter, is a function of the expected time paths of those variables (such as product prices) which determine the future profitability of j's operating environment.

The proportional-change form of (3.4.2) is:

$$i^{(t)}_{(32)j} = f_{(32)j} + \lambda^{(t)}_j G^{(t)}_j (p^{(t)}_{(4j)} - \pi^{(t)}_j) + (1 - G^{(t)}_j)\, v^{(t-1)}_{(32)j}$$

$$(j = 1,2) \quad (3.4.3),$$

where $G^{(t)}_j = (I^{(t)}_{(32)j} - d_j\, V^{(t-1)}_{(32)j}) / I^{(t)}_{(32)j}$, is the ratio of net to gross investment in industry j at t.

Each industry in MMO finances its fixed-capital investment by issuing new equity. This means, for industry j, that the actual amount of finance raised by additional shares issued at t must equal the cost of net investment undertaken in that period, i.e.:

$$\Pi^{(t)}_j (I^{(t)}_{(32)j} - d_j\, V^{(t-1)}_{(32)j}) = P^{(t)}_{(4j)} (Y^{(t)}_{(4j)j} - \sum_{k=3,5} V^{(t-1)}_{(4j)k})$$

$$(j = 1,2) \quad (3.4.4),$$

where $Y^{(t)}_{(4j)j}$ is the supply of equity by industry j at t. In proportional changes, (3.4.4) is written as:

$$\pi^{(t)}_j + i^{(t)}_{(32)j}/G^{(t)}_j - v^{(t-1)}_{(32)j}B^{(t)32}_j = p^{(t)}_{(4j)} + y^{(t)}_{(4j)j}B^{(t)33}_j$$

$$- \sum_{k=3,5} v^{(t-1)}_{(4j)k}B^{(t)34}_{(j)k} \quad (j = 1,2) \quad (3.4.5),$$

where the B coefficients appearing in (3.4.5) are explained in Appendix A.

3.5 Pricing equations

In this sub-section we deal with the relationships between the various types of commodity prices and with the linkages between prices and rates of return in the financial sector. As in ORANI, MMO recognizes several sets of commodity prices: basic prices, prices paid by purchasers, prices of fixed capital units, and foreign currency import and export prices (c.i.f. and f.o.b., respectively). Unlike ORANI, creation prices and second-hand prices of capital units can differ. The relationships between each are described below.

Under the assumption of zero pure-profits for all production activities, we have the following relationship for industry j's current production at t:

$$\sum_{i=1}^{2} P_{(i1)}^{(0)(t)} Y_{(i1)j}^{(t)} = \sum_{i=1}^{2} \sum_{z=1}^{2} P_{(iz)}^{(1)(t)} X_{(iz)j}^{(1)(t)} + P_{(31)}^{(1)(t)} X_{(31)j}^{(t)} + P_{(32)j}^{(t)} X_{(32)j}^{(t)} \qquad (j = 1,2) \quad (3.5.1).$$

Equation (3.5.1) says that the basic value of industry j's current production at t equals the total payments by industry j for intermediate and primary factor inputs to current production. In proportional changes:

$$\sum_{i=1}^{2} p_{(i1)}^{(0)(t)} H_{(i1)j}^{(0)(t)} = \sum_{i=1}^{2} \sum_{z=1}^{2} p_{(iz)}^{(1)(t)} H_{(iz)j}^{(1)(t)} + p_{(31)}^{(1)(t)} H_{(31)j}^{(t)} + p_{(32)j}^{(t)} H_{(32)j}^{(t)} \qquad (j = 1,2) \quad (3.5.2),$$

where the H's are revenue and cost shares. $H^{(0)(t)}_{(i1)j}$ is the share in industry j's total revenue at t accounted for by sales of good i. $H^{(1)(t)}_{(iz)j}$ is the share in j's total current production costs at t represented by the input of good i from source z; and $H^{(t)}_{(3z)j}$ is the cost share represented by the usage of primary factor z. The absence of all input and output quantities from (3.5.2) is due to each industry employing constant returns to scale production technology.

With respect to the creation of capital by each industry, the pure-profit condition implies (in proportional-changes):

$$\pi^{(t)}_j = \sum_{i=1}^{2} \sum_{z=1}^{2} p^{(2)(t)}_{(iz)} H^{(2)(t)}_{(iz)j} \qquad (j = 1,2) \quad (3.5.3),$$

where $H^{(2)(t)}_{(iz)j}$ is the share in industry j's total cost of creating capital at t accounted for by inputs of good i from source z and $\pi^{(t)}_j$ is the proportional change in the price of a unit of capital for industry j.

The third set of zero-pure profit conditions in MMO equalises the revenue from exporting to the relevant cost:

$$P^{(*)(t)}_{(i1)} \Phi^{(t)} = P^{(0)(t)}_{(i1)} - T^{(e)(t)}_{(i1)} P^{(*)(t)}_{(i1)} \Phi^{(t)} \qquad (i = 1,2) \quad (3.5.4),$$

where:

$\Phi^{(t)}$ is the nominal exchange rate expressed in terms of the domestic currency per unit of overseas currency;

and

$T^{(e)(t)}_{(i1)}$ is the ad valorem rate of subsidy on exports of good i (negative if a tax).

The LHS of (3.5.4) is the price paid by the overseas entity (in domestic currency) for good i at the domestic port of exit. The RHS is the basic price of good i plus the cost of the tax involved with transferring good i from the producer to the port (or less if a subsidy).

In proportional-change form, (3.5.4) can be written as:

$$p_{(i1)}^{(*)(t)} + \phi^{(t)} + \tau_{(i1)}^{(e)(t)} = p_{(i1)}^{(0)(t)} \qquad (i = 1,2) \quad (3.5.5),$$

where $\tau_{(i1)}^{(e)(t)}$ is the proportional-change in one plus the ad valorem rate of export subsidy (i.e., the proportional-change in the 'power' of the subsidy).[22]

The fourth set of pure profit conditions pertain to imports and equate their selling price to the costs of importing, i.e.:

$$P_{(i2)}^{(0)(t)} = P_{(i2)}^{(*)(t)} \ \Phi^{(t)} \left(1 + T_{(i2)}^{(m)(t)}\right) \qquad (i = 1,2) \quad (3.5.6),$$

where $T_{(i2)}^{(m)(t)}$ is the ad valorem rate of tariff on imported item i. $P_{(i2)}^{(0)(t)}$ is the basic price of i (i.e., the price received by domestic importers) and $P_{(i2)}^{(*)(t)}$ is the foreign currency c.i.f., price of imported units of good i.

The proportional-change form of (3.5.6) is:

$$p_{(i2)}^{(0)(t)} = p_{(i2)}^{(*)(t)} + \phi^{(t)} + \tau_{(i2)}^{(m)(t)} \qquad (i = 1,2) \quad (3.5.7),$$

where $\tau_{(i2)}^{(m)(t)}$ is the proportional-change in the <u>power</u> of the import tariff (i.e., the proportional-change in one plus the ad valorem rate of tariff).

The fifth set of price equations equate the prices paid by domestic agents for goods and labour to the basic prices of those items. In particular:

$$P^{(n)(t)}_{(iz)} = P^{(0)(t)}_{(iz)} \left(1 + T^{(n)(t)}_{(iz)}\right) \qquad (n = 1,2,3;\ i,z = 1,2)$$

and

$$P^{(1)(t)}_{(31)} = P^{(0)(t)}_{(31)} \left(1 + T^{(1)(t)}_{(31)}\right) \qquad (3.5.8),$$

where $T^{(n)(t)}_{(iz)}$ is the ad valorem rate of sales tax on good (iz) sold at t for purpose n (if i,z = 1,2), or the ad valorem payroll tax on the use of labour. According to (3.5.8) the sales of all goods and of labour attract a tax at an ad valorem rate which is specific to each flow.[23]

The proportional-change form of (3.5.8) is written as:

$$p^{(n)(t)}_{(iz)} = p^{(0)(t)}_{(iz)} + \tau^{(n)(t)}_{(iz)} \qquad (n = 1,2,3;\ i,z = 1,2),$$

and

$$p^{(1)(t)}_{(31)} = p^{(0)(t)}_{(31)} + \tau^{(1)(t)}_{(31)} \qquad (3.5.9),$$

where following our previous approach, $\tau^{(n)(t)}_{(iz)}$ is the proportional-change in the power of the sales tax.

The relationships between the price of, and the rate of return on, each financial asset in MMO are the final items to be considered under the heading of 3.5.1. With respect to the domestic price of each financial asset, we have:

$$P_{(4z)}^{(t)} = L_{(4z)}^{(t)}\ P_{(4z)}^{(t-1)} \qquad (z = 0, \ldots, 4) \quad (3.5.10),$$

in which the variable $L_{(4z)}^{(t)}$ is defined such that

$$R_{(4z)}^{(t)} = (L_{(4z)}^{(t)} - 1) \qquad (z = 0, \ldots, 4) \quad (3.5.11),$$

is the return for period t (i.e. the capital gain at t) on asset z calculated as a proportion of its initial market price, $P_{(4z)}^{(t-1)}$. Equations (3.5.10) and (3.5.11) are identities. Their only purpose is to introduce to the system the variables $R_{(4z)}^{(t)}$ which measure the yield on each financial asset in the domestic economy.[24]

In MMO, financial asset markets in the domestic economy are assumed to be perfectly integrated with those in the rest of the world. This assumption of perfect capital mobility implies that the price of financial asset z on the domestic market equals the foreign currency price converted to domestic currency via the nominal exchange rate, i.e.:

$$P_{(4z)}^{(t)} = P_{(4z)}^{(*)(t)}\ \Phi^{(t)} \qquad (z = 1, \ldots, 4) \quad (3.5.12).$$

In their computational—forms, equations (3.5.10), (3.5.11) and (3.5.12) become:

$$p_{(4z)}^{(t)} = \ell_{(4z)}^{(t)} + p_{(4z)}^{(t-1)} \qquad (z = 0, \ldots, 4) \quad (3.5.13),$$

$$dR_{(4z)}^{(t)} = \ell_{(4z)}^{(t)}\ L_{(4z)}^{(t)} \qquad (z = 0, \ldots, 4) \quad (3.5.14),$$

and

$$p_{(4z)}^{(t)} = p_{(4z)}^{(*)(t)} + \phi^{(t)} \qquad (z = 1, \ldots, 4) \quad (3.5.15),$$

3.6 Government budgetary rules

The government in MMO produces a pure public good which does not enter any utility or production function in the private sector. For simplicity, we assume that the only inputs to the government's production process at t are units of labour supplied by the household. Therefore, in proportional-changes:

$$y_4^{(t)} = x_{(31)4}^{(t)} \qquad (3.6.1),$$

where $y_4^{(t)}$ is the proportional-change in government production at t and $x_{(31)4}^{(t)}$ is the like-change in government demand for labour at t.

In its current stage of development, MMO contains no theory to explain government production at t. Instead, the output of the public good is merely posited to be a fixed proportion of gross national product in constant prices $(GNP_R{}^{(t)})$, i.e.:

$$y_4^{(t)} = gnp_R^{(t)} + f_4 \qquad (3.6.2),$$

where f_4 is a shift variable allowing an exogenous treatment of $y_4{}^{(t)}$.

Total government expenditure at t, $GEX^{(t)}$, consists solely of payments to labour used in the production process. Thus

$$GEX^{(t)} = P_{(31)}^{(o)(t)} \; X_{(31)4}^{(t)} \qquad (3.6.3),$$

where the superscript (o) on the wage variable indicates that the government does not impose a sales tax on its own purchases. The proportional-change form of (3.6.3) is written as:

$$gex^{(t)} = \left(p_{(31)}^{(0)(t)} + x_{(31)4}^{(t)}\right) \qquad (3.6.4).$$

The government's revenue, $GRE^{(t)}$, is given by:

$$GRE^{(t)} = \sum_{i=1}^{2} T^{(e)(t)}_{(i1)} P^{(*)(t)}_{(i1)} \Phi^{(t)} X^{(t)}_{(i1)5}$$

$$+ \sum_{j=1}^{2} \sum_{i=1}^{2} \sum_{n=1}^{2} T^{(m)(t)}_{(i2)} P^{(*)(t)}_{(i2)} \Phi^{(t)} X^{(n)(t)}_{(i2)j}$$

$$+ \sum_{i=1}^{2} T^{(m)(t)}_{(i2)} P^{(*)(t)}_{(i2)} \Phi^{(t)} X^{(t)}_{(i2)3}$$

$$+ \sum_{j=1}^{2} \sum_{i=1}^{2} \sum_{z=1}^{2} \sum_{n=1}^{2} T^{(n)(t)}_{(iz)} P^{(o)(t)}_{(iz)} X^{(n)(t)}_{(iz)j}$$

$$+ \sum_{i=1}^{2} \sum_{z=1}^{2} T^{(3)(t)}_{(iz)} P^{(o)(t)}_{(iz)} X^{(t)}_{(iz)3}$$

$$+ \sum_{j=1}^{2} T^{(1)(t)}_{(31)} P^{(o)(t)}_{(31)} X^{(t)}_{(31)j}$$

$$+ T^{(t)}_{3} P^{(o)(t)}_{(31)} Y^{(o)(t)}_{(31)3} \qquad (3.6.5).$$

The first, second and third terms on the RHS of (3.6.5) measure total revenues from taxes on exports and imports. The fourth and fifth items represent revenues from taxes imposed on the sale of goods to industries and the consumer, respectively. The sixth term measures total payroll tax receipts; while the seventh term measures revenue accruing from the direct tax imposed on the household's labour income. In proportional changes:

$$gre^{(t)} = \sum_{i=1}^{2} (dT^{(e)(t)}_{(i1)} / T^{(e)(t)}_{(i1)} + p^{(*)(t)}_{(i1)} + \phi^{(t)} + x^{(t)}_{(i1)5}) H^{(e)(t)}_{(i1)}$$

$$+ \sum_{j=1}^{2} \sum_{i=1}^{2} \sum_{n=1}^{2} (dT^{(m)(t)}_{(i2)} / T^{(m)(t)}_{(i2)} + p^{(*)(t)}_{(i2)} + \phi^{(t)} + x^{(n)(t)}_{(i2)j}) H^{(m)(n)(t)}_{(i2)j}$$

$$+ \sum_{i=1}^{2} (dT^{(m)(t)}_{(i2)} / T^{(m)(t)}_{(i2)} + p^{(*)(t)}_{(i2)} + \phi^{(t)} + x^{(t)}_{(i2)3}) H^{(m)(t)}_{(i2)3}$$

$$+ \sum_{j=1}^{2} \sum_{i=1}^{2} \sum_{z=1}^{2} \sum_{n=1}^{2} (dT^{(n)(t)}_{(iz)} / T^{(n)(t)}_{(iz)} + p^{(o)(t)}_{(iz)} + x^{(n)(t)}_{(iz)j}) H^{(n)(t)}_{(iz)j}$$

$$+ \sum_{i=1}^{2} \sum_{z=1}^{2} (dT^{(3)(t)}_{(iz)} / T^{(3)(t)}_{(iz)} + p^{(o)(t)}_{(iz)} + x^{(t)}_{(iz)3}) H^{(3)(t)}_{(iz)}$$

$$+ \sum_{j=1}^{2} (dT^{(1)(t)}_{(31)} / T^{(1)(t)}_{(31)} + p^{(o)(t)}_{(31)} + x^{(t)}_{(31)j}) H^{(1)(t)}_{(31)j}$$

$$+ (dT^{(t)}_{(3)} / T^{(t)}_{(3)} + p^{(o)(t)}_{(31)} + y^{(o)(t)}_{(31)3}) H^{(3)(t)} \qquad (3.6.6),$$

where H coefficients are shares in total government receipts at t. For example $H^{(3)(t)}_{(iz)} = (T^{(3)(t)}_{(iz)} \; P^{(o)(t)}_{(iz)} \; X^{(t)}_{(iz)3}) / GRE^{(t)}$.

The government budget deficit at t, $GBD^{(t)}$, is defined as:

$$GBD^{(t)} \equiv GEX^{(t)} - GRE^{(t)} \qquad (3.6.7),$$

or in computational-form:

$$dGBD^{(t)} \equiv gex^{(t)} \, GEX^{(t)} - gre^{(t)} \, GRE^{(t)} \qquad (3.6.8),$$

where, because $GBD^{(t)}$ can change sign, the proportional-change form of (3.6.8) is avoided. This deficit is financed by monetarisation and the sale of bonds (the only instrument of government debt). It is assumed that the proportions of the deficit to be financed by each asset, are determined such that:

$$P^{(t)}_{(4z)} \left(Y^{(t)}_{(4z)4} - \sum_{j=3,5} V^{(t-1)}_{(4z)j}\right) = M^{(t)}_{(4z)}\ GBD^{(t)} \qquad (z = 0,3) \quad (3.6.9),$$

in which

$$M^{(t)}_{(40)} + M^{(t)}_{(43)} = 1 \qquad (3.6.10),$$

and where $Y^{(t)}_{(4z)4}$ is the supply by the government of financial asset z at t, and the M's are positive variables reflecting the government's current policy towards the financing of its deficit. According to (3.6.9), the supply of money (z = 0) or of bonds (z = 3) by the central government at t is a simple proportion ($M^{(t)}_{(4z)}$) of the current budget deficit. How these proportions are determined by the policy maker is not explained.

Extending (3.6.9) to take explicit account of the government's reaction to changes in any number of policy targets (inflation, the rate of unemployment, etc.), as in Feltenstein (1986), is not too difficult. The cleanest approach is to specify the M's as functions of the target variables themselves so that, for example, an increase in inflation will cause the proportion of the deficit financed by debt to increase relative to the proportion financed by monetary expansion.

The computational-forms of (3.6.9) and (3.6.10) are written as:

$$p^{(t)}_{(4z)}\ B^{(t)35}_{(z)} + y^{(t)}_{(4z)4}\ B^{(t)36}_{(z)} - \sum_{j=3,5} dV^{(t-1)}_{(4z)j}\ B^{(t)37}_{(z)j} / V^{(t-1)}_{(4z)j}$$

$$= m^{(t)}_{(4z)}\ GBD^{(t)} + dGBD^{(t)} \qquad (z = 0,3) \quad (3.6.11),$$

and

$$m^{(t)}_{(40)} = -m^{(t)}_{(43)}\ B^{(t)38} \qquad (3.6.12),$$

where the B coefficients are defined in Appendix A.

3.7 Market clearing equations

For the two domestically produced goods, we have:

$$\sum_{j=1}^{2} Y^{(t)}_{(i1)j} = \sum_{j=1}^{2} \sum_{n=1}^{2} X^{(n)(t)}_{(i1)j} + X^{(t)}_{(i1)3} + X^{(t)}_{(i1)5} \qquad (i = 1,2) \quad (3.7.1).$$

Equation (3.7.1) equates the total supply of domestically produced good i at t to its demand at t. Total demand consists of demand for intermediate inputs to current production and capital creation, and final consumption and export demands.

The market clearing equations for the two primary factors are:

$$Y^{(t)}_{(31)3} = \sum_{j=1}^{2} X^{(t)}_{(31)j} + X^{(t)}_{(31)4} \qquad (3.7.2a),$$

and

$$V^{(t-1)}_{(32)j} = X^{(t)}_{(32)j} \qquad (j = 1,2) \quad (3.7.2b).$$

Equations (3.7.2a and b) require little explanation. Equation (3.7.2b) merely reflects the assumptions that fixed capital is industry-specific and that it takes until the beginning of the next period for investment in the current period to be installed.

To derive the market clearing equations for domestic financial assets, we first define the demand variables $X^{(t)}_{(4z)3}$ and $X^{(t)}_{(4z)5}$, such that:

$$X^{(t)}_{(4z)3} = Q^{(t)}_{(4z)3}\, W^{(t)}_{3} / \, P^{(t)}_{(4z)} \qquad (z = 0, \ldots, 4) \qquad (3.7.3a),$$

and

$$X^{(t)}_{(4z)5} = Q^{(*)(t)}_{(4z)5} W^{(*)(t)}_{5} / \, P^{(*)(t)}_{(4z)} \qquad (z = 1, \ldots, 4) \qquad (3.7.3b).$$

With this done, we can immediately write down the relevant market clearing relations:

$$Y^{(t)}_{(40)4} + NMM^{(t)} = X^{(t)}_{(40)3} \qquad (3.7.4a),$$

$$Y^{(t)}_{(4j)j} = X^{(t)}_{(4j)3} + X^{(t)}_{(4j)5} \qquad (j = 1,2) \qquad (3.7.4b),$$

and

$$Y^{(t)}_{(43)4} = X^{(t)}_{(43)3} + X^{(t)}_{(43)5} \qquad (3.7.4c)$$

where $NNM^{(t)}$ is the 'official monetary movements' item of the Balance of Payments accounts. Equation (3.7.4a) is the market clearing equation for money. The supply of this asset is written as the sum of two terms: the first is the domestic component of the money supply; while the second is the foreign exchange component. The remaining equations, (3.7.4b and c), equate the supplies of equity and government bonds to the corresponding demands from domestic and overseas sources.

In their computational-forms, equations (3.7.1) through (3.7.4) are written as:

$$y^{(t)}_{(i1)\cdot} = \sum_{j=1}^{2} \sum_{n=1}^{2} x^{(n)(t)}_{(i1)j} \; N^{(n)(t)}_{(i1)j} + x^{(t)}_{(i1)3} \, N^{(t)}_{(i1)3} + x^{(t)}_{(i1)5} \, N^{(t)}_{(i1)5} \qquad (i = 1,2) \quad (3.7.5),$$

$$y^{(t)}_{(31)3} = \sum_{j=1}^{2} x^{(t)}_{(31)j} \, N^{(t)}_{(31)j} + x^{(t)}_{(31)4} \, N^{(t)}_{(31)4} \qquad (3.7.6a),$$

$$v^{(t-1)}_{(32)j} = x^{(t)}_{(32)j} \qquad (j = 1,2) \quad (3.7.6b),$$

$$dX^{(t)}_{(4z)3} = dQ^{(t)}_{(4z)3} \, K^{(t)}_{(4z)3} + \left(w^{(t)}_{3} - p^{(t)}_{(4z)}\right) X^{(t)}_{(4z)3} \qquad (z = 0,\ldots, 4) \quad (3.7.7a),$$

$$dX^{(t)}_{(4z)5} = dQ^{(*)(t)}_{(4z)5} \; K^{(t)}_{(4z)5} + \left(w^{(*)(t)}_{5} - p^{(*)(t)}_{(4z)}\right) X^{(t)}_{(4z)5} \qquad (z = 1,\ldots, 4) \quad (3.7.7b),$$

$$dY^{(t)}_{(40)4} + dNMM^{(t)} = dX^{(t)}_{(40)3} \qquad (3.7.8a),$$

$$dY^{(t)}_{(4j)j} = dX^{(t)}_{(4j)3} + dX^{(t)}_{(4j)5} \qquad (j = 1,2) \quad (3.7.8b),$$

and

$$dY^{(t)}_{(43)4} = dX^{(t)}_{(43)3} + dX^{(t)}_{(43)5} \qquad (3.7.8c),$$

in which

$$y^{(t)}_{(i1)\cdot} = \sum_{j=1}^{2} y^{(t)}_{(i1)j} \; K^{(t)}_{(i1)j} \qquad (i = 1,2) \quad (3.7.9),$$

is the proportional-change in the total supply of domestically produced good i, $K^{(t)}_{(i1)j} = Y^{(t)}_{(i1)} / Y^{(t)}_{(i1)}$, $K^{(t)}_{(4z)3} = W^{(t)}_{3} / P^{(t)}_{(4z)}$ and $K^{(t)}_{(4z)5} = W^{(*)(t)}_{5} / P^{(*)(t)}_{(4z)}$. The N's are shares. For example $N^{(n)(t)}_{(i1)j}$ is the share in total sales of the domestically produced good i at t accounted for by sales to industry j for purpose n.

3.8 Macroeconomic equations

We deal first with the balance of payments. Aggregate domestic demand for imported commodity i at t, $X^{(t)}_{(i2)\cdot}$, consists of demands by industry and the consumer, i.e.

$$X^{(t)}_{(i2)\cdot} = \sum_{j=1}^{2} \sum_{n=1}^{2} X^{(n)(t)}_{(i2)j} + X^{(t)}_{(i2)3} \qquad (i = 1,2) \quad (3.8.1),$$

or in proportional—changes:

$$x^{(t)}_{(i2)\cdot} = \sum_{j=1}^{2} \sum_{n=1}^{2} x^{(n)(t)}_{(i2)j} N^{(n)(t)}_{(i2)j} + x^{(t)}_{(i2)3} N^{(t)}_{(i2)3} \qquad (i = 1,2) \quad (3.8.2),$$

where, as before, the N's are sales shares (e.g., $N^{(t)}_{(i2)3}$ is the share in total sales of imported good i represented by sales of that item to the consumer).

The total value of imports at t, $IM^{(t)}$, valued in the domestic currency exclusive of tariffs (i.e.,at c.i.f. prices), is given by:

$$IM^{(t)} = \sum_{i=1}^{2} \Phi^{(t)} P^{(*)(t)}_{(i2)} X^{(t)}_{(i2)\cdot} \qquad (3.8.3),$$

or in proportional-change form:

$$im^{(t)} = \sum_{i=1}^{2} \left(\phi^{(t)} + p^{(*)(t)}_{(i2)} + x^{(t)}_{(i2)\cdot}\right) IM^{(t)}_{i} \qquad (3.8.4),$$

where $IM^{(t)}_{i}$ is the share of $IM^{(t)}$ represented by the value of imported commodity i.

In the same fashion, we can define the following relationships for exports. The total value of exports at t, $EX^{(t)}$, is defined as follows:

$$EX^{(t)} = \sum_{i=1}^{2} X_{(i1)5}^{(t)} \, P_{(i1)}^{(*)(t)} \, \phi^{(t)} \qquad (3.8.5),$$

or in proportional-changes:

$$ex^{(t)} = \sum_{i=1}^{2} \left(x_{(i1)5}^{(t)} + p_{(i1)}^{(*)(t)} + \phi^{(t)}\right) EX_i^{(t)} \qquad (3.8.6),$$

where $EX_i^{(t)}$ is the share in $EX^{(t)}$ represented by the exported value of good i. Note that exports have been valued at their foreign currency f.o.b. prices, $P_{(i1)}^{(*)(t)}$, converted to domestic currency via the nominal exchange rate.

With $IM^{(t)}$ given by (3.8.3) and $EX^{(t)}$ by (3.8.5), the balance of trade at t, $BT^{(t)}$, can be written as:

$$BT^{(t)} = EX^{(t)} - IM^{(t)} \qquad (3.8.7),$$

or in computational-form:

$$dBT^{(t)} = ex^{(t)} \, EX^{(t)} - im^{(t)} \, IM^{(t)} \qquad (3.8.8),$$

where the absolute-change in $BT^{(t)}$ has been defined, rather than the proportional-change, because $BT^{(t)}$ can pass through zero.

The balance on the capital account at t, $CAP^{(t)}$, equals the value of domestic financial assets purchased by overseas interests less the value of 'overseas financial assets' purchased by the domestic consumer, all at t. This means, in algebraic terms, that:

$$CAP^{(t)} = -\sum_{z=1}^{3} P_{(4z)}^{(t)} \left(X_{(4z)5}^{(t)} - V_{(4z)5}^{(t-1)}\right)$$

$$- \; P_{(44)}^{(t)} \left(X_{(44)3}^{(t)} - V_{(44)3}^{(t-1)}\right) \qquad (3.8.9),$$

or in computational—form:

$$dCAP^{(t)} = - \sum_{z=1}^{3} \left[p_{(4z)}^{(t)} \; B_{(z)5}^{(t)39} + (dX_{(4z)5}^{(t)} - dV_{(4z)5}^{(t)}) \; P_{(4z)}^{(t)} \right]$$

$$- \; p_{(44)}^{(t)} \; B_{(4)3}^{(t)39} - (dX_{(44)3}^{(t)} + dV_{(4z)5}^{(t-1)}) \; P_{(44)}^{(t)} \qquad (3.8.10),$$

where the B's contained in (3.8.10) are defined in Appendix A.

We are now in the position to write down the Balance of Payments identity, the computational—form of which is:

$$dBT^{(t)} \; + \; dCAP^{(t)} \; = dNMM^{(t)} \qquad (3.8.11),$$

where $NMM^{(t)}$ is the sum of official monetary movements at t.

The final macroeconomic variable to be defined here is Gross National Product (GNP) in constant prices at t. GNP in current prices can be written as:

$$GNP^{(t)} = \text{consumption}^{(t)} + \text{investment}^{(t)} + \text{exports}^{(t)} - \text{imports}^{(t)}.$$

In terms of variables already defined:

$$GNP^{(t)} = \sum_{i=1}^{2} \sum_{z=1}^{2} P_{(iz)}^{(3)(t)} \; X_{(iz)3}^{(3)(t)} + \sum_{j=1}^{2} \Pi_j^{(t)} \; I_{(32)j}^{(t)} + EX^{(t)} - IM^{(t)} \qquad (3.8.12).$$

The proportional—change form of (3.8.12) is:

$$gnp^{(t)} = \sum_{i=1}^{2} \sum_{z=1}^{2} (p_{(iz)}^{(3)(t)} + x_{(iz)3}^{(3)(t)}) \; GNP_{(iz)3}^{(t)} + \sum_{j=1}^{2} (\pi_j^{(t)} + i_{(32)j}^{(t)}) GNP_{(32)j}^{(t)} + ex^{(t)} \; GNP_{(3)}^{(t)} - im^{(t)} \; GNP_{(4)}^{(t)} \qquad (3.8.13),$$

where $GNP^{(t)}_{(iz)3}$, $GNP^{(t)}_{(32)j}$,etc. are component shares (e.g., $GNP^{(t)}_{(32)j}$ is the share in $GNP^{(t)}$ at t represented by the value of investment demand in industry j).

To convert current price GNP into constant price GNP at t, we need to define the relevant price deflator. Following Horridge (1985), the GNP deflator is defined here as a weighted sum of the price deflators for consumption and investment. The proportional-change in the consumer price index, $\zeta^{(t)}_3$ at t, is given by:

$$\zeta^{(t)}_3 = \sum_{i=1}^{2} \sum_{z=1}^{2} p^{(3)(t)}_{(iz)} U^{(3)(t)}_{(iz)} \qquad (3.8.14),$$

where $U^{(3)(t)}_{(iz)}$ is the share of aggregate consumer spending devoted to good i from source z. Similarly, the proportional-change in the capital goods price index at t, $\zeta^{(t)}_2$, is given by:

$$\zeta^{(t)}_2 = \sum_{j=1}^{2} \pi^{(t)}_j U^{(2)(t)}_j \qquad (3.8.15),$$

where $U^{(2)(t)}_j$ is the share of aggregate private investment at t accounted for by industry j. The proportional-change in the GNP deflator, $\zeta^{(t)}$, is thus:

$$\zeta^{(t)} = \sum_{i=1}^{2} \sum_{z=1}^{2} GNP^{(t)}_{(iz)3} \zeta^{(t)}_3 + \sum_{j=1}^{2} GNP^{(t)}_{(32)j} \zeta^{(t)}_2 \qquad (3.8.16).$$

Given GNP in current prices (3.8.13) and the GNP price deflator (3.8.16), we can now define

$$gnp^{(t)}_R = gnp^{(t)} - \zeta^{(t)} \qquad (3.8.17).$$

This then completes our specification of MMO. In total, 143 equations in 216 variables (190 current and 26 lagged) have been specified. These equations and variables are set out in Appendices B and C, respectively.

4 Concluding Remarks and Some Areas for Further Research

In this paper, a small stylized CGE model solved using Johansen's method sequentially is presented. This model illustrates an alternative approach to the so-called Impact paradigm for dealing with ORANI's lack of short-run macroeconomic closure. The approach embodied in the model does not rely upon the assumption of micro/macro separability. Instead, it achieves macroeconomic closure by introducing financial assets directly into the decision-making processes of individuals and by specifying market clearing relationships for financial stocks as well as for monetary and real commodity flows.

The model is called MMO and has four quite distinct building blocks. The first is a complete specification along Walrasian lines of real flows of goods and services between industries and households in the domestic economy, and between the domestic economy and the rest of the world. This specification makes allowance for multi-product industries and multi-industry products, and for imperfect substitution between domestically produced and imported items with the same name. The second building block is a balance sheet framework developed along the lines of Tobin (1969) which envelopes (in principle) any number of financial assets and liabilities held by the domestic and overseas investors. The third is the explicit allowance for risk and uncertainty introduced by assuming that returns on all financial assets are, from the perspective of each portfolio holder, subject to unpredictable variations. The final building block is the assumption that the portfolio preferences of domestic and overseas agents are derived by maximization of direct intertemporal utility functions subject to stock and flow wealth constraints, and to dynamical equations for the evolution of asset prices over each agent's plan.

Clearly, MMO is a model still in its infancy. Many parts of its specification need refinement, in particular:

(i) while there is a detailed treatment of portfolio behaviour in the household sector, there is only a crude specification of such behaviour in the industrial sectors (applied models dealing explicitly with dividend behaviour and corporate saving are quite rare; some of the competing theories can be found in Auerbach and King, 1982 and Feldstein and Green, 1983);

(ii) the role played by institutional intermediaries in the financial sector is completely ignored as are the effects of interest rate controls and credit rationing policies;

(iii) while allowance has been made for taxation on the sales of goods and on the flow of wage income, no allowance has been made for any form of taxation on the sales or purchases of financial assets;

and

(iv) finally, the linkages between important policy targets and policy instruments (both fiscal and monetary) are neglected (the importance of such linkages within a CGE setting are emphasised in Feltenstein, 1986).

Work designed to extend MMO's specification in each of the four areas listed above is now underway.

Appendix A: The B Coefficients

$$B^{(t)1} = W_3^{(t-1)}/[W_3^{(t-1)} - \sum_{i=1}^{2} P_{(i.)}^{(3)(t)} X_{(i.)3}^{(t)} + (1-T_3^{(t)})(P_{(31)}^{(0)(t)} Y_{(31)3}^{(t)})]$$

$$B_{(z)}^{(t)2} = W_3^{(t)} W_3^{(t-1)} (P_{(4z)}^{(t)} - P_{(4z)}^{(t-1)})/P_{(4z)}^{(t)} B^{(t)1}$$

$$B_{(z)}^{(t)3} = W_3^{(t)} W_3^{(t-1)} Q_{(4z)3}^{(t)} \; P_{(4z)}^{(t-1)}/P_{(4z)}^{(t)} \; B^{(t)1}$$

$$B_{(i.)}^{(t)4} = W_3^{(t-1)} P_{(i.)}^{(3)(t)} X_{(i.)3}^{(t)}/B^{(t)1}$$

$$B^{(t)5} = W_3^{(t-1)} P_{(31)}^{(0)(t)} Y_{(31)3}^{(t)}/B^{(t)1}$$

$$B^{(t)6} = W_3^{(t-1)} (1 - T_3^{(t)}) P_{(31)}^{(0)(t)} Y_{(31)3}^{(t)}/B^{(t)1}$$

$$B_{(i.)}^{(t)7} = \beta_i \; (\xi^{(t)} - \sum_{j=1}^{2} P_{(j.)}^{(3)(t)} \bar{X}_{(j.)})/P_{(i.)}^{(3)(t)} X_{(i.)3}^{(t)}$$

$$B_{(i.)}^{(t)8} = \beta_i \mu^{(t)}/P_{(i.)}^{(3)(t)} X_{(i.)3}^{(t)}$$

$$B_{(ij)}^{(t)9} = \beta_i \mu^{(t)} \; (\xi^{(t)} - P_{(j.)}^{(3)(t)} \bar{X}_{(j.)})/P_{(i.)}^{(3)(t)} X_{(i.)3}^{(t)}$$

$$B_{(ij)}^{(t)10} = \beta_i \mu^{(t)} P_{(j.)}^{(3)(t)} \bar{X}_{(j.)}/P_{(i.)}^{(3)(t)} X_{(i.)3}^{(t)}$$

$$B_{(iz)}^{(t)11} = \omega_3^{(iz)} (\xi^{(t)} - \sum_{j=1}^{2} P_{(j.)}^{(3)(t)} \bar{X}_{(j.)})/A_{(0)3}^{(t)} \; W_3^{(t)}$$

$$B_{(iz)}^{(t)12} = A_{(i)3}^{(t)} \; B_{(iz)}^{(t)11}/A_{(0)3}^{(t)}$$

$$B_{(z)}^{(t)13} = \sum_{i=1}^{4} \omega_3^{(iz)} (A_{(i)3}^{(t)} - A_{(0)3}^{(t)})/A_{(0)3}^{(t)} \; W_3^{(t)}$$

$$B^{(t)14}_{(jz)} = P^{(3)(t)}_{(j.)} \bar{X}_{(j.)} B^{(t)13}_{(z)}$$

$$B^{(t-1)15}_{(z)} = P^{(t-1)}_{(4z)} (1 + \gamma_{(z)3}) / P^{(t-2)}_{(4z)}$$

$$B^{(t-2)16}_{(z)} = B^{(t-1)15}_{(z)} + P^{(t-2)}_{(4z)} \gamma_{(z)3} / P^{(t-3)}_{(4z)}$$

$$B^{(t-3)17}_{(z)} = \gamma_{(z)3} P^{(t-2)}_{(4z)} / P^{(t-3)}_{(4z)}$$

$$B^{(t)18}_{(z)} = P^{(t)}_{(4z)} V^{(t)}_{(4z)3} / W^{(t)}_{3}$$

$$B^{(t)19} = \delta / (A^{(t)}_{(0)3})^{2}$$

$$B^{(t)20} = A^{(t)}_{(0)3} W^{(t)}_{3}$$

$$B^{(t)21} = P^{(0)(t)}_{(31)} Y^{(t)}_{(31)3}$$

$$B^{(t)22} = (1 - T^{(t)}_{3}) P^{(0)(t)}_{(31)} Y^{(t)}_{(31)3}$$

$$B^{(t)23}_{(z)} = F_{(4z)5} \; [\sum_{i=1}^{3} \omega^{(iz)}_{5} (A^{(*)(t)}_{(i)5} - A^{(*)(t)}_{(4)5}) \eta^{(*)(t)} / W^{(*)(t)}_{5} A^{(*)(t)}_{(4)5}$$

$$B^{(t)24}_{(z)} = Q^{(*)(t)}_{(4z)5} / F_{(4z)5}$$

$$B^{(t)25}_{(iz)} = F_{(4z)5} \omega^{(iz)}_{5} \; (1 + \eta^{(*)(t)} / A^{(*)(t)}_{(4)5} W^{(*)(t)}_{5})$$

$$B^{(t)26}_{(iz)} = F_{(4z)5} \omega^{(iz)}_{5} (1 + \eta^{(*)(t)} A^{(*)(t)}_{(i)5} / (A^{(*)(t)}_{(4)5})^{2} \; W^{(*)(t)}_{5})$$

$$B^{(t)27}_{(z)} = F_{(4z)5} \sum_{i=1}^{3} \omega^{(iz)}_{5} (A^{(*)(t)}_{(i)5} - A^{(*)(t)}_{(4)5}) / A^{(*)(t)}_{(4)5} W^{(*)(t)}_{5}$$

$$B^{(t-1)28}_{(z)} = P^{(*)(t-1)}_{(4z)} (1 + \gamma_{(z)5}) / P^{(*)(t-2)}_{(4z)}$$

$$B^{(t-2)29}_{(z)} = B^{(t-1)28}_{(z)} + P^{(*)(t-2)}_{(4z)} \gamma_{(z)5} / P^{(*)(t-3)}_{(4z)}$$

$$B^{(t-3)30}_{(z)} = \gamma_{(z)5} P^{(*)(t-2)}_{(4z)} / P^{(*)(t-3)}_{(4z)}$$

$$B^{(t)31}_{(z)} = P^{(*)(t)}_{(4z)} V^{(t)}_{(4z)5} / W^{(*)(t)}_{5}$$

$$B^{(t)32}_{j} = d_j V^{(t-1)}_{(32)j} / (\pi^{(t)}_{j} (I^{(t)}_{(32)j} - d_j V^{(t-1)}_{(32)j}))$$

$$B^{(t)33}_{j} = P^{(t)}_{(4j)} Y^{(t)}_{(4j)j} B^{(t)32}_{j} / d_j V^{(t-1)}_{(32)j}$$

$$B^{(t)34}_{(j)k} = P^{(t)}_{(4j)} V^{(t-1)}_{(4j)k} B^{(t)32}_{j} / d_j V^{(t-1)}_{(32)j}$$

$$B^{(t)35}_{(z)} = P^{(t)}_{(4z)} (Y^{(t)}_{(4z)4} - \sum_{j=3,5} V^{(t-1)}_{(4z)j}) / M^{(t)}_{(4z)}$$

$$B^{(t)36}_{(z)} = P^{(t)}_{(4z)} Y^{(t)}_{(4z)4} / M^{(t)}_{(4z)}$$

$$B^{(t)37}_{(z)j} = P^{(t)}_{(4z)} V^{(t-1)}_{(4z)j} / M^{(t)}_{(4z)}$$

$$B^{(t)38} = M^{(t)}_{(43)} / M^{(t)}_{(40)}$$

$$B^{(t)39}_{(z)j} = P^{(t)}_{(4z)} (X^{(t)}_{(4z)j} - V^{(t-1)}_{(4z)j})$$

Appendix B : The MMO Equations

3.2 Final demand equations and portfolio rules

3.2.1 Consumer demand for goods and financial assets

$$w_3^{(t)} = w_3^{(t-1)}B^{(t)1} + \sum_{z=0}^{4}\left[dQ_{(4z)3}^{(t)}\, B_{(z)}^{(t)2} + (p_{(4z)}^{(t)} - p_{(4z)}^{(t-1)})B_{(z)}^{(t)3}\right]$$

$$- \sum_{i=1}^{2}(p_{(i\cdot)}^{(3)(t)} + x_{(i\cdot)3}^{(t)})B_{(i\cdot)}^{(t)4} - dT_3^{(t)}\, B^{(t)5}$$

$$+ (p_{(31)}^{(0)(t)} + y_{(31)3}^{(t)})B^{(t)6} \qquad (3.2.23)$$

$$x_{(i\cdot)3}^{(t)} = d\mu^{(t)}B_{(i\cdot)}^{(t)7} + d\zeta^{(t)}B_{(i\cdot)}^{(t)8} - p_{(i\cdot)}^{(3)(t)}B_{(ij)}^{(t)9}$$

$$- p_{(j\cdot)}^{(3)(t)}B_{(ij)}^{(t)10} \qquad (i = 1,2;\ j \neq i) \quad (3.2.24)$$

$$dQ_{(40)3}^{(t)} = -\sum_{i=1}^{4} dQ_{(4i)3}^{(t)} \qquad (3.2.25)$$

$$dQ_{(4z)3}^{(t)} + w_3^{(t)}Q_{(4z)3}^{(t)} = \sum_{i=1}^{4}(dA_{(i)3}^{(t)}B_{(iz)}^{(t)11} - dA_{(0)3}^{(t)}B_{(iz)}^{(t)12})$$

$$+ d\zeta^{(t)}B_{(z)}^{(t)13} - \sum_{j=1}^{2} p_{(j\cdot)}^{(3)(t)}B_{(jz)}^{(t)14}$$

$$(z = 1,\ldots,4) \quad (3.2.26)$$

$$x^{(t)}_{(is)3} = x^{(t)}_{(i\cdot)3} - \sigma^{(3)}_{(i)} \left(p^{(3)(t)}_{(is)} - \sum_{j=1}^{2} p^{(3)(t)}_{(ij)} S^{(t)}_{(ij)3}\right)$$

$$(i,s = 1,2) \quad (3.2.27)$$

$$dA^{(t)}_{(z)3} = p^{(t-1)}_{(4z)} B^{(t-1)15}_{(z)} - p^{(t-2)}_{(4z)} B^{(t-2)16}_{(z)} + p^{(t-3)}_{(4z)} B^{(t-3)17}_{(z)}$$

$$(z = 0,\ldots, 4) \quad (3.2.28)$$

$$w^{(t)}_{3} = \sum_{z=0}^{4} \left(p^{(t)}_{(4z)} B^{(t)18}_{(z)} + dV^{(t)}_{(4z)3}\, B^{(t)18}_{(z)} / V^{(t)}_{(4z)3}\right) \quad (3.2.29)$$

$$d\mu^{(t)} = -dA^{(t)}_{(0)3}\, B^{(t)19} \quad (3.2.30)$$

$$d\zeta^{(t)} = dA^{(t)}_{(0)3} W^{(t)}_{3} + w^{(t)}_{3} B^{(t)20} - dT^{(t)}_{3} B^{(t)21}$$

$$+ \left(p^{(0)(t)}_{(31)} + y^{(t)}_{(31)3}\right) B^{(t)22} \quad (3.2.31)$$

$$p^{(3)(t)}_{(i\cdot)} = \sum_{s=1}^{2} S^{(t)}_{(is)3}\, p^{(3)(t)}_{(is)} \quad (i = 1,2) \quad (3.2.32)$$

3.2.2 Overseas demand for domestic goods and financial assets

$$p_{(i1)}^{(*)(t)} = -\gamma_{(i1)} x_{(i1)5}^{(t)} + f_{(i1)5} \qquad (i = 1,2) \quad (3.2.34)$$

$$dQ_{(4z)5}^{(*)(t)} + w_5^{(*)(t)} B_{(z)}^{(t)23} = dF_{(4z)5} B_{(z)}^{(t)24} + \sum_{i=1}^{3} dA_{(i)5}^{(*)(t)} B_{(iz)}^{(t)25}$$

$$- \sum_{i=1}^{3} dA_{(4)5}^{(*)(t)} B_{(iz)}^{(t)26} + d\eta^{(*)(t)} B_{(z)}^{(t)27}$$

$$(z = 1,\ldots, 3),$$

$$dQ_{(44)5}^{(*)(t)} = - \sum_{z=1}^{3} dQ_{(4z)5}^{(*)(t)} F_{(44)5} + dF_{(44)5} [1 - \sum_{i=1}^{3} Q_{(4i)5}^{(*)(t)}]$$

$$(3.2.41)$$

$$dA_{(z)5}^{(*)(t)} = p_{(4z)}^{(*)(t-1)} B_{(z)}^{(t-1)28} - p_{(4z)}^{(*)(t-2)} B_{(z)}^{(t-2)29} + p_{(4z)}^{(*)(t-3)} B_{(z)}^{(t-3)30}$$

$$(z = 1,\ldots, 4) \quad (3.2.42)$$

$$w_5^{(*)(t)} = \sum_{z=1}^{4} (p_{(4z)}^{(*)(t)} B_{(z)}^{(t)31} + dV_{(4z)5}^{(t)} B_{(z)}^{(t)31} / V_{(4z)5}^{(t)}) \quad (3.2.43)$$

3.2.4 Industry demand for inputs to the production of fixed capital

$$x^{(2)(t)}_{(iz)j} = i^{(t)}_{(32)j} - \sigma^{(2)}_{(i)j} \left(p^{(2)(t)}_{(iz)} - \sum_{q=1}^{2} S^{(2)(t)}_{(iq)j} \, p^{(2)(t)}_{(iq)}\right)$$

$$(i,z,j = 1,2) \quad (3.2.47)$$

3.3 Industry inputs and outputs for current production

3.3.1 Industry supplies

$$y^{(t)}_{(i1)j} = z^{(t)}_{j} - \beta_j \left[p^{(0)(t)}_{(i1)} - \sum_{q=1}^{2} R^{(t)}_{(q1)j} \, p^{(0)(t)}_{(q1)}\right]$$

$$(i,j = 1,2) \quad (3.3.3)$$

3.3.2 Industry demands

$$x^{(1)(t)}_{(iz)j} = z^{(t)}_{j} - \sigma^{(1)}_{(i)j} \left(p^{(1)(t)}_{(i1)} - \sum_{q=1}^{2} S^{(1)(t)}_{(iq)j} \, p^{(1)(t)}_{(iq)}\right)$$

$$(i,z,j = 1,2) \quad (3.3.5)$$

$$x^{(t)}_{(31)j} = z^{(t)}_{j} - \sigma^{(1)}_{(3)j} \, S^{(t)}_{(32)j} \left(p^{(1)(t)}_{(31)} - p^{(t)}_{(32)j}\right) \quad (j = 1,2) \quad (3.3.6)$$

$$x^{(t)}_{(32)j} = z^{(t)}_j - \sigma^{(1)}_{(3)j} S^{(t)}_{(31)j} \left(p^{(t)}_{(32)j} - p^{(1)(t)}_{(31)}\right) \quad (j = 1,2) \quad (3.3.7)$$

3.4 The creation of industry-specific fixed capital and the supply of equities

$$i^{(t)}_{(32)j} = f_{(32)j} + \lambda^{(t)}_j G^{(t)}_j \left(p^{(t)}_{(4j)} - \pi^{(t)}_j\right) + \left(1 - G^{(t)}_j\right) v^{(t-1)}_{(32)j}$$

$$(j = 1,2) \quad (3.4.3)$$

$$\pi^{(t)}_j + i^{(t)}_{(32)j}/G^{(t)}_j - v^{(t-1)}_{(32)j} B^{(t)32}_j = p^{(t)}_{(4j)} + y^{(t)}_{(4j)j} B^{(t)33}_j$$

$$- \sum_{k=3,5} v^{(t-1)}_{(4j)k} B^{(t)34}_{(j)k} \quad (j = 1,2) \quad (3.4.5)$$

3.5 Pricing equations

$$\sum_{i=1}^{2} p^{(0)(t)}_{(i1)} H^{(0)(t)}_{(i1)j} = \sum_{i=1}^{2} \sum_{z=1}^{2} p^{(1)(t)}_{(iz)} H^{(1)(t)}_{(iz)j}$$

$$+ p^{(1)(t)}_{(31)} H^{(t)}_{(31)j} + p^{(t)}_{(32)j} H^{(t)}_{(32)j} \quad (j = 1,2) \quad (3.5.2)$$

$$\pi^{(t)}_j = \sum_{i=1}^{2} \sum_{z=1}^{2} p^{(2)(t)}_{(iz)} H^{(2)(t)}_{(iz)j} \quad (j = 1,2) \quad (3.5.3)$$

$$p^{(*)(t)}_{(i1)} + \phi^{(t)} + \tau^{(e)(t)}_{(i1)} = p^{(0)(t)}_{(i1)} \qquad (i = 1,2) \quad (3.5.5)$$

$$p^{(0)(t)}_{(i2)} = p^{(*)(t)}_{(i2)} + \phi^{(t)} + \tau^{(m)(t)}_{(i2)} \qquad (i = 1,2) \quad (3.5.7)$$

$$p^{(n)(t)}_{(iz)} = p^{(0)(t)}_{(iz)} + \tau^{(n)(t)}_{(iz)} \qquad (n = 1,2,3;\ i,z = 1,2),$$

$$p^{(1)(t)}_{(31)} = p^{(0)(t)}_{(31)} + \tau^{(1)(t)}_{(31)} \qquad (3.5.9)$$

$$p^{(t)}_{(4z)} = \ell^{(t)}_{(4z)} + p^{(t-1)}_{(4z)} \qquad (z = 0, \ldots, 4) \quad (3.5.13)$$

$$dR^{(t)}_{(4z)} = \ell^{(t)}_{(4z)} L^{(t)}_{(4z)} \qquad (z = 0, \ldots, 4) \quad (3.5.14)$$

$$p^{(t)}_{(4z)} = p^{(*)(t)}_{(4z)} + \phi^{(t)} \qquad (z = 1, \ldots, 4) \quad (3.5.15)$$

3.6 Government budgetary rules

$$y^{(t)}_{4} = x^{(t)}_{(31)4} \qquad (3.6.1)$$

$$y_4^{(t)} = gnp_R^{(t)} + f_4 \tag{3.6.2}$$

$$gex^{(t)} = \left(p_{(31)}^{(0)(t)} + x_{(31)4}^{(t)}\right) \tag{3.6.4}$$

$$gre^{(t)} = \sum_{i=1}^{2} (dT_{(i1)}^{(e)(t)} / T_{(i1)}^{(e)(t)} + p_{(i1)}^{(*)(t)} + \phi^{(t)} + x_{(i1)5}^{(t)}) H_{(i1)}^{(e)(t)}$$

$$+ \sum_{j=1}^{2} \sum_{i=1}^{2} \sum_{n=1}^{2} (dT_{(i2)}^{(m)(t)} / T_{(i2)}^{(m)(t)} + p_{(i2)}^{(*)(t)} + \phi^{(t)} + x_{(i2)j}^{(n)(t)}) H_{(i2)j}^{(m)(n)(t)}$$

$$+ \sum_{i=1}^{2} (dT_{(i2)}^{(m)(t)} / T_{(i2)}^{(m)(t)} + p_{(i2)}^{(*)(t)} + \phi^{(t)} + x_{(i2)3}^{(t)}) H_{(i2)3}^{(m)(t)}$$

$$+ \sum_{j=1}^{2} \sum_{i=1}^{2} \sum_{z=1}^{2} \sum_{n=1}^{2} (dT_{(iz)}^{(n)(t)} / T_{(iz)}^{(n)(t)} + p_{(iz)}^{(o)(t)} + x_{(iz)j}^{(n)(t)}) H_{(iz)j}^{(n)(t)}$$

$$+ \sum_{i=1}^{2} \sum_{z=1}^{2} (dT_{(iz)}^{(3)(t)} / T_{(iz)}^{(3)(t)} + p_{(iz)}^{(o)(t)} + x_{(iz)3}^{(t)}) H_{(iz)}^{(3)(t)}$$

$$+ \sum_{j=1}^{2} (dT_{(31)}^{(1)(t)} / T_{(31)}^{(1)(t)} + p_{(31)}^{(o)(t)} + x_{(31)j}^{(t)}) H_{(31)j}^{(1)(t)}$$

$$+ (dT_{(3)}^{(t)} / T_{(3)}^{(t)} + p_{(31)}^{(o)(t)} + y_{(31)3}^{(o)(t)}) H^{(3)(t)} \tag{3.6.6}$$

$$dGBD^{(t)} \equiv gex^{(t)} GEX^{(t)} - gre^{(t)} GRE^{(t)} \tag{3.6.8}$$

$$p^{(t)}_{(4z)}\ B^{(t)35}_{(z)} + y^{(t)}_{(4z)4}\ B^{(t)36}_{(z)} - \sum_{j=3,5} dV^{(t-1)}_{(4z)j}\ B^{(t)37}_{(z)j} / V^{(t-1)}_{(4z)j}$$

$$= m^{(t)}_{(4z)}\ GBD^{(t)} + dGBD^{(t)} \qquad (z = 0,3) \quad (3.6.11)$$

$$m^{(t)}_{(40)} = -m^{(t)}_{(43)}\ B^{(t)38} \qquad (3.6.12)$$

3.7 Market clearing equations

$$y^{(t)}_{(i1).} = \sum_{j=1}^{2} \sum_{n=1}^{2} x^{(n)(t)}_{(i1)j}\ N^{(n)(t)}_{(i1)j} + x^{(t)}_{(i1)3}\ N^{(t)}_{(i1)3}$$

$$+ x^{(t)}_{(i1)5}\ N^{(t)}_{(i1)5} \qquad (i = 1,2) \quad (3.7.5)$$

$$y^{(t)}_{(31)3} = \sum_{j=1}^{2} x^{(t)}_{(31)j}\ N^{(t)}_{(31)j} + x^{(t)}_{(31)4}\ N^{(t)}_{(31)4} \qquad (3.7.6a)$$

$$v^{(t-1)}_{(32)j} = x^{(t)}_{(32)j} \qquad (j = 1,2) \quad (3.7.6b)$$

$$dX^{(t)}_{(4z)3} = dQ^{(t)}_{(4z)3}\ K^{(t)}_{(4z)3} + \left(w^{(t)}_{3} - p^{(t)}_{(4z)}\right)\ X^{(t)}_{(4z)3}$$

$$(z = 0,\ldots, 4) \quad (3.7.7a)$$

$$dX^{(t)}_{(4z)5} = dQ^{(*)(t)}_{(4z)5}\ K^{(t)}_{(4z)5} + \left(w^{(*)(t)}_{5} - p^{(*)(t)}_{(4z)}\right)X^{(t)}_{(4z)5}$$

$(z = 1, \ldots, 4)$ (3.7.7b)

$$dY^{(t)}_{(40)4} + dNMM^{(t)} = dX^{(t)}_{(40)3} \qquad (3.7.8a)$$

$$dY^{(t)}_{(4j)j} = dX^{(t)}_{(4j)3} + dX^{(t)}_{(4j)5} \qquad (j = 1,2) \quad (3.7.8b)$$

$$dY^{(t)}_{(43)4} = dX^{(t)}_{(43)3} + dX^{(t)}_{(43)5} \qquad (3.7.8c)$$

$$y^{(t)}_{(i1)\bullet} = \sum_{j=1}^{2} y^{(t)}_{(i1)j}\ K^{(t)}_{(i1)j} \qquad (i = 1,2) \quad (3.7.9)$$

3.8 Macroeconomic equations

$$x^{(t)}_{(i2)\bullet} = \sum_{j=1}^{2} \sum_{n=1}^{2} x^{(n)(t)}_{(i2)j}\ N^{(n)(t)}_{(i2)j} + x^{(t)}_{(i2)3}\ N^{(t)}_{(i2)3}$$

$(i = 1,2)$ (3.8.2)

$$im^{(t)} = \sum_{i=1}^{2} \left(\phi^{(t)} + p_{(i2)}^{(*)(t)} + x_{(i2).}^{(t)}\right) IM_i^{(t)} \tag{3.8.4}$$

$$ex^{(t)} = \sum_{i=1}^{2} \left(x_{(i1)5}^{(t)} + p_{(i1)}^{(*)(t)} + \phi^{(t)}\right) EX_i^{(t)} \tag{3.8.6}$$

$$dBT^{(t)} = ex^{(t)} EX^{(t)} - im^{(t)} IM^{(t)} \tag{3.8.8}$$

$$\begin{aligned} dCAP^{(t)} = &- \sum_{z=1}^{3} \left[p_{(4z)}^{(t)} B_{(z)5}^{(t)39} + (dX_{(4z)5}^{(t)} - dV_{(4z)5}^{(t)}) P_{(4z)}^{(t)} \right] \\ &- p_{(44)}^{(t)} B_{(4)3}^{(t)39} - (dX_{(44)3}^{(t)} + dV_{(4z)5}^{(t-1)}) P_{(44)}^{(t)} \end{aligned} \tag{3.8.10}$$

$$dBT^{(t)} + dCAP^{(t)} = dNMM^{(t)} \tag{3.8.11}$$

$$\begin{aligned} gnp^{(t)} = &\sum_{i=1}^{2} \sum_{z=1}^{2} (p_{(iz)}^{(3)(t)} + x_{(iz)3}^{(3)(t)}) GNP_{(iz)3}^{(t)} \\ &+ \sum_{j=1}^{2} \left(\pi_j^{(t)} + i_{(32)j}^{(t)}\right) GNP_{(32)j}^{(t)} \\ &+ ex^{(t)} GNP_{(3)}^{(t)} - im^{(t)} GNP_{(4)}^{(t)} \end{aligned} \tag{3.8.13}$$

$$\zeta_3^{(t)} = \sum_{i=1}^{2} \sum_{z=1}^{2} p_{(iz)}^{(3)(t)} U_{(iz)}^{(3)(t)} \qquad (3.8.14)$$

$$\zeta_2^{(t)} = \sum_{j=1}^{2} \pi_j^{(t)} U_j^{(2)(t)} \qquad (3.8.15)$$

$$\zeta^{(t)} = \sum_{i=1}^{2} \sum_{z=1}^{2} GNP_{(iz)3}^{(t)} \zeta_3^{(t)} + \sum_{j=1}^{2} GNP_{(32)j}^{(t)} \zeta_2^{(t)} \qquad (3.8.16)$$

$$gnp_R^{(t)} = gnp^{(t)} - \zeta^{(t)} \qquad (3.8.17)$$

Appendix C : The MMO Variables

Variable	Subscript range	Description
$dQ^{(t)}_{(4z)5}$	$z = 0,\ldots,4$	End of period component shares in aggregate consumer financial wealth
$dQ^{(*)(t)}_{(4z)5}$	$z = 1,\ldots,4$	End of period component shares in aggregate overseas financial wealth
$dV^{(h)}_{(4z)j}$	$h = t,\ t-1$ $z = 0,\ldots,4;\ j = 3$ $z = 1,\ldots,4;\ j = 5$	End of period stocks of financial assets
$v^{(t-1)}_{(32)j}$	$j = 1,2$	Real capital stocks, at end of period
$w^{(h)}_{3}$	$h = t,\ t-1$	End of period aggregate stock of consumer financial wealth
$w^{(*)(t)}_{5}$		End of period aggregate stock of overseas financial wealth, valued in foreign currency
$i^{(t)}_{(32)j}$	$j = 1,2$	Capital creation by using industry
$dX^{(t)}_{(4z)j}$	$z = 0,\ldots,4;\ j = 3$ $z = 1,\ldots,4;\ j = 5$	Demands for financial assets
$x^{(t)}_{(i\cdot)3}$	$i = 1,2$	Household demands for goods, undifferentiated by source
$x^{(t)}_{(iz)3}$	$i,z = 1,2$	Household demands for goods, differentiated by type and source
$x^{(t)}_{(i1)5}$	$i = 1,2$	Export volumes
$x^{(n)(t)}_{(iz)j}$	$n,i,z,j = 1,2$	Industry demands for inputs (domestic and imported) for current production and capital creation

Variable	Subscript range	Description
$x^{(t)}_{(3z)j}$	z,j = 1,2	Industry demands for labour and capital
$x^{(t)}_{(31)4}$		Government demand for labour
$x^{(t)}_{(i2)\cdot}$	i = 1,2	Total domestic demand for imports by type
$y^{(t)}_{(4z)4}$	z = 0,3	Government supplies of financial assets
$y^{(t)}_{(4j)j}$	j = 1,2	Industry supplies of equities
$y^{(t)}_{(44)5}$		Supply of overseas financial assets
$y^{(t)}_{(31)3}$		Household supply of labour
$y^{(t)}_{(i1)j}$	i = 1,2	Supplies of goods by industries
$y^{(t)}_{4}$		Government production
$y^{(t)}_{(i1)\cdot}$	i = 1,2	Total supplies of domestic goods by type
$z^{(t)}_{j}$	j = 1,2	Industry activity levels
$dA^{(t)}_{(z)3}$	z = 0,...,4	'Permanent' proportional changes in domestic financial asset prices, from the perspective of the consumer
$dA^{(*)(t)}_{(z)5}$	z = 1,...,4	'Permanent' proportional changes in foreign financial asset prices, from the perspective of the overseas entity
$p^{(h)}_{(4z)}$	h = t, t-1, t-2 z = 0,...,4	Domestic financial asset prices

Variable	Subscript range	Description
$p_{(4z)}^{(*)(h)}$	$h = t, t-1, t-2$ $z = 1,\ldots,4$	Overseas financial asset prices
$p_{(i\cdot)}^{(3)(t)}$	$i = 1,2$	Purchasers' prices for consumer goods by type, but not by source
$p_{(iz)}^{(3)(t)}$	$i,z = 1,2$	Purchasers' prices for consumer goods by type and by source
$p_{(31)}^{(0)(t)}$		Basic price of labour
$p_{(i1)}^{(*)(t)}$	$i = 1,2$	F.o.b. foreign currency export prices
$p_{(iz)}^{(n)(t)}$	$n,i,z = 1,2$	Purchasers' prices for produced inputs for current production and capital creation
$p_{(iz)}^{(0)(t)}$	$i,z = 1,2$	Basic prices of domestic goods and imports
$p_{(31)}^{(1)(t)}$		Purchasers' price for labour
$p_{(32)j}^{(t)}$	$j = 1,2$	Prices paid by each industry for rental of capital
$\pi^{(t)}$	$j = 1,2$	Costs of units of capital
$\phi^{(t)}$		The exchange rate, domestic currency per unit of overseas currency
$p_{(i2)}^{(*)(t)}$	$i = 1,2$	C.i.f. foreign currency import prices
$dT_{3}^{(t)}$		Average rate of household income tax
$dT_{(i1)}^{(e)(t)}$	$i = 1,2$	Ad valorem rates of export subsidies

Variable	Subscript range	Description
$dT^{(m)(t)}_{(i2)}$	$i = 1,2$	Ad valorem rates of import tariffs
$dT^{(n)(t)}_{(iz)}$	$n = 1,2,3$ $i,z = 1,2$	Ad valorem rates of sales taxes
$dT^{(1)(t)}_{(31)}$		Ad valorem rate of payroll tax
$\tau^{(e)(t)}_{(i1)}$	$i = 1,2$	Powers of export subsidies
$\tau^{(m)(t)}_{(i2)}$	$i = 1,2$	Powers of import tariffs
$\tau^{(n)(t)}_{(iz)}$	$n = 1,2,3$ $i,z = 1,2$	Powers of sales taxes
$\tau^{(1)(t)}_{(31)}$		Power of payroll tax
$f_{(i1)5}$	$i = 1,2$	Shifts in foreign export demands
$dF_{(4z)5}$	$z = 1,\ldots,4$	Shifts in foreign demands for domestic financial assets
$f_{(32)j}$	$j = 1,2$	Shifts terms for net investment
f_4		Shift in government production
$d\mu^{(t)}$		The ratio of the consumers' subjective rate of time discount to the 'safe' rate of return
$d\xi^{(t)}$		Consumer 'safe disposable income'
$d\eta^{(*)(t)}$		Foreign wage income net of total foreign 'subsistence expenditure'
$\ell^{(t)}_{(4z)}$	$z = 0,\ldots,4$	Ratios of current to previous financial asset prices

Variable	Subscript range	Description
$dR_{(4z)}^{(t)}$	$z = 0,\ldots,4$	Domestic market rates of return on financial assets
$gnp_R^{(t)}$		Gross national product in constant prices
$gnp^{(t)}$		Gross national product in current prices
$gex^{(t)}$		Total government expenditure
$gre^{(t)}$		Total government revenue
$dGBD^{(t)}$		The government's budget deficit
$m_{(4z)}^{(t)}$	$z = 0,3$	Proportions of budget deficit financed by each financial asset
$dNMM^{(t)}$		Net monetary movements
$im^{(t)}$		Foreign currency value of imports
$ex^{(t)}$		Foreign currency value of exports
dBT		The balance of trade
$dCAP^{(t)}$		The capital account balance
$\zeta_3^{(t)}$		The consumer price index
$\zeta_2^{(t)}$		The capital-goods price index
$\zeta^{(t)}$		The GNP price deflator

ENDNOTES

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1. The IMPACT Project is an economic and demographic research endeavour conducted by several agencies of the Australian government in association with the University of Melbourne, the Australian National University and La Trobe University. The standard ORANI structure is described fully in Dixon, Parmenter, Sutton and Vincent (1982). Other CGE models of the Johansen class include Taylor and Black (1974), Staelin (1976), and Deardorff and Stern (1986).

2. See Cooper and McLaren (1980, 1982 and 1983); Cooper (1983); Powell, Cooper and McLaren (1983); and the summary given in Cooper, McLaren and Powell (1985). For a more general discussion of the problems associated with the bringing together of two separately constructed models, one micro the other macro, see Robinson and Tyson (1984).

3. The particular macro model chosen was a slightly revised version of the Reserve Bank of Australia's RBII model (Jonson and Trevor, 1981; see also Cooper, 1983).

4. Theoretical support for this position can be found in Grandmont (1983, pp. 32-45).

5. See Dixon, Parmenter and Rimmer (1984); Horridge and Powell (1984); and Horridge (1985).

6. The latter expression, necessary for correct stock-flow accounting, introduces unavoidably a form of primitive dynamics hitherto unknown in ORANI. However, as shown in Horridge and Powell (1984, pp. 3-34), simulation results from the subsequent model are not overly sensitive to the choice of dynamic path for domestic saving.

7. This approach is illustrated by the CGE models developed in Dee (1983 and 1986), Slemrod (1983), Ezaki (1985), Feltenstein (1986), and Manne, Rutherford and Waelbroeck (1986).

8. As described in Dixon, Parmenter and Sutton (1978 pp. 6-11); and discussed further in Dee (1984).

9. 'Temporary equilibrium' is a concept developed in Hicks (1946) from earlier work undertaken by the Swedish economists Lindhal and Myrdal; see also Hicks (1965) for reflections on its use, and Grandmont (1977) for an up-to-date survey of the literature.

10. Note that for economy of exposition (2.1) contains only one vector of lagged variables (pertaining to (t-1)); in principle any number of such vectors, each corresponding to a different period prior to t, can be specified.

11. See also Dixon (1985) who explains the (non-sequential) Johansen method with the aid of a simple example.

12. The matrices $A_t(t)$ and $A_t(t-1)$ in (2.2) provide only a local representation of the model (2.1). For small changes in U(t) and U(t-1), (2.2) will be precise, however for larger changes (2.2) may be effected by unacceptably large linearization errors. In the latter case, the appropriate procedure is to breakdown U(t) and U(t-1) each into n vectors containing only 'small' changes and then employ a series of n Johansen-style computations with n updates of the $A_t(\tau)$ matrices. This Euler solution procedure is fully explained in the context of ORANI by Dixon, Parmenter, Sutton and Vincent (1982, ch.5).

13. Note that the predetermined variables at t consist of lagged endogenous variables, lagged exogenous variables and current exogenous variables; the first category covers all variables determined by the model in the period immediately prior to t.

14. If the partition of variables into x( ) and y( ) is 'meaningful' then this will ensure that C_1( ) is of full rank and is thus invertible.

15. As in the experience with ORANI, it may be

> '... that the difference between some well-documented base-period data base and a reasonable ceteris paribus data base for the [period] projected would have only minor effects on the results' (Powell, Cooper and McLaren, 1983, p. 3).

16. Alternative models to the ELESA are rare; one example is Clements (1976).

17. See Hasenkamp (1976, p. 19) and Dixon, Parmenter, Sutton and Vincent (1982, pp. 68-94).

18. Note that a 'basic' price is the price received by the producer excluding any sales tax (or any other margin) involved in the transfer of the good to customers.

19. Full details of the derivation of (3.3.3) are given in Dixon, Vincent and Powell (1976).

20. An application of Tobin's 'q' theory within a CGE framework can be found in Dee (1984). Yoshikawa (1980) demonstrates that the 'q' theory can be derived from considering an optimal capital accumulation problem which explicitly accommodates adjustment costs pertaining to investment.

21. Hyashi (1982) derives the exact relationship between marginal q and average q, and also establishes the necessary conditions under which the two measures are equal.

22. In (3.5.6), the proportional-change in the 'power' of the export subsidy is defined (i.e., the proportional change in one plus the ad valorem rate). By using powers rather than rates we minimize the risk of computing the proportional-change of a variable whose initial value is zero.

23. Dixon, Parmenter, Sutton and Vincent (1982, p. 115) emphasise that the (1 + T) terms appearing in (3.5.9) are taxes on sales not production. There is no facility in MMO for handling taxes on the latter.

24. In the present specification of MMO, all of the rental to capital, net of depreciation allowances, is treated as retained earnings of the industry and not as income of the consumer. This is a simplifying assumption which will be maintained until a formal model of corporate saving can be developed and then integrated into the full system. Dee (1983, p. 28) notes

> 'The balance sheet counterpart of corporate saving would be a change in firm liabilities or a loan by the firm itself.'

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