IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

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MODELS OF SKILL SUBSTITUTION AND TRANSFORMATION IN AN OCCUPATIONALLY DISAGGREGATED LABOUR MARKET

by

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by Dean J. Parham & G. J. Ryland*

1. INTRODUCTION

The primary focus of this paper is to provide an appropriate methodology for obtaining empirical estimates of skill substitution parameters which may be used as input to the labour demand specification for the IMPACT Project's ORANI module (Dixon _et al_. (1977)). The production technology assumed in ORANI may be represented as a three-level nested production function; the third level of which is the relationship between labour services of nine occupational skill groups and a single composite index of labour services.[1] Our concern is the theoretical and econometric specification of alternative models of labour demand which provide estimates of the ease of substitution among different skills or what we have termed the Allen-Uzawa partial elasticities of skill substitution (ESS).

Recently economists have shown increased interest in the form and magnitude of the ESS both in the context of a further disaggregation

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1. See Dixon, P.B. _et al_. (1977), p. 22-3.

of factor inputs in production function analysis and for manpower planning purposes at the firm and economy-wide level. At the firm level, a relatively high ESS implies that a firm's demand for a particular skill is more responsive to changes in relative wages than to changes in the amounts of skilled labour available. An infinite ESS implies that a firm's demand for skills is perfectly elastic at the given wage rate for that skill. At the economy-wide level, it implies that employers do not discriminate among skills in their labour demands and the assumed homogeneity of labour means that any re-training scheme is likely to be ineffective. Further, any social objective such as reducing income inequality through increased expenditure on technical and professional education would also be ineffective if the ESS is high or infinite.

In contrast, a low ESS suggests that the firm's manpower requirements are based largely on fixed labour/output coefficients, with little reference to relative wages. At the macro level, a low ESS could account for shortages and therefore higher relative wages in particular occupations, even in periods of relatively high unemployment.

Labour as a factor input in the neo-classical production model of the firm is usually specified as a single composite index of labour services derived by simply adding all forms of labour services utilized by the firm. The assumed homogeneity of diverse labour inputs implies that the ESS is infinite between pairs of occupational groups. The importance of disaggregating the labour component and hence the importance of the ESS has been generally overlooked until recently.[1]

Incorporation of disaggregated labour inputs in multi-sectoral production models has been a relatively recent development. Adelman (1966), in an education planning study of Argentina, recognized the importance of disaggregation and assumed the ESS to be infinite. Bowles (1970) used a nested two level CES production function to estimate the ESS. The high estimate obtained was used to support his assumption of infinite ESS in his earlier work (Bowles (1969)) on education and manpower planning systems. More recently, Manne (1973) noted the empirical necessity of a non-zero ESS in order to generate meaningful shadow wage differentials in a linear programming model of development strategies for Mexico. Manne assumed the ESS to be infinite. Finally, Dixon et al. (1977) has specified imperfect, but non-zero, substitution between different categories of labour in the context of a multi-sectoral general equilibrium model.

Despite the recognition of the importance of skill substitution, econometric studies have been rather limited in scope. Generally, substitution possibilities are considered between broadly defined labour groups differentiated on the basis of nature of work (e.g. production and non-production workers) or level of schooling (e.g. graduate and other manpower). Tinbergen (1975) surveyed the literature, and found wide disparity in published estimates of the ESS.

1. Berndt and Christensen (1973) have shown that simple addition of less than perfectly substitutable occupations produces an inconsistent index of labour services. This will bias the estimate of the elasticity of substitution between labour and capital.

Differences in estimates of the ESS revealed by Tinbergen can be atrributed to three main factors. The first relates to the differences in the nature of the analysis conducted, in terms of the type of data used (time series or cross-section within or between countries), and the nature of the production function explicitly or implicitly specified.

The second factor (the main concern of Tinbergen's paper) is the problem of identifying the labour demand relationship. Tinbergen forwarded the criticism that studies based simply on the marginal productivity conditions for cost minimization have failed to identify the labour demand function, since they ignore considerations of labour supply. Tinbergen observed that these studies which implicitly assume a perfectly elastic supply curve for labour produced overestimates of the ESS. His own cross-section analysis, incorporating both labour demand and supply considerations in the context of an equilibrium labour market, produced an estimate of the ESS, which was not rejected as being significantly different from unity at appropriate confidence levels.[1]

The third factor which may account for differences in estimates of the ESS, is the assumed form of separability between labour and capital specified in the production function. For example, some researchers, e.g. Bowles (1970), followed a procedure which implies that labour and capital are strongly separable, i.e. disaggregated labour

1. It should be noted that Groenveld and Kuipers (1976) found Tinbergen's results to be sensitive to the exogenous variables specified in the demand and supply functions used to identify the system.

demands are independent of the level of capital services employed. Other researchers, such as Fallon and Layard (1975) have taken account of the level of capital services in influencing the demand for different skills. Berndt and Christensen (1974), following on from the work of Welch (1969) and Griliches (1969) did not reject the hypothesis of complementarity between capital services and services of non-production workers.

We aim to incorporate these and other considerations in the theoretical and econometric specification of the labour market. The major task of this paper is to present the theoretical and econometric specification of the demand side of an occupationally disaggregated labour market (Sections 2 to 5). In Section 6, we develop in general terms our labour supply specification. This foreshadows a more detailed development of the supply side at a later date. A research strategy which includes a synthesis of demand and supply under alternative market clearing hypotheses, is outlined in Section 7. We also state our intention to report empirical estimates of skill substitution and transformation parameters at various stages of development.

In the next section, we present in general terms a nested production model and the separability assumptions which are used for analysing disaggregated labour demands and the ease of substitution among different occupations. We then present theoretical specifications of two alternative models of labour demand based on different production technologies; one which preserves constant <u>ratios</u> of elasticities of substitution and one which preserves constant <u>differences</u> in elasticities of substitution for any aggregate level of demand. Section 3 presents an expansion path representation of factor demands and Section 4 presents the first order condition representation of factor demands. In Section 5 we discuss alternative stochastic specifications and derive estimatable forms.

2. AGGREGATION, SEPARABILITY AND DUALITY IN MODELS OF SKILL SUBSTITUTION

Assume that a representative firm's production function can be represented by[1] :

$$Y = f(K, L_1, L_2, \ldots, L_n) \quad , \tag{1}$$

where

Y = value added per unit time ,

K = level of capital services ,

L_i = labour services provided by the ith skill group .

Assume that the production technology is such that value added can be written as a function of two aggregate index numbers, one each for capital and labour. Further, partition the arguments of the labour index into as many subsets as there are different categories of labour. We assume strong separability of the labour index with respect to this partition. We also assume the analogous strong separability property for the capital index. This means that the demands for services of different skill groups is fully determined by the aggregate level of labour services and is tantamount to assuming that the Allen-Uzawa

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1. In the ORANI framework, equation (1) would be

$$P = f(K, L_1, L_2, \ldots, L_n)$$

where

P = index of 'effective' input of primary factors.

partial elasticities of substitution[1] (σ_{KLi}) are constant when capital is paired with each of the different skill groups; i.e. $\sigma_{KLi} = \sigma_{KLj}$.[2]

With the separability assumptions, the production function has been partitioned into a nested two level function of the form :

$$Y = f\left[K, h(L_1, L_2, \ldots, L_n)\right] \tag{2}$$

or, equivalently,

$$Y = f(K,L) \quad , \tag{3a}$$

and

$$L = h(L_1, L_2, \ldots, L_n) \quad , \tag{3b}$$

where

L = index of labour services provided by all skill groups.

It is from the labour aggregation function, equation (3b) that we derive demands for each skill group using the first order conditions for a solution to a constrained minimization problem. The

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1. In the context of a production function $\{Y = f(x_1, \ldots, x_N)\}$ the Allen-Uzawa partial elasticity of substitution between factors i and j is defined by

$$\sigma_{ij} = \frac{C_H(f_{ij})}{x_i x_j |H|} \sum_{k=1}^{N} x_k f_k \quad ,$$

in which f_k, f_{ij} respectively are the first and second derivatives of f, and where H is the Hessian of f bordered by marginal products f_i, and where $C_H(f_{ij})$ is the cofactor of f_{ij} in H.

2. See Berndt & Christensen (1974) p. 392.

problem can be stated as :

$$\text{Minimize} \quad \sum_{i=1}^{n} W_i L_i \tag{4}$$

where

W_i = exogenously specified wage rate payable to skill group i ,

subject to the production function (3b).

The skill group demand equations are used to estimate the ESS.

The specification and use of the production function of the form (3b) to estimate the ESS can be described as the direct production function approach. We will also make use of aspects of Duality Theory to derive the skill group demand equations from an indirect production function.

Duality Theory states that, under certain regularity conditions (concerning homogeneity and concavity), a production function defines a unique cost function satisfying the same regularity conditions, and <u>vice versa</u>. Further, using either Shepherd's Lemma in the case of a cost function or Roy's Identity for an indirect production function, a set of factor demand equations can be derived by differentation (as opposed to solving a constrained minimization problem involving the production function).[1]

This method of deriving factor demand relationships can be described as the indirect production function approach. Corresponding to some production function of the form (3b) there exists by Duality Theory a cost function of the form :

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1. See, for example, Diewert (1974) and Woodland (1976).

$$C = H(W_1,W_2,\ldots,W_n;L) \tag{5}$$

where

$$C = \text{total labour costs} ,$$

which, for fixed L, is concave and homogenous of degree one in wages, W_i, and for fixed wages, is homogeneous of degree one in L.

The identity

$$H(W_1/C,W_2/C,\ldots,W_n/C;L) = 1 \tag{5a}$$

defines an indirect reciprocal production :

$$\frac{1}{L} = g\left(\frac{W_1}{C},\ldots,\frac{W_n}{C}\right) \tag{5b}$$

which is uniquely associated with the cost function (5) .

In what follows we make use of both approaches. We derive skill group demand relationships by :

(a) defining a production function and solving a constrained minimization problem, and

(b) defining a reciprocal indirect production function (corresponding to some other production function) and using Roy's Identity.

3. THEORETICAL SPECIFICATION OF TWO ALTERNATIVE MODELS OF LABOUR AGGREGATION

To estimate the various ESS, we use two generalizations of the CES model proposed by Hanoch. These are the Constant Ratio of Elasticity of Substitution Homothetic (CRESH) model (Hanoch (1971), a generalized CES production function and the Constant Difference Elasticity of Substitution Homothetic (CONDESH) model (Hanoch (1975), a generalized CES cost function.

Although the CRESH and CONDESH models exhibit the same regularity conditions (homogeneity, homotheticity, concavity) and their functional forms are similar, the production technology subsumed in each model is not the same. This is manifested in different substitution restrictions imposed in each of the models. Whereas CRESH imposes constant ratios of elasticities of substitution, CONDESH imposes constant differences between elasticities of substitution.

The CONDESH model is the dual of a production function whose explicit functional form is not known exactly. However, we do gain some idea from the regularity conditions and from the substitution restrictions. The CONDESH model has the advantage over the CRESH model that it is more flexible in its representation of substitution possibilities (at least with respect to the representation of complementarity), but at the same time preserves equal parsimony in parameter requirement.[1]

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1. Other approaches based on the cost function such as the translog cost function are even more flexible in terms of substitution restrictions but suffer the disadvantages of much greater uncertainty about the production technology subsumed and more severe parameter requirements. Whereas a flexible functional form would require estimation of $\frac{1}{2}n(n-1)$ separate substitution parameters in an n factor model, CRESH and CONDESH require only n .

In this paper, we specify the production function of the form (3b) as being CRESH and the recriprocal indirect production function of the form (5b) as being CONDESH. We now derive the skill group demand relationships implicit in each model.

A methodology for obtaining a unique set of n substitution parameters for an n factor input CRESH production model has been developed recently by Ryland and Vincent (1977). The CRESH production function can be defined implicitly as :

$$\sum_{i}^{n} A_i \left[{}^{L_i}\!/_{L}\right]^{q_i} \equiv 1.0 \qquad (6)$$

subject to the parameter restrictions

$$A_i \geq 0 \qquad (6a)$$

and

either $q_i \leq 0$ all i or $0 < q_i < 1$ all i . (6b)

where

$\{A_i\}$ = distribution parameters ,

$\{q_i\}$ = substitution parameters .[1]

Assuming cost minimization on the part of producers, the objective function (4) is minimized subject to the production function (6).

The solution to this constrained minimization problem provides the following n first order conditions.

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1. The parameter restrictions include the possibility of a zero substitution parameter ($q_i = 0$) for any factor, in which case the corresponding term of (6) is of the form $B_i \log(L_i/L)$ where B_i is a composite term with the same sign as $A_i q_i$. (See Dixon, Vincent and Powell (1976.))

$$W_i = \lambda\, q_i\, A_i \left[\frac{L_i}{L}\right]^{q_i - 1} . \, 1/L \qquad (i = 1,\ldots,n) \quad , \tag{7}$$

where

λ = Lagrangean multiplier or shadow wage costs.

Eliminating λ, the (n-1) expansion path equations may be represented as

$$\frac{W_i}{W_j} = \frac{q_i}{q_j} . \frac{A_i}{A_j} \left[\frac{L_i}{L}\right]^{q_i - 1} \left[\frac{L_j}{L}\right]^{-(q_j - 1)} \qquad (i \neq j) \quad ; \tag{8}$$

or, by taking logarithmic differentials and denoting a lower case letter as the percentage change in its upper case counterpart,

$$\ell_i = \left[\frac{1}{q_i - 1}\right](w_i - w_j) + \left[\frac{q_j - 1}{q_i - 1}\right]\ell_j + \left[1 - \frac{q_j - 1}{q_i - 1}\right]\ell \quad . \tag{9}$$

Linearization of (6) by taking log differentials gives the identity[1]

$$\sum_{i=1}^{n} S_i^* \ell_i = \ell \quad , \tag{10}$$

where

$$S_i^* = \frac{W_i L_i}{\sum_{j=1}^{n} W_j L_j} = \frac{q_i A_i (L_i/L)^{q_i}}{\sum_{j=1}^{n} q_j A_j (L_j/L)^{q_j}} \quad . \tag{11}$$

1. See Dixon, Vincent and Powell (1976).

The (n-1) linear expansion path equations in (9) and the identity (10) form a closed CRESH production system.[1]

The n substitution parameters, q_i, provide the basis for estimating the ESS. The ESS between occupation i and occupation j (σ_{ij}) is calculated as

$$\sigma_{ij} = -\frac{1}{q_i-1} \cdot \frac{1}{q_j-1} \frac{1}{\sum_{k=1}^{n} \frac{1}{q_k-1} S_k^*} . \tag{12}$$

The constant ratio property is given by the fact that

$$\frac{\sigma_{ij}}{\sigma_{km}} = \frac{(q_k-1)}{(q_i-1)} \frac{(q_m-1)}{(q_j-1)} , \tag{13}$$

which, because it contains only i,j,k,m substitution parameters, implies that the ratio of the elasticities of substitution between any two pairs of factors is constant at every point in the input-output space.

Turning now to the CONDESH model, we follow Hanoch (1975) and hypothesize the following functional form for the indirect reciprocal production function :

$$L.g(W_i^*) = G(W_i^*,L) = \sum_i^n B_i L^{\varepsilon_i(1-\alpha_i)} W_i^{*(1-\alpha_i)} \equiv 1 , \tag{14}$$

1. In choosing among the $\binom{n}{2}$ possible expansion paths, care is needed to select an appropriate set of (n-1) paths. For guidance on this point, see Vincent, Dixon and Powell (1977).

subject to the parameter restrictions

$$B_i \; \varepsilon_i \geq 0 \quad , \tag{14a}$$

and either

$$\alpha_i \leq 0 \text{ for all } i \text{ or } 0 < \alpha_i < 1 \text{ for all } i \; , \tag{14b}$$

where

$\{B_i\}$ = distribution parameters ;

$\{\alpha_i\}$ = substitution parameters ;

$\{\varepsilon_i\}$ = expansion parameters ;

$\left\{W_i^*\right\} = \left\{W_{i/C}\right\}$ = normalized wages in skill group i .

Since we are concerned with homothetic generalizations of CES which exhibit constant returns to scale, we impose $\varepsilon_i = 1$, in which case (14) assumes the CONDESH functional form. This function possesses the same regularity conditions as the CRESH function.

Rather than solve a constrained minimization problem we apply Roy's Identity to (14) to produce n labour demand equations in n substitution parameters. Roy's Identity gives

$$L_i \left\{W_i^*, L\right\} = \frac{G'(W_i^*, L)}{\sum_{i=1}^{n} W_i^* \, G'(W_i^*, L)} \tag{15}$$

where

$L_i \left\{W_i^*, L\right\} = L_i$ expressed as a function of W_i^* and L ,

and

$$G'(W_i^*, L) = \frac{\partial \left[G(W_i^*, L)\right]}{\partial W_i^*} \quad . \tag{16}$$

Combining (14), (15), (16),

$$L_i = \frac{B_i \; L^{(1-\alpha_i)} \; (1-\alpha_i) \; W_i^{*(-\alpha_i)}}{\sum_{j=1}^{n} B_j \; L^{(1-\alpha_j)} \; W_j^{*(1-\alpha_j)} (1-\alpha_j)} \qquad (i = 1,\ldots,n) \; . \qquad (17)$$

In differential log form, (17) becomes

$$\ell_i = (1-\alpha_i)\,\ell - \alpha_i w_i^* - D \quad , \qquad (18)$$

where

$$D = d\,\ell n \left[\sum_{j=1}^{n} B_j \; L^{(1-\alpha_j)} \; W_j^{*(1-\alpha_j)} (1-\alpha_j) \right] \; . \qquad (18a)$$

D can be eliminated by subtraction to give :

$$\ell_i - \ell_j = (\alpha_j - \alpha_i)\,\ell - \alpha_i w_i^* + \alpha_j w_i^* \; . \qquad (19)$$

We can make a further substitution for w_i^* :

$$w_i^* = d\,\ell n\, W_i^* = d\,\ell n \left(W_i / C \right) = w_i - c \; .$$

Thus the $(n-1)$ expansion path equations are of the form :

$$\ell_i = \ell_j + (\alpha_j - \alpha_i)\,\ell - \alpha_i w_i + \alpha_j w_j + (\alpha_i - \alpha_j) c \, . \; . \qquad (20)$$

The specification of the nth equation to close the system (20) is more difficult in the case of CONDESH, since the explicit (direct) production function is not known. However, we would be astounded if (10) did not provide an adequate approximation to the appropriate index number for L in most samples likely to be encountered in practice. Accordingly, in pragmatic spirit, we use (10) to close the system, but as a stochastic equation rather than as an identity.

The constant difference of substitution elasticities property can be shown after first presenting the formula for the Allen-Uzawa partial elasticity of substitution (from Hanoch (1975) p. 409).

$$\sigma_{ij} = \alpha_i + \alpha_j - \sum_{k=1}^{n} \alpha_k S_k^* \quad , \tag{21}$$

for $i \neq j$.

The difference (D_{ij}^{km}) between the substitution elasticities between any two pairs of factors $\{i,j\}$ and $\{k,m\}$ is given by

$$D_{ij}^{km} = \alpha_i + \alpha_j - \alpha_k - \alpha_m \tag{22}$$

at all points in the input-output space.

Because of the constant difference rather than constant ratio property, CONDESH is considerably more flexible than CRESH. As a specific example, it more readily allows the case of complementarity between factors for $n \geq 3$. This would arise when the sum of α_i and α_j is smaller than the weighted mean of substitution parameters

$$\left(\sum_{k=1}^{n} \alpha_k S_k^*\right) .$$

4. ALTERNATIVE SPECIFICATION OF DISAGGREGATED LABOUR DEMAND

In the above discussion we have focused on the expansion path equations (9) in the case of CRESH and (19) in the case of CONDESH. It is also possible to obtain final forms of the systems suitable for estimation directly from the first-order conditions.

In log differential form the first-order conditions for the CRESH model may be written as :

$$w_i = \lambda^* + (q_i - 1) \quad (\ell_i - \ell) \quad . \tag{23}$$

On multiplying (23) through by $\sum_i S^*_{i/(q_i-1)}$ and using (10), Dixon, Vincent and Powell (1976) show that

$$\lambda^* - \ell = \sum_i \frac{S^*_{i/q_i-1} \, w_i}{\sum_i S^*_{j/q_j-1}} = \sum_i S_i \, w_i$$

where

$$S_i = \frac{S^*_{i/(q_i-1)}}{\sum_j S^*_{j/(q_i-1)}}$$

so that (23) becomes the n equation system :

$$\ell_i = \ell + \frac{1}{q_i-1} \left(w_i - \sum_j^n S_j \, w_j \right) \quad . \tag{24}$$

The last term of the CONDESH model in log differential form from Roy's Identity (18) may be written as :

$$D = d\,\ell n \left[\sum_i B_i L^{(1-\alpha_i)} \quad (1-\alpha_i) \quad W_i^{*(1-\alpha_i)} \right] \tag{25}$$

$$= \sum_i \frac{B_i L^{(1-\alpha_i)} \quad (1-\alpha_i) \quad W_i^{*(1-\alpha_i)} \quad (\ell + w_i^*)}{\sum_j B_j L^{(1-\alpha_j)} \quad (1-\alpha_j) \quad W_j^{*(1-\alpha_j)}} \quad ,$$

$$= \sum_i S_i^* \, (1-\alpha_i) \quad (\ell + w_i^*) \quad ;$$

and since from the indirect production function (14) ,

$$\sum_i S_i^* \, (\ell + w_i^*) \quad = \quad 0 \quad , \tag{26}$$

Roy's Identity in log differential form (18) becomes :

$$\ell_i \; = \; (1-\alpha_i)\,\ell - \alpha_i\, w_i^* + \sum_j S_j^* \, \alpha_j \, (\ell + w_j^*)$$

$$= \ell - \alpha_i\, \ell - \alpha_i\, w_i^* + \sum_j S_j^* \, \alpha_j \, \ell + \sum_j S_j^* \, \alpha_j \, w_j^*$$

$$= \ell \left\{ 1 + \sum_j S_j^* \, \alpha_j - \alpha_i \right\} + \sum_j S_j^* \, \alpha_j \, (w_j - c) - \alpha_i \, (w_i - c). \tag{27}$$

Using a substitution for ℓ based on (26), (27) can be written as :

$$\ell_i = \ell + \left[S_i^* \left(2\alpha_i - \sum_k S_k^* \alpha_k\right) - \alpha_i \right] w_i^*$$

$$+ \sum_{j \neq i} S_j^* \left[\alpha_i + \alpha_j - \sum_k S_k^* \alpha_k \right] w_j^* \quad . \qquad (28)$$

The coefficients of ℓ , w_i^* and w_j^* $(j \neq i)$ are respectively unity and the output-compensated own price elasticity $(\eta_{ii} \equiv S_i^* \sigma_{ii})$ and cross price elasticities $(\eta_{ij} \equiv S_j^* \sigma_{ij};\ j=1,\ldots, n;\ j \neq i)$.

In the CONDESH system proportionate changes in both total costs, c , and aggregate labour demand, ℓ , appear as right hand variables in the system. Clearly the factor demand system is over-determined since only one of either total factor costs or output may be treated exogenously in a competitive factor demand analysis. Where below both c and ℓ appear in the list of 'exogenous variables' it is to be understood that whichever of the two is (strictly speaking) endogenous will be purged of unwanted covariation with structural form errors before estimation is carried out. This will overcome the simultaneity bias which would otherwise result.

5. ECONOMETRIC SPECIFICATION

The (n-1) expansion path equations of the CRESH model (9) together with its production function (10) constitute a system of n simultaneous equations in which logarithmic differentials of labour demands for each of the n occupational groups are endogenous. Similarly the (n-1) expansion path equations of the CONDESH model (20) together with (10) constitute a similar system if we are prepared to use the latter as an approximation to the (unknown) functional form of the production function underlying (17). These equations may be written using matrix notation as follows :

$$B \underline{\ell}_p + C\underline{z}_p = \underline{u}_p \quad (p = 1,\ldots, \text{ K observations}) \; ; \; B \neq 0 \; ; \qquad (29)$$

where the variables underlined are column vectors and where

$$\underline{\ell}_p = (\ell_{1p}, \ell_{2p}, \ldots, \ell_{np})'$$

is a n:1 column vector of endogenous labour demands ;

$$\underline{z}_p = (\dot{\ell}_p, w_{1p}, w_{2p}, \ldots, w_{np}, c_p)'$$

is a n+2:1 column vector of exogenous variables and

$$\underline{u}_p = (u_{1p}, u_{2p}, \ldots, u_{np})'$$

is a n:1 column vector of residuals .[1]

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1. In certain cases the production function (10) may be treated as an identity and entered as the first row of equation (29). In this case, u_{1p} vanishes.

For both the CRESH and CONDESH specifications the first rows of the coefficient matrix $[B|C]$ are the same, representing the equality between the factor-share weighted sum of the endogenous variables and the proportional change in total labour requirements (as in (10)). Under the CRESH specification of the expansion paths, the matrix C becomes

$$C^{EP}_{CRESH} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & \cdots 0 & 0 & 0 \\ \Theta_1-1 & -\beta & \beta & 0 & 0 & \cdots 0 & 0 & 0 \\ \Theta_2-1 & 0 & -\beta/\Theta_1 & \beta/\Theta_1 & 0 & \cdots 0 & 0 & 0 \\ \Theta_3-1 & 0 & 0 & -\beta/\Theta_1\Theta_2 & \beta/\Theta_1\Theta_2 & \cdots 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \\ \Theta_{n-1}-1 & 0 & 0 & 0 & 0 & \cdots \frac{-\beta}{\Theta_1\Theta_2\cdots\Theta_n} & \frac{-\beta}{\Theta_1\Theta_2\cdots\Theta_n} & 0 \end{bmatrix} \quad (30)$$

which is a $n{:}n+2$ matrix of parameters in which $\beta = 1/(q_1-1)$,

$$\Theta_j = \frac{q_{j+1} - 1}{q_j - 1} \quad (j = 1,\ldots,n-1)$$ and in which the last column is zero.

In the CONDESH case,

$$
C^{EP}_{CONDESH} = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
-(\alpha_2-\alpha_1) & \alpha_1 & -\alpha_2 & 0 & 0 & \cdots & 0 & 0 & (\alpha_2-\alpha_1) \\
-(\alpha_3-\alpha_2) & 0 & \alpha_2 & -\alpha_3 & 0 & \cdots & 0 & 0 & (\alpha_3-\alpha_2) \\
-(\alpha_4-\alpha_3) & 0 & 0 & \alpha_3 & -\alpha_4 & \cdots & 0 & 0 & (\alpha_4-\alpha_3) \\
\cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot \\
-(\alpha_n-\alpha_{n-1}) & 0 & 0 & 0 & 0 & \cdots & \alpha_{n-1} & -\alpha_n & (\alpha_n-\alpha_{n-1})
\end{bmatrix} \tag{31}
$$

is a n:n+2 matrix of parameters associated with the expansion paths (20) .

In both models B is a square n:n matrix with

$$
B^{EP}_{CRESH} = \begin{bmatrix}
S^*_1 & S^*_2 & \cdots & S^*_{n-1} & S^*_n \\
1 & -\Theta_1 & & \underline{\underline{0}} & \\
 & 1 & -\Theta_2 & & \\
 & \underline{\underline{0}} & & & \\
 & & & 1 & -\Theta_{n-1}
\end{bmatrix} \tag{32}
$$

and

$$B^{EP}_{CONDESH} = \begin{bmatrix} S^*_1 & S^*_2 & \cdots\cdots & S^*_{n-1} & S^*_n \\ 1 & -1 & & \underline{\underline{0}} & \\ & 1 & -1 & & \\ & \underline{\underline{0}} & & & \\ & & \cdots\cdots & 1 & -1 \end{bmatrix} , \quad (33)$$

where $S^*_j = W_j L_j / \sum_j W_j L_j$, $S^*_j > 0$, $\sum_j S^*_j = 1$.

If the contemporaneous variance-covariance matrix $\Sigma_n = E\left[\underline{u}_p \underline{u}'_p\right]$ is assumed to be stationary and non-singular and $\underline{u}_p$ follows a multivariate normal distribution, then application of FIML methods to the structural form (29) is straightforward. On the other hand if the production function (10) is assumed to hold as an identity, Σ_n is singular but the likelihood function in this case can be expressed in terms of its (non-singular) leading principal minor. Consequently the parameter estimates will vary depending upon the particular stochastic specification adopted.

The factor demand equations in logarithmic differentials can also be written in the matrix format (29). In this notation the coefficient matrix (B) corresponding to the CRESH and CONDESH model is an identity matrix, while the matrix C for CRESH is given by :

$$C^{FD}_{[CRESH]} = \begin{bmatrix} -1 & -\phi_1\ (1-S_i\ ) & \phi_1\ S_2 & \cdots & \phi_n\ S_n\ , & 0 \\ -1 & \phi_2\ S_1 & -\phi_2(1-S_2\ ) & \cdots & \phi_n\ S_n\ , & 0 \\ \vdots & & & & & \\ -1 & \phi_n\ S_1 & \phi_n\ S_2 & \cdots & -\phi_n(1-S_n\ ), & 0 \end{bmatrix} \qquad (34)$$

where

$$\phi_j = \frac{1}{q_j-1} \qquad (34a)$$

and

$$S_j = \frac{S^*_j\ \phi_j}{\sum_k S^*_k\ \phi_k}\ . \qquad (34b)$$

In CONDESH the analagous restrictions from (27) are :

$$C^{FD}_{[CONDESH]} = \left[\begin{array}{ccccc:c} -(1-\alpha_1+K) & \alpha_1(1-S^*_1) & -S^*_2\ \alpha_2 & \cdots & -S^*_n\ \alpha_n & (K-\alpha_1) \\ -(1-\alpha_2+K) & -S^*_1\ \alpha_1 & \alpha_2(1-S^*_2) & & -S^*_n\ \alpha_n & (K-\alpha_2) \\ \vdots & & & & & \\ -(1-\alpha_n+K) & -S^*_1\ \alpha_1 & -S^*_2\ \alpha_2 & \cdots & \alpha_n(1-S^*_n) & (K-\alpha_n) \end{array}\right] \qquad (35)$$

where

$$K = \sum_j S_j^* \alpha_j . \quad . \qquad (35a)$$

The coefficient matrices C for both models contain shares and parameters. However, for the CRESH model the modified shares S_j are functionally dependent upon the parameters. This functional dependence is accommodated by explicit non-linear restrictions mapping the parameters to the coefficients of the structural form (29). The FIML package coded by Wymer (1973) permits maximization of the likelihood function subject to a wide variety of differentiable constraints between coefficients and parameters. Consequently little difficulty should be experienced computationally in obtaining parameter estimates using both CRESH and CONDESH models.

6. LABOUR SUPPLY CONSIDERATIONS

In the previous sections we have analysed the demand for labour of different skills from the viewpoint of producers who are concerned with cost minimization. In this section our concern is the aggregate actions of individuals who supply labour and so determine the supply schedules for labour of different skills and hence the skill composition of the workforce. It may be argued that, even if in the long run different skills are not perfectly substitutable on the demand side, they nevertheless can always be produced in the long run with relatively high supply elasticities. Given the long leads and lags in the production and utilization of different skills however, a short-run supply schedule would probably exhibit relatively low supply elasticities for labour, particularly at an occupationally disaggregated level.

The production processes which lead to the acquisition of different skills or amounts of human capital relate to the inherent gene pool endowment of the nation, outputs of the public and private education systems, migration, occupational choice and post-school investment in human capital or 'on-the-job' training. All these input processes in the short run operate simultaneously to define a labour market opportunity locus or skill transformation frontier for the economy as a whole. The positions actually adopted <u>ex-post</u> on this locus are manifested in the skill composition of the workforce. While changing attitudes to different types of work and the accumulation of particular skills through public and private investments in education shift the skill transformation frontier over time, it is our maintained hypothesis that at each point in time, the myriad of individual occupational choices being made throughout the workforce results in the real after-tax aggregate wage bill attaining a maximum. This is tantamount to modelling the economy's response in terms of that of a representative labour supplier who is introduced as a convenient fiction in order to generate a plausible model of aggregate labour market behaviour. The individual agent, however, has limited scope for occupational diversification; he cannot, for example, work 0.2 hours as a brain surgeon, 2.0 as an office manager, 3.5 as a teacher and so on. The representative agent, by contrast, can allocate his total working hours in whatever proportions are necessary to maximize his synthetic welfare index. The total time available for work is obtained from the standard model of labour leisure choice.[1]

From this point we shall treat the total supply of labour, $\bar{\bar{L}}$, as pre-determined. It is therefore our task to determine how average wage income, $\Pi = \sum_i W_i \bar{L}_i + \bar{Y}_o$, can be maximized by the representative agent by allocating his fixed hours of work, $\bar{\bar{L}}$, among alternative skills, $\bar{L}_i$, in order to determine the income maximizing skill composition.

Thus the problem confronting the representative agent is

$$\text{Max} \quad \Pi = \bar{Y}_o + \sum_i W_i \bar{L}_i \tag{37}$$

1. See, for example, Walter Oi (1975) for a survey of the neo-classical theory of labour supply.

subject to

$$\bar{\bar{L}} = h(\bar{L} = h(\bar{L}_1, \bar{L}_2, \ldots, \bar{L}_n) \quad . \tag{38}$$

The form of the constraint (38) needs some explanation. A simple form for h might be

$$\bar{\bar{L}} = h(\bar{L}) = \sum_{i=1}^{n} \bar{L}_i \quad , \tag{39}$$

in which case the total supply of hours worked is just the sum of the individual numbers of hours worked in each occupation. Such a simple linear transformation surface would give corner solutions and would hold no interest for our paradigm. It follows that $\bar{\bar{L}}$ should not be measured in nominal man-hours, but in units reflecting the relative short run marginal social costs of securing the services of workers in different occupations. Lacking, as we do, information on the n-dimensional shape of the skill transformation frontier h( ), it is important to use a functional form which is at the same time both flexible enough to accommodate a relatively wide range of empirical possibilities and sufficiently parsimonious in its use of parameters to allow the prospect of econometric estimation. One apparently suitable candidate is the CRETH functional form,

$$\sum_i A_i \, (\bar{L}_i/\bar{\bar{L}})^{k_i} \equiv 1 \qquad A_i \geq 0, \quad k_i > 1 \quad . \tag{40}$$

The first order conditions for a maximum of (40) subject to (42) may be expressed as :

$$W_i = \lambda k_i A_i (\bar{L}_i/\bar{\bar{L}})^{k_i-1} (1/\bar{\bar{L}}) \quad ; \qquad (41)$$

and (following Dixon, Powell and Vincent (1976) equation (41) may be expressed in log differential form as :

$$\bar{\ell}_i^{\,s} = \bar{\bar{\ell}} + 1/(k_i-1)\left(W_i - \sum_j G_j W_j\right) \quad , \qquad (42)$$

where

$$G_i = \frac{\dfrac{W_i \bar{L}_i}{k_i-1} \Big/ \sum_j W_j \bar{L}_j}{\sum_i \left(\dfrac{W_i \bar{L}_i}{k_i-1} \Big/ \sum_j W_j \bar{L}_j\right)}$$

In this form $\bar{\ell}_i^{\,s}$ is the log differential of the <u>average</u> time spent by the representative agent in occupation i and since the CRETH function is homogenous of degree one the log differential of total changes in the time spent by the workforce in occupation i is

$$\ell_i^{\,s} = \ell + 1/(k_i-1)\left(W_i - \sum_j G_j W_j\right) \quad . \qquad (43)$$

Apart from the construction of the shares, equation (43) is similar to the CRESH demand specification (24) and the CRETH supply parameters k_i may be estimated in a similar fashion to that discussed in previous sections.

Since CRETH is a direct transformation function analagous to the CRESH production function it is natural to enlarge the set of candidates for the functional form of h ( ) . We specify a reciprocal indirect transformation function called CONDETH analagous to the CONDESH reciprocal indirect production function as :

$$\sum_i C_i \,\bar{L}^{(1-\beta_i)}\, \bar{W}_i^{(1-\beta_i)} \equiv 1 \qquad (C_i \geq 0,\ \beta_i > 1) \tag{44}$$

where

$$\bar{W}_i = W_i/\Pi \quad .$$

Applying Roy's Identity to (44) gives the set of labour supply relationships similar to (18) which in log differential form are :

$$\bar{\ell}_i^{\,s} = (1-\beta_i)\bar{\ell} - \beta_i\bar{w}_i - d\log\left[\sum \bar{W}_i C_i \bar{\bar{L}}^{(1-\beta_i)}(1-\beta_i)\,\bar{W}_i^{\beta_i}\right] \tag{45}$$

and since the CONDETH function is homothetic, implying that $\left(\bar{\ell}_i^{\,s} - \bar{\bar{\ell}}\right)$ $=(\ell_i^{\,s} - \ell^s)$, (42) may be written as

$$\ell_i^{\,s} = (1 - \beta_i + \sum_j G_j^*\beta_j)\ell + \sum_j G_j^*\beta_j\bar{w}_j - \beta_i\bar{w}_i \quad . \tag{46}$$

where

$$G_i^* = W_i\bar{L}_i \;{}_j\; W_j\bar{L}_i$$

Equation (46) is similar to (27) apart from the construction of shares G_j and the normalization $\bar{w}_i = w_i - \pi^*$ where π^* is average income. Hence the CONDETH supply function similar to the CONDESH demand function for each occupation may be expressed as

$$\ell_i^s = (1 - \beta_i + \sum_j G_j^* \beta_j)\ell + \sum_j G_j^* \beta_j w_j$$

$$- \beta_i w_i + (\beta_i - \sum_j G_j^* \beta_j)\pi^* \quad . \qquad (47)$$

The coefficients $\beta_i (i = 1,\ldots,n)$ define n skill transformation parameters from which may be derived $\frac{1}{2} n(n - 1)$ skill transformation elasticities (STE). These elasticities may be used to infer changes in relative supplies of each occupational groups resulting from changes in relative earnings.

A high STE between occupations indicates that the representative labour supplier responds readily to changes in relative earnings, because of the ease with which particular skills may be acquired or with which existing skills may be transformed into others. By contrast a low STE between occupations indicates that relative earnings do not significantly influence labour supplies because other considerations such as job security, opportunity costs of skill acquisition, availability of re-training schemes and institutionalized barriers to entry in selected skills diminish the ease with which alternative skills are created.[1]

1. At this point it is worth noting that our view of the labour market emphasizes variables associated with the orthodox human investment theory of supply of labour in contrast to the segmented labour market (SLM) theories recently reviewed by Cain (1976). The core of the SLM theories is that wages are rigid and nearly irrelevant for explaining the number and type of job positions actually filled. In the SLM theories, job competition, discrimination and workers' attitude to work in particular occupations are more important determinants of skill mix.

For purposes of man-power planning, skills with relatively high STE may be acquired easily through re-training and/or on the job training. In order to project labour demand by occupation reliably, man-power specialists will need to know ESS. If the ESS are high it will be necessary to project relative wages before occupational demands can be projected. On the other hand, skills with relatively low STE and ESS indicate that relative earnings may be a poor guide for projecting the occupational distribution.

7. RESEARCH PERSPECTIVES

Our task in this paper has been to specify operationally oriented systems of demand and supply equations suitable for the analysis of an occupationally disaggregated labour market.

The question of whether the labour market clears or not - and if it does not, how the disequilibrium should be handled - has been left completely open in this paper.[1] Irrespective of how the disequilibrium may be accommodated, the functional specifications for labour demand and supply posited above should prove useful. This is because they embody considerable flexibility for substitution and transformation among skills on the demand and supply sides respectively without involving an unmanageably large number of parameters.

Empirical implementation might proceed in stages. Our research strategy in increasing order of ambition might be -

(i) Estimation of the demand or supply side alone assuming an equilibrium market clearing hypothesis with sufficiently strong priors on the side of the market not estimated in order to identify the parameters of whichever structural relationships (supply/demand) is actually estimated.

(ii) Estimation of a complete labour demand/supply system of structural equations assuming an equilibrium market.

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1. Several approaches to handling disequilibrium in an aggregate labour market have recently been discussed by Rosen and Quandt (1977). Two basic approaches (in the context of commodity markets) have been suggested in earlier works by Fair and Jaffee (1972) and Maddala and Nelson (1974). The former study proposes deterministic and the latter stochastic methods of partitioning the sample space into regions of excess supply and excess demand.

(iii) Estimation of a complete labour supply/demand system of structural equations in the light of a behavioural specification of disequilibrium dynamics in the labour market.

Various refinements and extensions of course are possible even within the individual demand and supply equations themselves. Our maintained separability and aggregation hypotheses avoid the important issue of the extent to which substitution elasticities differ between classes of labour and capital at our level of aggregation. While a more sophisticated specification of the production technology could lead to a more general (and therefore more plausible) set of labour demand functions it is doubtful whether a data base sufficiently comprehensive for estimation exists in Australia.

On the supply side, risk (principally risk of unemployment) could be introduced perhaps by replacing the relative wage variables by their certainty equivalents. In our supply theory we have not addressed the longer run issue of the shift of the skill transformation frontier over time. Whilst for short-run analysis the location of the frontier might be treated exogenously, long-run analysis would require a model of the behaviour of educational decision-makers, pressure groups and others who are responsible for determining the amounts and allocation of investments among the various streams of academic and vocational training. Hopefully these (and other) issues can be taken up in later work.

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