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THE THEORETICAL STRUCTURE OF ORANI 78

by

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1. Introduction[1]

The principal purpose of ORANI is to provide projections of the effects of various economic policy changes on a wide variety of economic variables. A typical ORANI result is of the form : given a policy change A in the macroeconomic environment B, then in the short-run variable C will differ by x per cent from the value it would have had in the absence of the policy change, while in the long-run it will differ by y per cent. Among the policy changes, A, which can be considered are tariff changes, exchange rate movements, changes in the level and composition of government expenditure, variations in indirect taxes and changes in wage policy. Examples of variables, C, for which projections can be made are rates of output by industry, demand for labor by occupation, rates of investment by industry, the balance of trade, aggregate employment and the rate of inflation.

ORANI recognizes that the effects of any policy change depend on the macroeconomic environment in which it takes place. For example, the effects of a tariff cut in an economy with full wage

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* This is a draft chapter for a book on the ORANI model being prepared by P.B. Dixon, B.R. Parmenter, J.M. Sutton and D.P. Vincent. Consequently there are occasional references to other chapters. The material is, however, self-contained.

1. This chapter contains some minor repetition of material from Chapter 2. This is unavoidable given our objective that both chapters be capable of standing alone.

indexation and a fixed exchange rate will differ from the effects of the same tariff cut in an economy with partial wage indexation and a flexible exchange rate. Hence, ORANI is designed so that users must specify the economic environment, B, for which their projections apply. ORANI also recognizes a distinction between the long-run and short-run effects of a policy change. The short-run effects are measured by the change in the equilibrium values of variables in a situation of fixed industry capital stocks, whereas for long-run calculations, ORANI allows policy changes to affect capital stocks.

Schematically, the ORANI model can be written as

$$\left.\begin{array}{l} F_1(X_t,\ K_{t+1},\ K_t) \ = \ 0 \\ \quad\vdots \\ F_m(X_t,\ K_{t+1},\ K_t) \ = \ 0 \end{array}\right\} , \qquad (1.1)$$

where K_t, the vector of industry capital stocks existing at time t, is usually treated as being predetermined. It depends on investment in the past. Current investment plans are viewed as determining the capital stock vector, K_{t+1}, for time $t+1$. X_t is the vector of all other ORANI variables, including instruments such as tariff rates, and targets such as employment, rates of output and prices. F_i, $i=1,\ldots,m$ are m differentiable functions. The equations (1.1) impose m "equilibrium" relationships on the ORANI variables. These relationships imply, for example, that demands equal supplies, that future capital stocks equal current capital stocks plus current net investment, and that costs equal revenues.

ORANI contains many more variables than equations. Where n is the total number of components in the vector (X_t, K_{t+1}, K_t), n is greater than m. Hence, to solve ORANI, $(n-m)$ variables must be treated as exogenously given. The selection of the $(n-m)$ exogenous variables is largely user-determined. For example, if a user wishes to project the effects of a tariff cut under fixed real wages, he will include the real wage rate in the exogenous category, but will probably let the aggregate level of employment be endogenous. On the other hand, if he were interested in the effects of a tariff cut with full employment, he would treat the level of employment as exogenous and the real wage rate as endogenous. The partitioning of the variables into the exogenous and endogenous sets is an important part of the mechanism by which the economic environment, B, is specified.

Once we have settled the economic environment via our selection of $(n-m)$ variables for the exogenous category and by our setting of various parameters,[1] we can in principle solve (1.1) to obtain

$$\begin{pmatrix} X_t(1) \\ \\ K_{t+1} \end{pmatrix} = H(X_t(2), K_t) \tag{1.2}$$

where $(X_t(1), K_{t+1})$ and $(X_t(2), K_t)$ are respectively the endogenous and exogenous subvectors of (X_t, K_{t+1}, K_t) and H is a vector function of order m. (We assume here that K_{t+1} is treated endogenously

1. The relevant parameters allow for the introduction of indexing assumptions. See sections 13 and 14.

and K_t is treated exogenously. Whilst these are the assumptions under which most ORANI simulations have been run, neither is obligatory.) Then on the basis of (1.2) we can compute the effects, both long run and short run, of changes in any of the exogenous variables on any of the endogenous variables.

The short run effects of changes in $X_t(2)$ can be computed as

$$\begin{pmatrix} dX_t(1) \\ \\ dK_{t+1} \end{pmatrix} = \nabla H \begin{pmatrix} dX_t(2) \\ \\ 0 \end{pmatrix} , \qquad (1.3)$$

where ∇H is the $m \times (n-m)$ matrix of first order partial derivatives of H. The interpretation of the $dX_t(1)$ is as follows. It is the vector of changes in short-run equilibrium prices levels, in short-run equilibrium rates of production, etc., which can be attributed to the exogenous shock $dX_t(2)$. $dX_t(1)$ is a change in equilibrium values because its computation implies the restoration of the equilibrium conditions (1.1), but it is a short-run change because its computation does not allow for the impact of changes in industry capital stocks. $dX_t(1)$ should not be interpreted as a forecast of actual changes in price levels, rates of output, etc. over any actual time period. It is a projection of the effects of a particular set of exogenous (typically, policy initiated) changes alone.

If we wish to project the effects of a 25 per cent cut in all tariffs, we set

$$dX_t(2) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\,dT \\ 0 \\ \vdots \\ 0 \end{bmatrix} ,$$

where $-\,dT$ is the proposed change in the tariffs. Then (1.3) generates the short-run equilibrium impact of the tariff cut. It says that as a result of the tariff cut, the rate of output, the level of prices, etc., will, in the short-run, be α, β, etc., per cent different from what they would have been in the absence of the tariff cut.

One obvious, but very difficult question is : how long is the short-run? The short-run must be long enough for local prices of imports to fully adjust to tariff increases, for major import users to decide whether or not to switch to domestic suppliers, for domestic suppliers to hire labor and to expand output with their existing plant, for new investment plans to be made but not completed, and for price increases to be passed onto wages and for wage increases to be passed back to prices. In attaching a calendar time period to the interpretation of an ORANI short-run result, both the policy change under consideration and the set of adjustments of interest are relevant. A given variable might adjust at different rates under the influence of different policy changes and certainly different variables will adjust at different rates even under the influence of the same policy change. If the important adjustments associated with a given policy change are believed to take 12 months, for example, then the appropriate interpretation of the ORANI short-run results is that 12 months after the policy change footwear output, say, will be running at a rate of 5 per cent higher than it would have been in the absence of the policy change. Of course that rate would be 10 per cent lower than the rate at the time of the policy change if it would have been 15 per cent lower without the policy change.

For long-run projections, we can use (1.3) sequentially. We introduce the policy shock, $dX_t(2)$, and compute dK_{t+1}. Then we use dK_{t+1} in the computation of $dX_{t+1}(1)$ and dK_{t+2}, etc.. An alternative to sequential solution for long-run projections involves swapping the initial capital stocks, K_t, and the rates of return, R_t, between the lists of endogenous and exogenous variables. More specifically, we interpret t as being a point of time five years (say) after the introduction of a proposed policy change. We assume that after such a time the sizes of industry capital stocks will have adjusted to the policy change so as to restore rates of return to their initial (or some other exogenously given) levels. Thus we compute the change in the capital stocks, dK_t, which would be consistent with an assumed change (perhaps zero change) in industry rates of return given the policy change under consideration.

The aims of this chapter are (i) to describe the ORANI system (1.1), and (ii) to set out its linearized form. Linearization of (1.1) is carried out by totally differentiating each equation to obtain

$$(\nabla F_i) \begin{pmatrix} dX_t \\ dK_{t+1} \\ dK_t \end{pmatrix} = 0 \ , \qquad i=1,\dots,m, \tag{1.4}$$

where ∇F_i is the vector of first order partial derivatives of F_i. Then, assuming that zero is not a "relevant" value for any of the ORANI variables,[1] we can rewrite (1.4) in the form

$$(\nabla F_i) \begin{pmatrix} \hat{X}_t & & \underset{=}{0} \\ & \hat{K}_{t+1} & \\ \underset{=}{0} & & \hat{K}_t \end{pmatrix} \begin{pmatrix} x_t \\ k_{t+1} \\ k_t \end{pmatrix} = 0 \ , \qquad i=1,\dots,m, \tag{1.5}$$

where x_t, k_{t+1} and k_t are the vectors of <u>percentage</u> changes in the elements of the vectors X_t, K_{t+1} and K_t, and $\hat{(\cdot)}$ is the diagonal matrix formed from the vector $(\cdot)$. In simpler notation, we represent (1.5) as

$$A\upsilon = 0 \ , \tag{1.6}$$

where A is an $m \times n$ matrix and υ' is the vector (x_t, k_{t+1}, k_t). The end product of this chapter (Table 1 in section 14) is a complete listing of the ORANI equations in the linearized form (1.6).

By using (1.6), we can arrive at systems such as (1.3) without going via (1.2), i.e., we can express the changes or percentage

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1. To avoid the "zero" problem, variables may be redefined. For example, the power of the tariff, i.e., one plus the <u>ad valorem</u> rate, can be used as a variable rather than the <u>ad valorem</u> rate.

changes in our endogenous variables as linear functions of the changes or percentage changes in our exogenous and predetermined variables without finding the explicit forms for the functions H. We simply rearrange (1.6) in the form

$$A_1 \upsilon(1) + A_2 \upsilon(2) = 0 ,$$

where $\upsilon(1)$ is the vector of percentage changes in those variables chosen to be endogenous and $\upsilon(2)$ is the vector of percentage changes in the predetermined variables and those chosen to be exogenous. A_1 and A_2 are matrices formed by selecting appropriate columns of A. Then we obtain (1.3), or its equivalent, by computing[1]

$$\upsilon(1) = - A_1^{-1} A_2 \upsilon(2) . \tag{1.7}$$

It is certainly fortunate that (1.2) can be avoided. The ORANI system (1.1) is extremely large and contains many nonlinearities. Explicit derivation of (1.2) would be quite impractical.

Our description of (1.1) and our derivation of the linearized system (1.6) are arranged in 13 sections. Most of these sections are concluded with summaries, and their key equations, those which form part of the linearized system (1.6), are enclosed in boxes. These equations, together with the ORANI variables, are listed in section 14.[2]

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1. Our experience suggests that A_1 is singular only for an economically meaningless choice for the exogenous-endogenous classification. For example, an attempt to include all wages and employment levels in the exogenous category would be likely to end with a singular A_1.

2. Section , Chapter 4, contains a table of definitions of ORANI parameters.

Readers may find it helpful to "skim" the section headings. They will see that sections 2-4 describe industry decisions : the commodity composition of output, demands for current inputs and demands for inputs for capital creation. Household demands, export demands, government and other demands and demands for margin services are handled in sections 5-8. Section 9 imposes the zero pure profits conditions while section 10 describes the allocation of investment funds across industries. Section 11 lists the market clearing conditions and sections 12 and 13 define various aggregates, e.g., aggregate employment, the aggregate price level, the balance of trade and several others. Section 14 contains brief comments on the total linearized system (1.6).

An apology should be offered in advance for our notation. In some places we are forced to carry up to six subscripts and superscripts. Also, we must rely on the reader taking seriously the indicated subscript and superscript ranges which follow most equations. One notational convention which we have attempted to follow throughout is the use of upper case letters for variable levels and the use of the corresponding lower case letters for their percentage changes.[1] It is worth mentioning that the notation of this introductory section is not continued in the subsequent sections. X, F, A, etc. will all have new meanings.

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1. The reader may find that we sometimes omit the word "percentage", i.e., we refer to percentage changes as changes.

2. The Production Functions for Current Goods

The production technology available to each of our h industries can be described in two parts, (i) the relationship between the industry's inputs and its activity level and (ii) the relationship between the industry's activity level and its commodity outputs. In Chapter 2, section 2.2, we discussed these relationships in general terms and provided an illustrative example. This section sets out the ORANI theory in detail.

The material falls naturally into two parts. In terms of the notation of Chapter 2, section 2.2, we first specify the H_j functions. We then turn to the G_j functions.

2.1 Inputs and activity level

We assume, for each industry j, that

$$\underset{i=1,g+2}{\text{LEONTIEF}} \left\{ \frac{X_{ij}^{(1)}}{A_{ij}^{(1)}} \right\} = A_j^{(1)} Z_j \quad , \quad j=1,\ldots,h \tag{2.1}$$

where

$$\underset{i=1,r}{\text{LEONTIEF}} \ \{f_i\} \equiv \text{minimum} \ \{f_1, f_2, \ldots, f_r\} \quad , \tag{2.2}$$

and where $X_{ij}^{(1)}$ is the effective input[1] of good or factor i into current production, Z_j is industry j's activity level and the $A_{ij}^{(1)}$'s and $A_j^{(1)}$ are technological coefficients. If $A_j^{(1)}$ is 1, then $A_{ij}^{(1)}$ is the input-output coefficient showing the minimum

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1. The concept of effective inputs is defined in (2.3) below.

effective input of i required to support a unit of activity in industry j. Although it is true that a reduction of x per cent in $A_j^{(1)}$ describes exactly the same change in technology as a uniform reduction of x per cent in all the coefficients $A_{ij}^{(1)}$, it will be convenient to retain $A_j^{(1)}$ in our model.[1] We can use changes in $A_j^{(1)}$ to simulate the effects of neutral technical progress and reserve the $A_{ij}^{(1)}$'s for handling various types of biased technical progress.

The superscript (1)'s used in equation (2.1) denote inputs into current production. Later we will be using a superscript (2) to denote inputs to capital formation, a superscript (3) for commodity flows to household consumption, a (4) for exports and a (5) for other demands. Also it should be emphasized that $X_{ij}^{(1)}$ refers to <u>directly</u> used inputs. When industry j buys steel, say, the price will include the transportation, wholesale and other margins involved in the delivery of steel. However, the input-activity function (2.1) includes the steel only. The treatment of margins is described in section 8.

In equation (2.1) we have $g+2$ inputs. The first g of these are to be interpreted as being "produced" intermediate inputs, e.g., steel, petroleum, etc.. For these, two sources of supply are identified, namely domestic production and imports. By distinguishing

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1. It is also true, of course, that the initial levels of the $A_{ij}^{(1)}$'s can only be determined in relation to an arbitrarily determined value for $A_j^{(1)}$.

these sources, we follow Armington (1969, 1970)[1] in allowing for the possibility that imported commodities may not be perfectly substitutable for the corresponding domestic product. It is a commonplace observation that significant changes in the relative prices of domestic and imported cars (say), can take place without the elimination of either from the local market.

To capture the idea of imperfect substitutability, we assume that units of a given input, differentiated by source, are combined to provide a unit of effective input according to the equation

$$X_{ij}^{(1)} = \underset{s=1,2}{\mathrm{CES}} \left\{ \frac{X_{(is)j}^{(1)}}{A_{(is)j}^{(1)}} ;\ \rho_{ij}^{(1)},\ b_{(is)j}^{(1)} \right\}, \qquad (2.3)$$

$$i=1,\dots,g, \quad j=1,\dots,h,$$

where $X_{(is)j}^{(1)}$ is the input of i from source s to the production of industry j or more simply the input of (is) to j. In the context of produced inputs, $s=1$ refers to the domestic source, while $s=2$ refers to imports. Thus, for example, $X_{(i2)j}^{(1)}$, $i=1,\dots,g$, is the use of imported good i as an intermediate input to the production of industry j. The notation $\underset{s=1,2}{\mathrm{CES}} \{f_s, \rho, b_s\}$ means that the variables f_s, $s=1,2$, are to be aggregated according to a

1. See also Artus and Rhomberg (1973), Goodman, et al. (1973), Dixon (1976) and Dervis (1978).

CES function[1] with parameters ρ and b_s, i.e.,

$$\underset{s}{\mathrm{CES}}\ \{f_s;\ \rho,\ b_s\} \equiv (\sum_s f_s^{-\rho}\ b_s)^{-(1/\rho)} \qquad (2.4)$$

with the b_s being non-negative and ρ being greater than -1 but not equal to zero.[2] (In general, we will omit the parameter list and simply write the LHS of (2.4) as $\mathrm{CES}_s(f_s)$.) The $A^{(1)}_{(is)j}$'s are positive coefficients. Their role is to allow for technical change. For example, a reduction in $A^{(1)}_{(i1)j}$, $i=1,\ldots,g$, would simulate the impact of (i1)-augmenting technical change[3] in production in industry j. Once more we note an overlap in the role of the A's. An x per cent reduction in $A^{(1)}_{ij}$ describes exactly the same change in technology as a uniform x per cent reduction in each of the $A^{(1)}_{(is)j}$, $s=1,2$.

The remaining two inputs distinguished in equation (2.1) are primary factors (subscript $g+1$) and what we will call other cost tickets (subscript $g+2$). Other cost tickets are a useful device for allowing for production taxes, costs of holding liquidity, costs of holding inventories and other miscellaneous production costs. In order to achieve a unit level of activity, industry j must buy $A^{(1)}_{g+2,j}$ other cost tickets. The effects of changes in production taxes, etc., can be simulated by introducing the appropriate change in

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1. CES (Constant Elasticity of Substitution) functions were first applied by Arrow et al. (1961).

2. As s approaches zero, (2.4) collapses to a Cobb-Douglas form, see, for example Arrow et al. (1961).

3. We follow the definitions of technical change in Allen (1967, pp.236-258).

the price of other cost tickets. The effects of technological improvements which allow reductions in inventory holdings, etc., can be simulated via changes in $A^{(1)}_{g+2,j}$.

In the case of primary factors, ORANI distinguishes three sources, labor, fixed capital (buildings, plant and machinery) and agricultural land. These three elements are combined to form effective units of primary factor inputs according to the equation

$$X^{(1)}_{g+1,j} = \underset{s=1,3}{\mathrm{CRESH}} \left\{ \frac{X^{(1)}_{(g+1,s)j}}{A^{(1)}_{(g+1,s)j}} ;\ h^{(1)}_{(g+1,s)j} ,\ Q^{(1)}_{(g+1,s)j} ,\ \kappa^{(1)}_{g+1,j} \right\} \tag{2.5}$$

where $X^{(1)}_{(g+1,s)j}$ is the input of primary factor of type s to production in industry j and the $A^{(1)}_{(g+1,s)j}$'s are positive coefficients used in simulations of the effects of technical change. In the context of primary factors, $s=1$ refers to labor, $s=2$ refers to capital and $s=3$ refers to agricultural land. Thus for example, $X^{(1)}_{(g+1,2)j}$ is the use of fixed capital in the production of industry j . The notation $\underset{s=1,3}{\mathrm{CRESH}} \{f_s, h_s, Q_s, \kappa\}$ means that the variables f_s, $s=1,2,3$ are to be aggregated according to a CRESH function[1] with parameters h_s, Q_s, $s=1,2,3$ and κ, i.e.,

$$X = \underset{s=1,3}{\mathrm{CRESH}} \{f_s ; h_s, Q_s, \kappa\} \tag{2.6}$$

implies that

$$\sum_{s=1}^{3} \left(\frac{f_s}{X}\right)^{h_s} \frac{Q_s}{h_s} = \kappa \tag{2.7}$$

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1. CRESH (Constant ratios of elasticities of substitution, homothetic), functions were introduced by Hanoch (1971).

where each h_s is less than 1 (but not equal to zero),[1] each Q_s is positive and the Q_s's and κ are normalized so that $\sum_s Q_s = 1$. In general κ can have either sign, but clearly if each of the (Q_s/h_s) has the same sign, then κ must have their common sign. As with our CES notation, we will normally omit the parameter list and write the RHS of (2.6) as $\underset{s=1,3}{\mathrm{CRESH}}(f_s)$.

While most readers will be familiar with CES functions, CRESH functions are less widely known. Some brief explanation may therefore be helpful. First, we note that (2.7) implies constant returns to scale. If we multiply f_s by $\lambda > 0$, then we can multiply X by λ and continue to satisfy (2.7). Second, we check that (2.7) implies positive marginal products and diminishing marginal rates of substitution (i.e., the number of units of input r required to replace 1 unit of input t at a given level of X declines as we increase f_t/f_r). To find the marginal product of input r, we allow f_r to change, but hold all other input levels constant. Then the change in the left side of (2.7) is given by

$$\frac{Q_r f_r^{h_r-1}}{X^{h_r}} df_r - \sum_s \frac{f_s^{h_s} Q_s}{X^{h_s+1}} dX = 0 . \qquad (2.8)$$

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1. The difficulty of h_s passing through zero can be handled by appending logarithmic terms to the LHS of (2.7), see Hanoch (1971, p.697). For convenience, however, this refinement will be ignored.

Hence, the marginal production of input r is

$$\frac{dX}{df_r} = Q_r\left(\frac{f_r}{X}\right)^{h_r-1} \Bigg/ \sum_s Q_s\left(\frac{f_s}{X}\right)^{h_s} \tag{2.9}$$

and the marginal rate of substitution of input r for input t is

$$\begin{aligned} MRS_{tr} &\equiv \frac{\partial X/\partial f_t}{\partial X/\partial f_r} \\ &= \frac{Q_t\left(f_t/X\right)^{h_t-1}}{Q_r\left(f_r/X\right)^{h_r-1}} \quad . \end{aligned} \tag{2.10}$$

With (h_t-1) and (h_r-1) both negative, MRS_{tr} will decrease as we increase f_t and decrease as we reduce f_r. Thus, (2.9) and (2.10) establish that (2.7) implies positive marginal products and diminishing marginal rates of substitution. Finally, we note that CRESH is a generalization of CES. If $h_s = h$ for all s, then (2.7) implies that

$$X = \underset{s}{CES}\ \{f_s; -h, Q_s/h\kappa\} \quad .$$

The potential advantage CRESH over CES is that it allows the elasticity of substitution between labor and capital to differ from that between capital and agricultural land. In turn, both these elasticities can differ from the elasticity of substitution between labor and agricultural land. Under CES, on the other hand, all substitution elasticities are given by

$$\sigma = 1/(1 + \rho) \quad , \tag{2.11}$$

where ρ is the parameter appearing in (2.4). Where only two sources of supply are distinguished (e.g., domestic and foreign as in the case of produced inputs, or labor and capital as in the case of primary factor use by non-agricultural industries)[1] then the generalization to a more flexible function form than CES may not seem profitable. However, given the importance of the agricultural industries in Australia, and the availability of data on their three types of primary inputs, we thought it worthwhile to attempt the estimation of separate values for the different agricultural substitution elasticities. As will be seen in Chapter 4, section , this attempt was unsuccessful and (2.5) has in practice been implemented for each industry j with $h^{(1)}_{(g+1,s)j} = h^{(1)}_{g+1,j}$ for all s, i.e., our econometric work has supported the use of CES functions only. Nevertheless, in setting up the ORANI theory and computer programmes, we have taken an optimistic view and allowed for the possibility of more successful estimation in the future. Of course, CRESH is not the only available generalization of CES. If estimates of the parameters of other forms of production functions for Australian industries should become available, then there would be little difficulty in modifying the model so as to make use of them.

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1. We have written the ORANI theory and computer programmes as though each industry uses three primary factors. It is easier to treat the non-agricultural industries as though they use agricultural land at close to the zero level than to treat the agricultural industries as being exceptional. In fact, the allowance for the third primary factor is occasionally useful in non-agricultural industries. For example, in mining industries we may wish to introduce supply constraints associated with the diminishing quality of ore bodies.

ORANI recognizes one further disaggregation on the input side. The primary factor labor is disaggregated into M skill categories. The effective input of labor into industry j is given by

$$X^{(1)}_{(g+1,1)j} = \underset{m=1,M}{\text{CRESH}} \left\{ \frac{X^{(1)}_{(g+1,1,m)j}}{A^{(1)}_{(g+1,1,m)j}} \right\} , \qquad j=1,\ldots,h, \tag{2.12}$$

where $X^{(1)}_{(g+1,1,m)j}$ is the input of primary factor $(g+1)$ from source 1 (i.e. labor) of skill group m used in current production in industry j. Again the A's are positive coefficients which can be used in simulations of the effects of changes in technology.

According to (2.12), industry j's labor requirements can be met by a variety of combinations of labor inputs from different skill groups. Equation (2.12) allows us to recognize that inputs of labor hours of one type will be imperfectly substitutable for inputs of labor hours of another type. In addition, (2.12) is sufficiently flexible to encompass variations in the substitution elasticities between different pairs of skill groups, e.g., the elasticity of substitution in industry j between skilled blue collar workers and professionals may differ from that between workers classified to the groups skilled white collar and unskilled blue collar.

In summary, our specification of input technology is as follows : the input-activity functions exhibit constant returns to scale and are of a three level form. At the top level we have adopted the Leontief assumption. There is no substitution between different materials (chemicals, steel, etc.) or between them and primary factors. This assumption is reasonable in view of the numerous studies which have failed to establish relative input prices as a major determinant of changes in input-output coefficients. For example, Sevaldson (1976) examines a long time series of Norwegian input-output tables. He rejects the idea that the changes in coefficients are closely related to changes in relative prices. His results imply that attempts to model material-material or material-primary factor substitution are likely to have minor payoffs. This is not to say that input-output coefficients are fixed through time. With ORANI, we can analyse the effects of changes in the input-output coefficients $\left\{A_{ij}^{(1)},\ i=1,\ldots,g+2\right\}$ but we cannot explain these changes.

At the second level, we have CES and CRESH functions describing substitution possibilities between imported and domestic goods of the same type and among primary factors; i.e., domestic and imported chemicals are substitutes as are capital, labor and agricultural land.

At the third level, which applies only to labor inputs, we again have CRESH. This allows us to recognize that labor possessing one skill can be substituted for labor of another skill, although imperfectly. The CRESH specification also gives us sufficient flexibility to introduce variations in substitution elasticities across different pairs of labor types. On the other hand, the particular nesting (or separability) assumptions used in the current version of

ORANI imply that marginal rates of substitution between different types of labor are independent of the level of capital inputs. Model modifications would be required if it were found, for example, that professionals were complementary with capital while unskilled blue collar workers were competitive.

2.2 Activity level and outputs

We assume, for each industry j, that

$$\underset{t=1,\ N(j)}{\mathrm{CRETH}} \left\{ X^{(0)}_{(t^*)j}\ A^{(0)}_{(t^*)j};\ h^{(0)}_{(t^*)j},\ Q^{(0)}_{(t^*)j},\ \kappa^{(0)}_{j} \right\} = Z_j / A^{(0)}_j \tag{2.13}$$

where $X^{(0)}_{(t^*)j}$ is the output of composite commodity t by industry j. The superscript (0) denotes output while the $*$ denotes composite commodity.[1]

The notation $\underset{t=1,\ N}{\mathrm{CRETH}} \{f_t;\ h_t,\ Q_t,\ \kappa\}$ means that the variables f_t, $t=1,\ldots,N$ are to be combined according to a CRETH function[1] with parameters h_t, Q_t and κ, i.e.,

$$X = \underset{t=1,\ N}{\mathrm{CRETH}} \left\{ f_t;\ h_t,\ Q_t,\ \kappa \right\} \tag{2.14}$$

implies that

$$\sum_{t=1}^{N} \left(\frac{f_t}{X}\right)^{h_t} \frac{Q_t}{h_t} = \kappa \tag{2.15}$$

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1. The concept of composite commodities is defined in (2.16) below.

2. CRETH (Constant ratios of elasticities of transformation, homothetic) functions are used in Vincent, Dixon and Powell (1978). More detail on their properties is supplied in Dixon, Powell and Vincent (1976).

where each of the h_t's is greater than 1, κ and each of the Q_t's is positive and these latter parameters are normalized so that $\sum_t Q_t = 1$. Thus, apart from the restrictions on the parameters, the CRETH function is identical to CRESH. The parameter restrictions for CRESH ensure isoquants which are convex to the origin whereas the parameter restrictions for CRETH ensure transformation surfaces which are concave to the origin. By using formula (2.10), we see that under CRETH (because (h_t-1) and (h_r-1) are both positive) the marginal rate of transformation of r for t (i.e., the number of units of r which can be produced if we reduce the output of t by 1 unit holding all other product outputs and X constant) will increase as we increase f_t/f_r. We also note that CRETH is a generalization of Powell and Gruen's (1968) CET (Constant elasticity of transformation) function. Equation (2.15) reduces to CET if $h_t = h$ for all t. Just as CES implies a common value for all pairwise substitution elasticities, CET implies a common value for all pairwise transformation elasticities. Under CRETH, however, the transformation elasticity between wheat and sheep, say, can differ from that between sheep and cattle, etc..

The remaining notation in (2.13) -- $A_j^{(0)}$ and the $A_{(t^*)j}^{(0)}$ for $t=1,\ldots,N(j)$ -- are technological change coefficients. An x per cent reduction in $A_j^{(0)}$ simulates a uniform x per cent outward shift in the transformation frontier, i.e., at any given commitment of inputs (or value of Z_j), x per cent more of each composite commodity, t, can be produced. This type of neutral technical improvement could also be simulated by an x per cent reduction in each of the $A^{(0)}_{(t^*)j}$. Non-uniform changes in the $A^{(0)}_{(t^*)j}$'s simulate biased shifts in industry j's transformation frontier. For example, a reduction in $A^{(0)}_{(t^*)j}$, holding the other $A^{(0)}$'s constant, would simulate a (t^*)-augmenting shift in industry j's production possibilities at all levels of factor input.

The production of composite commodities is related to the production of commodities by

$$X^{(0)}_{(t^*)j} = - \underset{i \varepsilon G(t,j)}{\text{LEONTIEF}} \left\{ -X^{(0)}_{(i1)j} \, A^{(0)}_{(i1)j} \right\} , \qquad (2.16)$$

$$t=1,\ldots,N(j), \quad j=1,\ldots,h \quad .$$

where $X^{(0)}_{(i1)j}$ is the production of good $(i1)$ by industry j .[1,2] For each industry j, the g domestically produced goods, $(1,1)$, $(2,1),\ldots,(g,1)$ are partitioned into $N(j)$ non-overlapping sets, $G(1,j),\ldots,G(N(j),j)$. Composite commodity t for industry j is a Leontief combination of all the commodities i where $i \varepsilon G(t,j)$. The $A^{(0)}_{(i1)j}$'s are technological change coefficients. An x per cent reduction in the $A^{(0)}_{(i1)j}$ for all i belonging to $G(t,j)$ would be equivalent to an x per cent reduction in $A^{(0)}_{(t^*)j}$. That is, we would have an x per cent (t^*)-augmenting shift in j's production possibilities frontiers. On the other hand, a reduction in $A^{(0)}_{(i1)j}$, alone, would simulate the effects of an $(i1)$-augmenting shift.

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1. The two minus signs on the RHS of (2.16) imply that $X^{(0)}_{(t^*)j}$ is the maximum of the $X^{(0)}_{(i1)j} \, A^{(0)}_{(i1)j}$ over the relevant i's .

2. Domestic industries produce only the domestic commodities.

As will become apparent in Chapter 4, section , the role of composite commodities is to cover some data deficiencies. For example, in ORANI's industry 1, pastoral zone farming, data was insufficient to allow estimation of transformation elasticities between wheat (commodity 3), barley (4) and other grains (5). We formed these commodities into the composite commodity grains, which, in the event, was composite commodity 3 for industry 1. Hence

$$G(3,1) = \{3,4,5\} .$$

Under (2.16) we are assuming that in pastoral zone farming, wheat, barley and other grains are produced in fixed proportions. We do, however, estimate the transformation elasticities between the composite commodity, grains, and the other composite commodities produced by pastoral zone farming.

In summary, our specification of output technology for each industry is as follows : output-activity functions exhibit constant returns to scale and are of a two level form. At the top level we have CRETH functions describing transformation possibilities between composite commodities. At the second level we assume that the commodities within each composite commodity are produced in fixed proportions. This assumption is a reflection of data inadequacies and is discussed further in Chapter 4, section .

3. Input Demands and Commodity Supplies : Solutions to Cost Minimization and Revenue Maximization Problems

We will assume that producers are competitive and efficient. They are competitive in that they treat all input and output prices as exogenously given. They are efficient in the sense that at any given level of activity, Z_j, the producers in industry j select the combination of inputs which minimizes their costs and the combination of outputs which maximizes their revenue. We start by solving the cost minimizing problem. Then we move to the revenue maximizing problem.

3.1 Demands for directly used inputs for the production of current goods : a cost minimizing problem

We assume that producers treat all factors of production as variable. In particular, they act as if they rent their fixed capital and agricultural land. We will see in section 11 that both capital and agricultural land are treated as though they are non-shiftable between industries, i.e., industry specific. In effect we are assuming that there is a rental market for the capital and agricultural land of each industry and that each producer in industry j treats the rental prices of capital and agricultural land of type j as given. The rental rates adjust so that for each j, the sum of the demands from all producers in industry j equals the available supplies of capital and agricultural land of type j. Alternatively, we could assume that producers in industry j are also the owners of the fixed factors of type j . In this case we would assume that inputs and outputs are chosen so as to maximize profits subject to the availability of capital and agricultural land. The two approaches, cost minimization and profit maximization, yield identical results. The market-clearing rental per unit of fixed factor in the first approach is the profit per unit of fixed factor in the second.[1]

Under the cost minimization assumption, we must solve the following problem for $j=1,\ldots,h$: choose the input levels

$X_{ij}^{(1)}$, $i=1,\ldots,g+2$, (effective intermediate and primary inputs)

$X_{(is)j}^{(1)}$, $i=1,\ldots,g$, $s=1,2$, (intermediate inputs, domestic and imported)

$X_{(g+1,s)j}^{(1)}$, $s=1,2,3$, (overall labor input, capital and agricultural land)

and $X_{(g+1,1,m)j}^{(1)}$, $m=1,\ldots,M$, (labor by different skill groups; we assume there are M groups)

to minimize

$$\sum_{i=1}^{g}\sum_{s=1}^{2} P_{(is)j}^{(1)} X_{(is)j}^{(1)} + \sum_{m=1}^{M} P_{(g+1,1,m)j}^{(1)} X_{(g+1,1,m)j}^{(1)}$$

$$+ \sum_{s=2}^{3} P_{(g+1,s)j}^{(1)} X_{(g+1,s)j}^{(1)} + P_{g+2,j}^{(1)} X_{g+2,j}^{(1)} , \qquad (3.1)$$

subject to (2.1), (2.3), (2.5) and (2.12), where Z_j and the P's are treated as exogenous variables. $P_{(is)j}^{(1)}$ is the cost to industry j per unit of intermediate input i from source s . The superscript

1. See Taylor and Black (1974) for a model in which the profit maximizing story is told.

(1) allows us to recognize that the cost per unit of good i from source s to industry j may depend on the particular use to which it will be put. For example, taxes on capital inputs may be different from taxes on current inputs.[1] Similarly, the subscript j is necessary because different industries will pay different prices for the same commodities because of, among other reasons, differences in taxes, transportation, wholesale, retail and other margins. $p^{(1)}_{(g+1,1,m)j}$ is the price to industry j of a unit of labor of skill m. Again we include the superscript (1) and the subscript j. In fact the superscript (1) is superfluous in the case of primary factors and is included only because it proves to be notationally convenient. The only use of primary factors recognized in ORANI is in current production. The subscript j, however, is required. ORANI can be used to simulate the effects of changes in the wage rates payable in one industry while wage rates payable in other industries are held constant. $p^{(1)}_{(g+1,s)j}$, $s=2,3$ are the rental costs to industry j of units of capital and agricultural land. The j subscripts appear because (as indicated above) we assume that units of capital and agricultural land are industry specific, i.e., each industry uses a particular type of capital and a particular type of agricultural land and these are non-transferable between industries. $p^{(1)}_{g+2,j}$ is the price to industry j of other cost tickets.

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1. Prices of inputs to capital creation are identified by a superscript (2). Superscripts (3), (4) and (5) are used for the prices paid by households, by users of Australian exports and by other users respectively.

The solution of the cost minimization problem can be carried out in stages. First, from (2.1) and (2.3), we note that for each i, $i=1,\ldots,g$, $X^{(1)}_{(i1)j}$ and $X^{(1)}_{(i2)j}$ will be chosen so as to minimize

$$\left.\begin{aligned} &P^{(1)}_{(i1)j}\, X^{(1)}_{(i1)j} + P^{(1)}_{(i2)j}\, X^{(1)}_{(i2)j} \ , \\ \text{subject to} \qquad & \\ &A^{(1)}_{j}\, A^{(1)}_{ij}\, Z_j \ = \ \mathrm{CES}_s \left(X^{(1)}_{(is)j} \big/ A^{(1)}_{(is)j} \right) \end{aligned}\right\} \tag{3.2}$$

i.e., the effective input of good i, $A^{(1)}_{j}\, A^{(1)}_{ij}\, Z_j$, required to sustain the activity level Z_j will be provided by the cost minimizing combination of imported and domestic inputs of good i .

Problem (3.2) can be rewritten conveniently as : choose $\overline{X}^{(1)}_{(is)j}$, $s=1,2$, to minimize

$$\left.\begin{aligned} &\sum_{s=1}^{2} \overline{P}^{(1)}_{(is)j}\, \overline{X}^{(1)}_{(is)j} \\ \text{subject to} \qquad & \\ &\overline{Z}_j \ = \ \mathrm{CES}_s \left(\overline{X}^{(1)}_{(is)j} \right) \end{aligned}\right\} \tag{3.3}$$

where

$$\overline{X}^{(1)}_{(is)j} \ = \ X^{(1)}_{(is)j} \big/ A^{(1)}_{(is)j} \ , \tag{3.4}$$

$$\overline{P}^{(1)}_{(is)j} \ = \ A^{(1)}_{(is)j}\, P^{(1)}_{(is)j} \tag{3.5}$$

and

$$\overline{Z}_j \ = \ A^{(1)}_{j}\, A^{(1)}_{ij}\, Z_j \quad . \tag{3.6}$$

The first order conditions for a solution of problem (3.3) are

$$\overline{P}^{(1)}_{(is)j} - \Lambda \ \frac{\partial \mathrm{CES}_s \left(\overline{X}^{(1)}_{(is)j} \right)}{\partial\, \overline{X}^{(1)}_{(is)j}} \ = \ 0 \quad , \quad s=1,2, \tag{3.7}$$

and
$$\overline{Z}_j - \mathrm{CES}_s\left(\overline{X}^{(1)}_{(is)j}\right) = 0 \qquad (3.8)$$

where Λ is the Lagrangian multiplier. The first of these conditions may be rewritten as

$$\overline{P}^{(1)}_{(is)j} - \Lambda b^{(1)}_{(is)j}\left(\frac{\overline{X}^{(1)}_{(is)j}}{\overline{Z}_j}\right)^{-\rho^{(1)}_{ij} - 1} = 0\ , \quad s=1,2\ . \qquad (3.9)$$

Then by using (3.8) and (3.9) we could derive functions of the form

$$\overline{X}^{(1)}_{(is)j} = f^{(1)}_{(is)j}\left(\overline{Z}_j, \overline{P}^{(1)}_{(i1)j}, \overline{P}^{(1)}_{(i2)j}\right)\ . \qquad (3.10)$$

By substitution from (3.4) - (3.6) into (3.10) we would obtain demand functions for imported and domestic intermediate inputs to industry j. However, rather than finding the explicit form for $f^{(1)}_{(is)j}$, it will be sufficient for our purposes to work in percentage changes, i.e., we will express the percentage change in $X^{(1)}_{(is)j}$ in terms of percentage changes in Z_j, $P^{(1)}_{(i1)j}$, $P^{(1)}_{(i2)j}$, $A^{(1)}_j$, $A^{(1)}_{(i1)j}$ and $A^{(1)}_{(i2)j}$. We proceed by expressing the first order conditions (3.8) and (3.9) in terms of percentage changes.

From (3.9), we obtain

$$\bar{p}^{(1)}_{(is)j} - \lambda + \left(\rho^{(1)}_{ij} + 1\right)\left(\bar{x}^{(1)}_{(is)j} - \bar{z}_j\right) = 0\ , \quad s=1,2\ , \qquad (3.11)$$

where the lower case symbols denote percentage changes in the variables represented by the corresponding upper case symbols. In general we will use the notation[1]

$$y = 100 \left(\frac{dY}{Y}\right) .$$

Next, by totally differentiating (3.8), and using (3.7), we find that

$$d\overline{Z}_j = \sum_s \left(\frac{\partial CES_s}{\partial \overline{X}^{(1)}_{(is)j}}\right) d\overline{X}^{(1)}_{(is)j} ,$$

i.e.,

$$d\overline{Z}_j = \sum_s \left(\frac{\overline{P}^{(1)}_{(is)j}\ d\overline{X}^{(1)}_{(is)j}}{\Lambda}\right)$$

and

$$z_j = \sum_s \left(\frac{\overline{P}^{(1)}_{(is)j}\ \overline{X}^{(1)}_{(is)j}}{\Lambda\ \overline{Z}_j}\right) \bar{x}^{(1)}_{(is)j} . \qquad (3.12)$$

(3.12) can be written in more convenient notation as

$$z_j = \sum_s S^{(1)}_{(is)j}\ \bar{x}^{(1)}_{(is)j} , \qquad (3.13)$$

where

$$S^{(1)}_{(is)j} = \frac{\overline{P}^{(1)}_{(is)j}\ \overline{X}^{(1)}_{(is)j}}{\Lambda\ \overline{Z}_j} . \qquad (3.14)$$

Equations (3.11) and (3.13) are linear in $\bar{x}^{(1)}_{(is)j}$, $\bar{p}^{(1)}_{(is)j}$, s=1,2 , λ and $\bar{z}_j$. λ may be eliminated so that we can express the $\bar{x}^{(1)}_{(is)j}$'s as linear functions of the percentage

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1. The question arises as to what happens if $Y = 0$. In this chapter we will simply assume that none of the ORANI variables has a base period value of precisely zero. If in reality the base period value for Y is precisely zero, then we will assume that reality can be approximated by setting Y at an arbitrarily small non-zero value. For further discussion of this point, see section of Chapter .

changes in the $\bar{p}^{(1)}_{(is)j}$'s and $\bar{z}_j$. However, before doing that, we will interpret the $S^{(1)}_{(is)j}$.

We can show that

$$S^{(1)}_{(is)j} = P^{(1)}_{(is)j} X^{(1)}_{(is)j} \Big/ \sum_s P^{(1)}_{(is)j} X^{(1)}_{(is)j} , \qquad (3.15)$$

i.e., $S^{(1)}_{(is)j}$ is the share of good i from source s in the total cost of the inputs of i to industry j . To establish (3.15), we multiply (3.7) through by $\overline{X}^{(1)}_{(is)j}$ and aggregate over s . This gives

$$\sum_s \overline{P}^{(1)}_{(is)j} \overline{X}^{(1)}_{(is)j} - \Lambda \sum_s \frac{\partial CES_s}{\partial \overline{X}^{(1)}_{(is)j}} \overline{X}^{(1)}_{(is)j} = 0 . \qquad (3.16)$$

Next we apply Euler's theorem, so that (3.16) reduces to

$$\sum_s \overline{P}^{(1)}_{(is)j} \overline{X}^{(1)}_{(is)j} - \Lambda \overline{Z}_j = 0 . \qquad (3.17)$$

(3.17) together with (3.14), (3.4) and (3.5), implies (3.15).

We return to the main task, the derivation of equations for the $x^{(1)}_{(is)j}$ in terms of z_j, $p^{(1)}_{(is)j}$, s=1,2, $a^{(1)}_j$ and $a^{(1)}_{(is)j}$, s=1,2 . By multiplying (3.11) through by $S^{(1)}_{(is)j}$, aggregating over s, and using (3.13), we find that

$$\lambda = \sum_s \bar{p}^{(1)}_{(is)j} S^{(1)}_{(is)j} . \qquad (3.18)$$

Now, we substitute back into (3.11), to obtain

$$\bar{x}^{(1)}_{(is)j} = \bar{z}_j - \sigma^{(1)}_{ij} \left(\bar{p}^{(1)}_{(is)j} - \sum_s \bar{p}^{(1)}_{(is)j} S^{(1)}_{(is)j}\right) , \quad s=1,2 , \qquad (3.19)$$

where $\sigma_{ij}^{(1)} = 1\Big/\left[1 + \rho_{ij}^{(1)}\right]$ is the elasticity of substitution in industry j between intermediate inputs of good i from domestic and foreign sources. Finally, we note from (3.4) - (3.6) that

$$\bar{x}_{(is)j}^{(1)} = x_{(is)j}^{(1)} - a_{(is)j}^{(1)} \tag{3.20}$$

$$\bar{p}_{(is)j}^{(1)} = p_{(is)j} + a_{(is)j}^{(1)} \tag{3.21}$$

and

$$\bar{z}_j = a_j^{(1)} + a_{ij}^{(1)} + z_j \quad . \tag{3.22}$$

On substituting (3.20) - (3.22) into (3.19) we obtain our intermediate input demand functions

$$\boxed{\begin{aligned} x_{(is)j}^{(1)} &= z_j - \sigma_{ij}^{(1)} \left(p_{(is)j}^{(1)} - \sum_s S_{(is)j}^{(1)} \, p_{(is)j}^{(1)}\right) \\ &\quad + a_j^{(1)} + a_{ij}^{(1)} + a_{(is)j}^{(1)} - \sigma_{ij}^{(1)} \left(a_{(is)j}^{(1)} - \sum_s S_{(is)j}^{(1)} \, a_{(is)j}^{(1)}\right) \\ &\qquad i=1,\ldots,g \; , \\ &\qquad s=1,2 \; , \\ &\qquad j=1,\ldots,h \; . \end{aligned}} \tag{3.23}$$

To interpret (3.23) we first consider the case in which there is no technical change, i.e., all the $a_j^{(1)}$, $a_{ij}^{(1)}$ and $a_{(is)j}^{(1)}$ are zero. Under these conditions, (3.23) says that if there are no changes in the relative prices of good i from alternative sources, then a one per cent increase in Z_j leads to a one per cent increase in each of the $X^{(1)}_{(is)j}$, $i=1,\ldots,g$ and $s=1,2$. This reflects the assumption of constant returns to scale. If, however, $P^{(1)}_{(i1)j}$ rises relative to the price of imported good i, then $X^{(1)}_{(i1)j}$ will increase less rapidly than Z_j . There will be substitution against the domestic source of good i in favour of imports. The strength of this substitution effect will depend on the value of the parameter $\sigma^{(1)}_{ij}$.

Now consider the case in which the input prices and j's activity level are constant but in which technical change is taking place. If the change is Hicks neutral across all inputs at the rate of one per cent, i.e.,

$$a^{(1)}_j = -1 \ ,$$

then (3.23) implies that j's requirements for all intermediate inputs[1] fall by 1 per cent. If the change is i-augmenting at the rate of one per cent,

$$\text{i.e.,} \qquad a^{(1)}_{ij} = -1 \ ,$$

then j's requirements for inputs of good i from both domestic and foreign sources fall by 1 per cent. If the change is (is)-augmenting at the rate of one per cent, i.e.,

$$a^{(1)}_{(is)j} = -1 \ , \tag{3.24}$$

then j's requirement for inputs of good i from source s fall by

1. It is also true that j's requirements for primary factor inputs fall by 1 per cent (see (3.25) and (3.64)).

$\left[1 - \sigma_{ij}^{(1)} \left(1 - S_{(is)j}^{(1)}\right)\right]$ per cent, i.e., less than one per cent. The (is)-augmenting technical progress induces some substitution in favour of input (is) and away from input (ir), $r \neq s$. Notice that under (3.24), j's requirements for input (ir), $r \neq s$, fall by $\sigma_{ij}^{(1)} S_{(is)j}^{(1)}$ per cent.

Of the remaining inputs, g+1 and g+2, the demand functions for g+2 can be written immediately. The demand for input g+2, other cost tickets, by industry j is

$$X_{g+2,j}^{(1)} = A_j^{(1)} A_{g+2,j}^{(1)} Z_j$$

and in percentage change form we have

$$\boxed{x_{g+2,j}^{(1)} = z_j + a_j^{(1)} + a_{g+2,j}^{(1)} \qquad j=1,\ldots,h \ .} \tag{3.25}$$

The demand functions for primary factors, on the other hand, are more complex. Industry j will choose its inputs of labor, fixed capital and agricultural land to minimize

$$\sum_{m=1}^{M} \overline{P}_{(g+1,1,m)j}^{(1)} \overline{X}_{(g+1,1,m)j}^{(1)} + \overline{P}_{(g+1,2)j}^{(1)} \overline{X}_{(g+1,2)j}^{(1)} + \overline{P}_{(g+1,3)j}^{(1)} \overline{X}_{(g+1,3)j}^{(1)} \tag{3.26}$$

subject to

$$\overline{Z}_j^{(1)} = \underset{s=1,3}{\text{CRESH}} \left(\overline{X}_{(g+1,s)j}^{(1)}\right) \tag{3.27}$$

and

$$\overline{X}^{(1)}_{(g+1,1)j} = \underset{m=1,M}{\text{CRESH}} \left(\overline{X}^{(1)}_{(g+1,1,m)j}\right) , \qquad (3.28)$$

where

$$\overline{P}^{(1)}_{(g+1,1,m)j} = P^{(1)}_{(g+1,1,m)j} \, A^{(1)}_{(g+1,1)j} \, A^{(1)}_{(g+1,1,m)j} , \quad m=1,\ldots,M , \qquad (3.29)$$

$$\overline{X}^{(1)}_{(g+1,1,m)j} = X^{(1)}_{(g+1,1,m)j} \Big/ \left(A^{(1)}_{(g+1,1)j} \, A^{(1)}_{(g+1,1,m)j}\right) , \quad m=1,\ldots,M , \qquad (3.30)$$

$$\overline{P}^{(1)}_{(g+1,s)j} = P^{(1)}_{(g+1,s)j} \quad A^{(1)}_{(g+1,s)j} , \quad s=1,2,3 , \qquad (3.31)^1$$

$$\overline{X}^{(1)}_{(g+1,s)j} = X^{(1)}_{(g+1,s)j} \Big/ A^{(1)}_{(g+1,s)j} , \quad s=1,2,3 \qquad (3.32)$$

and

$$\overline{Z}^{(1)}_j = A^{(1)}_j \quad A^{(1)}_{g+1,j} \quad Z_j \quad . \qquad (3.33)$$

That is, industry j chooses the combination of capital, agricultural land and labor of different skill categories to minimize the costs of providing the effective input, $A^{(1)}_j \, A^{(1)}_{g+1,j} \, Z_j$, of primary factors required to sustain the activity level Z_j .

The first order conditions for a solution of problem (3.26) - (3.28) are

$$\overline{P}^{(1)}_{(g+1,1,q)j} - \overline{P}^{(1)}_{(g+1,1)j} \frac{\partial \text{CRESH}_m\left(\overline{X}^{(1)}_{(g+1,1,m)j}\right)}{\partial \overline{X}^{(1)}_{(g+1,1,q)j}} = 0 , \quad q=1,\ldots,M ; \qquad (3.34)$$

$$\overline{X}^{(1)}_{(g+1,1)j} - \text{CRESH}_m \left(\overline{X}^{(1)}_{(g+1,1,m)j}\right) = 0 , \qquad (3.35)$$

---

1. Neither $P^{(1)}_{(g+1,1)j}$ nor $\overline{P}^{(1)}_{(g+1,1)j}$ have yet been defined. The latter, however, is defined in (3.34). Then (3.31) for $s=1$ can be thought of as a defining equation for $P^{(1)}_{(g+1,1)j}$.

$$\overline{P}^{(1)}_{(g+1,v)j} - \Lambda \frac{\partial \mathrm{CRESH}_s \left(\overline{X}^{(1)}_{(g+1,s)j}\right)}{\partial \overline{X}^{(1)}_{(g+1,v)j}} = 0 \ , \quad v=1,2,3 \tag{3.36}$$

and

$$\overline{Z}^{(1)}_j - \mathrm{CRESH}_s \left(\overline{X}^{(1)}_{(g+1,s)j}\right) = 0 \ , \tag{3.37}$$

where Λ and $\overline{P}^{(1)}_{(g+1,1)j}$ are the Lagrangian multipliers. Notice that the Lagrangian multiplier on the constraint (3.28) is the shadow value of a unit increase in $\overline{X}^{(1)}_{(g+1,1)j}$; hence the notation $\overline{P}^{(1)}_{(g+1,1)j}$.

We move from the first order conditions to our primary factor demand equations in much the same way as we went from (3.7) - (3.8) to the intermediate input demand equations (3.23). That is, we express the first order conditions (3.34) - (3.37) in linear percentage change form. First we work with equations (3.34) and (3.35) and focus attention on industry j's demand for labor by occupational group. Later we will be deriving industry j's demand equations for labor in general, capital and agricultural land by looking at (3.36) and (3.37).

Before we attempt any algebraic manipulations we rewrite equations (3.34) and (3.35) as

$$\overline{P}_q - \overline{P} \frac{\partial \mathrm{CRESH}_m (\overline{X}_m)}{\partial \overline{X}_q} = 0 \ , \quad q=1,\ldots,M \tag{3.38}$$

and

$$\overline{X} - \mathrm{CRESH}_m (\overline{X}_m) = 0 \ . \tag{3.39}$$

What we have done is to drop those subscripts and superscripts which are unessential to the present purposes. In particular we omit the subscripts g+1, 1 and j and the superscript (1) . The resulting notation has no role outside the current context.

From (2.9) we know that

$$\frac{\partial CRESH_m\,(\bar{X}_m)}{\partial \bar{X}_q} = Q_q \left(\frac{\bar{X}_q}{\bar{X}}\right)^{h_q-1} \Big/ \sum_{u=1}^{M} Q_u \left(\frac{\bar{X}_u}{\bar{X}}\right)^{h_u} \tag{3.40}$$

Hence, in percentage change form (3.38) becomes

$$\bar{p}_q = \bar{p} + (h_q-1)(\bar{x}_q-\bar{x}) - \sum_{u=1}^{m} h_u(\bar{x}_u-\bar{x})S_u \quad , \quad q=1,\ldots,M \quad , \tag{3.41}$$

where

$$S_u = Q_u \left(\frac{\bar{X}_u}{\bar{X}}\right)^{h_u} \Big/ \sum_{u=1}^{M} Q_u \left(\frac{\bar{X}_u}{\bar{X}}\right)^{h_u} \quad . \tag{3.42}$$

It follows from (3.40) and (3.42) that

$$S_q = \frac{\partial CRESH_m\,(\bar{X}_m)}{\partial \bar{X}_q} \frac{\bar{X}_q}{\bar{X}} \quad . \tag{3.43}$$

Thus, from (3.39) we see that

$$\bar{x} = \sum_{u=1}^{m} S_u\,\bar{x}_u \quad . \tag{3.44}$$

Notice, also, that if we substitute from (3.43) into (3.38) we obtain

$$S_q = \frac{\bar{P}_q\,\bar{X}_q}{\bar{P}\,\bar{X}} \quad ,$$

i.e.,

$$S_q = \frac{P_q\,X_q}{P\,X} \quad . \tag{3.45}$$

Since from (3.42) we know that

$$\sum_u S_u = 1$$

it follows that

$$PX = \sum_q P_q X_q \quad ,$$

or using the full notation

$$P^{(1)}_{(g+1,1)j} \; X^{(1)}_{(g+1,1)j} = \sum_{q=1}^{m} P^{(1)}_{(g+1,1,q)j} \; X^{(1)}_{(g+1,1,q)j} \quad . \tag{3.46}$$

Consequently, in full notation, we see that (3.45) implies that

$$S^{(1)}_{(g+1,1,q)j} = P^{(1)}_{(g+1,1,q)j} \; X^{(1)}_{(g+1,1,q)j} \Big/ \sum_q P^{(1)}_{(g+1,1,q)j} \; X^{(1)}_{(g+1,1,q)j}$$

$$q=1,\ldots,M \quad , \tag{3.47}$$

i.e., $S^{(1)}_{(g+1,1,q)j}$ is the share of skill q in the total labor costs of industry j .

Returning to our short-hand notation, we rearrange (3.41) as

$$\bar{x}_q = \frac{1}{(h_q-1)} (\bar{p}_q - \bar{p}) + \bar{x} + \frac{1}{(h_q-1)} \sum_u h_u(\bar{x}_u - \bar{x})S_u \; , \quad q=1,\ldots,M \; . \tag{3.48}$$

On multiplying (3.48) through by S_q, aggregating over all q and applying (3.44), we obtain

$$\sum_u h_u(\bar{x}_u - \bar{x})S_u = - \sum_q S^*_q \, (\bar{p}_q - \bar{p}) \tag{3.49}$$

where S^*_q is the "modified" cost share defined by

$$S^*_q = \frac{S_q}{(h_q-1)} \Big/ \sum_{q=1}^{M} \frac{S_q}{(h_q-1)} \quad . \tag{3.50}$$

Then substituting from (3.49) into (3.48) we see that

$$\bar{x}_q = \bar{x} + \frac{1}{(h_q - 1)} \left[\bar{p}_q - \sum_q S_q^* \bar{p}_q\right] \quad , \quad q=1,\ldots,M \quad . \tag{3.51}$$

We continue to use our short-hand notation, and we write the percentage change forms for (3.29), (3.30) and (3.32) for $s=1$, as

$$\bar{p}_q = p_q + a + a_q \quad , \quad q=1,\ldots,M \quad , \tag{3.52}$$

$$\bar{x}_q = x_q - a - a_q \quad , \quad q=1,\ldots,M \tag{3.53}$$

and

$$\bar{x} = x - a \quad . \tag{3.54}$$

Now we substitute into (3.51), obtaining

$$x_q = x - \left(\frac{1}{1-h_q}\right)\left(p_q - \sum_q S_q^* p_q\right) + a_q - \left(\frac{1}{1-h_q}\right)\left(a_q - \sum_q S_q^* a_q\right) \quad . \tag{3.55}$$

Finally, we reintroduce our full notation and rewrite (3.55) as

$$\boxed{\begin{aligned} x^{(1)}_{(g+1,1,q)j} = {} & x^{(1)}_{(g+1,1)j} - \sigma^{(1)}_{(g+1,1,q)j}\left(p^{(1)}_{(g+1,1,q)j} - \sum_q S*^{(1)}_{(g+1,1,q)j}\, p^{(1)}_{(g+1,1,q)j}\right) \\ & + a^{(1)}_{(g+1,1,q)j} - \sigma^{(1)}_{(g+1,1,q)j}\left(a^{(1)}_{(g+1,1,q)j} - \sum_q S*^{(1)}_{(g+1,1,q)j}\, a^{(1)}_{(g+1,1,q)j}\right), \\ & \qquad q=1,\ldots,M \ , \quad j=1,\ldots,h \end{aligned}}$$

(3.56)

where

$$\sigma^{(1)}_{(g+1,1,q)j} = 1\Big/\left(1 - h^{(1)}_{(g+1,1,q)j}\right) \quad , \quad q=1,\ldots,M \tag{3.57}$$

and

$$S^{*(1)}_{(g+1,1,q)j} = \sigma^{(1)}_{(g+1,1,q)j} \, S^{(1)}_{(g+1,1,q)j} \Big/ \sum_{q=1}^{M} \sigma^{(1)}_{(g+1,1,q)j} \, S^{(1)}_{(g+1,1,q)j} \, , \quad q=1,\ldots,M \; . \qquad (3.58)$$

(This last equation merely rewrites (3.50) in full notation.)

Equation (3.56) relates each industry's demands for labor of particular types to the industry's demand for labor in general, to the costs of the different types of labor and to various technical change variables. The interpretation of (3.56) is similar to that of (3.23). If there is no technical change and no change in the relative prices of the different types of labor, then the occupational composition of industry j's workforce remains unchanged. However, if $P^{(1)}_{(g+1,1,q)j}$ increases relative to a weighted average of all the occupational wage rates payable by industry j, then j's use of labor of type q will increase more slowly than j's use of labor in general.[1] Because we have adopted CRESH rather than CES to describe labor-labor substitution possibilities (see (2.12)), the weights used in (3.56) to compute the average of the labor prices are not simply cost shares. They are the "modified" cost shares defined by (3.58). More importantly, however, (3.56) generalizes the input demand functions derivable in the CES case by allowing the responsiveness of $X^{(1)}_{(g+1,1,q)j}$ to changes in $P^{(1)}_{(g+1,1,q)j}$ relative to the average wage rate payable by industry j to depend on q. That is,

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1. Recall that the parameter restrictions in CRESH ensure that the coefficient, $\sigma^{(1)}_{(g+1,1,q)j}$, on the relative price term in (3.56) is positive.

the coefficient, $\sigma^{(1)}_{(g+1,1,q)j}$, on the relative price term in (3.56) has a q subscript.

The technical change terms in (3.56) indicate that if we hold all other variables on the RHS constant, then skill q augmenting technical progress at the rate of one per cent, i.e.,

$$a^{(1)}_{(g+1,1,q)j} = -1 \quad , \tag{3.59}$$

will reduce industry j's requirements for labor of type q by less than one per cent. Under (3.59), industry j substitutes towards skill q and away from other skills. Perhaps, one slightly surprising aspect of (3.56) is the absence of other technical change variables describing primary factor augmenting and Hicks neutral technical change. These more general types of technical change have their impact on industry j's demand for labor by occupational group via their impact on industry j's overall requirements for labor, $X^{(1)}_{(g+1,1)j}$.

Our final task for this subsection is the derivation of industry j's demand functions for labor in general, capital and agricultural land, i.e., we must explain $X^{(1)}_{(g+1,s)j}$, s=1,2,3 . We return to the first order conditions (3.36) and (3.37). These equations are analogous to (3.34) and (3.35). Hence, by reference to (3.51), we can immediately write

$$\bar{x}^{(1)}_{(g+1,v)j} = \bar{z}^{(1)}_{j} - \sigma^{(1)}_{(g+1,v)j} \left(\bar{p}^{(1)}_{(g+1,v)j} - \sum_{v=1}^{3} S^{*(1)}_{(g+1,v)j}\, \bar{p}^{(1)}_{(g+1,v)j}\right) ,$$

$$v=1,2,3 \quad , \tag{3.60}$$

where

$$\sigma^{(1)}_{(g+1,v)j} = 1 \Big/ \left(1 - h^{(1)}_{(g+1,v)j}\right) , \quad v=1,2,3 , \tag{3.61}$$

$$S^{*(1)}_{(g+1,v)j} = \sigma^{(1)}_{(g+1,v)j} S^{(1)}_{(g+1,v)j} \Big/ \sum_{v=1}^{3} \sigma^{(1)}_{(g+1,v)j} S^{(1)}_{(g+1,v)j} , \quad v=1,2,3 \tag{3.62}$$

and

$$S^{(1)}_{(g+1,v)j} = P^{(1)}_{(g+1,v)j} X^{(1)}_{(g+1,v)j} \Big/ \sum_{v=1}^{3} P^{(1)}_{(g+1,v)j} X^{(1)}_{(g+1,v)j} , \quad v=1,2,3 . \tag{3.63}$$

$P^{(1)}_{(g+1,v)j} X^{(1)}_{(g+1,v)j}$, $v=2,3$, are industry j's rental payments for capital and agricultural land. It is also true that $P^{(1)}_{(g+1,v)j} X^{(1)}_{(g+1,v)j}$ is industry j's expenditure on labor (see (3.46)). Hence the $S^{(1)}_{(g+1,v)j}$, $v=1,2,3$, are the shares of labor, capital and agricultural land in industry j's primary factor payments while the $S^{*(1)}_{(g+1,v)j}$'s are modified shares.

The last step is to apply the percentage change forms of (3.31), (3.32) and (3.33) to (3.60). This yields the primary factor demand equations,

$$\boxed{\begin{aligned} x^{(1)}_{(g+1,v)j} &= z_j - \sigma^{(1)}_{(g+1,v)j}\left(p^{(1)}_{(g+1,v)j} - \sum_{v=1}^{3} S^{*(1)}_{(g+1,v)j}\, p^{(1)}_{(g+1,v)j}\right) \\ &+ a^{(1)}_{j} + a^{(1)}_{g+1,j} + a^{(1)}_{(g+1,v)j} - \sigma^{(1)}_{(g+1,v)j}\left(a^{(1)}_{(g+1,v)j} - \sum_{v} S^{*(1)}_{(g+1,v)j}\, a^{(1)}_{(g+1,v)j}\right) \\ &\qquad v=1,2,3 , \quad j=1,\dots,h . \end{aligned}} \tag{3.64}$$

The interpretation of (3.64) presents no special problems apart from the variable $p^{(1)}_{(g+1,1,)j}$. For the moment we simply interpret $p^{(1)}_{(g+1,1)j}$ as the percentage change in the cost to industry j of a unit of labor. Then looking separately at the effects of each term in (3.64) we see (a) that a one per cent increase in industry j's activity level leads to a one per cent increase in the industry's requirements for labor in general, capital and agricultural land, (b) that increases in the cost to industry j of any particular factor relative to a weighted average over the costs of all three factors leads to substitution away from that factor in favour of the other two, and (c) that Hicks neutral or general primary factor augmenting technical change at the rate of one per cent, i.e.,

$$a^{(1)}_{j} = -1 \quad \text{or} \quad a^{(1)}_{g+1,j} = -1$$

leads to a one per cent reduction in j's demands for labor, capital and agricultural land. On the other hand, (g+1,v)-augmenting technical change induces substitution in favour of factor v and away from the other two primary factors. Hence,

$$a^{(1)}_{(g+1,v)j} = -1$$

generates a less than one per cent reduction in j's demand for factor v .

The one difficulty which arises in interpreting (3.64) is that unlike the rentals on capital and agricultural land $\left(p^{(1)}_{(g+1,v)j},\ v=2,3\right)$, $p^{(1)}_{(g+1,1)j}$ is not exogenous to our cost minimizing problem (3.26) - (3.28). Our interpretation of $p^{(1)}_{(g+1,1)j}$ as the cost to industry j of a unit of labor is justified by (3.46). Equation (3.46) implies that $P^{(1)}_{(g+1,1)j}$ is the ratio of total labor costs in industry j to the industry's aggregate labor input where aggregate labor input is defined by (2.12). The solution to the cost minimizing problem is not complete, however, until $P^{(1)}_{(g+1,1)j}$ is expressed in terms of variables which are exogenous to the problem (3.26) - (3.28). This can be done by writing (3.46) in linear percentage change form as

$$p^{(1)}_{(g+1,1)j} = \sum_{q=1}^{M} p^{(1)}_{(g+1,1,q)j} \, S^{(1)}_{(g+1,1,q)j} + \sum_{q=1}^{M} x^{(1)}_{(g+1,1,q)j} \, S^{(1)}_{(g+1,1,q)j} - x^{(1)}_{(g+1,1)j} \, . \tag{3.65}$$

On writing (3.44) in full notation and substituting from (3.53) and (3.54), we see that

$$x^{(1)}_{(g+1,1)j} - a^{(1)}_{(g+1,1)j} = \sum_{q=1}^{M} x^{(1)}_{(g+1,1,q)j} \, S^{(1)}_{(g+1,1,q)j} - a^{(1)}_{(g+1,1)j} - \sum_{q=1}^{M} a^{(1)}_{(g+1,1,q)j} \, S^{(1)}_{(g+1,1,q)j} \, .$$

Thus, (3.65) reduces to

$$\boxed{p^{(1)}_{(g+1,1)j} = \sum_{q=1}^{M} p^{(1)}_{(g+1,1,q)j} \, S^{(1)}_{(g+1,1,q)j} + \sum_{q=1}^{M} a^{(1)}_{(g+1,1,q)j} \, S^{(1)}_{(g+1,1,q)j} \, , \quad j=1,\ldots,h \, .} \tag{3.66}$$

One sensible convention would be to set

$$\sum_{q=1}^{M} a^{(1)}_{(g+1,1,q)j} \; S^{(1)}_{(g+1,1,q)j} = 0 \quad . \tag{3.67}$$

Under such a convention we would use changes in $A^{(1)}_{(g+1,1)j}$ to simulate the effects of general labor augmenting technical change and we would reserve $A^{(1)}_{(g+1,1,q)j}$ for studying the effects of technical change being relatively skill q augmenting. If convention (3.67) were adopted, then (3.66) would imply that the percentage change in $P^{(1)}_{(g+1,1)j}$ is a weighted average of the percentage changes in the costs to industry j of units of labor from the different skill groups, the weights being the shares of each skill group in j's total labor costs.

Equation (3.66) not only completes our solution to industry j's problem of choosing the cost minimizing combination of primary factor inputs, but it also completes our solution to the overall cost minimizing problem (3.1). We have solved this problem in parts. First, we considered the choice between domestic and imported intermediate inputs and we derived (3.23). Second, we noted that industry j has no substitute inputs for other cost tickets. Hence we obtained (3.25). Third, we expressed industry j's demand for labor of type q as a function of occupational wage rates, technical change variables and the industry's demand for labor in general, see equation (3.56). Subsequently, in equation (3.64), we explained j's demand for labor in general (together with the industry's demands for capital and agricultural land) in terms of the industry's activity level, the general price to the industry of labor, the rental rate for capital, the rental rate for agricultural land and various technical change variables. Finally, we derived equation (3.66) to define the general price of labor to industry j .

3.2 Commodity supplies : a revenue maximizing problem

At any given activity level, Z_j, producers in industry j will choose the commodity output combination which maximizes their revenue. That is, for each industry j we will assume that

$$X^{(0)}_{(t^*)j}\ , \quad t=1,\ldots,N(j)\ , \qquad \text{(outputs of composite commodities)}$$

and

$$X^{(0)}_{(i1)j}\ , \quad i=1,\ldots,g\ , \qquad \text{(outputs of commodities)}$$

are chosen to maximize

$$\sum_{i=1}^{g} P^{(0)}_{(i1)}\ X^{(0)}_{(i1)j} \tag{3.68}$$

subject to (2.13) and (2.16), where Z_j and the P's are treated as exogenous variables. $P^{(0)}_{(i1)}$ is the basic price of domestically produced good i, i.e., it is the price received by producers. It excludes sales taxes and transport and other margin costs involved in the transfer of good $(i1)$ from producers to users. We append no user or producer subscripts to the basic prices. Hence, we are assuming that basic prices are uniform across producing industries and across users. Further discussion of this assumption is in section 9.

To solve problem (3.68) we first define the basic price of composite commodity t in industry j by

$$P^{(0)}_{(t^*)j} = \sum_{i\varepsilon G(t,j)} P^{(0)}_{(i1)} \Big/ A^{(0)}_{(i1)j} \ , \quad t=1,\ldots,N(j) \quad j=1,\ldots,h \ . \tag{3.69}$$

According to (2.16), a unit of composite good t in industry j consists of $1/A^{(0)}_{(i1)j}$ units of good $(i1)$ for all $i \varepsilon G(t,j)$. Thus, (3.69) defines the revenue received by industry j per unit sale of composite good t. We can now reduce the revenue maximizing problem for industry j to one of choosing

$$\overline{X}^{(0)}_{(t^*)j} \ , \quad t=1,\ldots,N(j)$$

to maximize

$$\sum_{t=1}^{N(j)} \overline{P}^{(0)}_{(t^*)j} \ \overline{X}^{(0)}_{(t^*)j} \tag{3.70}$$

subject to

$$\underset{t=1,N(j)}{\text{CRETH}} \left\{\overline{X}^{(0)}_{(t^*)j}\right\} = \overline{Z}^{(0)}_j \tag{3.71}$$

where

$$\overline{P}^{(0)}_{(t^*)j} = P^{(0)}_{(t^*)j} \Big/ A^{(0)}_{(t^*)j} \ , \quad t=1,\ldots,N(j) \ , \tag{3.72}$$

$$\overline{X}^{(0)}_{(t^*)j} = X^{(0)}_{(t^*)j} \ A^{(0)}_{(t^*)j} \ , \quad t=1,\ldots,N(j) \ . \tag{3.73}$$

and

$$\overline{Z}^{(0)}_j = Z_j \Big/ A^{(0)}_j \ . \tag{3.74}$$

The first order conditions for this problem are

$$\bar{p}^{(0)}_{(r^*)j} - \Lambda \frac{\partial \underset{t=1,N(j)}{\text{CRETH}} \left(\bar{X}^{(0)}_{(t^*)j}\right)}{\partial \bar{X}^{(0)}_{(r^*)j}} = 0 \,, \quad r=1,\ldots,N(j) \tag{3.75}$$

and

$$\underset{t=1,N(j)}{\text{CRETH}} \left(\bar{X}^{(0)}_{(t^*)j}\right) - \bar{Z}^{(0)}_{j} = 0 \quad . \tag{3.76}$$

As was pointed out in our discussion of (2.14) and (2.15), the CRETH function is identical to CRESH apart from the restrictions on the parameters. Therefore, by analogy to the derivation of (3.51) from (3.38) and (3.39), we see that (3.75) and (3.76) imply

$$\bar{x}_{(r^*)j} = \bar{z}^{(0)}_{j} + \sigma^{(0)}_{(r^*)j} \left(\bar{p}^{(0)}_{(r^*)j} - \sum_{r=1}^{N(j)} H^{*(0)}_{(r^*)j}\, \bar{p}^{(0)}_{(r^*)j}\right) \quad r=1,\ldots,N(j) \,, \tag{3.77}$$

where

$$\sigma^{(0)}_{(r^*)j} = 1 \Big/ \left(h^{(0)}_{(r^*)j} - 1\right) , \quad r=1,\ldots,N(j) \,, \tag{3.78}$$

$$H^{*(0)}_{(r^*)j} = \sigma^{(0)}_{(r^*)j}\, H^{(0)}_{(r^*)j} \Big/ \sum_{r=1}^{N(j)} \sigma^{(0)}_{(r^*)j}\, H^{(0)}_{(r^*)j} \,, \quad r=1,\ldots,N(j) \tag{3.79}$$

and

$$H^{(0)}_{(r^*)j} = P^{(0)}_{(r^*)j}\, X^{(0)}_{(r^*)j} \Big/ \sum_{r=1}^{N(j)} P^{(0)}_{(r^*)j}\, X^{(0)}_{(r^*)j} \,, \quad r=1,\ldots,N(j) \quad . \tag{3.80}$$

By applying the percentage change forms of (3.72) - (3.74) to (3.77) we obtain

$$x^{(0)}_{(r^*)j} = z_j + \sigma^{(0)}_{(r^*)j}\left(p^{(0)}_{(r^*)j} - \sum_{r=1}^{N(j)} H^{*(0)}_{(r^*)j}\, p^{(0)}_{(r^*)j}\right)$$
$$- a^{(0)}_j - a^{(0)}_{(r^*)j} - \sigma^{(0)}_{(r^*)j}\left(a^{(0)}_{(r^*)j} - \sum_{r=1}^{N(j)} H^{*(0)}_{(r^*)j}\, a^{(0)}_{(r^*)j}\right) \quad (3.81)$$
$$r=1,\ldots,N(j)\ , \quad j=1,\ldots,h\ .$$

Equation (3.81) relates each industry's supplies of composite commodities to the industry's overall activity level, to the relative prices of the different composite commodities and to various technical change variables. If other variables on the RHS of (3.81) are held constant, then a one per cent increase in industry j's activity level generates a one per cent increase in the supplies of each of industry j's composite commodities. If, on the other hand, there is an increase in the price of composite commodity r relative to a weighted average of the prices of all j's composite commodities, then j transforms the commodity composition of its output in favour composite commodity r and away from the other composite commodities. (Notice that the restrictions on the CRETH parameters ensure that the coefficients, $\sigma^{(0)}_{(r^*)j}$, on the price transformation terms in (3.81) are positive.) The weights used to compute the average of the percentage changes in the prices of the composite commodities are the modified revenue shares defined by (3.79) and (3.80). If $h^{(0)}_{(r^*)j}$ were equal to $h^{(0)}_j$ for all r (the CET case) then the modified revenue shares would equal the revenue shares, $H^{(0)}_{(r^*)j}$. This simplification, however, would imply that $\sigma^{(0)}_{(r^*)j}$ would equal $\sigma^{(0)}_{j}$ for all r , and we would lose our ability to recognize variations across composite commodities in the strength of the price transformation terms.

The technical progress terms in (3.81) permit both product-neutral and biased shifts in industry j's production possibilities frontiers. The effects of a one per cent neutral change can be simulated by setting

$$a^{(0)}_{j} = -1 \quad . \tag{3.82}$$

Under (3.82), industry j can increase its output of each composite commodity by one per cent at any given level of inputs (i.e., at any given value of Z_j). We could simulate precisely the same change in j's production technology from the input side. We would set

$$a^{(1)}_{j} = -1 \quad .$$

For any given set of inputs, this would allow a one per cent increase in Z_j and, in consequence, a one per cent increase in each of the $X^{(0)}_{(r^*)j}$. Of course, we could also simulate the effects of neutral technical changes by imposing uniform values for all the $a^{(0)}_{(r^*)j}$'s or for all the $a^{(1)}_{ij}$'s , etc.. The effects of biased shifts in j's production possibilities frontiers can be obtained by using non-uniform values for $a^{(0)}_{(r^*)j}$, $r=1,\ldots,N(j)$. For example, if we set

$$a^{(0)}_{(r^*)j} = -1 \quad ,$$

with all other technical change variables held constant, then we are simulating the effects of an (r^*)-expanding shift in j's production possibilities frontier. Equation (3.81) implies that under such a shift, industry j is induced to increase its output of composite commodity r by more than one per cent. There is transformation in favour of (r^*).

To complete this section, all that remains is to relate the percentage changes in the outputs and prices of composite commodities to those of commodities. Under the Leontief assumption, (2.16), the assumption of revenue maximization ensures that

$$X^{(0)}_{(t^*)j} = X^{(0)}_{(i1)j} \, A^{(0)}_{(i1)j} \quad , \quad \text{for all } i \in G(t,j) \; ,$$
$$t=1,\ldots,N(j) \; , \quad j=1,\ldots,h \; .$$

Hence

$$\boxed{\begin{aligned} x^{(0)}_{(i1)j} &= x^{(0)}_{(t^*)j} - a^{(0)}_{(i1)j} \; , \\ &\text{for all } i \in G(t,j) \; , \\ &\qquad t=1,\ldots,N(j) \; , \\ &\qquad j=1,\ldots,h \; . \end{aligned}} \tag{3.83}$$

Shifts in the commodity composition of composite commodities can be introduced via non-zero values for the $a^{(0)}_{(i1)j}$'s . If, however, these are set at zero, then the output of $(i1)$ by industry j will change by the same percentage as the output of the relevant composite commodity.

In the case of composite commodity prices, (3.69) implies

$$\boxed{P^{(0)}_{(t^*)j} = \sum_{i\varepsilon G(t,j)} P^{(0)}_{(i1)} S^{(0)}_{(i1)j} - \sum_{i\varepsilon G(t,j)} a^{(0)}_{(i1)j} S^{(0)}_{(i1)j} , \quad t=1,\ldots,N(j) , \quad j=1,\ldots,h ,} \tag{3.84}$$

where

$$S^{(0)}_{(i1)j} = \frac{P^{(0)}_{(i1)}}{A^{(0)}_{(i1)j}} \Bigg/ \sum_{i\varepsilon G(t,j)} \frac{P^{(0)}_{(i1)}}{A^{(0)}_{(i1)j}} , \tag{3.85}$$

$$\text{for all } i \varepsilon G(t,j) , \quad t=1,\ldots,N(j) , \quad j=1,\ldots,h .$$

Recalling that composite commodity t in industry j consists of $\left(1 \Big/ A^{(0)}_{(i1)j}\right)$ units of good $(i1)$ for all $i \varepsilon G(t,j)$, we see that $S^{(0)}_{(i1)j}$, $i \varepsilon G(t,j)$, is the share of $(i1)$ in the total receipts by industry j for its composite commodity t .

4. Demands for Inputs for the Production of Fixed Capital

In this section we are concerned with the demands for inputs to the construction of units of fixed capital. We assume that a unit of fixed capital for use in industry j can be created according to the production function

$$A_j^{(2)} Y_j = \underset{i=1,g}{\text{LEONTIEF}} \left\{ \frac{X_{ij}^{(2)}}{A_{ij}^{(2)}} \right\} , \quad j=1,\ldots,h \quad , \tag{4.1}$$

where Y_j is the number of units of fixed capital created for industry j, $X_{ij}^{(2)}$, $i=1,\ldots,g$, is the direct[1] effective input of good i into creating capital for industry j and $A_j^{(2)}$ and $A_{ij}^{(2)}$, $i=1,\ldots,g$, are positive coefficients. Changes in these $A^{(2)}$'s can be used to simulate various changes in the technology of making units of capital for industry j.

The effective inputs, $X_{ij}^{(2)}$, $i=1,\ldots,g$, are defined by

$$X_{ij}^{(2)} = \underset{s=1,2}{\text{CES}} \left(X_{(is)j}^{(2)} \Big/ A_{(is)j}^{(2)} \right) \tag{4.2}$$

where $X_{(i1)j}^{(2)}$ and $X_{(i2)j}^{(2)}$ are the inputs of good i, from domestic and imported sources respectively, to the production of capital for industry j and the $A_{(is)j}^{(2)}$ are a further set of positive technological coefficients. Just as we allowed imported and domestic

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1. Transportation margins, etc., associated with the delivery of inputs for capital construction are included in the prices paid, but not in the production function. See the discussion following (2.1).

supplies of good i to be imperfect substitutes as inputs in current production, we allow them to be imperfect substitutes in the production of units of capital. On the other hand, a point of contrast between the technologies for current production and those described by (4.1) - (4.2) is that capital creation requires no inputs of primary factors or other costs tickets. The use of labor, capital and agricultural land,[1] the payment of production taxes and the costs of holding liquidity and inventories associated with the creation of capital is recognised via the inputs of "construction", i.e., the construction industries use primary inputs and pay production taxes, etc., and the creation of fixed capital requires heavy inputs of construction.

We assume that fixed capital is created competitively and efficiently. Producers of capital for industry j treat input prices as beyond their control, and for any given level of capital creation, Y_j, they choose

$$X^{(2)}_{(is)j} \quad , \quad i=1,\ldots,g \ , \quad s=1,2$$

to minimize

$$\sum_{i=1}^{g} \sum_{s=1}^{2} P^{(2)}_{(is)j} \ X^{(2)}_{(is)j} \tag{4.3}$$

subject to (4.1) and (4.2) where $P^{(2)}_{(is)j}$ is the price of good i from source s when it is used as an input for creating capital

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1. ORANI does not explicitly recognize the factory site as part of fixed capital. For most industries, this is a small fraction of the costs of fixed capital.

for industry j . The j subscript and the (2) superscript again play the role of allowing us to introduce variations across users and uses in the purchasers' price of good i from source s -- see the discussion following (3.1).

The solution to the cost minimization problem (4.3) follows the steps (3.2) to (3.23). We obtain

$$\boxed{\begin{aligned} x^{(2)}_{(is)j} &= y_j - \sigma^{(2)}_{ij}\left(p^{(2)}_{(is)j} - \sum_{s=1}^{2} S^{(2)}_{(is)j}\, p^{(2)}_{(is)j}\right) + a^{(2)}_j \\ &\quad + a^{(2)}_{ij} + a^{(2)}_{(is)j} - \sigma^{(2)}_{ij}\left(a^{(2)}_{(is)j} - \sum_{s=1}^{2} S^{(2)}_{(is)j}\, a^{(2)}_{(is)j}\right), \\ &\qquad i=1,\ldots,g\ , \\ &\qquad s=1,2\ , \\ &\qquad j=1,\ldots,h\ , \end{aligned}} \tag{4.4}$$

where $S^{(2)}_{(is)j}$ is the share of good i from source s in the total cost of good i used for creation of capital in industry j and $\sigma^{(2)}_{ij} = 1\Big/\left(1 + \rho^{(2)}_{ij}\right)$ is the elasticity of substitution between imported and domestic good i as inputs for creation of capital of type j .

In summary, we have made the following assumptions concerning the construction of fixed capital :

(1) Units of fixed capital for industry j are created by combining effective units of produced (i.e., non-primary) inputs according to a Leontief production function.

(2) The effective input of good i to construction of capital for industry j is a CES combination of inputs from domestic and foreign sources.

(3) The composition of units of capital varies across industries. There is a separate production function (4.1) - (4.2) for capital in each industry j . Hence, we are able to recognize, for example, that a given dollar increase in investment in agriculture brings forth a greater increase in demand for tractors than a similar dollar increase in investment in the chemical industry.

(4) We have not at this stage explained how the investment level, Y_j, in each industry is determined. That is done in section 10. However, given the level for Y_j, we have, on the basis of the competition and cost minimizing assumptions, derived the demand functions, (4.4), for inputs to capital creation.

5. Household Demands

Household demands are explained by a utility maximizing model. Letting Q be the number of households, we assume that the consumption bundle of effective inputs, $X_i^{(3)}/Q$, $i=1,\ldots,g$, for the "average" household is chosen to maximize

$$U\left(\overline{X}_1^{(3)},\ldots,\overline{X}_g^{(3)}\right) \tag{5.1}$$

subject to

$$\overline{X}_i^{(3)} = \underset{s=1,2}{\mathrm{CES}}\left(\overline{X}_{(is)}^{(3)}\right), \quad i=1,\ldots,g, \tag{5.2}$$

and

$$\sum_{s=1}^{2}\sum_{i=1}^{g}\overline{P}_{(is)}^{(3)}\,\overline{X}_{(is)}^{(3)} = C, \tag{5.3}$$

where

$$\overline{X}_i^{(3)} = X_i^{(3)}\Big/\left(A_i^{(3)}\,Q\right), \quad i=1,\ldots,g \tag{5.4}$$

$$\overline{X}_{(is)}^{(3)} = X_{(is)}^{(3)}\Big/\left(A_i^{(3)}\,A_{(is)}^{(3)}\,Q\right), \quad i=1,\ldots,g, \quad s=1,2, \tag{5.5}$$

and

$$\overline{P}_{(is)}^{(3)} = P_{(is)}^{(3)}\,A_i^{(3)}\,A_{(is)}^{(3)}\,Q, \quad i=1,\ldots,g, \quad s=1,2. \tag{5.6}$$

$X_{(is)}^{(3)}$ and $P_{(is)}^{(3)}$, $i=1,\ldots,g$, $s=1,2$, are the quantities consumed and prices paid by households for units of good i from source s, with $s=1$ referring to domestic sources and $s=2$ referring to imports. C is the aggregate consumer budget and the $A^{(3)}$'s are positive coefficients introduced to allow for changes in tastes.

In problem (5.1) - (5.3) we assume that C is explained elsewhere in the model or simply remains exogenous. In this section we explain the allocation of C across commodities by assuming that, whatever its level might be, it will be allocated so as to maximize utility for the "average" household. The $A^{(3)}$'s are also exogenous to the household utility maximizing problem. The effects of a one per cent good i augmenting change in tastes could be simulated by an exogenously imposed one per cent reduction in $A_i^{(3)}$, i.e.,

$$a_i^{(3)} = -1 ,$$

while the effects of a one per cent good (is) augmenting change in tastes could be simulated by a one per cent reduction in $A_{(is)}^{(3)}$.

In adopting (5.1) - (5.3), we are allowing consumers to satisfy their demands for any good i by drawing on imported and domestic sources, with the two sources providing imperfect substitutes. Also, by using the (3) superscript on prices, we are recognizing that households may pay a different amount (e.g., because of taxes and distribution costs) for good i from source s than is paid by other users. On the other hand, we have introduced the simplification that consumer demands can be treated as though they arise from the maximization of a single utility function subject to an aggregate budget constraint, i.e., we have assumed that aggregation across consumers is legitimate. While the inclusion of different types of consumers would be an obvious model improvement, the payoff in terms of more accurate simulation of aggregate consumer behaviour might be quite small. Only in very long-run simulations, allowing large demographic changes, or in simulations which introduce major changes in income distribution, is the single consumer assumption likely to be inadequate.[1]

The first order conditions for a solution of problem (5.1) - (5.3) can be written as

$$\frac{\partial U}{\partial \overline{X}_i^{(3)}} - \Gamma \overline{P}_i^{(3)} = 0 \ , \quad i=1,\ldots,g \ , \tag{5.7}$$

$$\overline{P}_i^{(3)} \frac{\partial CES_s\left(\overline{X}_{(is)}^{(3)}\right)}{\partial \overline{X}_{(is)}^{(3)}} - \overline{P}_{(is)}^{(3)} = 0 \ , \quad i=1,\ldots,g \ , \quad s=1,2 \ , \tag{5.8}$$

$$\overline{X}_i^{(3)} - CES_s\left(\overline{X}_{(is)}^{(3)}\right) = 0 \ , \quad i=1,\ldots,g \ , \tag{5.9}$$

and

$$\sum_{s=1}^{2} \sum_{i=1}^{g} \overline{P}_{(is)}^{(3)} \overline{X}_{(is)}^{(3)} = C \ , \tag{5.10}$$

where Γ is the Lagrangian multiplier on constraint (5.3) and the $\overline{P}_i^{(3)}$'s are defined so that $\Gamma\overline{P}_i^{(3)}$, $i=1,\ldots,g$, are the Lagrangian multipliers on the constraints (5.2).

It follows from (5.8) and (5.9), by analogy with the argument by which we derived (3.19) and (3.18) from (3.7) and (3.8), that

$$\bar{x}_{(is)}^{(3)} = \bar{x}_i^{(3)} - \sigma_i^{(3)}\left(\bar{p}_{(is)}^{(3)} - \sum_{s=1}^{2} S_{(is)}^{(3)} \bar{p}_{(is)}^{(3)}\right) \ ,$$

$$i=1,\ldots,g \ , \quad s=1,2 \ , \tag{5.11}$$

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1. For empirical evidence on this point see Dixon (1975, Ch. 2), and for a multi-consumer, long-run model see Dixon, Harrower and Powell (1976).

and

$$\bar{p}_i^{(3)} = \sum_{s=1}^{2} \bar{p}_{(is)}^{(3)} S_{(is)}^{(3)} , \quad i=1,\ldots,g , \tag{5.12}$$

where $\sigma_i^{(3)}$, $i=1,\ldots,g$, is the elasticity of substitution in consumption between domestic and imported good i and $S_{(is)}^{(3)}$ is the share of total consumer spending on good i which is devoted to good i from source s .

Next, we derive equations for the $\bar{x}_i^{(3)}$, $i=1,\ldots,g$. We start by multiplying (5.8) through by $\overline{X}_{(is)}^{(3)}$, aggregating over s and applying Euler's theorem, establishing that

$$\overline{P}_i^{(3)} \overline{X}_i^{(3)} = \sum_s \overline{P}_{is}^{(3)} \overline{X}_{is}^{(3)} , \quad i=1,\ldots,g . \tag{5.13}$$

Hence, (5.10) may be rewritten as

$$\sum_{i=1}^{g} \overline{P}_i^{(3)} \overline{X}_i^{(3)} = C . \tag{5.14}$$

On combining (5.7) and (5.14), we see that we have the first order conditions for the problem of choosing $\overline{X}_i^{(3)}$, $i=1,\ldots,g$, to maximize

$$U\left(\overline{X}_1^{(3)},\ldots,\overline{X}_g^{(3)}\right) \tag{5.15}$$

subject to

$$\sum_{i=1}^{g} \overline{P}_i^{(3)} \overline{X}_i^{(3)} = C . \tag{5.16}$$

This means that the choice of the $\overline{X}_i^{(3)}$'s can be handled by the conventional un-nested utility maximizing model. Hence, we may conclude that

$$\bar{x}_i^{(3)} = \varepsilon_i \; c + \sum_{k=1}^{g} \eta_{ik} \; \bar{p}_k^{(3)} \quad , \qquad i=1,\ldots,g \quad , \tag{5.17}$$

where ε_i, η_{ik}, $i,k=1,\ldots,g$, may be interpreted as expenditure and own and cross price elasticities satisfying the usual restrictions -- homogeneity, symmetry and Engel's aggregation.[1]

Before providing a more detailed interpretation of the ε_i and η_{ij}, it will be convenient to translate (5.17), together with (5.11) and (5.12), into relationships between unbarred variables. To do this we need the percentage change forms of (5.4) - (5.6), i.e.,

$$\bar{x}_i^{(3)} = x_i^{(3)} - a_i^{(3)} - q \quad , \qquad i=1,\ldots,g \quad , \tag{5.18}$$

$$\bar{x}_{(is)}^{(3)} = x_{(is)}^{(3)} - a_i^{(3)} - a_{(is)}^{(3)} - q \; , \quad i=1,\ldots,g \; , \quad s=1,2 \tag{5.19}$$

and

$$\bar{p}_{(is)}^{(3)} = p_{(is)}^{(3)} + a_i^{(3)} + a_{(is)}^{(3)} + q \; , \quad i=1,\ldots,g \; , \quad s=1,2 \; . \tag{5.20}$$

On using (5.18) - (5.20) in (5.11) we find that

$$\boxed{\begin{aligned} x_{(is)}^{(3)} = \; & x_i^{(3)} - \sigma_i^{(3)} \left(p_{(is)}^{(3)} - \sum_{s=1}^{2} S_{(is)}^{(3)} \; p_{(is)}^{(3)} \right) \\ & + a_{(is)}^{(3)} - \sigma_i^{(3)} \left(a_{(is)}^{(3)} - \sum_{s=1}^{2} S_{(is)}^{(3)} \; a_{(is)}^{(3)} \right) \; , \\ & i=1,\ldots,g \; , \quad s=1,2 \; . \end{aligned}} \tag{5.21}$$

---

1. See, for example, Powell (1974, Ch. 1) and Dixon, Bowles and Kendrick (1980, Ch. 2).

Next, we use (5.20) in (5.12). This gives

$$\bar{p}_i^{(3)} = p_i^{(3)} + a_i^{(3)} + \sum_{s=1}^{2} a_{(is)}^{(3)} S_{(is)}^{(3)} + q \,, \quad i=1,\ldots,g \,, \tag{5.22}$$

where

$$\boxed{p_i^{(3)} = \sum_{s=1}^{2} S_{(is)}^{(3)} p_{(is)}^{(3)} \,, \quad i=1,\ldots,g \,.} \tag{5.23}$$

Finally, we substitute from (5.18) and (5.22) into (5.17), obtaining

$$\boxed{\begin{aligned} x_i^{(3)} - q = {} & \varepsilon_i(c-q) + \sum_{k=1}^{g} \eta_{ik} p_k^{(3)} \\ & + a_i^{(3)} + \sum_{k=1}^{g} \eta_{ik} \left(a_k^{(3)} + \sum_{s=1}^{2} S_{(ks)}^{(3)} a_{(ks)}^{(3)}\right) , \\ & \qquad i=1,\ldots,g \,. \end{aligned}} \tag{5.24}$$

In deriving (5.24), we made use of the homogeneity restriction[1] (implied by the problem (5.15) - (5.16)), namely

$$\sum_{k=1}^{g} \eta_{ik} = -\varepsilon_i \,. \tag{5.25}$$

More generally, (5.24) confirms our interpretation of the ε_i's and η_{ik}'s as expenditure and own and cross price elasticities. ε_i and

1. Equation (5.25) reflects the fact that in (5.15) - (5.16) a one per cent reduction in C will have precisely the same effect on the solution for $\overline{X}_i^{(3)}$ as a one per cent increase in all the $\overline{P}_i^{(3)}$'s .

η_{ik} are the effects on per household consumption of effective units of good i arising from respectively, a one per cent increase in average household expenditure and a one per cent increase in the general price of good k. The general price of good k is an index, defined by (5.23) of the prices of the locally produced and imported commodities.

In assigning values for the elasticities ε_i, η_{ik}, $i,k=1,\ldots,g$, we are free to draw on the extensive literature on the systems approach to applied demand theory.[1] However, in view of our resource constraints, we have currently adopted one of the empirically least challenging specifications. We have assumed that the utility function (5.15) is of the Klein-Rubin form,[2] i.e.,

$$U\left(\overline{X}_1^{(3)},\ldots,\overline{X}_g^{(3)}\right) = \sum_{i=1}^{g} \delta_i \, \ell n\left(\overline{X}_i^{(3)} - \Theta_i\right) \qquad (5.26)$$

where δ_i and Θ_i, $i=1,\ldots,g$ are parameters with $\delta_i > 0$ for all i and $\sum_{i=1}^{g} \delta_i = 1$. On solving problem (5.15) - (5.16) under (5.26) we obtain the well-known linear expenditure system,

$$\overline{X}_i^{(3)} = \Theta_i + \delta_i \left(C - \sum_{k=1}^{g} \overline{P}_k^{(3)} \Theta_k\right) \Big/ \overline{P}_i^{(3)}, \quad i=1,\ldots,g \; . \qquad (5.27)$$

---

1. For a survey which highlights both the economic theory and the econometrics of the systems approach to applied demand theory, see Powell (1974).

2. See Klein and Rubin (1948-49). Utility functions of the form (5.26) are also referred to as Stone-Geary functions in recognition of the contributions of Stone (1954) and Geary (1950-51).

Then on the basis of (5.27) we have

$$\varepsilon_i = \delta_i \Big/ S_i^{(3)} , \quad i=1,\ldots,g , \tag{5.28}$$

$$\eta_{ik} = - \delta_i \; S_k^{*(3)} \Big/ S_i^{(3)} \quad \text{for all} \quad i \neq k \tag{5.29}$$

and

$$\eta_{ii} = - \varepsilon_i - \sum_{k \neq i} \eta_{ik} , \tag{5.30}$$

where

$$S_i^{(3)} = \overline{P}_i^{(3)} \; \overline{X}_i^{(3)} \Big/ \sum_k \overline{P}_k^{(3)} \; \overline{X}_k^{(3)} , \quad i=1,\ldots,g , \tag{5.31}$$

and

$$S_i^{*(3)} = \overline{P}_i^{(3)} \; \Theta_i^{(3)} \Big/ \sum_k \overline{P}_k^{(3)} \; \overline{X}_k^{(3)} , \quad i=1,\ldots,g . \tag{5.32}$$

In view of (5.13), (5.5) and (5.6) we may interpret $S_i^{(3)}$ as the share of household expenditure devoted to good i . The interpretation of $S_i^{*(3)}$, however, is less clear. It depends firstly on the interpretation of Θ_i . This is the minimum (often called the subsistence level)[1] for $\overline{X}_i^{(3)}$. Unless $\overline{X}_i^{(3)} > \Theta_i$ for all i, i.e., unless

$$X_i^{(3)} \Big/ Q > \Theta_i \; A_i^{(3)} \quad \text{for all } i ,$$

then the utility function (5.26) is undefined. Thus we interpret $\Theta_i \; A_i^{(3)}$ as the current subsistence level for the consumption of good i per household. Now we rewrite (5.32) as

$$S_i^{*(3)} = S_i^{(3)} \; \frac{\Theta_i \; A_i^{(3)}}{X_i^{(3)}/Q} , \quad i=1,\ldots,g . \tag{5.33}$$

---

1. Since there is no requirement that the Θ_i's are non-negative, their interpretation as subsistence quantities frequently breaks down in applied work.

We see that $S_i^{*(3)}$ is the product of two shares: the share of good i in household expenditure and the share that the subsistence level for the consumption of good i represents in the average household's consumption of good i .

In summary, our household demand specification can be described as follows :

(1) Consumers behave as if they maximize a single utility function subject to a budget constraint, see (5.1) - (5.3).

(2) The utility function is assumed to be Klein-Rubin, see (5.26). This assumption simplifies the estimation of the "outside" demand elasticities, i.e., the elasticities which measure the response of demand for good i, in general, to changes in the expenditure level of the average household and changes in the general price of good j . There is, of course, nothing in the theory presented here which precludes the implementation of (5.24) with the ε_i and η_{ik} derived from more general utility specifications than (5.26).

(3) The consumption of good i, in general, is defined by a CES aggregate of the consumption of good i from domestic and foreign sources, see (5.2). Consumers will substitute between the two sources of supply of good i in response to changes in the relative prices of good i from each source, see (5.21). They will substitute between good i, in general, and good j, in general,

in response to changes in the relative general prices of i and j, see (5.24). The change in the general price of i is a weighted average of the changes in the prices of i from the alternative sources, see (5.23).

(4) The effects of changes in household preferences can be simulated via quantity-augmenting variables. The treatment of changes in preferences closely parallels that of changes in technology.

(5) Our demand specification could be usefully improved by the implementation of (a) a non-additive utility function in place of the additive form (5.26) and (b) a more general aggregation function over domestic and foreign sources than the CES form (5.2). Notice that under the additive specification (5.26), we have

$$\frac{\partial^2 U}{\partial \bar{X}_i^{(3)}\ \partial \bar{X}_j^{(3)}} = 0 \quad \text{for all} \quad i \neq j \ . \tag{5.34}$$

Literally, (5.34) means that consumers behave as if their marginal utility for good i is independent of their consumption of good j for all $i \neq j$. Hence, the additivity assumption is not suitable for studies which distinguish "apples" and "pears" as separate commodities. The marginal utility of apples will depend on the consumption of pears. On the other hand, the additivity assumption has been successfully applied in many studies where the commodities are "food", "clothing", "transport", etc..[1] In ORANI there are several commodities distinguished within each of the "food", "clothing", "transport", etc., groups. Hence, the additivity assumption must be considered an expedient only. By using it, we are failing to recognise that two food items such as "milk products" and "bread, cakes and biscuits" are probably more closely substitutable for each other than they are for "clothing", say.[2]

The main limitation of the CES specification (5.2) is that it implies the perhaps unsatisfactory restriction that

$$\varepsilon_{(is)} = \varepsilon_i \ , \qquad i=1,\ldots,g \ , \qquad s=1,2 \ , \qquad (5.35)$$

i.e., the expenditure elasticity of demand for imported good i is the same as that for domestically produced good i. A utility function which avoids (5.35), but retains additivity across goods in general is provided by Brown and Heien (1972). A combination of the "almost-additivity" approach of Barten (1964), which allows some specific substitution effects, with the Brown-Heien method, would provide a potentially fruitful avenue for future empirical research.[3]

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1. For pioneering theoretical work on the additivity assumption, see Frisch (1959) and Houthakker (1960). For early empirical applications, see Johansen (1960), Barten (1964) and Powell (1966).

2. Additivity rules out the so-called <u>specific</u> substitution effects. See for example Theil (1975, section 1.4).

3. Some further guidance on this approach is obtainable from Powell (1974, chapters 4 and 5).

6. Foreign Demand for Australian Exports

We assume that

$$P^e_{(i1)} = g_i \left(X^{(4)}_{(i1)}\right) F^{(4)}_{(i1)} \quad , \quad i=1,\ldots,g \quad , \tag{6.1}$$

where $P^e_{(i1)}$ is the foreign currency receipt per unit of export of good i . $P^e_{(i1)}$ includes payments for transport and other margins involved in the delivery of exports to Australian ports, but it excludes shipping costs from the Australian ports to the final destination, i.e., $P^e_{(i1)}$ is the f.o.b. price in foreign currency. g_i is a non-increasing function of $X^{(4)}_{(i1)}$ and $X^{(4)}_{(i1)}$ is the volume of exports of good i .[1] Finally, $F^e_{(i1)}$ is a shift variable which will increase if there is an increase in overseas demand for good i from Australia.

In percentage change form, (6.1) becomes

$$\boxed{\begin{aligned} p^e_{(i1)} &= -\gamma_i \, x^{(4)}_{(i1)} + f^e_{(i1)} \quad , \\ &\qquad i=1,\ldots,g \quad , \end{aligned}} \tag{6.2}$$

where

$$-\gamma_i \equiv \frac{\partial g_i}{\partial X^{(4)}_{(i1)}} \; \frac{X^{(4)}_{(i1)}}{g_i} \quad ,$$

---

1. The 1 subscript is not strictly required on $X^{(4)}_{(i1)}$. We assume that all exports are domestically produced, i.e., no imports are exported without flowing through a domestic industry.

i.e., γ_i is non-negative and is the reciprocal of the foreign elasticity of demand for Australian exports of good i .

Equation (6.2) can be used flexibly. For example, for commodities where Australia supplies a very small fraction of the world market, γ_i can be set at zero. Then (6.2) implies that the foreign currency f.o.b. export price of good i is independent of the volume of Australian exports of i . For commodities in which Australian export volumes and prices are determined largely by government agreement, we might fix γ_i at zero, and then set $f^e_{(i1)}$ and $x^{(4)}_{(i1)}$ exogenously. (Note that by merely writing (6.2), we have not explained $x^{(4)}_{(i1)}$; certainly we have not excluded the possibility of setting it exogenously.) For a commodity i, in which there are well developed world markets and in which Australia is a major participant, e.g., wool, we might set γ_i at 0.4, say. Then the achievement of a one per cent increase in Australian exports of good i would require a 0.4 per cent decrease in their foreign currency f.o.b. price. Finally, it should be pointed out that it is only for algebraic and computing convenience that we allow (6.2) to apply over all i, $i=1,\ldots,g$. Obviously there is no f.o.b. export price for the numerous non-exported services. For non-exportables, we set γ_i and $f^e_{(i1)}$ arbitrarily to zero (in fact, any other numbers would do). No harm is done because for non-exportables, $p^e_{(i1)}$ and $f^e_{(i1)}$ have no impact on any variable of economic interest within the model.[1]

1. Apart from (6.2), $p^e_{(i1)}$ enters equations (9.14) and (12.6). When we examine these equations, it will be obvious that the existence of arbitrary and meaningless values for the $p^e_{(i1)}$ corresponding to non-export industries has no distorting effects on model results.

7. Other Demands

The final category of direct demands is "other". This is mainly government demands for both imported and domestically produced goods and services.[1]

At its present stage of development, ORANI contains no theory for "other" demands. We merely include the equations

$$\boxed{x_{(is)}^{(5)} = c_R\, h_{(is)}^{(5)} + f_{(is)}^{(5)}\,, \quad i=1,\ldots,g\,, \quad s=1,2\,.} \tag{7.1}$$

$x_{(is)}^{(5)}$ is the percentage change in "other" demands for good i from source s, c_R is the percentage change in real aggregate household expenditure, i.e.,

$$\boxed{c_R = c - \xi^{(3)}\,,} \tag{7.2}$$

where $\xi^{(3)}$ is the ORANI consumer price index (see (13.1)), the $f_{(is)}^{(5)}$'s are shift variables and the $h_{(is)}^{(5)}$'s are parameters.

---

1. Governments are viewed as buying goods and services only. There are no direct "other" demands for labor or other primary factors, The government buys from the defence industry, the public administration industry, the education industry, etc., and these employ primary factors.

In typical ORANI calculations we set all the $h^{(5)}$ parameters at one and the $f^{(5)}$ shift variables at zero. This has the effect of forcing other demands to move in line with real household expenditure. However, (7.1) means that it is very easy to introduce alternative scenarios for other demands. For example, if we wish to compute the effects of a 10 per cent cut in all real current government spending, then (7.1) is included in the model with the $h^{(5)}$'s set at zero and the $f^{(5)}$'s set at approximately -10 .[1]

1. $f^{(5)}_{(is)}$ would be set at precisely -10 only if other demands for good i from source s consisted entirely of government demands. In fact, as explained in Chapter 4, section , some artificial entries are included in other demands. Care must be taken to allow for these items in simulations of changes in government spending.

8. Demands for Margins

In the previous five sections we have considered the direct demands for goods and services by producers, investors, households, foreigners and governments. The satisfaction of these direct demands creates demands for "margins", i.e., transport, wholesale and retail services, etc.. In 1968-69 about 20 per cent of the Australian GDP was generated by the margins industries in their role of facilitating the flow of goods and services from producers to users. Hence, despite the rather tedious algebra, computing and data manipulation which are involved, we decided that explicit modelling of the demands for margins should be attempted.

In general, demands for margins cannot be handled satisfactorily within the theoretical framework developed to explain direct demands. For example, consider the implications of treating retail trade margins as just another consumer good. If we included them as an argument of our additive utility function (5.26), we would be likely to generate some strange distortions; consumers would substitute retail trade for food in response to an increase in the price of food. A more plausible story would be that the increase in the price of food reduces the demand for food and the associated demand for retail services. A second possibility is to treat retail trade as a cost of production, i.e., it would appear as an argument in the production functions (2.1). That approach fails when the amount of retail trade involved in delivering good i depends on the purchaser. For example, retail trade margins associated with the sale of petrol to householders are much higher than those associated with petrol sales to industrial users. Hence, a change in household demand for petrol has a more marked effect on retailing activity than an equal change in industrial demand for petrol. This phenomenon will not be simulated if retailing is treated as an input to petrol production.

For margins associated with the delivery of inputs for production and capital construction, we assume that

$$X_{(r1)}^{(is)jk} = A_{(r1)}^{(is)jk} X_{(is)j}^{(k)} , \tag{8.1}$$

$$i,r=1,\ldots,g , \quad j=1,\ldots,h ,$$
$$k,s=1,2 ,$$

where $X_{(r1)}^{(is)jk}$ is the quantity of good $(r1)$ used as a margin to facilitate the flow of good i from source s to industry j for purpose k, $A_{(r1)}^{(is)jk}$ is a positive coefficient and $X_{(is)j}^{(k)}$ is, as previously defined in sections 2 and 4, the quantity of good i from source s used in industry j for purpose k.

In the absence of changes in the $A_{(r1)}^{(is)jk}$'s, equation (8.1) forces margin flows to be proportional to commodity flows. If the amount of domestically produced chemicals used by the fertilizer industry in current production is doubled, then the amount of "transport" involved in transferring domestically-produced chemicals from the chemical producers to the fertilizer manufacturers is doubled. Notice also that we have assumed that all margins are domestically produced. Perhaps it should be emphasized that we are not concerned with transport costs, etc., associated with transferring commodities between foreign ports and Australia. In the case of imports (s=2), (8.1) is intended to represent the demand for margins associated with deliveries from Australian ports to the users within Australia. This point is further elaborated in section 9. Finally, in (8.1) we have allowed r to run over all values from 1 to g. In fact, there are only 8 commodities (services) used as margins among the 115 commodities in the model. Where r is not a margin service, we can set $A_{(r1)}^{(is)jk}$ equal to zero.[1]

In percentage change form, (8.1) is

$$\boxed{\begin{array}{l} x_{(r1)}^{(is)jk} = x_{(is)j}^{(k)} + a_{(r1)}^{(is)jk} , \\ \qquad i,\ r=1,\ldots,g , \\ \qquad j=1,\ldots,h , \quad k,s=1,2 . \end{array}} \qquad (8.2)$$

Changes in the $A_{(r1)}^{(is)jk}$'s, are allowed so that we can simulate the effects of changes in the amounts of margin service associated with various commodity flows. For example, some of the effects of containerization at Australian ports might be studied by setting the $a_{(r1)}^{(i2)jk}$'s at negative values for r = "transport and handling". This would simulate the effects of reductions in the transport and handling requirements associated with unit flows of imported inputs to domestic industries.

---

1. Because we allow percentage changes in the $A_{(r1)}^{(is)jk}$'s, it may be better to think of these coefficients as having arbitrarily small, but non-zero, values.

Margin flows associated with the delivery of commodities to households and other users and to Australian ports prior to export are handled by equations similar to (8.1). Thus we have

$$X_{(r1)}^{(is)k} = A_{(r1)}^{(is)k} X_{(is)}^{(k)} , \quad k=3,5 , \quad s=1,2 , \quad r,i=1,\ldots,g , \tag{8.3}$$

and

$$X_{(r1)}^{(i1)4} = A_{(r1)}^{(i1)4} X_{(i1)}^{(4)} , \quad i,r=1,\ldots,g , \tag{8.4}$$

where (8.3) describes the margin flows associated with the delivery of commodities to households and other users, and (8.4) describes the margins flows associated with the delivery of commodities from Australian producers to Australian ports prior to export.

In percentage change form, (8.3) and (8.4) generate

$$\boxed{x_{(r1)}^{(is)k} = x_{(is)}^{(k)} + a_{(r1)}^{(is)k} , \quad k=3,5, \quad r,i=1,\ldots,g, \quad s=1,2,} \tag{8.5}$$

and

$$\boxed{x_{(r1)}^{(i1)4} = x_{(i1)}^{(4)} + a_{(r1)}^{(i1)4} , \quad i, \quad r=1,\ldots,g .} \tag{8.6}$$

9. The Price Systems

ORANI uses several sets of commodity prices: purchasers' prices, basic values, prices of capital units, f.o.b. foreign currency export prices and c.i.f. foreign currency import prices. In this section we set out the relationships between them.

Two initial assumptions are (1) there are no pure profits in any economic activity (producing, importing, exporting, transporting, etc.), and (2) basic values are uniform across users and across producing industries in the case of domestic goods and importers in the case of foreign goods. As was explained in section 3, basic values for domestic goods are the prices received by producers, i.e., basic values exclude sales taxes and margin costs. For imports, basic values are the prices received by the importers. Sales taxes and margin costs associated with deliveries from the ports to domestic users are excluded.

In interpreting assumption (2) as applied to domestic goods, a possible picture to have in mind is one in which all industries producing wheat (say) receive a common price per bushel from a central pool. No margin costs are involved in the transfer of wheat from producers to the pool. (If there were transfer costs, they could be treated as a cost of production.) However, margin costs are involved in the transfer of wheat from the pool to the final users. These costs can differ across users and thus purchasers' prices for wheat can differ across users. For foreign goods, the story is the same except that we replace industries with importers.

Under our two assumptions, zero pure profits and uniform basic values, we can write

$$\left.\begin{aligned}\sum_{i=1}^{g} P^{(0)}_{(i1)}\, X^{(0)}_{(i1)j} &= \sum_{i=1}^{g}\sum_{s=1}^{2} X^{(1)}_{(is)j}\, P^{(1)}_{(is)j}\\ &\quad + \sum_{m=1}^{M} P^{(1)}_{(g+1,1,m)j}\, X^{(1)}_{(g+1,1,m)j} + \sum_{s=2}^{3} P^{(1)}_{(g+1,s)j}\, X^{(1)}_{(g+1,s)j}\\ &\quad + P^{(1)}_{g+2,j}\, X^{(1)}_{g+2,j} \quad , \qquad j=1,\ldots,h \quad ,\end{aligned}\right\} \tag{9.1}$$

where the notation has already been introduced in sections 2 and 3. The left side of (9.1) is the basic value of the output of industry j and right side is the total payment for inputs. The equality is implied by the assumption of no pure profits.

In percentage change form, (9.1) reduces to

$$\boxed{\begin{aligned}\sum_{i=1}^{g} p^{(0)}_{(i1)}\, H^{(0)}_{(i1)j} &= \sum_{i=1}^{g}\sum_{s=1}^{2} p^{(1)}_{(is)j}\, H^{(1)}_{(is)j} + \sum_{m=1}^{M} p^{(1)}_{(g+1,1,m)j}\, H^{(1)}_{(g+1,1,m)j}\\ &\quad + \sum_{s=2}^{3} p^{(1)}_{(g+1,s)j}\, H^{(1)}_{(g+1,s)j} + p^{(1)}_{g+2,j}\, H^{(1)}_{g+2,j} + a(j) \quad ,\\ &\qquad j=1,\ldots,h \quad ,\end{aligned}} \tag{9.2}$$

where

$$\boxed{\begin{aligned} a(j) = {} & a_j^{(0)} + \sum_{r=1}^{N(j)} a_{(r^*)j}^{(0)} H_{(r^*)j}^{(0)} + \sum_{i=1}^{g} a_{(i1)j}^{(0)} H_{(i1)j}^{(0)} \\ & + a_j^{(1)} + \sum_{i=1}^{g+2} a_{ij}^{(1)} H_{ij}^{(1)} + \sum_{i=1}^{g} \sum_{s=1}^{2} a_{(is)j}^{(1)} H_{(is)j}^{(1)} \\ & + \sum_{s=1}^{3} a_{(g+1,s)j}^{(1)} H_{(g+1,s)j}^{(1)} + \sum_{m=1}^{M} a_{(g+1,1,m)j}^{(1)} H_{(g+1,1,m)j}^{(1)} , \\ & \qquad j=1,\ldots,h \quad . \end{aligned}} \tag{9.3}$$

The H's are revenue and cost shares. $H_{(i1)j}^{(0)}$ and $H_{(r^*)j}^{(0)}$ are the shares of industry j's revenue accounted for by its sales of commodity (i1) and composite commodity r . $H_{(is)j}^{(1)}$, $H_{ij}^{(1)}$ and $H_{(g+1,1,m)j}^{(1)}$ are the shares of j's costs accounted for by inputs of (is), by inputs of i from all sources and by inputs of labor of skill m . The remaining notation in (9.2) and (9.3) follows from section 3.

In deriving (9.2) and (9.3) from (9.1) we first write

$$\begin{aligned} \sum_{i=1}^{g} \left(p_{(i1)}^{(0)} + x_{(i1)j}^{(0)}\right) H_{(i1)j}^{(0)} = {} & \sum_{i=1}^{g} \sum_{s=1}^{2} \left(x_{(is)j}^{(1)} + p_{(is)j}^{(1)}\right) H_{(is)j}^{(1)} \\ & + \sum_{m=1}^{M} \left(p_{(g+1,1,m)j}^{(1)} + x_{(g+1,1,m)j}^{(1)}\right) H_{(g+1,1,m)j}^{(1)} \\ & + \sum_{s=2}^{3} \left(p_{(g+1,s)j}^{(1)} + x_{(g+1,s)j}^{(1)}\right) H_{(g+1,s)j}^{(1)} \\ & + \left(p_{g+2,j}^{(1)} + x_{g+2,j}^{(1)}\right) H_{g+2,j}^{(1)} , \qquad j=1,\ldots,h \quad . \end{aligned} \tag{9.4}$$

Then we express each of the $\sum x\, H$ terms in (9.4) in terms of z_j and various technical change variables (the a's). For example, using (3.83), we have

$$\sum_{i=1}^{g} x_{(i1)j}^{(0)} H_{(i1)j}^{(0)} = \sum_{t=1}^{N(j)} x_{(t^*)j}^{(0)} \sum_{i\varepsilon G(t,j)} H_{(i1)j}^{(0)} - \sum_{i=1}^{g} a_{(i1)j}^{(0)} H_{(i1)j}^{(0)}$$

$$= \sum_{t=1}^{N(j)} x_{(t^*)j}^{(0)} H_{(t^*)j}^{(0)} - \sum_{i=1}^{g} a_{(i1)j}^{(0)} H_{(i1)j}^{(0)} \quad ,$$

and on applying (3.81) we find that

$$\sum_{i=1}^{g} x_{(i1)j}^{(0)} H_{(i1)j}^{(0)} = z_j - a_j^{(0)} - \sum_{r=1}^{N(j)} H_{(r^*)j}^{(0)} a_{(r^*)j}^{(0)} - \sum_{i=1}^{g} a_{(i1)j}^{(0)} H_{(i1)j}^{(0)} \quad . \qquad (9.5)$$

Expressions similar to (9.5) can be generated for $\sum_i \sum_s x_{(is)j}^{(1)} H_{(is)j}^{(1)}$, etc.. When these, together with (9.5), are substituted into (9.4), we find that z_j can be eliminated and we eventually obtain (9.2) and (9.3). The disappearance of the x's and z_j in the translation from (9.1) to (9.2) and (9.3) can be traced back to the assumption of constant returns to scale in production. Under constant returns to scale, both revenue and costs per unit of activity are independent of the activity level. They are influenced only by changes in prices and technology.

Equations (9.2) and (9.3) make obvious intuitive sense. Consider, first, the situation in which there is no technical change, i.e., $a(j) = 0$. Then for each industry, j, (9.2) implies that a weighted average (the weights being revenue shares) of the percentage changes in the basic prices of outputs equals a weighted average (the weights being cost shares) of the percentage changes in the relevant purchaser's prices of inputs.

Now consider the technical change term, $a(j)$. According to (9.3), $a(j)$ is a weighted sum of the percentage changes in all the technical change coefficients for industry j appearing in the production specification (2.1), (2.3), (2.5), (2.12), (2.13) and (2.16). The weights reflect the effect of each type of technical change on either j's costs or revenue per unit of activity at the initial input and output prices. For example, $a_j^{(0)}$ has a weight of one. If $a_j^{(0)}$ is -10 with all other technical change coefficients held constant, then, at any given level of activity, industry j can produce 10 per cent more of each commodity, see (2.13). At the initial prices this would mean a 10 per cent increase in revenue per unit of activity. However, because $a_j^{(0)}$ has a weight of one in (9.3), $a(j)$ will enter equation (9.2) with a value of -10. This will ensure that input and output prices adjust so as to reduce the revenue to cost ratio per unit of activity by 10 per cent, thus restoring zero pure profits. A similar adjustment will be required if $a_j^{(1)}$ is -10, with all other technical coefficients held constant. In this case, industry j can sustain any given level of activity with 10 per cent less of each input, see (2.1). If there were no adjustment in input and output prices then revenue per unit of activity in industry j

would be left 10 per cent higher than costs per unit of activity. As a final example, consider a situation where $a^{(1)}_{(g+1,1)j} = -10$ with all other technical change coefficients held constant. Under this condition, industry j could maintain any given level of activity with 10 per cent less labor. At the initial input prices, the industry's costs per unit of activity would change by $-10 \; H^{(1)}_{(g+1,1)j}$ per cent. From (9.2) and (9.3) we see that $a(j)$ would enter into equation (9.2) with this value and cause a compensating change in j's input and output prices.

Our second price relationship is

$$\boxed{\begin{aligned} \pi_j = & \sum_{i=1}^{g} \sum_{s=1}^{2} p^{(2)}_{(is)j} \; H^{(2)}_{(is)j} + a^{(2)}_j + \sum_{i=1}^{g} a^{(2)}_{ij} \; H^{(2)}_{ij} \\ & + \sum_{i=1}^{g} \sum_{s=1}^{2} a^{(2)}_{(is)j} \; H^{(2)}_{(is)j} \; , \qquad j=1,\ldots,h \; , \end{aligned}} \tag{9.6}$$

where π_j is percentage change in the price of a unit of capital for industry j and the $H^{(2)}_{(is)j}$, $H^{(2)}_{ij}$ are cost shares. They are, respectively, the share of good i from source s and the share of good i from all sources in the costs of constructing a unit of capital from industry j .

The derivation of (9.6) from the equation

$$\Pi_j \; Y_j = \sum_{i=1}^{g} \sum_{s=1}^{2} P^{(2)}_{(is)j} \; X^{(2)}_{(is)j} \; , \qquad j=1,\ldots,h \; , \tag{9.7}$$

is similar to the derivation of (9.2) - (9.3) from (9.1). ((9.7) imposes the zero profits condition: the value of new capital equals the cost of its production.) The logic of (9.6) is also similar to that underlying (9.2) - (9.3). If there is no technical change, then (9.6) implies that the percentage change in the cost of a unit of capital for industry j is a weighted average of the percentage changes in the prices of the inputs, the weights being cost shares. If technical change takes place, then the cost of a unit of capital can move independently of input prices. Percentage changes in each of the technical coefficients appearing in the production specification (4.1) - (4.2) enter (9.6) and the weights on these terms reflect the importance of each type of technical change (at the one per cent level) in reducing the costs of a unit of capital to industry j .

To avoid any possible confusion, it might be useful to emphasize the difference between Π_j and $P^{(1)}_{(g+1,2)j}$. $P^{(1)}_{(g+1,2)j}$, which was introduced in section 3, is the cost of <u>using or renting</u> a unit of capital for industry j, while Π_j is the cost of <u>buying or producing</u> a unit of capital for industry j . $P^{(1)}_{(g+1,2)j} / \Pi_j$ can be thought of as the gross (i.e., before depreciation) rate of return on units of capital of type j . In section 10, we will use rates of return as a key element in the ORANI investment theory, i.e., the determination of the Y_j , $j=1,\ldots,g$.

Our third set of price equations is

$$P^{(0)}_{(i2)} = P^{m}_{(i2)} \Phi + G(i2,0) , \qquad i=1,\ldots,g , \tag{9.8}$$

where $P^{(0)}_{(i2)}$ is the basic price of imported good i (i.e., the price received by Australian importers, excluding transport and other margin costs involved in transferring imports from Australian ports to final users), $P^{m}_{(i2)}$ is the foreign currency c.i.f. price of imported units of good i, Φ is the exchange rate ($A per unit of foreign currency) and G(i2,0) is the tariff in $A per unit import of good i . A broad interpretation is intended for the G(i2,0). For example, if the import of good i is subject to quota restrictions, then G(i2,0) is the "tariff equivalent" of the quota.

So that users of ORANI can model tariffs in a variety of ways, we add the equation

$$G(i2,0) = \left(\overline{G}(i2,0)\ \Xi^{(3)}\right)^{h_1(i2,0)} \left(T(i2,0)\ P^{m}_{(i2)}\ \Phi\right)^{h_2(i2,0)} \left(V(i2,0)\right)^{h_3(i2,0)} \qquad i=1,\ldots,g \ . \tag{9.9}$$

where the h's and $\overline{G}(i2,0)$ are parameters. ($\overline{G}(i2,0)$ has no role beyond that of a scaling parameter.) $\Xi^{(3)}$ is the ORANI consumer price index. (The percentage change in this variable is $\xi^{(3)}$ and has already been encountered in (7.2).) T(i2,0) and V(i2,0) are variables used to reflect <u>ad valorem</u> and specific rates of protection and $P^{m}_{(i2)}$ and Φ are as defined in (9.8). If we set $h_1(i2,0)$ at one and the other two h's at zero, then our model will simulate a situation in which tariff charges per unit of imports are fixed in real terms, i.e., they move with the consumer price index. If $h_2(i2,0)$ is one, with the other h's being zero, then the tariff on i is <u>ad valorem</u> at the rate T(i2,0). Finally, if $h_1(i2,0)$ and $h_2(i2,0)$ are both zero while $h_3(i2,0)$ is one, then the tariff on i is specific at the rate V(i2,0) .

In percentage change form, (9.8) and (9.9) can be written as

$$\boxed{p^{(0)}_{(i2)} = \left(p^{m}_{(i2)} + \phi\right) \zeta_1(i2,0) + g(i2,0)\ \zeta_2(i2,0) , \quad i=1,\ldots,g ,} \tag{9.10}$$

and

$$\boxed{g(i2,0) = h_1(i2,0)\ \xi^{(3)} + h_2(i2,0) \left(t(i2,0) + p^{m}_{(i2)} + \phi\right) + h_3(i2,0)\ v(i2,0) , \quad i=1,\ldots,g ,} \tag{9.11}$$

where $\zeta_1(i2,0)$ and $\zeta_2(i2,0)$ are, respectively, the shares in the basic price of $(i2)$ accounted for by the foreign currency price in \$A $\left(P^{m}_{(i2)}\ \Phi\right)$ and the tariff $(G(i2,0))$.

The fourth set of price equations relates prices of domestic goods to f.o.b. export prices. We assume that

$$P^{e}_{(i1)}\ \Phi = P^{(0)}_{(i1)} + G(i1,4) + \sum_{r=1}^{g} A^{(i1)4}_{(r1)}\ P^{(0)}_{(r1)} , \qquad i=1,\ldots,g , \tag{9.12}$$

where the only new notation is $G(i1,4)$. $G(i1,4)$ is the export tax per unit export of $(i1)$. In the case of an export subsidy $G(i1,4)$ will be negative.

The left side of (9.12) is the Australian currency price paid by foreigners for units of good (i1) at Australian ports, i.e., it is the foreign currency f.o.b. price, $P^e_{(i1)}$, converted to local currency via the exchange rate, Φ . The right side is the basic price of good (i1) plus the costs of the taxes and margins involved in delivering good (i1) to the foreigners at the Australian ports of exit. (In section 8, $A^{(i1)4}_{(r1)}$ was defined as the quantity of domestically produced good r which is required as a margin per unit of export of good (i1) .) It will be noticed that in calculating the margin costs, we have used basic prices. We assume that there are no margins on margins.[1]

The export taxes, G(i1,4) , i=1,...,g , are handled in the same way as the tariffs, G(i2,0) . We write

$$G(i1,4) = \left[\overline{G}(i1,4)\ \Xi^{(3)}\right]^{h_1(i1,4)} \left[T(i1,4)\ P^e_{(i1)}\ \Phi\right]^{h_2(i1,4)} \left[V(i1,4)\right]^{h_3(i1,4)} \qquad i=1,\ldots,g \quad . \tag{9.13}$$

The notation in (9.13) follows from that in (9.9). As we have already explained, (9.9) allows a choice between treating tariffs as determined in real, ad valorem or specific terms. Equation (9.13) allows the same flexibility with respect to export taxes and subsidies.

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1. In effect, we treat the transport costs, etc., associated with the provision of retail margins, say, as a cost of production of retailing. This prevents us from allowing the transportation costs to vary across uses of retail margins. It does, however, seem to be a harmless simplification and is in any case imposed upon us by our data base.

In percentage change form, (9.12) and (9.13) can be written as

$$\boxed{\begin{aligned}\left(p^{e}_{(i1)} + \phi\right) &= p^{(0)}_{(i1)}\,\zeta_1(i1,4) + g(i1,4)\,\zeta_2(i1,4)\\ &+ \left(\sum_{r=1}^{g} M^{(i1)4}_{(r1)}\,p^{(0)}_{(r1)}\right)\zeta_3(i1,4)\\ &+ \left(\sum_{r=1}^{g} M^{(i1)4}_{(r1)}\,a^{(i1)4}_{(r1)}\right)\zeta_3(i1,4)\ , \qquad i=1,\ldots,g\ ,\end{aligned}} \tag{9.14}$$

and

$$\boxed{\begin{aligned}g(i1,4) &= h_1(i1,4)\,\xi^{(3)} + h_2(i1,4)\left(t(i1,4) + p^{e}_{(i1)} + \phi\right)\\ &+ h_3(i1,4)\;v(i1,4)\ , \qquad i=1,\ldots,g\ ,\end{aligned}} \tag{9.15}$$

where $\zeta_1(i1,4)$, $\zeta_2(i1,4)$ and $\zeta_3(i1,4)$ are, respectively, the shares accounted for by the basic value, the export tax and the margins in the Australian currency price paid by foreigners for units of good (i1) at Australian ports. $M^{(i1)4}_{(r1)}$ is the share in the total cost of margin services involved in transferring good (i1) from domestic producers to the ports of exit represented by the use of good (r1) .

Before leaving (9.14) and (9.15), we should consider the case of non-export products. Some readers may object to (9.14) and (9.15) on the grounds that they do not make sense for construction and services, etc.. However, where i is a non-export product, we can set $h_1(i1,4)$ and $h_2(i1,4)$ at zero, $h_3(i1,4)$ at one and allow $v(i1,4)$ and $g(i1,4)$ to be endogenous. Then (9.14) and (9.15) can be written as

$$g(i1,4) = v(i1,4) = \left((p^e_{(i1)} + \phi) - p^{(0)}_{(i1)}\, \zeta_1(i1,4) - \ldots\right) \Big/ \zeta_2(i1,4) \ ,$$

and may be thought of as simply defining $g(i1,4)$ and $v(i1,4)$. Since these variables appear in no model equations apart from (9.14) and (9.15), no harm is done by allowing them to take on whatever values are produced via (9.14) and (9.15). The advantage of our procedure over one which handles export and non-export products asymmetrically is that it simplifies both our computing and algebraic manipulations.

The fifth and final set of price equations relates the various purchasers' prices paid by domestic users of good i from source s to its basic value. The equations are

$$p^{(k)}_{(is)j} = p^{(0)}_{(is)} + G(is,jk) + \sum_{r=1}^{g} A^{(is)jk}_{(r1)}\, p^{(0)}_{(r1)} \ ,$$

$$i=1,\ldots,g \ , \quad j=1,\ldots,h \ , \quad s,k=1,2 \ , \qquad (9.16)$$

and

$$p^{(k)}_{(is)} = p^{(0)}_{(is)} + G(is,k) + \sum_{r=1}^{g} A^{(is)k}_{(r1)}\, p^{(0)}_{(r1)}$$

$$i=1,\ldots,g \ , \quad s=1,2 \ , \quad k=3,5 \ , \qquad (9.17)$$

where the G's are tax terms, e.g., G(is,jk) is the tax (or subsidy) associated with the sale of good i from source s for purpose k to industry j . The remaining notation in (9.16) - (9.17) has already been defined (see especially section 8).

Equation (9.16) equates the price paid in industry j for good (is) to be used for purpose k, to the sum of the basic value of good (is) and the costs of the relevant taxes and margins. (Notice, again, that we do not allow margins on margins.) (9.17) describes the purchasers' prices of good (is) when used by households and in other demands. One feature of (9.16) and (9.17) which is worth emphasizing is the interpretation of the G's . The G's are taxes on sales, not production. The effects of production taxes can be simulated via other cost tickets (see section 2). Unlike production taxes, we can allow the G's to vary across users. For example, agricultural enterprises might be subsidized for their purchases of chemical fertilizers whereas industrial buyers might pay a tax.[1] This situation is modelled by having negative G's associated with the sales from chemical fertilizers to agriculture and positive G's for the sales to non-agricultural users.

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1. This particular example of variation in sales taxes arises from heterogeneity across products within the chemical fertilizer industry. The products purchased from the chemical fertilizer industry by agricultural users are subsidized, whereas the other products are taxed.

In percentage change form (9.16) and (9.17) are

$$p_{(is)j}^{(k)} = p_{(is)}^{(0)}\,\zeta_1(is,jk) + g(is,jk)\,\zeta_2(is,jk) + \left(\sum_{r=1}^{g} M_{(r1)}^{(is)jk}\,p_{(r1)}^{(0)}\right)\zeta_3(is,jk) + \left(\sum_{r=1}^{g} M_{(r1)}^{(is)jk}\,a_{(r1)}^{(is)jk}\right)\zeta_3(is,jk)\ ,$$

$$i=1,\ldots,g\ ,\quad j=1,\ldots,h\ ,\quad s,k=1,2\ ,$$

(9.18)

and

$$p_{(is)}^{(k)} = p_{(is)}^{(0)}\,\zeta_1(is,k) + g(is,k)\,\zeta_2(is,k) + \left(\sum_{r=1}^{g} M_{(r1)}^{(is)k}\,p_{(r1)}^{(0)}\right)\zeta_3(is,k) + \left(\sum_{r=1}^{g} M_{(r1)}^{(is)k}\,a_{(r1)}^{(is)k}\right)\zeta_3(is,k)\ ,$$

$$i=1,\ldots,g\ ,\quad s=1,2\ ,\quad k=3,5\ ,$$

(9.19)

where the definitions of the share coefficients (the ζ's and M's) follow the pattern established in (9.14). For example, $\zeta_1(is,3)$ is the share in the purchasers price to households of good (is) accounted for by the basic price. $M_{(r1)}^{(i1)5}$ is the share in the total cost of margin services involved in transferring good (i1) from producers to other users accounted for by the use of good (r1), etc..

Finally, we add equations to allow flexible handling of the tax terms appearing in (9.18) and (9.19). Following the approach and notation of (9.15) we have[1]

$$\boxed{\begin{aligned} g(is,jk) \;=\; & h_1(is,jk)\,\xi^{(3)} + h_2(is,jk)\left[t(is,jk) + p^{(0)}_{(is)}\right] \\ & + h_3(is,jk)\;\; v(is,jk)\;, \\ & i=1,\ldots,g\;,\quad j=1,\ldots,h\;,\quad s,k=1,2\;, \end{aligned}} \qquad (9.20)$$

and

$$\boxed{\begin{aligned} g(is,k) \;=\; & h_1(is,k)\,\xi^{(3)} + h_2(is,k)\left[t(is,k) + p^{(0)}_{(is)}\right] \\ & + h_3(is,k)\;\; v(is,k)\;, \\ & i=1,\ldots,g\;,\quad s=1,2\;,\quad k=3,5\;. \end{aligned}} \qquad (9.21)$$

We conclude the section with a summary of the material covered. Our first system of price equations, (9.2), relates changes in the basic prices of outputs to changes in the purchaser's prices of inputs and to changes in technology for current production. The second system, (9.6), relates changes in the costs of capital units

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1. One minor difference between the treatment of taxes in (9.20) and (9.21) compared with (9.15) is in the ad valorem terms. In (9.15), the t is an ad valorem rate calculated on the purchasers' price whereas in the other two equations, the t is an ad valorem rate on the basic price.

to changes in the purchasers' prices of inputs to capital creation and to changes in the technologies for capital creation. The third system, (9.10) and (9.11), defines the changes in the basic prices of imports in terms of changes in their c.i.f. foreign currency prices, the exchange rate and the tariff rates. Tariff rates can be handled as though they are set in real, ad valorem or specific terms. (This is also true of the taxes and subsidies appearing in the fourth and fifth sets of price equations.) The fourth set of price equations, (9.14) and (9.15), relates basic prices of domestic commodities to export prices. We assume that changes in the prices payable at Australian ports by foreigners for Australian products reflect changes in the basic price, the export tax or subsidy and the costs of the margin services involved in transferring commodities from producers to the ports of exit. Changes in the costs of these margin services depend on changes in the basic prices per unit of the various margin services and on changes in the quantities of margin services required per unit of exports. The fifth system of price relationships, (9.18) - (9.21), relates changes in the prices paid by domestic users for all goods to changes in their basic prices, to changes in the relevant taxes and to changes in the costs of margin services. Again, changes in the costs of these services reflect changes in the basic prices of margin services and changes in the technology for transferring commodities from producers to users.

The principal simplifying assumptions employed in our specification of the price systems are as follows. (1) There are no pure profits in production, capital creation importing, exporting or distribution. (2) Basic values are uniform across users and producing industries. Differences across users in purchasers' prices are accounted for entirely by taxes and payments for margins. (3) There are no margins on margins.

10. The Allocation of Investment across Industries

Section 4 described the technology for creating units of capital. It left open, however, the question of how many units will be created for each industry. We consider that problem in this section, i.e., we describe the ORANI theory for the y_j, $j=1,\ldots,h$.

A possible approach to the problem of investment is to set the y_j's exogenously.[1] While we have left that option available to model users, we feel that there are valid reasons for attempting to be more ambitious. ORANI is largely concerned with simulating the effects on industrial activity and the balance of trade of tariff changes and other disturbances in commodity and factor prices. One of the ways in which tariff changes can affect industrial activity and the balance of trade is through generating a reallocation of investment across industries. For example, if investment is shifted towards industries whose capital structure is relatively import intensive, then this will have an adverse effect on the balance of trade and on the activity of domestic suppliers of capital goods.

One important limitation of our investment theory should be understood from the outset. We do not attempt to explain aggregate private investment in fixed plant, machinery and buildings; only how this investment is allocated across using industries. Our view is that aggregate investment is best explained in a macroeconomic model which captures the effects of monetary phenomena and government

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1. See for example Taylor and Black (1974).

macroeconomic policy. Consequently, without an appended macro model, ORANI results must be interpreted as showing the effects of a tariff change, etc., under the assumption that macroeconomic policy leads to a given (exogenously specified) change, perhaps zero change, in the aggregate level of investment. ORANI does not reveal what macroeconomic policy leads to the assumed exogenous change in aggregate investment. The implied assumption is that such a policy does exist.

The first step in our theory of the allocation of investment across industries is to note that the current rate of return on fixed capital in industry j is

$$R_j(0) = \frac{P^{(1)}_{(g+1,2)j}}{\Pi_j} - d_j \tag{10.1}$$

where d_j is the rate of depreciation (assumed fixed) and $P^{(1)}_{(g+1,2)j}$ and Π_j are, as previously defined, the rental value and the cost of a unit of capital in industry j.

The second step is to assume that capital in industry j takes one period to install. For the present purposes it is not important whether a period is two years or three years or some other length of time. A period may be different lengths of time for different industries. It would only become important to associate a period with an exact calendar time if our theory of investment were to be a vehicle for dynamizing our model. However, this is not what we are aiming to do. In particular, we do not regard the theory to be developed here as a means of explaining the time paths of capital stocks. We are concerned only with the effects of shocks (e.g., tariff changes) on the allocation of current investment expenditure across industries.[1]

The third step is to assume that investors are cautious in assessing the effects of expanding the capital stock in industry j. They behave as if they expect that industry j's rate of return schedule in one period's time will have the form

$$R_j(1) = R_j(0) \left(\frac{K_j(1)}{K_j(0)}\right)^{-\beta_j} ,$$

where β_j is a positive parameter, $K_j(0)$ is the current level of capital stock in industry j and $K_j(1)$ is the level at the end of one period. The situation is illustrated in figure 1

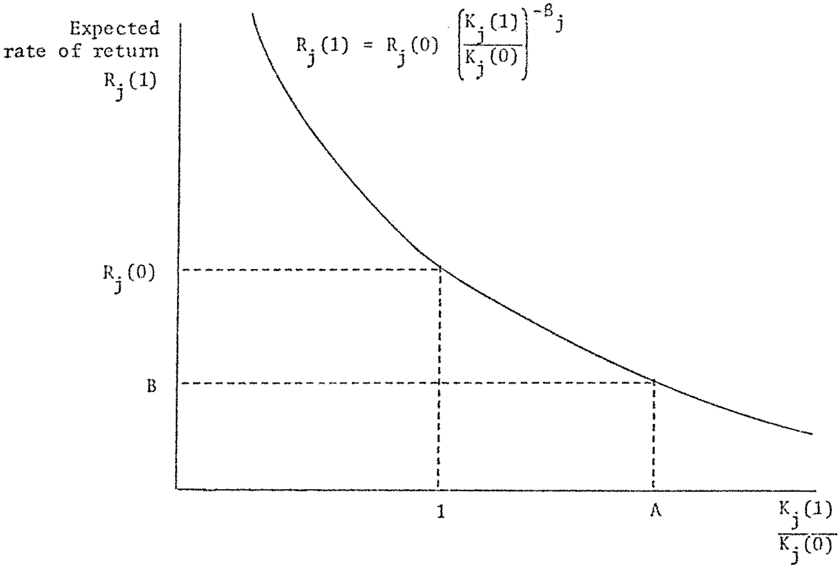

Figure 1 : Expected rate of return schedule for industry j

1. Our approach to long-run simulations is explained in section 14.

where the horizontal axis measures the ratio of future capital stock (i.e., capital stock existing in one period's time) to current capital stock, and the vertical axis shows the expected rate of return. If the capital stock were maintained at the existing level, then the expected rate of return would be the current rate of return, $R_j(0)$. However, if investment plans were set so that $K_j(1)/K_j(0)$ would reach the level A, then businessmen would behave as if they expected the rate of return to fall to B .

The fourth step is to assume that total private[1] investment expenditure (I) is allocated across industries so as to equate the expected rates of return. This means that there exists some rate of return Ω such that[2]

$$\left(\frac{K_j(1)}{K_j(0)}\right)^{-\beta_j} R_j(0) = \Omega \quad , \qquad j \in J \quad , \tag{10.2}$$

where J is the subset of $\{1,2,\ldots,h\}$ which contains the identifying numbers of those industries whose investment is treated as endogenous within ORANI. The user of ORANI has a choice over the elements of J . In most applications, J would exclude industries which are dominated by government activity.

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1. Model users can treat investment in some (or all) industries as exogenous to ORANI. The word "private" simply refers to those industries for which investment is explained within ORANI.

2. We assume that (10.2) will not imply disinvestment in any industry j beyond that which would occur via depreciation at a zero level of gross investment.

The fifth step is to set out defining equations for $K_j(1)$ and I. We assume that

$$K_j(1) = K_j(0)\ (1 - d_j) + Y_j \quad , \qquad j=1,\ldots,h \tag{10.3}$$

and

$$I = \sum_{j \varepsilon J} \Pi_j\ Y_j \quad . \tag{10.4}$$

Equation (10.3) introduces the simplification that the only variables which influence the capital stock at the end of one period are the current capital stock and the current level of investment. It is assumed that the effects of past investment decisions are fully incorporated in the current capital stock. Equation (10.4) simply defines the private investment budget, I, as the sum of investment expenditure across those industries, j, for which $j \varepsilon J$.

The sixth step is to provide for investment in those industries $(j \notin J)$ for which the rate of return theory is considered inappropriate. We use the equations

$$Y_j = \left(I_R\right)^{h_j^{(2)}} \quad F_j^{(2)} \quad , \qquad j \notin J \quad , \tag{10.5}$$

where

$$I_R = I \Big/ \Xi^{(2)} \quad . \tag{10.6}$$

$\Xi^{(2)}$ is the ORANI capital goods price index, I_R is the real level of private investment and $F_j^{(2)}$ (defined for $j \notin J$) is a shift variable. If we set the parameter $h_j^{(2)}$ in (10.5) equal to one, then the percentage changes in $F_j^{(2)}$ reflect the growth in investment in industry

j relative to that of the entire private sector. Usually we allow no changes in the $F_j^{(2)}$'s, in which case Y_j moves with I_R for all $j \notin J$.

These six steps are sufficient to achieve what we set out to do in this section. Equations (10.1) - (10.6) tie down the allocation of investment across industries. On the one hand investment in industry j $(j \notin J)$ can either be set exogenously or determined mechanically according to a rule which forces Y_j to be a simple function of I_R. On the other hand, ORANI users can allow investment in industry j to be affected by what the model implies about relative rates of return. Consider for example, the effect on investment by industry of an increase in tariff protection for the commodities produced by industry j, $j \varepsilon J$. Such a change will tend to increase the demand for capital of type j relative to the demand for capital in other industries. This will tend to increase the rental rate $P_{(g+1,2)j}^{(1)}$ relative to other rental rates, thus increasing $R_j(0)$ relative to rates of return in other industries. In terms of figure 1, we would expect a vertical upward shift in j's rate of return schedule. For other industries the schedule might move either up or down depending on whether the industry is a supplier or customer of industry j. In any case, (10.2) - (10.4) will ensure that industries for which the upward movements in their rate of return schedules are most pronounced will receive increased shares of the investment budget whereas industries suffering pronounced downward shifts will receive smaller shares.

In percentage change form, equations (10.1) - (10.6) may be expressed as follows :

$$r_j(0) = Q_j\left(p^{(1)}_{(g+1,2)j} - \pi_j\right) \quad , \quad j=1,\ldots,h \quad , \qquad (10.7)$$

$$-\beta_j\left[k_j(1) - k_j(0)\right] + r_j(0) = \omega \quad , \quad j\varepsilon J \quad , \qquad (10.8)$$

$$k_j(1) = k_j(0)\,(1 - G_j) + y_j\,G_j \quad , \quad j=1,\ldots,h \quad , \qquad (10.9)$$

$$\sum_{j\varepsilon J}\left(\pi_j + y_j\right) T_j = \left(\sum_{j\varepsilon J} T_j\right) i \quad , \qquad (10.10)$$

$$y_j = h_j^{(2)}\, i_R + f_j^{(2)} \quad , \quad j\not\varepsilon J \quad , \qquad (10.11)$$

and

$$i_R = i - \xi^{(2)} \quad , \qquad (10.12)$$

where $Q_j = \left(R_j(0) + d_j\right)\Big/R_j(0)$, i.e., Q_j is the ratio of the gross rate of return in industry j to the net rate of return. $G_j = Y_j\Big/K_j(1)$, i.e., G_j is the ratio of gross investment in industry j to its future capital stock, and T_j is the share of total aggregate fixed investment accounted for by industry j, i.e.,

$$T_j = \frac{Y_j\,\Pi_j}{\sum\limits_{j=1}^{h} Y_j\,\Pi_j} \quad , \qquad j=1,\ldots,h \quad .$$

Notice that the T_j sum to one over all j, not just over $j \varepsilon J$. It is convenient to define the T_j so that they are independent of the user-determined choice of the elements in J.

11. The Market Clearing Equations

This section lists the equations which ensure that demand equals supply for domestically produced commodities and for the primary factors of production, labor, capital and agricultural land. The equations are

$$\left.\begin{aligned} X^{(0)}_{(r1)} = & \sum_{j=1}^{h} X^{(1)}_{(r1)j} + \sum_{j=1}^{h} X^{(2)}_{(r1)j} + X^{(3)}_{(r1)} + X^{(4)}_{(r1)} + X^{(5)}_{(r1)} \\ & + \sum_{i=1}^{g}\sum_{s=1}^{2}\sum_{j=1}^{h}\sum_{k=1}^{2} X^{(is)jk}_{(r1)} + \sum_{i=1}^{g}\sum_{s=1}^{2}\sum_{k=3,5} X^{(is)k}_{(r1)} \\ & + \sum_{i=1}^{g} X^{(i1)4}_{(r1)} \quad , \qquad r=1,\ldots,g \quad , \end{aligned}\right\} \qquad (11.1)$$

where

$$X^{(0)}_{(r1)} = \sum_{j=1}^{h} X^{(0)}_{(r1)j} \quad , \qquad r=1,\ldots,g \quad , \qquad (11.2)$$

$$L_m = \sum_{j=1}^{h} X^{(1)}_{(g+1,1,m)j} \quad , \qquad m=1,\ldots,M \quad , \qquad (11.3)$$

$$K_j(0) = X^{(1)}_{(g+1,2)j} \quad , \qquad j=1,\ldots,h \quad , \qquad (11.4)$$

and

$$N_j = X^{(1)}_{(g+1,3)j} \quad , \qquad j=1,\ldots,h \quad . \qquad (11.5)$$

Equation (11.1) equates supply $\left(X^{(0)}_{(r1)}\right)$ and demand for each of the domestically produced goods, (r1), r=1,...,g . Total supply is the sum over the outputs of (r1) by each of the industries (see (11.2)). Total demand is made up of

(i) demand for intermediate inputs to the production of current goods $\left(X^{(1)}_{(r1)j}\right)$;

(ii) demand for inputs to the production of capital equipment $\left(X^{(2)}_{(r1)j}\right)$;

(iii) demand for consumption goods $\left(X^{(3)}_{(r1)}\right)$;

(iv) export demand $\left(X^{(4)}_{(r1)}\right)$;

(v) other demands, e.g., government purchases $\left(X^{(5)}_{(r1)}\right)$;

(vi) demand for margins on the delivery of goods to industrial users for current production and capital creation $\left(X^{(is)jk}_{(r1)}\right)$;

(vii) demand for margins on the delivery of goods to households and to other users $\left(X^{(is)k}_{(r1)}\right)$;

(viii) demand for margins on the delivery of exports from Australian producers to the ports of exit $\left(X^{(i1)4}_{(r1)}\right)$.

The only noteworthy feature of (11.1) is the absence of imports. This is explained by the fact that ORANI treats imports of good i as a distinct product from domestically produced i .

Equation (11.3) equates the supply of labor of skill m, L_m , to the demand for it. It implies that labor is homogenous within each skill group and is shiftable between industries. It does not imply, however, that ORANI is necessarily a "full-employment" model. Full employment could be imposed by setting the L_m exogenously at their full employment levels. Alternatively, wages might be set exogenously and the L_m would become endogenous. ORANI would then generate the employment levels, L_m , corresponding to the given wage rates.

Equation (11.4) equates supply and demand for capital in each industry. Unlike labor, capital is assumed to be industry specific or non-shiftable between industries. The non-shiftability assumption rules out the possibility of dismantling capital units and reallocating the components to other industries. For example, we could imagine buildings to be shiftable across industries. Except in the simulation of policies which could produce very rapid declines in the outputs of some industries, the non-shiftability assumption is unlikely to significantly distort model results; large scale shifting of capital from industry j to other industries will occur only if industry j is declining at a more rapid rate than is allowed by depreciation. We also note that the non-shiftability assumption has been very popular in other models emphasizing international trade.[1] General equilibrium trade models have often tended to imply unrealistic levels of industrial specialization, and the industry specific capital assumption plays a useful role in imposing some diversification.[2]

The last of the balance equations (11.5) ensures equality between the demand for, and supply of, agricultural land for each industry. As was the case for capital, we assume that agricultural land is non-shiftable between industries. This approach makes sense in terms of our industrial definitions for agriculture. ORANI divides the agricultural sector into industries mainly on a regional basis. Industry 1, for example, consists of farms in Australia's pastoral zone. Thus, the land of the pastoral zone is specific to the pastoral zone industry. Pastoral zone land is not, of course, specific to the production of a particular product. The pastoral zone industry is modelled as producing a variety of products. Details of the empirical specification of the agricultural sector are in Chapter 4, sections

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1. See for example Evans (1972) and Taylor and Black (1974). Johansen (1960) allowed shiftable capital; but for Johansen, trade was largely exogenous.

2. This point is further discussed in Dixon and Butlin (1977).

In percentage change form, equations (11.1) - (11.5) are

$$x^{(0)}_{(r1)} = \sum_{j=1}^{h} x^{(1)}_{(r1)j}\, B^{(1)}_{(r1)j} + \sum_{j=1}^{h} x^{(2)}_{(r1)j}\, B^{(2)}_{(r1)j} + x^{(3)}_{(r1)}\, B^{(3)}_{(r1)}$$

$$+ x^{(4)}_{(r1)}\, B^{(4)}_{(r1)} + x^{(5)}_{(r1)}\, B^{(5)}_{(r1)} + \sum_{i=1}^{g}\sum_{s=1}^{2}\sum_{j=1}^{h}\sum_{k=1}^{2} x^{(is)jk}_{(r1)}\, B^{(is)jk}_{(r1)}$$

$$+ \sum_{i=1}^{g}\sum_{s=1}^{2}\sum_{k=3,5} x^{(is)k}_{(r1)}\, B^{(is)k}_{(r1)} + \sum_{i=1}^{g} x^{(i1)4}_{(r1)}\, B^{(i1)4}_{(r1)}\ , \quad r=1,\ldots,g\ , \tag{11.6}$$

$$x^{(0)}_{(r1)} = \sum_{j=1}^{h} x^{(0)}_{(r1)j}\, B^{(0)}_{(r1)j}\ , \qquad r=1,\ldots,g\ , \tag{11.7}$$

$$\ell_m = \sum_{j=1}^{h} x^{(1)}_{(g+1,1,m)j}\, B^{(1)}_{(g+1,1,m)j}\ , \qquad m=1,\ldots,M\ , \tag{11.8}$$

$$k_j(0) = x^{(1)}_{(g+1,2)j}\ , \qquad j=1,\ldots,h\ , \tag{11.9}$$

and

$$n_j = x^{(1)}_{(g+1,3)j}\ , \qquad j=1,\ldots,h\ . \tag{11.10}$$

where the B's appearing in (11.6) are the shares of the sales of domestically produced goods which are absorbed by the various types of demands identified on the RHS of (11.1). For example,

$B^{(1)}_{(r1)j}$ is the share of the total sales of domestically produced good r accounted for by direct sales to industry j for current production. $B^{(is)jk}_{(r1)}$ is the share of the total sales of domestically produced good r which is used as margin in facilitating the flow of good i from source s to industry j for purpose k. The B's in equation (11.7) are production shares. $B^{(0)}_{(r1)j}$ is the share of industry j in the economy's output of (r1). Finally, the B's in (11.8) are employment shares. $B^{(1)}_{(g+1,1,m)j}$ is the share of the total employment of labor of type m which is accounted for by industry j.

12. Aggregate Imports, Exports and the Balance of Trade

Aggregate demand for imported good r, $r=1,\ldots,g$, is denoted by $X^{(0)}_{(r2)}$ and computed as

$$X^{(0)}_{(r2)} = \sum_{k=1}^{2} \sum_{j=1}^{h} X^{(k)}_{(r2)j} + X^{(3)}_{(r2)} + X^{(5)}_{(r2)} , \quad r=1,\ldots,g . \tag{12.1}$$

In percentage change form, this equation is

$$\boxed{x^{(0)}_{(r2)} = \sum_{k=1}^{2} \sum_{j=1}^{h} x^{(k)}_{(r2)j} \, B^{(k)}_{(r2)j} + \sum_{k=3,5} x^{(k)}_{(r2)} \, B^{(k)}_{(r2)} , \quad r=1,\ldots,g ,} \tag{12.2}$$

where the B's are shares of total import flows. For example, $B^{(k)}_{(r2)j}$ is the share of the economy's imported good r which is absorbed by industry j for purpose k.

In terms of foreign currency cost[1] the aggregate value of imports, M, is given by

$$M = \sum_{r=1}^{g} P^{m}_{(r2)} X^{(0)}_{(r2)} , \tag{12.3}$$

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1. Foreign currency cost can be computed in terms of any currency, including Australian dollars. When Australian dollars are used, it must be understood that $P^{m}_{(r2)}$ is the Australian dollar cost, at a fixed base period exchange rate between Australian dollars and SDR, say, of the foreign currency required to import a unit of good r. ORANI assumes that $P^{m}_{(r2)}$ is independent of Australian exchange rate movements.

and in percentage change form, this is

$$\boxed{m = \sum_{r=1}^{g} \left(p^{m}_{(r2)} + x^{(0)}_{(r2)}\right) M_{(r2)} ,} \tag{12.4}$$

where $M_{(r2)}$ is the share of the aggregate foreign currency cost of commodity imports which is accounted for by imports of good r.

Next, we define the aggregate foreign currency receipts, E, from commodity exports by

$$E = \sum_{r=1}^{g} P^{e}_{(r1)} X^{(4)}_{(r1)} . \tag{12.5}$$

In percentage form we have

$$\boxed{e = \sum_{r=1}^{g} \left(p^{e}_{(r1)} + x^{(4)}_{(r1)}\right) E_{(r1)} ,} \tag{12.6}$$

where $E_{(r1)}$ is good r's share in aggregate export receipts.

Finally, we define the balance of trade on commodity account as

$$B = E - M . \tag{12.7}$$

This gives

$$\boxed{100 \Delta B = Ee - Mm ,} \tag{12.8}$$

where ΔB is the change (not percentage change) in B. Because B can change sign, we avoid the percentage change form of (12.7). Thus ΔB becomes the only variable in the model which requires units. In the computations reported in later chapters, ΔB has the units "millions of 1968-9 Australian dollars at the 1968-9 Australian exchange rate". Hence, if a particular ORANI experiment produced a value of 1 for ΔB, we would interpret this to mean that the balance of trade improves by the 1968-9 foreign currency equivalent of 1 million 1968-9 Australian dollars. For example, if the 1968-9 exchange rate were \$US1.1 per \$A, then $\Delta B = 1$ can be interpreted as an improvement in the balance of trade of 1.1 million 1968-9 U.S. dollars. This in turn could be translated into U.S. dollars of some other year by applying a suitable price index.

13. Macro Indices and Wage Indexation

In sections 7 and 10, we introduced the variables $\xi^{(3)}$ and $\xi^{(2)}$, the percentage changes in the ORANI consumer and capital goods price indices. $\xi^{(3)}$ is defined by

$$\boxed{\xi^{(3)} = \sum_{s=1}^{2} \sum_{i=1}^{g} w_{(is)}^{(3)} p_{(is)}^{(3)} ,} \tag{13.1}$$

where $w_{(is)}^{(3)}$ is the share of aggregate consumer spending devoted to good i from source s, i.e., $\xi^{(3)}$ is a weighted average of the percentage changes in the purchasers' prices of consumer goods. The capital goods price index, $\xi^{(2)}$, is defined by

$$\boxed{\xi^{(2)} = \sum_{j\varepsilon J} \tilde{T}_j \pi_j ,} \tag{13.2}$$

where $\tilde{T}_j = T_j \Big/ \sum_{j\varepsilon J} T_j$, i.e., $\tilde{T}_j$ is the share of total private investment expenditure accounted for by industry j.

Other useful macroeconomic indices included in ORANI are aggregate employment in labor hours (L), aggregate capital stock in base period value units $(K(0))$, and the ratio of real private investment expenditure to real private consumption (F_R) . Percentage changes in these are given by

$$\boxed{\ell = \sum_{m=1}^{M} \ell_m \psi_{1m} \quad ,} \tag{13.3}$$

$$\boxed{k(0) = \sum_{j=1}^{h} k_j(0) \psi_{2j} \; ,} \tag{13.4}$$

and

$$\boxed{f_R = i_R - c_R \quad ,} \tag{13.5}$$

where ψ_{1m} is the share of skill m in total hours of employment and ψ_{2j} is the share of capital of type j (valued at base period prices) in the total value of fixed capital for the economy. For macro indices not explicitly included in the model, projections can often be made, as required, by additional computations following the ORANI solution.

Our final pair of equations allows wages $\left(p^{(1)}_{(g+1,1,m)j}, \; m=1,\ldots,M, \; j=1,\ldots,h\right)$ and the prices of other cost tickets $\left(p^{(1)}_{g+2,j} \; , \; j=1,\ldots,h\right)$ to be indexed to the ORANI consumer price index. In percentage change form we write

$$\boxed{\begin{aligned} p^{(1)}_{(g+1,1,m)j} &= h^{(1)}_{(g+1,1,m)j}\,\xi^{(3)} + f^{(1)}_{(g+1,1)} + f^{(1)}_{(g+1,1)j} \\ &\quad + f^{(1)}_{(g+1,1,m)} + f^{(1)}_{(g+1,1,m)j}\quad , \\ &\qquad m=1,\ldots,M,\quad j=1,\ldots,h\quad , \end{aligned}} \tag{13.6}$$

and

$$\boxed{\begin{aligned} p^{(1)}_{g+2,j} &= h^{(1)}_{g+2,j}\,\xi^{(3)} + f^{(1)}_{g+2,j}\quad , \\ &\qquad j=1,\ldots,h\quad , \end{aligned}} \tag{13.7}$$

where the f's are variables and the h's are parameters. If $h^{(1)}_{(g+1,1,m)j}$ is set at one for all m and j , and all the f's in (13.6) are set at zero, then we have full wage indexation, i.e., real wages are fixed in all occupations and industries. Of course, alternative treatments are possible. In a "full employment" calculation we could set $f^{(1)}_{(g+1,1)}$, $f^{(1)}_{(g+1,1)j}$ for all j and $f^{(1)}_{(g+1,1,m)j}$ for all m and j, at zero. The $f^{(1)}_{(g+1,1,m)}$, $m=1,\ldots,M$ could be left as endogenous variables. If we continued to set $h^{(1)}_{(g+1,1,m)j}$

at one for all m and j, then the $f^{(1)}_{(g+1,1,m)}$'s would indicate the percentage changes in real occupational wage rates which would be required to achieve exogenously given employment levels in each occupation under a tariff change or some other specified shock. Another possibility would be to impose full employment in aggregate, not by occupation. Then $f^{(1)}_{(g+1,1)}$ would be left as endogenous. If the other f's in (13.6) were set exogenously at zero and the h's remained at one, then solutions for $f^{(1)}_{(g+1,1)}$ would reveal the overall change in real wages which would be required for the achievement of an exogenously given aggregate employment level (L). In general, $f^{(1)}_{(g+1,1)}$ plays a role in simulations where we wish to vary the overall level of real wages while holding occupational and industrial wage relativities fixed. The $f^{(1)}_{(g+1,1)j}$'s can be used in simulations involving variations in industrial wage relativities while the $f^{(1)}_{(g+1,1,m)}$'s reflect changes in occupational relativities. The $f^{(1)}_{(g+1,1,m)j}$'s can be used in simulations involving changes in the cost of labor of type m in industry j both in relation to other wages paid in industry j and in relation to wages paid for labor of type m in other industries.

Equation (13.7) indexes the prices of other cost tickets. Normally, we set the $h^{(1)}_{g+2,j}$'s at 1 and treat the $f^{(1)}_{g+2,j}$'s as exogenous variables with value zero. However, alternative treatments are possible if we wish to introduce changes in the real costs of holding working capital or changes in taxes on production.

14. The Complete Model

Tables 1 and 2 list the ORANI equations and variables in percentage change form.[1] The equations are linear and could be arranged as in (1.6), i.e., they could be written as

$$A\upsilon = 0 \quad ,$$

where υ is the vector of variables and A is a rectangular matrix (the number of columns exceeds the number of rows). The variables, υ , are percentage changes in outputs, prices, demands, employment levels, tariffs, subsidies, capital stocks, the exchange rate, several macro aggregates and numerous "shift" and technological change terms. The only variable which is not a percentage change is ΔB , the change in the balance of trade (see section 12).

Energetic readers can check that the number of variables exceeds the number of equations by D , where

$$D = 4g^2h + 5g^2 + 15gh + 2Mh + 12h + 19g + M + \sum_{j=1}^{h} N(j) + 5 + (h-J^*) \quad .$$

Each ORANI solution requires D variables to be set exogenously. In our standard data base (see Chapter 4) h , the number of industries is 113, g the number of commodities is 115, M the number of labor groups is 9, and $(h-J^*)$, the number of industries whose investment is handled exogenously, is 12. Hence, D is a

1. Table 4 in Chapter 4, section , contains a complete list of the ORANI parameters. The reader can use this table as a definition list without reading Chapter 4. However, the information in the "source" column of Table 4 does rely on an understanding of parts of Chapter 4.

Table 1 : The ORANI Equations, A Linear System in Percentage Changes

Identifier	Equation	Subscript Range	Number	Description
(3.23)	$x^{(1)}_{(is)j} = z_j - \sigma^{(1)}_{ij}\left[p^{(1)}_{(is)j} - \sum_s S^{(1)}_{(is)j}\, p^{(1)}_{(is)j}\right] + a^{(1)}_j + a^{(1)}_{ij} + a^{(1)}_{(is)j} - \sigma^{(1)}_{ij}\left[a^{(1)}_{(is)j} - \sum_s S^{(1)}_{(is)j}\, a^{(1)}_{(is)j}\right]$	i=1,...,g, j=1,...,h, s=1,2.	2gh	Demands for intermediate inputs, domestic and imported.
(3.25)	$x^{(1)}_{g+2,j} = z_j + a^{(1)}_j + a^{(1)}_{g+2,j}$	j=1,...,h.	h	Demands for other cost tickets.
(3.56)	$x^{(1)}_{(g+1,1,q)j} = x^{(1)}_{(g+1,1)j} - \sigma^{(1)}_{(g+1,1,q)j}\left[p^{(1)}_{(g+1,1,q)j} - \sum_q S^{*(1)}_{(g+1,1,q)j}\, p^{(1)}_{(g+1,1,q)j}\right] + a^{(1)}_{(g+1,1,q)j} - \sigma^{(1)}_{(g+1,1,q)j}\left[a^{(1)}_{(g+1,1,q)j} - \sum_q S^{*(1)}_{(g+1,1,q)j}\, a^{(1)}_{(g+1,1,q)j}\right]$	q=1,...,M, j=1,...,h.	Mh	Demands for labor by industry and skill group.
(3.64)	$x^{(1)}_{(g+1,v)j} = z_j - \sigma^{(1)}_{(g+1,v)j}\left[p^{(1)}_{(g+1,v)j} - \sum_{v=1}^{3} S^{*(1)}_{(g+1,v)j}\, p^{(1)}_{(g+1,v)j}\right] + a^{(1)}_j + a^{(1)}_{g+1,j} + a^{(1)}_{(g+1,v)j} - \sigma^{(1)}_{(g+1,v)j}\left[a^{(1)}_{(g+1,v)j} - \sum_v S^{*(1)}_{(g+1,v)j}\, a^{(1)}_{(g+1,v)j}\right]$	v=1,2,3, j=1,...,h.	3h	Industry demands for primary factors.
(3.66)	$p^{(1)}_{(g+1,1)j} = \sum_{q=1}^{M} p^{(1)}_{(g+1,1,q)j}\, S^{(1)}_{(g+1,1,q)j} + \sum_{q=1}^{M} a^{(1)}_{(g+1,1,q)j}\, S^{(1)}_{(g+1,1,q)j}$	j=1,...,h.	h	Price to each industry of labor in general.
(3.81)	$x^{(0)}_{(r^*)j} = z_j + \sigma^{(0)}_{(r^*)j}\left[p^{(0)}_{(r^*)j} - \sum_{r=1}^{N(j)} H^{*(0)}_{(r^*)j}\, p^{(0)}_{(r^*)j}\right] - a^{(0)}_j - a^{(0)}_{(r^*)j} - \sigma^{(0)}_{(r^*)j}\left[a^{(0)}_{(r^*)j} - \sum_{r=1}^{N(j)} H^{*(0)}_{(r^*)j}\, a^{(0)}_{(r^*)j}\right]$	r=1,...,N(j), j=1,...,h.	$\sum_{j=1}^{h} N(j)$	Supplies of composite commodities by industries.
(3.83)	$x^{(0)}_{(i1)j} = x^{(0)}_{(t^*)j} - a^{(0)}_{(i1)j}$	$i\varepsilon G(t,j)$, t=1,...,N(j), j=1,...,h.	gh	Supplies of commodities by industries.
(3.84)	$p^{(0)}_{(t^*)j} = \sum_{i\varepsilon G(t,j)} p^{(0)}_{(i1)}\, S^{(0)}_{(i1)j} - \sum_{i\varepsilon G(t,j)} a^{(0)}_{(i1)j}\, S^{(0)}_{(i1)j}$	t=1,...,N(j), j=1,...,h.	$\sum_{j=1}^{h} N(j)$	Prices of composite commodities.

.... *continued*

Table 1 continued ....

Identi-fier	Equation	Subscript Range	Number	Description
(4.4)	$x^{(2)}_{(is)j} = y_j - \sigma^{(2)}_{ij}\left[p^{(2)}_{(is)j} - \sum_{s=1}^{2} S^{(2)}_{(is)j}\, p^{(2)}_{(is)j}\right] + a^{(2)}_{j} + a^{(2)}_{ij} + a^{(2)}_{(is)j} - \sigma^{(2)}_{ij}\left[a^{(2)}_{(is)j} - \sum_{s=1}^{2} S^{(2)}_{(is)j}\, a^{(2)}_{(is)j}\right]$	i=1,...,g, s=1,2, j=1,...,h.	2gh	Demand for inputs to capital creation.
(5.21)	$x^{(3)}_{(is)} = x^{(3)}_{i} - \sigma^{(3)}_{i}\left[p^{(3)}_{(is)} - \sum_{s=1}^{2} S^{(3)}_{(is)}\, p^{(3)}_{(is)}\right] + a^{(3)}_{(is)} - \sigma^{(3)}_{i}\left[a^{(3)}_{(is)} - \sum_{s=1}^{2} S^{(3)}_{(is)}\, a^{(3)}_{(is)}\right]$	i=1,...,g, s=1,2.	2g	Household demands for commodities classified by source.
(5.23)	$p^{(3)}_{i} = \sum_{s=1}^{2} S^{(3)}_{(is)}\, p^{(3)}_{(is)}$	i=1,...,g.	g	General price of each commodity to households.
(5.24)	$x^{(3)}_{i} - q = \varepsilon_i (c-q) + \sum_{k=1}^{g} \eta_{ik}\, p^{(3)}_{k} + a^{(3)}_{i} + \sum_{k=1}^{g} \eta_{ik}\left[a^{(3)}_{k} + \sum_{s=1}^{2} S^{(3)}_{(ks)}\, a^{(3)}_{(ks)}\right]$	i=1,...,g.	g	Household demands for commodities, undifferentiated by source.
(6.2)	$p^{e}_{(i1)} = -\gamma_i\, x^{(4)}_{(i1)} + f^{(e)}_{(i1)}$	i=1,...,g.	g	Export demand functions.
(7.1)	$x^{(5)}_{(is)} = c_R\, h^{(5)}_{(is)} + f^{(5)}_{(is)}$	i=1,...,g, s=1,2.	2g	Other demands for commodities classified by source.
(7.2)	$c_R = c - \xi^{(3)}$		1	Real household expenditure.
(8.2)	$x^{(is)jk}_{(r1)} = x^{(k)}_{(is)j} + a^{(is)jk}_{(r1)}$	i,r=1,...,g, j=1,...,h, k,s=1,2.	$4g^2h$	
(8.5)	$x^{(is)k}_{(r1)} = x^{(k)}_{(is)} + a^{(is)k}_{(r1)}$	k=3,5, i,r=1,...,g, s=1,2.	$4g^2$	Demands for margins to facilitate commodity flows to producers, to capital creators, to households, to other users and to domestic ports prior to export.
(8.6)	$x^{(i1)4}_{(r1)} = x^{(4)}_{(i1)} + a^{(i1)4}_{(r1)}$	i,r=1,...,g.	g^2	

.... *continued*

Table 1 continued ....

Identifier	Equation	Subscript Range	Number	Description
(9.2)	$\sum_{i=1}^{g} p_{(i1)}^{(0)} H_{(i1)j}^{(0)} = \sum_{i=1}^{g}\sum_{s=1}^{2} p_{(is)j}^{(1)} H_{(is)j}^{(1)} + \sum_{m=1}^{M} p_{(g+1,1,m)j}^{(1)} H_{(g+1,1,m)j}^{(1)} + \sum_{s=2}^{3} p_{(g+1,s)j}^{(1)} H_{(g+1,s)j}^{(1)} + p_{g+2,j}^{(1)} H_{g+2,j}^{(1)} + a(j)$	j=1,...,h.	h	Zero pure profits in production.
(9.3)	$a(j) = a_j^{(0)} + \sum_{r=1}^{N(j)} a_{(r^*)j}^{(0)} H_{(r^*)j}^{(0)} + \sum_{i=1}^{g} a_{(i1)j}^{(0)} H_{(i1)j}^{(0)} + a_j^{(1)} + \sum_{i=1}^{g+2} a_{ij}^{(1)} H_{ij}^{(1)} + \sum_{i=1}^{g}\sum_{s=1}^{2} a_{(is)j}^{(1)} H_{(is)j}^{(1)} + \sum_{s=1}^{3} a_{(g+1,s)j}^{(1)} H_{(g+1,s)j}^{(1)} + \sum_{m=1}^{M} a_{(g+1,1,m)j}^{(1)} H_{(g+1,1,m)j}^{(1)}$	j=1,...,h.	h	Weighted sums of the technical change terms affecting the production functions of each industry.
(9.6)	$\pi_j = \sum_{i=1}^{g}\sum_{s=1}^{2} p_{(is)j}^{(2)} H_{(is)j}^{(2)} + a_j^{(2)} + \sum_{i=1}^{g} a_{ij}^{(2)} H_{ij}^{(2)} + \sum_{i=1}^{g}\sum_{s=1}^{2} a_{(is)j}^{(2)} H_{(is)j}^{(2)}$	j=1,...,h.	h	Zero pure profits in capital creation.
(9.10)	$p_{(i2)}^{(0)} = (p_{(i2)}^{m} + \phi)\ \zeta_1(i2,0) + g(i2,0)\ \zeta_2(i2,0)$	i=1,...,g.	g	Zero pure profits in importing.
(9.11)	$g(i2,0) = h_1(i2,0)\ \xi^{(3)} + h_2(i2,0)(t(i2,0) + p_{(i2)}^{m} + \phi) + h_3(i2,0)\ v(i2,0)$	i=1,...,g.	g	Flexible handling of tariff rates.
(9.14)	$p_{(i1)}^{e} + \phi = p_{(i1)}^{(0)}\ \zeta_1(i1,4) + g(i1,4)\ \zeta_2(i1,4) + \left(\sum_{r=1}^{g} M_{(r1)}^{(i1)4}\ p_{(r1)}^{(0)}\right) \zeta_3(i1,4) + \left(\sum_{r=1}^{g} M_{(r1)}^{(i1)4}\ a_{(r1)}^{(i1)4}\right) \zeta_3(i1,4)$	i=1,...,g.	g	Zero pure profits in exporting.
(9.15)	$g(i1,4) = h_1(i1,4)\ \xi^{(3)} + h_2(i1,4)\left(t(i1,4) + p_{(i1)}^{e} + \phi\right) + h_3(i1,4)\ v(i1,4)$	i=1,...,g.	g	Flexible handling of export taxes.

.... *continued*

Table 1 continued ....

Identifier	Equation	Subscript Range	Number	Description
(9.18)	$p_{(is)j}^{(k)} = p_{(is)}^{(0)} \zeta_1(is,jk) + g(is,jk)\ \zeta_2(is,jk) + \left[\sum_{r=1}^{g} M_{(r1)}^{(is)jk}\ p_{(r1)}^{(0)}\right] \zeta_3(is,jk) + \left[\sum_{r=1}^{g} M_{(r1)}^{(is)jk}\ a_{(r1)}^{(is)jk}\right] \zeta_3(is,jk)$	$i=1,\ldots,g,$ $j=1,\ldots,h,$ $s,k=1,2.$	4gh	Zero pure profits in the distribution of goods to domestic users.
(9.19)	$p_{(is)}^{(k)} = p_{(is)}^{(0)} \zeta_1(is,k) + g(is,k)\ \zeta_2(is,k) + \left[\sum_{r=1}^{g} M_{(r1)}^{(is)k}\ p_{(r1)}^{(0)}\right] \zeta_3(is,k) + \left[\sum_{r=1}^{g} M_{(r1)}^{(is)k}\ a_{(r1)}^{(is)k}\right] \zeta_3(is,k)$	$i=1,\ldots,g,$ $s=1,2,$ $k=3,5.$	4g	
(9.20)	$g(is,jk) = h_1(is,jk)\ \xi^{(3)} + h_2(is,jk)\left[t(is,jk) + p_{(is)}^{(0)}\right] + h_3(is,jk)\ v(is,jk)$	$i=1,\ldots,g,$ $j=1,\ldots,h,$ $s,k=1,2.$	4gh	Flexible handling of taxes on sales to domestic users.
(9.21)	$g(is,k) = h_1(is,k)\ \xi^{(3)} + h_2(is,k)\left[t(is,k) + p_{(is)}^{(0)}\right] + h_3(is,k)\ v(is,k)$	$i=1,\ldots,g,$ $s=1,2,$ $k=3,5.$	4g	
(10.7)	$r_j(0) = Q_j\left[p_{(g+1,2)j}^{(1)} - \pi_j\right]$	$j=1,\ldots,h.$	h	Rates of return on capital in each industry.
(10.8)	$-\beta_j\left[k_j(1) - k_j(0)\right] + r_j(0) = \omega$	$j\varepsilon J.$	J^* 1	Equality of rates of return across industries.
(10.9)	$k_j(1) = k_j(0)\ (1 - G_j) + y_j\ G_j$	$j=1,\ldots,h.$	h	Capital accumulation.
(10.10)	$\sum_{j\varepsilon J} (\pi_j + y_j)\ T_j = \left[\sum_{j\varepsilon J} T_j\right] i$		1	Investment budget.
(10.11)	$y_j = h_j^{(2)}\ i_R + f_j^{(2)}$	$j\notin J.$	$h-J^*$	Equations for handling exogenous investment.
(10.12)	$i_R = i - \xi^{(2)}$		1	Real private investment expenditure.

.... *continued*

1. J^* is the number of elements in J.

Table 1 continued ....

Identifier	Equation	Subscript Range	Number	Description
(11.6)	$x^{(0)}_{(r1)} = \sum_{j=1}^{h} x^{(1)}_{(r1)j} B^{(1)}_{(r1)j} + \sum_{j=1}^{h} x^{(2)}_{(r1)j} B^{(2)}_{(r1)j} + x^{(3)}_{(r1)} B^{(3)}_{(r1)} + x^{(4)}_{(r1)} B^{(4)}_{(r1)}$ $+ x^{(5)}_{(r1)} B^{(5)}_{(r1)} + \sum_{i=1}^{g} \sum_{s=1}^{2} \sum_{j=1}^{h} \sum_{k=1}^{2} x^{(is)jk}_{(r1)} B^{(is)jk}_{(r1)}$ $+ \sum_{i=1}^{g} \sum_{s=1}^{2} \sum_{k=3,5} x^{(is)k}_{(r1)} B^{(is)k}_{(r1)} + \sum_{i=1}^{g} x^{(i1)4}_{(r1)} B^{(i1)4}_{(r1)}$	r=1,...,g.	g	Demand equals supply for domestically produced commodities.
(11.7)	$x^{(0)}_{(r1)} = \sum_{j=1}^{h} x^{(0)}_{(r1)j} B^{(0)}_{(r1)j}$	r=1,...,g.	g	Total output of good (r1).
(11.8)	$\ell_m = \sum_{j=1}^{h} x^{(1)}_{(g+1,1,m)j} B^{(1)}_{(g+1,1,m)j}$	m=1,...,M.	M	Demand equals supply for labor of each skill.
(11.9)	$k_j(0) = x^{(1)}_{(g+1,2)j}$	j=1,...,h.	h	Demand and supply for capital.
(11.10)	$n_j = x^{(1)}_{(g+1,3)j}$	j=1,...,h.	h	Demand and supply for agricultural land.
(12.2)	$x^{(0)}_{(r2)} = \sum_{k=1}^{2} \sum_{j=1}^{h} x^{(k)}_{(r2)j} B^{(k)}_{(r2)j} + \sum_{k=3,5} x^{(k)}_{(r2)} B^{(k)}_{(r2)}$	r=1,...,g.	g	Import volumes.
(12.4)	$m = \sum_{r=1}^{g} \left[p^{m}_{(r2)} + x^{(0)}_{(r2)}\right] M_{(r2)}$		1	Foreign currency value of imports
(12.6)	$e = \sum_{r=1}^{g} \left[p^{e}_{(r1)} + x^{(4)}_{(r1)}\right] E_{(r1)}$		1	Foreign currency value or exports.

.... *continued*

Table 1 continued ....

Identifier	Equation	Subscript Range	Number	Description
(12.8)	$100\ \Delta B = Ee - Mm$		1	The balance of trade.
(13.1)	$\xi^{(3)} = \sum_{s=1}^{2} \sum_{i=1}^{g} w_{(is)}^{(3)}\ p_{(is)}^{(3)}$		1	ORANI consumer price index.
(13.2)	$\xi^{(2)} = \sum_{j \in J} \bar{T}_j\ \pi_j$		1	ORANI capital goods price index.
(13.3)	$\ell = \sum_{m=1}^{M} \ell_m\ \psi_{1m}$		1	Aggregate employment
(13.4)	$k(0) = \sum_{j=1}^{h} k_j(0)\ \psi_{2j}$		1	Aggregate capital stock.
(13.5)	$f_R = i_R - c_R$		1	Ratio of real investment to real consumption.
(13.6)	$p_{(g+1,1,m)j}^{(1)} = h_{(g+1,1,m)j}^{(1)}\ \xi^{(3)} + f_{(g+1,1)}^{(1)} + f_{(g+1,1)j}^{(1)} + f_{(g+1,1,m)}^{(1)} + f_{(g+1,1,m)j}^{(1)}$	$m=1,\ldots,M,$ $j=1,\ldots,h.$	Mh	Flexible handling of wages by occupation and industry.
(13.7)	$p_{g+2,j}^{(1)} = h_{g+2,j}^{(1)}\ \xi^{(3)} + f_{g+2,j}^{(1)}$	$j=1,\ldots,h.$	h	Indexing of the prices of other cost tickets.

$$\text{Total} = 4g^2h + 5g^2 + 13gh + 2Mh + 14h + 22g + M + 2\sum_{j=1}^{h} N(j) + 11$$

Table 2 : The ORANI Variables

Variable	Subscript Range	Number	Description
			All variables are percentage changes with the exception of ΔB
z_j	j=1,...,h.	h	Industry activity levels
$x^{(k)}_{(is)j}$	i=1,...,g, j=1,...,h, s,k=1,2.	$4gh^{\#}$	Demands for inputs (domestic and imported) for current production and capital creation
$x^{(1)}_{g+2,j}$	j=1,...,h.	$h^{\#}$	Demands for other cost tickets
$x^{(1)}_{(g+1,1,q)j}$	q=1,...,M, j=1,...,h.	$Mh^{\#}$	Demands for labor inputs by skill group and industry
$x^{(1)}_{(g+1,v)j}$	v=1,2,3, j=1,...,h.	$3h^{\#}$	Industry demands for labor in general, capital and agricultural land
$x^{(0)}_{(r^*)j}$	j=1,...,h, r=1,...,N(j).	$\sum_{j=1}^{h} N(j)^{\#}$	Supplies of composite commodities by industry
$x^{(0)}_{(i1)j}$	j=1,...,h.	$gh^{\#}$	Supplies of commodities by industry
$x^{(k)}_{(is)}$	k=3,5, s=1,2, i=1,...,g.	$4g^{\int}$	Household and other demands for goods by type and source
$x^{(3)}_{i}$	i=1,...,g.	$g^{\#}$	Household demands for goods by type undifferentiated by source
$x^{(4)}_{(i1)}$	i=1,...,g.	g	Export demands
$x^{(is)jk}_{(r1)}$	j=1,...,h, i,r=1,...,g, k,s=1,2.	$4g^2h^{\#}$	Demands for margin services to facilitate commodity flows to production and capital creation
$x^{(is)k}_{(r1)}$	i,r=1,...,g, k=3,5, s=1,2.	$4g^{2\#}$	Demands for margin services to facilitate the flow of goods to households and other users
$x^{(i1)4}_{(r1)}$	r,i=1,...,g.	$g^{2\#}$	Demands for margin services to facilitate the flow of goods to ports for export

.... *continued*

Table 2 continued ....

Variable	Subscript Range	Number	Description
$x^{(0)}_{(r1)}$	r=1,...,g.	g	Total supplies of domestic commodities
y_j	j=1,...,h.	h	Capital creation by using industry
$p^{(k)}_{(is)j}$	i=1,...,g, j=1,...,h, k,s=1,2.	$4gh^{\#}$	Purchasers' prices for produced inputs for production and capital creation
$p^{(1)}_{(g+1,v)j}$	v=1,2,3, j=1,...,h.	$3h^{\int}$	Prices paid by each industry for their labor in general, rental of capital and rental of agricultural land
$p^{(1)}_{(g+1,1,m)j}$	m=1,...,M, j=1,...,h.	Mh	Prices paid by industries for units of labor of different skill categories
$p^{(3)}_{i}$	i=1,...,g.	$g^{\#}$	Purchasers' prices for consumer goods by type but not by source
$p^{(k)}_{(is)}$	i=1,...,g, s=1,2, k=3,5.	$4g^{\#}$	Purchasers' prices paid for commodities by households and other users
$p^{e}_{(i1)}$	i=1,...,g.	g	F.o.b. foreign currency export prices
$p^{(0)}_{(is)}$	i=1,...,g, s=1,2.	2g	Basic prices of both domestic goods and imports
$p^{(0)}_{(t^*)j}$	t=1,...,N(j), j=1,...,h.	$\sum_{j=1}^{h} N(j)^{\#}$	Prices of composite commodities
$p^{(1)}_{g+2,j}$	j=1,...,h.	$h^{\#}$	Prices of other cost tickets to each industry
π_j	j=1,...,h.	h	Costs of units of capital
$p^{m}_{(i2)}$	i=1,...,g.	g	C.i.f. foreign currency import prices
ϕ		1	The exchange rate, \$A per \$US, say
q		1	Number of households

.... *continued*

Table 2 continued ....

Variable	Subscript Range	Number	Description
$k_j(1)$	$j=1,\ldots,h.$	$h^{\#}$	Future capital stocks
$k_j(0)$	$j=1,\ldots,h.$	h	Current capital stocks
$r_j(0)$	$j=1,\ldots,h.$	h	Current rates of return on fixed capital
ω		1	Economy-wide expected rate of return on capital
ℓ_m	$m=1,\ldots,M.$	M	Employment of labor by skill group
n_j	$j=1,\ldots,h.$	h	Use of agricultural land in each industry
$x_{(r2)}^{(0)}$	$r=1,\ldots,g.$	g	Aggregate imports by commodity
m		1	Foreign currency value of imports
e		1	Foreign currency value of exports
ΔB		1	The balance of trade
$\xi^{(3)}$		1	ORANI consumer price index
$\xi^{(2)}$		1	ORANI capital goods price index
c		1	Aggregate household expenditure
c_R		1	Real aggregate household expenditure
i_R		1	Aggregate real private investment expenditure
i		1	Aggregate private investment expenditure
ℓ		1	Aggregate employment
k(0)		1	Aggregate capital stock
f_R		1	The ratio of real private investment expenditure to real household consumption expenditure

.... *continued*

Table 2 continued ....

Variable	Subscript Range	Number	Description
$f^{e}_{(i1)}$	i=1,...,g.	g	Shifts in foreign export demands
$f^{(5)}_{(is)}$	i=1,...,g, s=1,2.	2g	Shift terms for introducing changes in the ratio of real other demands to real household expenditure
$f^{(2)}_{j}$	$j \not\in J$.	$h-J^{*}$[1]	Exogenous investment terms. Can sometimes be interpreted as the ratio of real investment in particular industries to total real private investment
$f^{(1)}_{(g+1,1)}$		1	General wage shift variable. Can sometimes be interpreted as the change in the overall level of real wages
$f^{(1)}_{(g+1,1)j}$	j=1,...,h.	h.	Variable used for simulating the effects of changes in the wages payable by particular industries relative to other industries
$f^{(1)}_{(g+1,1,m)}$	m=1,...,M.	M	Variable used in simulations involving changes in occupational wage relativities
$f^{(1)}_{(g+1,1,m)j}$	m=1,...,M, j=1,...,h.	Mh	Variable allowing changes in both occupational and industrial wage relativities
$f^{(1)}_{g+2,j}$	j=1,...,h.	h	Shift terms for allowing for changes in the real price of other cost tickets
a(j)	j=1,...,h.	$h^{\#}$	Weighted sums of the technicalchange terms affecting the production functions for each industry
$a^{(1)}_{j}$	j=1,...,h.	$h^{\dagger}$	Neutral input augmenting technical change
$a^{(1)}_{ij}$	i=1,...,g+2, j=1,...,h.	$(g+2)h^{\dagger}$	Input i augmenting technical change
$a^{(1)}_{(is)j}$	i=1,...,g, s=1,2, j=1,...,h.	$2gh^{\dagger}$	Input (is) augmenting technical change

.... *continued*

1. J^{*} is the number of elements in J.

Table 2 continued ....

Variable	Subscript Range	Number	Description
$a^{(1)}_{(g+1,s)j}$	s=1,2,3, j=1,...,h.	$3h^{\dagger}$	Labor, capital and agricultural land augmenting technical change
$a^{(1)}_{(g+1,1,q)j}$	q=1,...,M, j=1,...,h.	$Mh^{\dagger}$	Specific skill augmenting technical change
$a^{(0)}_{j}$	j=1,...,h.	$h^{\dagger}$	Neutral output augmenting technical change
$a^{(0)}_{(r^*)j}$	r=1,...,N(j), j=1,...,h.	$\sum_{j=1}^{h} N(j)^{\dagger}$	Composite good augmenting technical change
$a^{(0)}_{(i1)j}$	i=1,...,g, j=1,...,h.	$gh^{\dagger}$	Augmenting technical change with respect to commodity outputs
$a^{(2)}_{j}$	j=1,...,h.	$h^{\dagger}$	Neutral input augmenting technical change with respect to capital creation
$a^{(2)}_{ij}$	i=1,...,g, j=1,...,h.	$gh^{\dagger}$	Input i augmenting technical change with respect to capital creation
$a^{(2)}_{(is)j}$	i=1,...,g, s=1,2, j=1,...,h.	$2gh^{\dagger}$	Input (is) augmenting technical change with respect to capital creation
$a^{(3)}_{i}$	i=1,...,g.	$g^{\dagger}$	Commodity i augmenting change in household preferences
$a^{(3)}_{(is)}$	i=1,...,g, s=1,2.	$2g^{\dagger}$	Commodity (is) augmenting change in household preferences
$a^{(is)jk}_{(r1)}$	r,i=1,...,g, s,k=1,2, j=1,...,h.	$4g^2h^{\dagger}$	Technical change associated with the use of services in facilitating input flows to industries for current production and capital creation
$a^{(is)k}_{(r1)}$	r,i=1,...,g, s=1,2, k=3,5.	$4g^{2\dagger}$	Technical change associated with the use of services in facilitating commodity flows to households and other users.
$a^{(i1)4}_{(r1)}$	r,i=1,...,g.	$g^{2\dagger}$	Technical change associated with the use of services in facilitating the flow of exports from producers to the ports of exit

continued

Table 2 continued ....

Variables	Subscript Range	Number	Description
g(i2,0), t(i2,0), v(i2,0)	i=1,...,g.	g#, g, g†	The g's are the tariffs per unit of imports. The t's and v's are variables allowing tariffs to be modelled as <u>ad valorem</u> or specific
g(i1,4), t(i1,4), v(i1,4)	i=1,...,g.	g#, g†, g	The g's are taxes per unit of exports. The t's and v's allow these taxes to be modelled as <u>ad valorem</u> or specific
g(is,jk), t(is,jk), v(is,jk)	i=1,...,g, s,k=1,2, j=1,...,h.	4gh#, 4gh†, 4gh†	The g's are taxes on the sales of inputs to industries for current production and capital creation. The t's and v's allow these taxes to be modelled as <u>ad valorem</u> or specific
g(is,k), t(is,k), v(is,k)	i=1,...,g, s=1,2, k=3,5.	4g#, 4g†, 4g†	The g's are taxes on the sales of commodities to households and other users. The t's and v's allow these taxes to be modelled as <u>ad valorem</u> or specific

$$\text{Total} = 8g^2h + 10g^2 + 28gh + 4Mh + 27h + 41g + 2M + 3\sum_{j=1}^{h} N(j) + 16 - J^*$$

\# These variables are eliminated in the condensed system, see section 17.

∫ These variables are partly eliminated in the condensed system.
$x^{(k)}_{(is)}$ is eliminated for $k=5$ and
$p^{(1)}_{(g+1,v)j}$ is eliminated for $v=1$.

† These variables are replaced by *b*-variables in the condensed system.

Table 3 : A Typical List of Exogenous Variables

Variable	Subscript Range	Number	Description
$p^{m}_{(i2)}$	i=1,...,g.	g	C.i.f. foreign currency import prices
t(i2,0), v(i2,0)	i=1,...,g.	2g	Tariff terms
t(is,jk), v(is,jk)	i=1,...,g, s,k=1,2, j=1,...,h.	8gh	*Ad valorem* and specific sales tax terms
t(is,k), v(is,k)	i=1,...,g, s=1,2, k=3,5.	8g	
v(i1,4)	$i \in G$[1].	g	Selection of specific export terms and complementary selection of export volumes
$x^{(4)}_{(i1)}$	$i \notin G$.		
t(i1,4)	i=1,...,g.	g	*Ad valorem* export tax terms
a's (excluding a(j))	subscript ranges can be read from Table 2.	$4g^2h + 5g^2 + 7gh + Mh + 8h + 3g + \sum_{j=1}^{h} N(j)$	Technological change and changes in household preferences
$k_j(0)$	j=1,...,h.	h	Current capital stocks
c_R		1	Real household aggregate expenditure
i_R		1	Aggregate real private investment
n_j	j=1,...,h.	h	Use of agricultural land in each industry
$f^{(1)}_{(g+1,1)}$		1	Wage shift variables
$f^{(1)}_{(g+1,1,m)}$	m=1,...,M.	M	

.... *continued*

1. G is a subset of {1,...,g}. Further explanation is in the text.

Table 3 continued ....

Variable	Subscript Range	Number	Description
$f^{(1)}_{(g+1,1)j}$	j=1,...,h.	h	
$f^{(1)}_{(g+1,1,m)j}$	m=1,...,M, j=1,...,h.	Mh	
$f^{(5)}_{(is)}$	i=1,...,g, s=1,2.	2g	Other demand shift terms
$f^{(2)}_{j}$	$j \notin J$.	h-J*	Exogenous investment
f^{e}_{i}	i=1,...,g.	g	Shifts in foreign export demands
$f^{(1)}_{g+2,j}$	j=1,...,h.	h	Shifts in the real price of other cost tickets
q		1	Number of households
ϕ		1	The exchange rate, \$A per \$US, say.

$$\text{Total} = 4g^2h + 5g^2 + 15gh + 2Mh + 13h + 19g + M + \sum_{j=1}^{h} N(j) + 5 - J^*$$

very large number. Nevertheless, this does not present any insurmountable difficulties. In applications of the model, the overwhelming majority of the exogenous variables have the value zero. In terms of the solution equation (1.7), this means that in each application we need to compute and store only a small proportion of the columns of A_2 .[1]

There are many ways of selecting the D variables for the exogenous list. In Table 3 we have shown one possibility. By working through Table 3 we can point out some of the other possible choices.[2]

The first group of exogenous variables given in Table 3 are the c.i.f. foreign currency prices of imports. ORANI contains no equations describing foreign supply conditions and therefore it is difficult to imagine an ORANI experiment in which the $p^m_{(i2)}$ would be endogenous. By placing the $p^m_{(i2)}$ in the exogenous category, we are adopting the "small country" assumption on the import side, i.e., world prices are independent of Australian demands. We are also allowing for the computation of answers to questions of the form : what were (or will be) the effects of past (or projected) changes in foreign import supply prices.

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1. Details of the computational techniques are in Chapter 5.

2. Some of these were indicated in Chapter 2, section 3. For completeness, however, we will repeat that earlier material in the context of the full model.

The second group of exogenous variables are concerned with tariffs or tariff equivalents of quantitative restrictions. The $t(i2,0)$ and $v(i2,0)$ are among the exogenous variables for any computation directed at the traditional effective protection question : which industries benefit and which lose from protection. Other questions might concern the effects of protection on employment and on the rate of inflation. Each of these questions could be analysed under exogenously given changes in either the $t(i2,0)$ or $v(i2,0)$ with suitable values assigned to the parameters $h_1(i2,0)$, $h_2(i2,0)$ and $h_3(i2,0)$ (see the discussion of (9.9) in section 9). On the other hand, it is possible to conduct ORANI experiments in which some, or all, the tariffs are endogenous. For example, we might wish to compute the change in the level of ad valorem protection which would be required to maintain current output in footwear, say, in the face of exogenously given movements in foreign prices, domestic wages and the exchange rate. For such a computation, footwear output would replace the ad valorem tariff variable for footwear in the exogenous list. That is, for i=footwear, we would add $x^{(0)}_{(i1)}$ to the exogenous list and remove $t(i2,0)$. $v(i2,0)$ would continue to be exogenous, $h_2(i2,0)$ would be set at one and the other two h's would be set at zero.

The third set of variables in Table 3 are the sales tax terms. One can imagine a computation in which the sales tax on household oil consumption (say) would be treated endogenously. The question might be to compute the increase in the tax which would be required to achieve an exogenously given decrease in household use of oil. In most simulations, however, the sales tax terms are exogenous and are set at zero. The standard value for the h_2 and h_3 parameters in equations (9.20) and (9.21) is zero, while the standard value for the h_1's is one. Thus, we normally treat sales taxes as fixed in real value per physical unit of sale.

Our fourth group of typical exogenous variables are a selection of export tax terms and export levels. G is a user-specified subset of $1,\ldots,g$, and contains the labels of those commodities for which ORANI is allowed to explain exports. For all other commodities, i.e., $i \notin G$, exports are exogenous and ORANI projects the export tax (or subsidy) required to achieve the given export level. Certainly, the non-export commodities, the services, construction, etc., always appear in the list given by $i \notin G$. Changes in the exports of these commodities are set exogenously at zero. The resulting endogenously determined tax rates are quite meaningless, but also harmless (see the discussion of (9.14) and (9.15) in section 9). In most ORANI computations, we have included in G those commodities for which exports are more than 20 per cent of total domestic output. This has given us approximately 13 of the 115 ORANI commodities and covers about 67 per cent of total Australian commodity exports.

The fifth group of exogenous variables are the ad valorem export tax terms $t(i1,4)$, $i=1,\ldots,g$. Table 3 implies that the $v(i1,4)$ are endogenous for i=non-export product. We could equally well have made the relevant t(i1,4) endogenous. Then rather than computing the required changes in the specific tax rates to achieve the exogenously given export volumes for the non-export products, we would compute the required changes in the ad valorem tax rates.

Sixth on the list of our exogenous variables are the technological change and change in preference terms, the a's .[1] ORANI does not explain technology or preferences. What the a's allow model users to do is to simulate the effects on the industrial composition of GDP, the occupational composition of the workforce, etc., of a wide variety of exogenously given changes in technology and household preferences. These changes may be specified at a very broad level : for example, what would be the effects on employment of a 10 per cent increase in productivity throughout the manufacturing sector. On the other hand the technological change under examination might be very narrowly defined. What, for example, would be the effects on industrial structure, employment, etc., of a 20 per cent reduction in the services required from the retail trade industry to facilitate the flow of any given volume of vegetables from producers to consumers. Such a question might arise in connection with the analysis of the effects of the replacement of greengrocers by fruit and vegetable sections of large supermarkets.

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1. The a(j)'s are determined by the other technological change variables, see (9.3). Thus, they would not appear on a list of exogenous variables containing the other a's .

The seventh group of variables in Table 3 are the $k_j(0)$, the current capital stocks. As was mentioned in section 1, the $k_j(0)$'s appear on the exogenous list in short-run simulations. For long-run simulations we can replace the $k_j(0)$'s with the rates of return, $r_j(0)$. Another possibility is to replace the $k_j(0)$'s with the rentals on capital, the $p_{(g+1,2)j}$'s. This would be appropriate in a situation of excess capacity and fixed markup pricing. If both wages and rentals on capital are set exogenously, then domestic commodity prices behave as if they are determined independently of demand and are set according to a fixed markup rule.[1]

The next two variables are c_R and i_R, the real aggregate levels of household expenditure and private investment. By placing these on the exogenous list, we are setting an economic environment in which real aggregate demands are controllable independently of the other variables appearing in Table 3. The underlying assumption is that policy makers have available macro instruments, not explained in ORANI, by which they can influence c_R and i_R. While ORANI cannot provide specific results concerning these macro instruments, it can provide some guidance. With c_R and i_R exogenous, ORANI will indicate the changes in the monetary aggregates, C and I, which would be required to achieve the exogenously specified changes in C_R and I_R in view of whatever other exogenous changes have been introduced. For example, ORANI will indicate that under an x per cent tariff cut, C and I should be changed by

1. Further discussion of fixed markup pricing in ORANI is given in Chapter , section .

u and v per cent from what they otherwise would have been if we are to achieve the exogenously given targets C_R and I_R. Alternatively, model users might set ΔB and f_R exogenously in place of c_R and i_R. In this case, ORANI would indicate the change in the level of domestic absorption which would need to accompany a tariff cut, say, if we were to maintain a given level for the balance of trade. The model would not explain the allocation of the change in absorption between investment and consumption. This is imposed exogenously. If f_R were set at zero, for example, our assumption would be that the tariff cut has no effect on the allocation of real private expenditure between consumption and investment.

On continuing down the list in Table 3, we reach n_j, the employment of the primary factor agricultural land in each industry. With the n_j's exogenous, ORANI will determine changes in the rental prices of units of agricultural land, the $p^{(1)}_{(g+1,3)j}$'s. On the other hand, for the primary factor labor, the inclusion of all the wage shift terms, $f^{(1)}_{(g+1,1)}$, $f^{(1)}_{(g+1,1,m)}$, $f^{(1)}_{(g+1,1)j}$ and $f^{(1)}_{(g+1,1,m)j}$ in the exogenous list indicates that the prices rather than the employment levels are treated exogenously. Other ways of handling labor markets, involving endogenous determination of some of the $f^{(1)}$'s, were discussed in section 13.

The presence of the shift variables $f^{(5)}_{(is)}$, $f^{(2)}_{j}$, f^{e}_{i} and $f^{(1)}_{g+2,j}$, and the number of households, q, in the exogenous list requires only brief discussion. Exogenous manipulation of the variables $f^{(5)}_{(is)}$, in combination with assumptions concerning the parameters $h^{(5)}_{(is)}$ (see equation (7.1)), can be used to force other demands to move in line with real aggregate household expenditure or in any other exogenously specified way. Similarly, the variables $f^{(2)}_j$, $j \notin J$, can be used in combination with the parameters, $h^{(2)}_j$ (see equation (10.11)), to force investment in selected industries to follow real aggregate private investment rather than be determined by rates of return. Exogenous changes in the variables f^e_i simulate the effects of shifts in foreign demands for exports from Australia, while the role of the variables, $f^{(1)}_{g+2,j}$, is to allow for exogenous changes in both production taxes and in the costs associated with holding working capital. Explanation of the number of households, q, requires demographic modelling outside ORANI. Hence, q will normally be exogenous to ORANI.

The last variable in Table 3 is the exchange rate, ϕ. In many ORANI simulations, ϕ simply acts as the numeraire. This is true, for example, in simulations where the exogenous variables are as in Table 3 and the various h parameters are set so that real wages, the real prices of other cost tickets and the real values of tariffs, export taxes and sales taxes per physical unit of the relevant flows are independent of the price level. More formally, under the exogenous variable list given in Table 3, and the indexing assumption,

$$h^{(1)}_{(g+1,1,m)j} = 1 \quad \text{for all } m \text{ and } j , \tag{14.1}$$

$$h^{(1)}_{g+2,j} = 1 \quad \text{for all } j \tag{14.2}$$

and[1]

$$h_1 = 1, \quad h_2 = h_3 = 0 \quad \text{for all the } h_1, h_2 \text{ and } h_3 \text{ parameters appearing in (9.11), (9.15), (9.20) and (9.21)}, \tag{14.3}$$

all real endogenous variables are homogeneous of degree zero with respect to the exchange rate and all domestic prices and monetary quantities are homogeneous of degree 1. This is a common property of neoclassical general equilibrium models.[2] It may be checked in ORANI by working through Tables 1 and 2 and testing the assertion that given $\phi=1$ and all other variables in Table 3 are set at zero, then

$$\left.\begin{array}{rcll} z_j & = & 0 & \text{for all } j, \\ x_{(is)j}^{(k)} & = & 0 & \text{for all } i, j, s \text{ and } k \\ & \vdots & & \\ p_{(is)j}^{(k)} & = & 1 & \text{for all } i, j, s \text{ and } k \\ & \vdots & & \\ \Delta B & = & 0 & \\ & \vdots & & \\ c & = & 1 & \\ & \vdots & & \end{array}\right\} \tag{14.4}$$

is a model solution.[3]

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1. ϕ would retain its property as a numeraire under various other settings for the h_1, h_2 and h_3's . For example, all or some of the h_2's could be set at one in place of all or some of the h_1's .

2. The property really is that almost all applied general equilibrium models can be rewritten (if necessary) so that the exchange rate becomes the numeraire.

3. Starting at (3.23), we substitute from (14.4) and Table 3, and check that left hand sides equal right hand sides. For example,

$$\text{LHS of (3.23)} = 0$$
$$\text{RHS of (3.23)} = 0 - \sigma_{ij}^{(1)}\left(1 - \Sigma_s S_{(is)j}^{(1)}\right) = 0 \ .$$

Other variables may be used as the numeraire. The consumer price index, $\xi^{(3)}$, would be a natural alternative to ϕ . Another possibility is to have no single-variable numeraire. If we include both $\xi^{(3)}$ and ϕ among our exogenous variables, then variations in one relative to the other are possible and will cause real (not just price level) effects. Of course, if $\xi^{(3)}$ were added to the exogenous list without removing ϕ , then a different variable on that list would have to be removed. An obvious candidate would be $f^{(1)}_{(g+1,1)}$. Thus we would be able to compute the change in the overall level of real wages which would be required to limit the inflationary effect of a devaluation to an exogenously given level.

We conclude with one final comment on the partitioning of variables into exogenous and endogenous categories. While our discussion of Table 3 indicates a wide variety of legitimate possibilities, it is not true that ORANI can be closed by the exogenous setting of _any_ D variables. For example, at least one monetary variable should be included in the exogenous list. If all domestic currency prices, the exchange rate, all wages and all monetary aggregates are treated as endogenous, then ORANI computations will fail since there is nothing to determine the absolute price level. Similarily, some care is necessary to avoid inconsistencies. For example, if an attempt were made to set all three variables, c_R , c and $\xi^{(3)}$ exogenously, then (7.2) would be violated. Although we can offer no formal theory to guide ORANI users in their choice of exogenous variables, as a working rule, if a price appears on the exogenous list, then a corresponding quantity should be on the endogenous list and *vice versa*. If wages are exogenous, then employment will be endogenous; if export taxes are endogenous, then export volumes will be exogenous; if tariffs are exogenous, then imports will be endogenous; and if sales taxes are endogenous, then consumption will be exogenous.

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