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A Commonwealth Government inter-agency project in co-operation with the University of Melbourne, to facilitate the analysis of the impact of economic demographic and social changes on the structure of the Australian economy

ESTIMATES OF THE CRETH SUPPLY SYSTEM IN AUSTRALIAN AGRICULTURE

by

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David P. Vincent, Peter B. Dixon & Alan A. Powell*

1. INTRODUCTION

In an earlier paper we described the theoretical properties of the CRETH/CRESH production system in detail. A strategy for estimation of the system from BAE Australian Sheep Industry Survey (ASIS) data, and salient features of those data, were also outlined in the earlier paper.[1] To recapitulate briefly, we chose the CRETH (constant ratio of elasticities of transformation, homothetic) transformation frontier as a suitable basis for product supply equations because such a specification makes it possible to deal realistically with joint production in the overwhelmingly multi-product environment which pertains in Australian agriculture. In the light of its greater flexibility relative to the CES alternative in a situation where more than two inputs are distinguished, we chose the CRESH[2] (constant ratio

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* We are especially grateful to the Director and officers of the Bureau of Agricultural Economics for their generous co-operation concerning the supply and interpretation of the data series.

1. Peter B. Dixon, David P. Vincent and Alan A. Powell, "Factor Demand and Product Supply Relations in Australian Agriculture : The CRESH/CRETH Production System", Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. OP-08, Industries Assistance Commission, Melbourne, November, 1976 (mimeo), pp. 41.

2. Giora Hanoch, "CRESH Production Functions", Econometrica, Vol. 39, No. 5 (September, 1971), pp. 695-712.

of elasticities of substitution, homothetic) specification as a suitable basis for modelling input demands. In the event, the amount of independent variation in inputs and input prices occurring in the sample period (1952-53 through 1973-74) proved insufficient to estimate the isoquant maps (i.e., CRESH), so that in this paper we report results for the product transformation frontiers (i.e., CRETH) alone.[1]

We report separate estimates for the Pastoral Zone, Wheat/Sheep Zone, and High Rainfall Zone of the ASIS. These involve respectively disaggregations of output into three, six, and four products; the corresponding numbers of different pair-wise partial transformation elasticities estimated in the three zones are three, fifteen and six, respectively.

The plan of the rest of the paper is as follows. In Section 2 we write down the structural and (approximate) reduced forms of CRETH, which are linear in logarithmic differentials of the variables, but highly non-linear in the behavioural parameters. In Section 3 we give a brief description of the time series data used. The variables proper to an economic theory of supply are planned output and expected prices. Our method for reconciling these concepts with observed data is also documented in this section. In Section 4 we comment briefly

1. In the context of Australia-wide aggregate data (rather than the ASIS zonal data considered here) and a three input split, Ryland and Vincent have successfully estimated CRESH. See G.J. Ryland and D.P. Vincent, "Empirical Estimation of the CRESH Production Function", Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. OP-12, Industries Assistance Commission, Melbourne, June, 1977 (mimeo) pp. 14. In that paper however, all factors were treated as variable in the short run, whereas in the current paper land and capital were treated as fixed in the short run.

This suggested that our model was quite incapable of estimating the input substitution parameter. To investigate the influence of the substitution parameter on the product transformation parameters, we have respecified the model treating h as a known constant with a 'neutral' value (0.5). The resultant k_i parameters were : k_1 (4.30), k_2 (4.68), k_3 (3.73), k_4(1.90), k_5 (2.80), k_6 (1.78). A likelihood ratio test was carried out on the two results. The test involves calculating $-2(L_1 - L_2)$ where L_1 is the log likelihood value from the result in which the h parameter is estimated and L_2 the log likelihood value when h is treated as a known constant. The test value of 0.021 (X_1^2 = 6.63 at 1 per cent) indicates that our estimates of the transformation parameters are fairly robust despite the instability in the input substitution parameter.

III High Rainfall Zone

Parameter	Estimate	Ratio of estimated parameter to asymptotic standard error
k_1 wool	16.86	0.97
k_2 sheep	9.67	0.94
k_3 cattle	3.67	1.94
k_4 'other'	1.26	1.06

II Wheat/Sheep Zone (cont'd)

	Parameter	Estimate	Ratio of estimated parameter to asymptotic standard error
W_1	wheat quota dummy on wool output 1968/69 → 1969/70	0.033	0.74
W_2	wheat quota dummy on sheep output 1968/69 → 1969/70	0.096	0.96
W_3	wheat quota dummy on cattle output 1968/69 → 1969/70	0.262	0.97
W_4	wheat quota dummy on wheat output 1968/69 → 1969/70	-0.183	1.06
W_5	wheat quota dummy on barley output 1968/69 → 1969/70	0.215	0.86
W_6	wheat quota dummy on 'other' output 1968/69 → 1969/70	-0.155	0.66
W_7	wheat quota dummy on wool output 1972/73 → 1973/74	-0.145	1.66
W_8	wheat quota dummy on sheep output 1972/73 → 1973/74	0.075	0.49
W_9	wheat quota dummy on cattle output 1972/73 → 1973/74	-0.006	0.02
W_{10}	wheat quota dummy on wheat output 1972/73 → 1973/74	0.376	1.81
W_{11}	wheat quota dummy on barley output 1972/73 → 1973/74	-0.249	0.98
W_{12}	wheat quota dummy on 'other' output 1972/73 → 1973/74	0.098	0.39
C_1	constant in equation 1	-0.016	0.37
C_2	constant in equation 2	-0.013	0.14
C_3	constant in equation 3	-0.070	0.27
C_4	constant in equation 4	-0.062	0.27
C_5	constant in equation 5	-0.044	0.26
C_6	constant in equation 6	-0.104	0.43

1. In the result reported, the estimate of h has a large negative value and an extremely high standard error. Our experience with the data showed that the k_i estimates were comparatively stable under a range of starting values for S^* ; the h estimate, however, fluctuated wildly with an extremely large standard error.

(footnote continued)

on the estimation technique used, and follow in Section 5 with tabulations of our empirical results. In the final section we comment on the directions which seem to us to be desirable for future work in the area.

2. THE FORM OF THE SUPPLY EQUATIONS

2.1 The CRETH Structural Form

The structural form of the CRETH supply system, expressed in differentials of logarithms, is[1] :

$$y_i = z + \frac{1}{k_i - 1}\left\{p_i - \sum_{j=1}^{N} p_j S_j^*\right\} \qquad (i = 1,\ldots,N) , \tag{1}$$

in which y_i is the proportional change in the output of the i^{th} product; z is the proportional change in an index of overall capacity to produce; k_i is a parameter of the CRETH system; p_j is the proportional change in the j^{th} product price; N is the number of different products produced; and S_j^* is a weighted share of product j in total revenue, computed according to the formula :

$$S_j^* = \frac{1}{k_j - 1} S_j \Big/ \left[\sum_{\ell=1}^{N} \frac{1}{k_\ell - 1} S_\ell\right] , \tag{2}$$

1. Dixon, Vincent and Powell, op. cit., p. 19.

in which S_j is the actual share of product j in total revenue. The index z is simply the share-weighted average movement in output :

$$z = \sum_{j=1}^{N} S_j y_j \quad . \qquad (3)$$

2.2 The CRETH Reduced Form

Whilst the shares S_j are, strictly speaking, both variable and endogenous, the recognition of both of these features would render the system inestimable with currently available software. In the case of the Pastoral and the High Rainfall zones, we recognize the variability, and to some extent the endogeneity, of the shares, and fit the system {(1), (3)} by full information maximum likelihood (FIML). In the case of the Wheat/Sheep Zone, we have exploited the relatively greater stability of the product mix by treating the shares as exogenous constants -- this does not, we believe, introduce serious error. Using this approximation, the reduced form supply equations of the CRESH/CRETH system are linear in the variables, and may be written[1] :

$$y_i = \sum_{j=1}^{M-M^*} \ell_{ij} q_{1j} + \sum_{j=M-M^*+1}^{M} m_{ij} \bar{x}_{2j} + \sum_{j=1}^{N} c_{ij} p_j \qquad (1 = 1,\ldots,N) \ , \qquad (4)$$

in which $\{\ell_{ij};\ m_{ij};\ c_{ij}\}$ is the set of coefficients of the variables; q_{1j} $(j = 1,\ldots,M-M^*)$ represents the prices of the $(M-M^*)$ variable inputs; $\bar{x}_{2j}$ $(j = M-M^*+1,\ldots,M)$ represents the quantities of the M^*

1. Dixon, Vincent and Powell, op. cit., p. 37.

APPENDIX

CRETH Supply System : Parameter Estimates

I Pastoral Zone

	Parameter	Estimate	Ratio of estimated parameter to asymptotic standard error
k_1	wool/sheep	10.61	1.10
k_2	cattle	1.62	4.36
k_3	'other'	1.22	5.78
C_1	constant in equation 1	0.024	0.37
C_2	constant in equation 2	0.155	0.42

II Wheat/Sheep Zone

	Parameter	Estimate	Ratio of estimated parameter to asymptotic standard error
k_1	wool	4.36	3.13
k_2	sheep	5.27	1.11
k_3	cattle	2.93	1.71
k_4	wheat	1.62	6.74
k_5	barley	2.92	2.39
k_6	'other' products	1.76	6.60
h	input substitution parameter[1]	-88.1	0.10
D_1	coefficient on drought variable in equation 2 (sheep)	-0.345	2.81
D_2	coefficient on drought variable in equation 3 (cattle)	-0.321	1.40
D_3	coefficient on drought variable in equation 6 ('other')	0.145	0.48

the various agricultural products are modelled as if they are produced according to traditional single output production functions. The incorporation of CRETH, involving multi-output production functions, will, therefore, require modifications to ORANI's theoretical structure.

inputs which are fixed in the short run (land, capital and owner-operator's labour); and y_i and p_j are as previously.

Because of the restrictions on the CRESH/CRETH system, the $NM+N^2$ coefficients, ℓ, m, and c may be written as functions of only (M+N) parameters (where M is the number of inputs and N is the number of products). Since in this study we are concentrating on the product supply side, simplifications on the factor demand side can be used to reduce even further the number of parameters to be estimated. In particular, for the Wheat/Sheep Zone we aggregated land and capital into a single fixed factor, and adopted the CES specification on the input side. The number of "free" reduced form supply parameters condenses, as a result, to (N+1) .

The parametric constraints which bind the coefficients under these assumptions are, first, that

$$\ell_{ij} = W_j/[(h-1)W_M] = \ell_j \qquad (i = 1,\ldots,N, j = 1,\ldots,M-1) , \tag{5}$$

in which the ordering of the inputs places the fixed factor last (i.e., in position M), and in which the W's are shares of non-specific inputs in total (non-specific) cost,[1] and h is the common substitution parameter (related to the elasticity of substitution by $\sigma = 1/(1-h)$) ;

1. Expenditure on factors of production (e.g., shearing) which are fully product specific is first deducted from both revenue and from cost before any shares are computed. See Dixon, Vincent and Powell, op. cit., p. 10.

(6) $$m_{iM} \equiv 1, \quad i = 1,\ldots,N \; ;$$

and finally, that

(7a) $$c_{ij} = \frac{\delta_{ij}}{k_i - 1} - \frac{S_j^*}{(k_i - 1)} + \mu S_j \quad ,$$

where

(7b) $$\mu = (1 - W_M)/\left[W_M(h - 1)\right] \quad ,$$

and in which δ_{ij} is Kronecker's delta. In these equations -- that is, (5) and (7a & b) -- the values of the shares W and S are all treated as constants, being set at their sample mean values.

3. BRIEF DESCRIPTION OF THE DATA

3.1 Zonal Characteristics

We have been provided with a time series of the average values across the sample of key variables of interest in the BAE's ASIS spanning the period 1952/53 to 1973/74. The data have been analysed on a zonal basis with the zones classified broadly according to rainfall. This disaggregation is particularly important in the Australian environment where, because of climatic and biological influences,

set of price combinations viz. beef \$0.68/kg, wheat \$51.42/tonne, wool \$2.32/kg, the APMAA model yielded the following elasticities :

wool with respect to price of wheat -0.19 (versus -0.15, our Table 8);
wool with respect to price of beef -0.21 (versus -0.02, our Table 8);
wool with respect to price of wool +0.25 (versus +0.26, our Table 8).

These long run results provide collaborating evidence of the fairly high transformation possibilities between competing products on Australian farms.

An important argument in favour of using systems (rather than single equations) framework and full (rather than limited) information estimators, is that these methods provide results which are, statistically speaking, consistent and relatively efficient. One expects such a framework to track better over time and in particular to do a better job in modelling the response to small (but cumulatively important) price signals interspersed with large amounts of noise (originating predominately, but not wholly, in the climate). To test the success of our endeavour in this regard it would be necessary to follow up with comparative simulation studies in which our estimated system is matched against the simpler alternatives.

A final line of development (and the one to which we are giving top priority) is the incorporation of CRETH within IMPACT's general equilibrium model ORANI.[1] In the current version of ORANI,

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1. Peter B. Dixon, B.R. Parmenter, G.J. Ryland and John Sutton, ORANI, A General Equilibrium Model of the Australian Economy : Current Specification and Illustrations of Use for Policy Analysis -- First Progress Report of the IMPACT Project, Vol. 2 (Melbourne : Industries Assistance Commission, March, 1977).

outward shift of the production frontier resulting from changes in investment (and from technology) has so far proved intractable, the problem remains of critical importance for long run analysis.

A further point of consideration involves the relationship between supply responses estimated by our technique and those obtained from other statistical methods. Since the original CET work of Powell and Gruen in the mix 1960's, the only multiproduct model of Australian agriculture recognizing joint production to emerge has been the University of New England's Aggregative Programming Model of Australian Agriculture, APMAA.[1] The APMAA study has the advantage of a great deal of regional detail consisting of a set of approximately 500 regional representative farms for each of which a complete activity analysis linear programming framework is developed. The APMAA model has been used to provide aggregate estimates of own and cross elasticities of supply for different product combinations. The methodology allows shifts in the land use between sheep, beef and wheat in response to changes in the gross margins of these activities.[2] For a number of reasons, the APMAA results are not strictly comparable with those from CRETH. An obvious difference is that the APMAA own and cross price elasticities differ according to the particular combination of beef, wheat, and wool prices chosen. Assuming a 'central'

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1. John R. Monypenny, "APMAA 1974: Model, Algorithm, Testing and Application", APMAA Report No. 7, Department of Agricultural Economics and Business Management, University of New England, November, 1975.

2. Long Run Supply Response of Australian Wool Production Relative to the Price of Wool, Beef and Wheat; A Report to the AWC Wool Economics Research Committee, by The APMAA Research Team , University of New England, February, 1977.

the same product may be produced using quite different technologies in different zones. The essential characteristics of the BAE zones are as follows :

Pastoral Zone : The pastoral zone is the largest in area of the three zones. It includes all of the arid and most of the semi-arid parts of Australia which support a sheep population. Apart from some fringe areas, cropping is not feasible in this zone because of the inadequate and unreliable rainfall. Livestock are generally grazed extensively on native vegetation. Hence stocking rates are low and property areas extremely large.

In our empirical analysis of the Pastoral Zone we distinguish three product categories : wool, cattle and 'other'. Our treatment of the wool category for the pastoral zone warrants a brief comment at this stage. Unlike in the other zones, sheep are grazed in the pastoral zone almost entirely for their wool. Although a substantial amount of income is also obtained from the sale of 'cast for age' sheep, such income is really part of the wool enterprise. Hence although the share of the wool enterprise in total income includes the contribution of sheep sales proceeds, the relevant decision variable in our supply analysis is simply sheep numbers shorn, and the relevant price variable is the expected price of wool. The 'other' category includes a small amount of crops which are successfully grown in fringe areas if seasonal conditions are favourable.

Wheat/Sheep Zone : The wheat/sheep zone includes portions of all main-land states. It is the largest of the BAE's three zones in terms of sheep numbers and sheep properties, accounting for nearly one half the national sheep flock and well over half the number of sheep properties in Australia. Rainfall is generally sufficient for crop production, and nearly all dryland cropping of cereals is carried out in this zone. As well as forming the basis for the Australian sheep and wheat industries, the zone has become an increasingly important source of cattle output in recent years. Climate, topography and soil type are such that opportunities for diversification into various cropping and livestock activities are large. The zone thus forms a classic example of a multiproduct agricultural region.

In our analysis we recognize six products for the zone; wool, sheep, cattle, wheat, barley and 'other'. The 'other' category is a miscellaneous one including small amounts of non-cereal crops and livestock products such as pigs and dairying.

High Rainfall Zone : This zone consists entirely of land within 200 miles of the coast and is located in the south-east and south-west corners of Australia (excluding Queensland, but including the whole of Tasmania). Although it is smallest in terms of area, the high rainfall zone is second (following the wheat/sheep zone) in importance with respect to sheep numbers. The sheep concentration per unit area is highest in this zone. Sheep are often grazed in association with beef cattle and, to a lesser extent, with crop production and other enterprises. As in the wheat-sheep zone, a large proportion of properties combine wool growing with prime lamb production. That is, sheep meat production is an enterprise within its own right. Four product categories are distinguished : wool, sheep, cattle and 'other'.

A third line of further development concerns our treatment of price expectations. There are a considerable number of methods we could have employed to filter the product price series to reflect producers' expectations. Strictly speaking, our results are conditional upon both the Koyck/Nerlove expectations specification (and the particular set of coefficients of expectation we have assumed within this specification), as well as upon the assumption of total output adjustment within one year. The earlier experience of Powell and Gruen suggested to us that sensitivity analysis of our results by varying the sets of coefficients of expectation should receive only a low priority. Nevertheless, sensitivity testing of the specification is needed to complete the story.

Fourth, there is the question of the time framework applying to the measured supply responses. Our estimates are one year elasticities. That is, we have modelled shifts around the various partial transformation frontiers rather than the shift of the frontiers over time. Our original theoretical CRESH-CRETH specification recognized land and capital as being fixed for one year. On this assumption, the theory allowed for the generation of shadow prices on both land and capital. It was anticipated that the estimation of these shadow prices would form a natural starting point for the specification of an investment function for each of the regions, thus establishing the mechanism by which product prices feed back into inputs and allowing shifts in the transformation frontier. Our failure so far to estimate the complete CRESH-CRETH model has meant at least the temporary abandonment of this line of work. Whilst empirical measurement of the

BAE's 1972/73 Wheat Industry Survey (drawn from a population of farms growing at least 40 hectares of wheat) showed that well over 90 per cent of farms ran sheep and 70 per cent ran cattle.[1] Hence while the ASIS may not be representative of Australian farming as a whole, it does encompass a very high proportion of the output of the cereals-livestock complex. We are therefore confident that our results are representative of the joint production features of Australian agriculture.

The second point relates to the 'other' products result. In the Pastoral Zone and in the High Rainfall Zone (but especially in the latter), this category was responsible for a high proportion of the total price responsiveness of the zone. The question arises as to whether this result is 'genuine' in terms of representing the alternative enterprise possibilities confronting farmers. Our instinct is to treat the 'other' product result with a great deal of caution. Our data base was not detailed enough to reflect accurately the minor component products, hence causing our lumping them into the 'other' category. We are therefore unsure of the extent to which the 'other' products price index constructed for each zone portrays the 'appropriate' price confronting the farmer for this composite category. Our analysis would have been more convincing had information been available also for the minor products produced in the zones.

1. Bureau of Agricultural Economics, The Australian Wheat Growing Industry: an Economic Survey: 1972-73, Australian Government Publishing Service, Canberra, 1975.

Our information on the output of the various crops in this zone is far from complete. The zone is not particularly suited to the growing of cereal crops and a considerable proportion of such output is retained on the farm as livestock feed. We have therefore included the minor cropping activities for wheat and other grains as part of the 'other' category.

The product categories distinguished in our analysis and their relative importance in total revenue for each zone are shown in Table 1.

Table 1

Composition of Output by Zone

Product	Average Share of Output (1952/53 to 1973/74)		
	Pastoral	Wheat/Sheep[(a)]	High Rainfall
Wool	0.858 (Wool and Sheep)	0.353	0.545
Sheep		0.120	0.158
Cattle	0.102	0.063	0.150
Wheat	0.040 (Wheat, Barley and Other)	0.346	0.147 (Wheat, Barley and Other)
Barley		0.053	
Other		0.065	

(a) A small amount of oats is grown on the 'average' farm. We have excluded oats from the analysis because of its often dual role on a cereal-livestock farm as both an output and an input (for the livestock enterprise). While recognizing that wheat and barley can also be used as inputs on such farms our assumption is that these products are essentially outputs.

3.2 Planned Output and Expected Prices

In both the reduced and structural form, the variables are all differentials of logarithms. Operationally we have used the simplest discrete analogue; i.e., $d \log (a_t)$ is replaced by $\left[(a_t - a_{t-1})/a_t\right]$. Hence in the following discussion of the construction of variables, it should be noted that all variables (with the exception of product and input shares) enter the estimating equations as proportional changes.

Raw data on prices and output are generally inappropriate for empirical supply analysis. Output data may not accurately reflect producers' intentions because of weather influences. In addition, account must be taken of underlying long-term productivity trends (which are unlikely to be responsive to prices). If output series are not adjusted for these influences, then allowances must be made within the supply model itself. Given the symmetry of our CRETH system, it seemed desirable to correct output indicators where possible before they enter the system. Such adjustments in quantity series have dictated complementary adjustments to price series. For example, if planned output of wool is measured by fleece numbers, then the appropriate price is dollars/fleece. The price series constructed in this fashion require further modification to reflect producers' expectations (to be discussed below, section 3.3).

In transforming the basic data series we have followed closely the procedure outlined in Powell and Gruen.[1] To simplify our

1. A.A. Powell and F.H. Gruen, "Problems in Aggregate Agricultural Supply Analysis, I: The Construction of Time Series for Analysis". Review of Marketing and Agricultural Economics, Vol. 34, No. 3 (September 1966), pp. 112-135.

(iii) It follows from the high responsiveness of product outputs to changes in the price of competing products, that attempts to manipulate the price of one product on its own will cause quite substantial changes in the output of competing products.

6. SUGGESTIONS FOR FURTHER WORK

In concluding, there are a number of issues concerning our approach which require further comment. Some of these can be viewed as a logical starting point for further research in this area.

The first relates to the representativeness of our results. Rather than follow Powell and Gruen and work with aggregate data, we have chosen to model joint production in the three agricultural zones of Australia as delineated by the BAE in their ASIS.[1] The ASIS is a stratified random sample of Australian farms with the eligibility criteria requiring that a property run at least 200 sheep, provide full time occupation for at least one person, and not be principally a stud, part of a multiple holding or used mainly for dealing.[2] Results of the

1. Our work reported here forms part of a larger project designed to model the Australian economy within a general equilibrium framework at a level of disaggregation which distinguishes about 100 different industry groups. If we had analysed aggregate data our results when incorporated into the general equilibrium model would have implied that all agricultural land was homogeneous. The zonal approach, in recognizing three types of agricultural land (classified according to rainfall), provides a far more plausible representation of the agricultural sector than that afforded by such an aggregate analysis.

2. The sample size and population size of the ASIS vary from year to year. In 1971/72 about 700 farms were drawn from seven size strata comprising a population of about 85,000 farms.

following planned output changes; wool +0.14 per cent, sheep +0.16 per cent, cattle +0.11 per cent, wheat -0.02 per cent, barley +0.12 per cent, 'other products' +0.03; in view of approximations inherent in the method, these estimates should be rounded to the first significant digit. Using the product shares set out in Table 1, these figures imply that the short run total agricultural supply elasticity in the Wheat/Sheep Zone with respect to the overall level of expected product prices (but with input prices and fixed inputs held constant), is about +0.01 (which is one tenth of the weighted sum of the above responses using the value shares given in Table 1 as weights).

In conclusion, our results suggest the following :

(i) Farming in the three zones, and especially in the Wheat/Sheep Zone, is characterised by multi-enterprise production possibilities reflecting generally high technical prospects for farmers to change their enterprise mix in response to changes in the expected prices of products.

(ii) Unless agricultural supply analysis is carried out within an econometric framework sufficiently comprehensive to allow modelling the joint production features of such regions, the resultant estimates of the responsiveness of various products to changes in own prices and prices of competing products is likely to be considerably understated. This has crucial implications for agricultural policy analysis which often involves situations in which product prices are manipulated for a range of policy purposes.

data requirements, we have assumed that product and variable input prices do not vary across zones and have therefore constructed the appropriate price series from aggregate data. While there is some variation across regions in farm-gate prices received by producers, our judgement is that these variations are too small to be of significance in our supply analysis.

Wool : The decision variable used to measure planned wool production is total number of sheep and lambs shorn. By using numbers rather than the weight of wool (which is the more conventional measure), we go some way towards removing the random variation in production due to climate. In computing the corresponding price series ($/fleece), we need to incorporate the upward trend in fleece weights due to genetic improvements that have occurred over the data period.[1] This relationship was quantified by fitting a linear trend to Australian aggregate data on fleece weights (including crutchings) for the period 1945/56 to 1973/74. The resultant equation,

$$\text{(8)} \qquad \text{Fleece-weight (Kg)} = 3.855 + 0.016\overset{*}{t}; \quad R^2 = 0.42; \quad (\text{*std error} = .004),$$

was used to determine 'normal' annual fleece weight. The normal fleece weight figures were combined with national average wool realizations

1. Our treatment implies that wool prices ($/fleece) are equivalent across regions. Since the tendency is for higher fleece weights to return lower prices per kg of wool we believe this approximation to be reasonable in the context in which it is used, that is, in constructing a series of proportional changes in expected prices facing producers in each zone.

figures to obtain a climatically adjusted series on expected average returns/fleece.

Sheep : In the case of the livestock enterprises (sheep and cattle), the farmer has considerable flexibility over the stage at which products are marketed. For example livestock can be withheld from the market if seasonal conditions are favourable. Such strategies involve changes in inventories which must be accounted for somewhere. Rather than modify the structure of the livestock supply equations to reflect inventory behaviour, we have defined the decision variable to represent output whether actually sold or in the form of a change in inventories. Sheep enterprise output (number of sheep) was computed as (sales - purchases) + changes in stock. Unfortunately the data does not allow us to distinguish between sheep sold as fat lambs and as 'cast for age' wool sheep.[1] Average price ($/sheep) was calculated as (trading gain ($) + operating gain ($)) / output.[2]

Cattle : Cattle output and price was calculated using analogous procedures to those used for sheep.

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1. For a comprehensive discussion and treatment of livestock inventory response in conjunction with supply response, see J.W. Freebairn, "Some Estimates of Supply and Inventory Response Functions for the Cattle and Sheep Sector of New South Wales", Review of Marketing and Agricultural Economics, Vol. 41, Nos. 2 and 3 (June and September 1973).

2. The definitions of trading gain and operating gain are those used by the BAE in their analyses of livestock trading accounts. Trading gain is the difference between sale and purchase value of livestock (plus the value of rations). Operating gain is the difference between opening and closing livestock numbers valued at the rates applied to closing numbers; i.e., it is that part of the change in livestock capital that is due to livestock operations.

response in all three zones. Although transformation elasticities are lowest in the high rainfall zone, they are relatively high in all zones when compared with the aggregate Powell/Gruen results.

Own price elasticities are lowest for wool in all zones. In the Pastoral and High Rainfall Zones (which have been traditionally dominated by wool production), wool supply elasticities are lower than is the case in the Wheat/Sheep Zone (where wool is less dominant). Cattle supply elasticities are comparatively high ($\geq$ 0.3) in all zones. So too are 'other' product supply elasticities ($\geq$ 0.6) in all zones, and supply elasticities for wheat and barley in the Wheat/Sheep Zone (0.8 and 0.5 respectively. (In the other zones, wheat and barley are components of 'other' products.)

In all zones the extent of cross price responsiveness is generally high. However, cross price responses are lowest for wool output with respect to changes in the price of each of the other products.

Because in our structural form the aggregate short run capacity to produce, z , is exogenous, we cannot obtain any information from our estimates for the Pastoral or High Rainfall Zones about supply responses to changes in absolute (as distinct from relative) product prices. This weakness is shared with the earlier (Powell/Gruen) methodology. The reduced form system fitted to the Wheat/Sheep Zone, however, does allow computation of short run expansion effects due to changes in the product price level. For example, according to Table 8, a 10 per cent increase in the expected price of all products would result one year later in the apply. The estimated (short run) expansion effects were sufficiently small, however, that ex post this equation applied as a first approximation among the estimated reduced form elasticities (see Table 8).

A third point to note is that the values obtained for the pairwise transformation elasticities from CRETH are considerably higher than those reported by Powell and Gruen from national time series aggregate data using their more restrictive (and theoretically less well articulated) CET model.[1]

Furthermore, our estimates of one year own and cross price elasticities of supply are considerably higher in absolute value than the estimates obtained from previous Australian agricultural supply studies which generally have employed conventional single equation techniques with the parameter space largely unconstrained.

Finally, we note a number of broad similarities across the three zones in the ease of product transformation and in own and cross price responsiveness of products. Transformation elasticities are lowest between wool and sheep, reflecting the dual purpose or fixed proportions element; they are generally lowest between pairs of livestock products and highest between crop products. The 'other' products category apparently adds considerably to the flexibility of the production

1. In a six product analysis of aggregate Australian agricultural data, Powell and Gruen reported (after constraining a number of pairwise elasticities to zero) the following transformation elasticities; wool/wheat -0.16, wheat/coarse grains -0.29, beef/dairy -0.29, lamb/dairy -0.26, and lamb/wool -0.13. The treatment of price expectations and other variables in the present analysis sufficiently replicates Powell and Gruen's as to allow direct comparison of estimates.

Wheat : The measure of planned output chosen was area sown. While area sown is likely to be considerably less influenced by weather than amount harvested, poor conditions at sowing can influence area sown. However, such conditions seldom occur over a wide area in the one year.

In constructing an appropriate wheat price series we have followed the procedure of Powell and Gruen to incorporate the effects of the Australian Wheat Board's stabilisation machinery on farmers' price expectations.[1] Price expectations should be a function only of variables whose values are already known at the time expectations are formulated. The wheat pool payments system generally implies that at the time of sowing a crop, more or less final information on receipts may only be available for crops sown two or more years previously. Powell and Gruen point out that by deferring judgement about total receipts until all payments were in, producers would discard information about initial payments received in respect of more recent crops, which hardly seems rational. They formed their expectational model in terms of the date by which an arbitrarily high percentage (80 per cent) of total receipts from wheat could be expected to accrue. They hypothesised that farmers inflate actual payments up to a certain cut off date by a factor of 1.25 . These cut off dates occur two years after planting for the years 1952/53 and 1953/54, and one year after planting for all subsequent years. The Powell/Gruen series was updated to 1973/74. A linear trend was fitted to Australian aggregate data on wheat yields. The resultant equation,

1. Op. cit., p. 10.

(9) yield (bushels/acre) = 15.0 + 0.126*t; R^2 = 0.11;

(* standard error = 0.07),

was used to convert the price series ($/bushel) into a dollar/acre sown basis.

Barley : Planned output was measured by area sown. A unit value series was constructed from aggregate information on gross value and volume of production. There were no trends apparent in our analysis of national average yield figures over the time period. Hence an average yield figure was used to convert the unit value ($/bushel) series to a ($/acre sown) basis.

'Other' Products : This category is a residual consisting of a number of products of minor importance. The exact year to year composition of the category cannot be discerned from our data series. The composition of the 'other' category does of course differ across the three zones. For each zone, a measure of 'other' products output was obtained by dividing the dollar value of 'other' products revenue by an 'appropriate' price index. The price indexes were constructed to incorporate at least to some extent the price movements of products thought to be more important components of the 'other' category. To avoid the necessity of constructing such price indexes from scratch, the indexes chosen represent composites of existing BAE product price indexes.[1] For example, the index constructed for the pastoral

1. See Bureau of Agricultural Economics, Trends in Australian Rural Production, Exports, Income and Prices, various issues, Canberra, for a tabulation of the various product price indexes constructed by BAE.

the matrices of cross price elasticities. That is, the cross elasticity of supply of product i with respect to changes in the price of product j, ξ_{ij}, does not equal the cross elasticity of supply of product j with respect to changes in the price of product i, ξ_{ji}. The difference between ξ_{ij} and ξ_{ji} is due to the relative sizes of enterprises j and i. It should be noted however, that the relative scales on which enterprises i and j are conducted does not in itself affect their basic ease of transformation (that is, their τ_{ij}-values). For example, consider the Wheat/Sheep Zone where our formulation ensures that since the relative share in income of wheat is large compared with that of cattle, a price rise for wheat will not require an unduly high $\tau_{wheat/cattle}$ value to 'explain' the decline in the output of cattle that takes place. Part of the response in the output of cattle to a change in the price of wheat must be attributed to the sheer size of the wheat enterprise compared with the cattle enterprise in the Wheat/Sheep Zone, rather than to the technological possibilities for the transformation of potential cattle output into wheat output. Table 8 shows that $\xi_{cattle/wheat}$ (-2.69) is greater in absolute value than $\xi_{wheat/cattle}$ (-1.14), while from Table 1 we note that the share of wheat in total output (0.346) exceeds the share of cattle in total output (0.063).

A second point to note is that the homogeneity of degree zero of the structural form supply system in relative prices, implies that, for any product, the sum of all of its cross price elasticities is equal to the negative of its own price elasticity; that is, $\sum_j \xi_{ij} = -\eta_{ii}$. In the reduced form system this condition does not

5.4 Non-Parametric Testing for Serial Correlation

A "Runs Test" was carried out on the residuals of each of the simultaneous equations.[1] For all stochastic equations in the three zones, the number of runs observed was considerably below the critical number. That is, in all equations we could not reject the hypothesis that the residuals are independent. Needless to say, we harbour no illusions about the power of this test; the volume of computing involved in a more suitable test,[2] and the poor prospect of being able to make meaningful corrections for autocorrelation within such a tightly constrained system to be estimated by full information methods, led us to settle for the simple non-parametric test.

5.5 Some Comments on the Results

There are a number of features of the results which warrant further discussion. The first relates to the assymetric structure of

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1. For a description of the test used see E.J. Kane, Economic Statistics: Econometrics: an Introduction to Quantitative Economics, (New York: Harper and Row, 1968, p. 364). Briefly, the test involved computing $Pr(n_r)$, the probability of observing n_r , the actually observed number of runs on the null hypothesis that the sequence of residuals is completely random. Whenever $Pr(n_r)$ is less than or equal to one minus the confidence coefficient, we reject the null hypothesis. $Pr(n_r)$ itself is calculated as $M(n_r)/M(S)$, where $M(n_r)$ equals the number of distinct ways in which n_r runs can occur in a sequence of N items in which n+ and n- are alike, and M(S) is the number of possible outcomes.

2. D.F. Hendry, "Maximum Likelihood Estimation of Systems of Simultaneous Regression Equations with Errors Generated by a Vector Autoregressive Process:, International Economic Review, Vol. 12, No. 2 (June 1971), pp. 257-72.

zone was a weighted average of the BAE's 'all crops excluding wheat' price index and the BAE's 'wheat' price index, where the weights used correspond to one minus the annual share Sw of wheat revenue in total 'other' revenue, and Sw respectively.

The price index used for the wheat/sheep zone was a weighted average of the BAE's 'pigs' price index and 'all crops excluding wheat' price index with weights of 0.3 and 0.7 respectively. Finally, for the high rainfall zone the 'other' products index represents a simple average of the BAE's 'total dairy, pigs, poultry and honey' price index and the BAE's 'all crops excluding wheat' price index.

3.3 Modifying Price Series to Reflect Producers' Expectations

The product price series (with the exception of wheat)[1] were filtered using the classic Koyck/Nerlove adaptive expectations model. All lags in observed response to prices, whether due to adjustment frictions or the updating of expectations, under our treatment are located on the expectational side. As our expectational model we have used :

$$\Pi_{it} = \sum_{\ell=1}^{\infty} \beta_i (1 - \beta_i)^{\ell-1} P_{i,t-\ell} \quad , \qquad (10)$$

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1. We follow the reasoning of Powell and Gruen in declaring the distributed lag model of price expectations to be inappropriate for wheat because of the greater confidence about wheat prices and greater wheat price stability resulting from the wheat stabilisation arrangements.

where Π_{it} is the price expected to prevail for product i in time t from the viewpoint of year $t-1$, β_i is the coefficient of expectations for product i, and $P_{i,t-1}$ is the actual price pertaining ℓ years ago from the viewpoint of year t. On pragmatic grounds we have truncated the distributed lag series after seven years so that prices occurring eight or more years previously are considered to have no influence on expectations.[1]

Our model is not geared towards simultaneously estimating the coefficients of expectation β_i. All results are computed as conditional estimates for given values of the β_i. We have assumed that coefficients of expectations vary inversely with the coefficient of variation of the associated price series. That is, the reliability of last year's price as an estimate of the longer run price level will be high for stable series but less so for erratic ones. Coefficients of expectations have been calculated conditionally on the basis of an assumed value of 0.5 for wool. Details are shown in Table 2.

1. Hence prices for all products except wheat must be constructed for the seven years preceding the start of the BAE data series. Wool, barley and 'other products' prices are based on aggregate data which is available for this period. However, the sheep and cattle price series are calculated from BAE data. The sheep price series was extrapolated to 1945/46 using a sheep unit value index constructed from aggregate information on quantity and value of sheep and lambs slaughtered for the period 1945/46 to 1952/53. The cattle price series was extrapolated to 1945/46 using the cattle price index in Powell and Gruen, op. cit., p. 22.

Own and cross product price elasticities for the high rainfall zone are shown in Table 10.

Table 10

Estimated Own and Cross Price Responsiveness :
High Rainfall Zone

Per cent response one year later in the planned output of (a)	Product whose expected price changes by 10 per cent			
	Wool	Sheep	Cattle	'Other'
Wool	0.60 (1.07)	-0.02 (0.20)	-0.05 (0.21)	-0.53 (1.20)
Sheep	-0.06 (0.20)	1.12 (0.90)	-0.10 (0.21)	-0.97 (0.91)
Cattle	-0.19 (0.21)	-0.10 (0.21)	3.43 (1.64)	-3.14 (1.25)
'Other'	-1.96 (1.20)	-1.04 (0.91)	-3.20 (1.25)	6.20 (1.67)

(a) Actual output may differ from planned output due to droughts, etc.
Ratio of estimated coefficient to asymptotic standard error in brackets.

Once again, in a predominantly woolgrowing region, own supply elasticities for wool and sheep are considerably less than those of the two other product categories distinguished. Similarly, cross price effects are lower for wool output with respect to changes in the price of competing products than for other output and price combinations. Nevertheless, estimated cross price responsiveness in the high rainfall zone is still quite large.

Although all parameters have the correct sign, standard errors are very high. As was the case in the other zones, the estimated ease of transformation is lowest between wool and sheep and highest when the 'other' products category is involved. Values for wool/cattle and sheep/cattle are lower than might have been expected given the inherent suitability of high rainfall zone pastures to the grazing of sheep and cattle. Traditionally, the high rainfall zone has been dominated by sheep grazing for both wool and sheepmeats. The emergence of a beef cattle industry on a significant scale has taken place only in more recent years. We suspect that our analysis, framed entirely in terms of relative prices, has failed to capture the rate of adoption or adjustment path towards the new cattle/sheep equilibrium. Even though relative prices may clearly suggest a transformation from sheep to cattle, it is likely that, because of the 'newness' of a cattle enterprise in a predominantly sheep grazing region, such a transformation occurs only slowly at first then at an accelerated rate. That is, the lag in acceptance of the relatively new cattle enterprise is progressively overcome as cattle gain greater acceptance among sheep men.

One way of allowing for this type of adjustment process is by the incorporation of a suitably shaped trend curve (for example the logistic curve, which is sigmoidal) into the cattle supply equation, and the mirror image of this curve into the wool supply equation. We are unable to do this at the present time because the resultant estimation problem exceeds the capacity of the available version of RESIMUL.

Table 2

Coefficients of Variation in Product Price Series and Expectational Coefficients Assumed

Product	Coefficient of variation 1945/46 to 1973/74†	Ratio relative to wool = 1	Inverse ratio	Coefficient of Expectations when β = 0.50 for wool*
Wool	0.40	1.00	1.00	0.50
Sheep	0.38	0.95	1.05	0.53
Cattle	0.51	1.28	0.78	0.39
Barley	0.23	0.58	1.74	0.87
Other products	0.24	0.60	1.67	0.83

† Coefficient of variation includes any variation due to trend.

* Product of number in preceding column and 0.5 .

3.4 Product Shares of Income

Product shares were computed from information on gross revenue (net of product-specific costs)[1] of individual enterprises and from total net revenue. The product specific costs recognized were:

1. The reader will recall (Dixon, Vincent and Powell, op. cit.) that the parameter restrictions inherent in our theoretical approach require that factors are either completely product specific (in which case their cost can be treated like an excise tax, and deducted from revenue) or else totally non-specific. The latter situation, which implies that no input has comparative advantage vis a vis any other input in the production of any particular product, is assumed to apply to all factors except those listed as (fully) product-specific in section 3.4.

(i) wool marketing, shearing and crutching (including store classing) charges, which were deducted from wool gross income; and
(ii) livestock selling charges apportioned between sheep and cattle enterprises according to their relative importance and then deducted from sheep and cattle revenue respectively.

3.5 Fixed Inputs

Our 'reduced form' approach (equation (4)), which we applied to the wheat/sheep zone, requires information on fixed inputs. The farm firm's land area and capital input were regarded as fixed in the short run. In this analysis we have also treated the labour inputs of the owner-operator and his family as part of the firm's fixed input.

Land inputs are represented by land area. For capital items we have assumed that service flow is proportional to the stock, and have constructed a stock series of the form $k_{t+1} = k_t + I_t - dk_t$ where k is capital stock (constant prices), I is gross investment net of sales of plant (constant prices) and d is the annual rate of depreciation. This procedure was adopted for four types of capital, water, fencing, buildings and plant. A total capital stock series was obtained by adding these four component series. In constructing the series we used BAE figures on opening stocks in 1952/53 and BAE information on net additions (additions - sales) in current prices. The net addition figures were converted to constant prices using relevant BAE indexes. The indexes, and the assumed depreciation rates are given in Table 3.

Although they are generally poorly determined, it is of interest to interpret the estimates of the wheat quota parameters for the Wheat/Sheep zone. Our results (see Appendix) indicate that following the imposition of wheat quotas, planned output of wheat declined sharply. This was offset by very small expansions in the planned outputs of wool and of sheep and much larger expansions in the planned outputs of cattle and of barley. For the period between 1972/73 and 1973/74 when wheat quotas became more or less inoperative, the results indicate a large expansion in planned output of wheat, a large contraction in the planned output of barley and a moderate contraction in the planned output of wool.

5.3 High Rainfall Zone

The six estimated pairwise transformation elasticites are shown in Table 9.

Table 9

Estimated Product Transformation Elasticities :
High Rainfall Zone

	Sheep	Cattle	'Other'
Wool	-0.01 (0.20)	-0.04 (0.21)	-0.36 (1.20)
Sheep		-0.07 (0.21)	-0.66 (0.91)
Cattle			-2.13 (1.25)

Ratio of estimated parameter to asymptotic standard error in brackets.

Table 8

Estimated Own and Cross Price Responsiveness :
Wheat/Sheep Zone

Per cent response one year later in the planned output of (a) :	Product whose expected price changes by 10 per cent						Sum of Cross Price Responses
	Wool	Sheep	Cattle	Wheat	Barley	'Other'	
Wool	2.56 (2.38)	-0.12 (0.52)	-0.20 (1.27)	-1.52 (1.52)	-0.14 (1.12)	-0.44 (1.82)	-2.42
Sheep	-0.31 (0.41)	2.25 (0.91)	-0.15 (0.71)	-1.18 (0.78)	-0.11 (0.67)	-0.34 (0.83)	-2.09
Cattle	-0.77 (0.78)	-0.23 (0.72)	4.83 (1.13)	-2.69 (1.06)	-0.25 (0.99)	-0.78 (1.09)	-4.72
Wheat	-2.54 (2.05)	-0.75 (1.96)	-1.14 (2.42)	7.66 (2.59)	-0.80 (2.37)	-2.45 (2.51)	-7.68
Barley	-0.78 (0.99)	-0.23 (0.90)	-0.36 (1.42)	-2.70 (1.49)	4.97 (1.57)	-0.78 (1.54)	-4.85
'Other'	-2.04 (2.06)	-0.61 (1.93)	-0.92 (2.63)	-6.87 (2.71)	-0.64 (2.56)	11.11 (2.87)	-11.08

(a) Actual output may differ from planned output due to droughts, etc. Ratio of estimated coefficient to apparent asymptotic standard error in brackets. (See footnote to Table 7.)

Both own and cross elasticities of supply are high. For example, in the Wheat/Sheep Zone, a 10 per cent rise in the expected price for wheat, with all other product and input prices held constant, leads one year later to an 8 per cent increase in the output of wheat, and to the following percentage contractions in the output of competing products : sheep, 1.2 per cent; barley, 2.7 per cent; wool, 1.5 per cent; cattle, 2.7 per cent; and 'other' products, 7 per cent.

Table 3

Price Indexes and Depreciation Rates
used to Construct Indexes of Farm Capital

Capital Type	BAE price index used	Annual Rate of Depreciation Assumed, d
Water	Machinery prices paid index	0.05
Fencing	Fencing materials prices paid index	0.03
Buildings	Building materials prices paid index	0.03
Plant	Machinery prices paid index	0.125

Source : Bureau of Agricultural Economics, Trends in Australian Rural Production, Exports, Income and Prices, various issues, Canberra.

We have combined the capital and the land series to form a single fixed input variable using an aggregation procedure of the form, $\frac{dF}{F} = \alpha\frac{dK}{K} + (1 - \alpha)\frac{dL}{L}$ where $\frac{dF}{F}$ is the proportional change in aggregate fixed factor, dK/K and dL/L are the proportional changes in the capital and land inputs respectively; and where α and $(1-\alpha)$ are the relative shares of capital and land respectively in their combined rental.[1]

1. It is notoriously difficult to obtain reliable splits of the rental accruing to capital and to land. For the purpose of obtaining aggregate weights, we have calculated the rental on land from its market price (capitalised rent). This represents the discounted present value of expected returns from the land. Under the assumption that land yields an inexhaustible flow of services over time, rental payments to land represent the perpetuity associated with its market value. Payments to capital and owner-operator and family labour are then computed as a residual component of income after subtracting payments to land and to the variable factors.

3.6 Variable Input Prices

Variable input prices appear as explanatory variables in our reduced form equation (4). In our treatment of the wheat/sheep zone we have recognized five variable input categories; fuel, fertilizer, maintenance, hired labour, and 'other' inputs. The first three categories are self explanatory. The hired labour variable refers to labour hired by the firm, excluding product specific contract labour such as contract shearing and crutching. The 'other' input category is a residual consisting of all inputs not distinguished separately.

We have assumed that prices of variable inputs are anticipated accurately. For fuel, maintenance and the 'other input' category, we have used the relevant BAE indexes of prices paid for these inputs without modification. The price of hired labour is measured by the Federal Pastoral Award for Station Hands.

The BAE's fertilizer price index is unsuitable for this study because it does not include the impact of the fertilizer bounty which has applied in most years. We have therefore used a price series to represent 'ex works' prices net of bounty. This series was compiled from data obtained by Waring and Morris.[1]

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1. E.J. Waring and J.G. Morris, "Usage of Superphosphate on Pastures: an Exploratory Analysis", Quarterly Review of Agricultural Economics, Vol. 27, No. 1, January 1974.

conditions. Highest transformation elasticities occur among pairs of the cropping activities, wheat, barley and 'other' products. In the Wheat/Sheep Zone, one can envisage few technical problems in changing the relative size of these enterprises in response to changes in relative product prices. Finally, transformation elasticities between product pairs which involve a crop enterprise and a livestock enterprise are intermediate in magnitude between those involving pairs of livestock products and pairs of crop products.

In Table 8, we report a matrix of own and cross supply elasticities associated with the reduced form CRETH system. This matrix corresponds to the elements c_{ij} in equation (4). The reduced form elasticities incorporate both transformation and expansion effects. That is, they are derived under the assumption that the scale of output is free to change as transformation from one product to another occurs. For example, an increase in the price of one product is likely to lead to an increase in the overall scale of output while at the same time leading to a reduction in the quantity of a competing product. Hence the size reduction in the competing product is likely to be smaller than would have been the case had the scale of output been held constant.[1]

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1. The relationship between the structural form (SF) and reduced form (RF) cross price elasticities is as follows :

$$\underset{(RF)}{\xi_{ij}} = \underset{(SF)}{\xi_{ij}} + \xi_{iz}\xi_{zpj} ,$$

where ξ_{iz} is the elasticity of output of product i with respect to the overall capacity variable z (which is unity for all products) and ξ_{zpj} is the elasticity of z with respect to the price of product j . Estimates of ξ_{zpj} can be deduced from the reduced form analysis via the parameter h (see Appendix for details). They range from 0.001 to 0.006. Hence for all practical purposes, the SF and RF price elasticities can be regarded as equivalent.

Table 7

Estimated Product Transformation Elasticities : Wheat/Sheep Zone

	Sheep	Cattle	Wheat	Barley	'Other'
Wool	-0.08 (0.81)	-0.19 (1.09)	-0.58 (2.57)	-0.19 (1.34)	-0.47 (1.82)
Sheep		-0.15 (0.72)	-0.45 (0.94)	-0.15 (0.77)	-0.37 (0.85)
Cattle			-1.00 (1.13)	-0.32 (0.95)	-0.81 (1.12)
Wheat				-1.01 (1.57)	-2.52 (2.81)
Barley					-0.81 (1.39)

Ratio of estimated parameter to apparent asymptotic standard error in brackets. Ratio may be overstated due to iterative treatment of S^*s external to FIML package.

The estimated transformation elasticities are rather high for a number of product pairs. This is consistent with the view that Wheat/Sheep Zone farmers have considerable possibilities for changing their output mix in response to relative price changes. As one might expect from the nature of the products, the lowest value is for wool:sheep. There is a sizeable fixed proportions element between wool and sheep. (It is difficult to produce wool without at the same time producing some sheep and vice versa). Values for wool/cattle and sheep/cattle are next lowest. It is generally acknowledged that there is some complementary behaviour between these products in Australian grazing

3.7 Input Shares of Costs

The shares of variable inputs for the wheat/sheep zone were computed from information on the total value of output and expenditure on the various input categories designated. The fixed input share is obtained by subtracting the total variable input share from unity. The average cost shares for the wheat/sheep zone are shown in Table 4.

Table 4

Average Shares of Non-Specific Inputs in Total Costs: Wheat/Sheep Zone, 1952-53 through 1973-74

Input	Average Share of Total Cost
Fuel	0.055
Fertilizer	0.049
Maintenance	0.086
Hired labour	0.050
Other variable inputs	0.149
Capital : land : owner/family labour	0.611

Source : Derived from Bureau of Agricultural Economics, Australian Sheep Industry Survey, various issues, Australian Government Publishing Service, Canberra.

3.8 Index of Capacity

An index of the proportional change in the overall capacity to produce (z) is an explanatory variable in our CRETH structural form. This index is constructed using equation (3) with the planned output and product share variables discussed in sections 3.2 and 3.4 respectively.

3.9 Weather Influences

We were unable to modify the variables representing planned outputs of sheep, cattle and 'other' products to abstract from the influences of weather. An alternative approach to that of assembling 'weather-free' variables is to add an explanatory variable constructed to incorporate the influences of weather on the output of a particular product. In our structural form specification the influence of weather would enter the system automatically via its impact on the righthand variable z in equation (1).

Thus a weather variable is required only on the RHS of our reduced form equation (4). We have used the ratio : (Sheep deaths)/(total sheep numbers) as an indicator of seasonal influences in the Wheat/Sheep Zone. Poor seasonal conditions in Australian agriculture are generally reflected in a rise in sheep mortality (and vice versa). Before use in estimation of the CRETH reduced form, the weather variable (i.e., mortality ratio) was transformed into differences of logarithms. After the addition of this variable to the right of equation (4), our

Table 6

Estimated Own and Cross Price Responsiveness :
Pastoral Zone

Percentage response one year later in the planned output (a) of :	Product whose expected price changes by 10 per cent :		
	Wool	Cattle	'Other'
Wool	0.83 (1.25)	-0.39 (1.05)	-0.43 (0.96)
Cattle	-3.32 (1.05)	10.08 (2.38)	-6.76 (1.46)
'Other'	-9.29 (0.96)	-17.24 (1.46)	26.54 (1.75)

(a) Actual output may differ from planned output due to droughts, etc.
Ratio of estimated parameter to asymptotic standard error in brackets.

Planned production of wool in the pastoral zone is considerably less responsive to its expected price than are cattle or 'other' output to their respective prices. This result appears reasonable in view of the traditionally wool oriented activities of the pastoral zone.

5.2 Wheat/Sheep Zone

The fifteen estimated pairwise transformation elasticities, which all have the correct sign, are shown in Table 7.

Table 5

Estimated Partial Transformation Elasticities :
Pastoral Zone

	Cattle	Other
Wool	-0.39 (1.05)	-1.08 (0.96)
Cattle		-16.91 (1.46)

Ratio of estimated parameter to asympotic standard error in brackets.

All transformation elasticities have the expected (negative) sign, although the standard errors are disappointingly large. As would be expected, the ease of transformation is highest when the miscellaneous 'other' category is one of the product pairs. Nevertheless, for a one-year response, the value obtained for wool : cattle is quite large, implying a cross elasticity of cattle supply with respect to expected wool price of about -0.3 . The matrix of own and cross product supply elasticities for the pastoral zone is shown in Table 6.[1]

1. The own product supply elasticities (η_{ii}) can be deduced from the CRETH structural form equation (1) as :

$$\eta_{ii} = (1-S_i^*)/(k_i-1) \quad .$$

CRETH, in common with any regular transformation surface, has elasticities with the following property : $E_{i\ell} = \tau_{i\ell}S_\ell$ where $E_{i\ell}$ is the scale-of-output-compensated cross elasticity of supply of product i with respect to changes in the price of product ℓ . In applying these formulae, the Ss and S^*s have been evaluated at sample mean values of S .

Wheat/Sheep Zone supply system states that the proportional change in output of product i is a linear function of the proportional changes in product and input prices, fixed factor levels and weather influences.

3.10 Institutional Restrictions on Output

For the period 1969/70 to 1973/74, the quantity of wheat that growers were permitted to deliver to the Australian Wheat Board (the sole purchaser of wheat under Australian legislation) was determined by delivery quotas imposed on growers following the large carry over of wheat stocks from the 1968/69 harvest.

The aggregate quota has differed somewhat in each of the five years in which quotas operated. Furthermore, the extent to which the quota itself (rather than other factors such as poor seasonal conditions) has restricted the planned output of wheat has also differed. The influence of quotas on planned output of wheat needs to be taken into account in our Wheat/Sheep Zone supply system. A comprehensive treatment of quotas would require the addition of five dummy variables, one for each of the annual changes in quotas that have taken place, and the estimation of six quota parameters -- one in each supply equation -- for each dummy variable. This multiplicity of parameters is a result of the implication of the CRETH theory that a constriction on the output of one product would lead to expansion in the output of the other five products competing for inputs. Rather than estimate all thirty wheat quota parameters, we have, after inspection of the data, confined the modelling of quota influences to two data points : (i) in order to reflect the effect on output of the introduction of quotas, 1968/69 → 1969/70; and (ii) 1972/73 → 1973/74 when quotas became, in effect, inoperative. Our assumption is that in the intervening period, the small annual changes in quota level did not affect planned output of wheat and competing products to any significant extent.

4. ESTIMATION METHOD

Inspection of the data revealed that except for fertilizer (whose price has been influenced by Government bounties), variations in the relative prices of inputs have been small, thus making it unlikely that different pairwise substitution elasticities could be estimated. In fact, early attempts to estimate the full CRESH/CRETH system failed because we could not obtain covergence in the estimates of the substitution parameters. At this point we decided to concentrate on the establishment of a product supply system of equations. The prices of the different products are, for the most part, determined in quite different markets, possessing different demand characteristics. The observed variations in relative product prices over time seemed to be large enough to give our efforts to establish the shape of the relevant transformation frontiers a reasonable chance of succeeding.

(12) and (14) and then tested the validity of this assumption *ex post* using a non-parametric procedure (outlined below).

5. THE RESULTS[1]

We report the results of our CRETH analysis in the form of the following matrices for each zone :

(i) a symmetric matrix of pairwise product transformation elasticities;

(ii) an assymetric matrix of own and cross price elasticities of supply.

5.1 Pastoral Zone

Estimation of the three product system yielded the transformation elasticities[2] shown in Table 5.

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1. A complete listing of parameter estimates for each of the equations systems is contained in the Appendix.

2. It will be recalled (see Dixon, Powell & Vincent, *op. cit.*, p. 20) that the (Allen) partial transformation elasticity $\tau_{i\ell}$ between products i and ℓ under CRETH is :

$$\tau_{i\ell} = -\frac{1}{(k_i-1)}\frac{1}{(k_\ell-1)}\frac{1}{\sum_{i'=1}^{N}\hat{S}_{i'}} \qquad (i\neq\ell),\text{ where } \hat{S}_i = S_i/(k_i-1) \quad .$$

In applying this formula, the values of $\hat{S}_i$ have been evaluated at sample mean values of S_i .

to Wheat/Sheep Zone data, was equation (4). This equation, we remind the reader, is an approximation based on a treatment of the product shares as exogenous <u>constants</u>. From equations (5) and (7) it can be seen that although the reduced form system is linear in the variables y_i, q_{1j}, $\bar{x}_{2j}$ and p_j, the coefficients ℓ_j and c_{ij} are highly non-linear functions of the parameters (k_i) and h. Having decided to fix the variables S_i and W_j for all i and j at their sample mean values, we can know the explicit numerical form of these functions. However, without a further reduction in the extent of the non-linearity of these functions, our prospects for successful estimation would have been small. We have therefore treated not only the S_i and W_j as constants but initially also the S_i^* components of (5) and (7). That is, after obtaining estimates for (k_i) based on provisional values of S_i^*, we updated the S_i^* estimates and re-estimated iteratively till convergence was achieved. This treatment complicates the interpretation of the asymptotic standard errors.

For the Pastoral and Wheat/Sheep zones, slightly better results were obtained when regression constants were included in the supply equations.

4.2 Serial Properties

The mapping of the serial properties of the CRETH system as we move from levels to differential changes to discrete annual percentage changes is obviously very complicated. We have simply appended white noise errors to the operational versions of our equations

4.1 The Form of the Equations Fitted

For reasons of computational convenience, and especially to avoid the iterative procedure necessitated by the presence of the S^* in equation (1),[1] we have estimated the CRETH structural form for the High Rainfall and Pastoral Zones using a slightly different form from equation (1). First we restate (1) as :

$$(11) \qquad y_i = z + \frac{1}{k_i - 1}(p_i - p'S^*) \quad ,$$

in which $p'S^*$ is shorthand for $\sum_{j=1}^{N} p_j S_j^*$. Then

$$(12) \qquad y_i = (k_j - 1)\left\{\frac{z}{(k_j-1)} + \frac{(p_i - p'S^*)}{(k_i-1)\,(k_j-1)}\right\}$$

$$= \frac{(k_j-1)}{(k_i-1)}\left\{z + \left[\frac{(k_i-1)}{(k_j-1)} - 1\right] z + \frac{1}{(k_j-1)}(p_j - p'S^*) + \frac{(p_i-p_j)}{(k_j-1)}\right\}$$

$$= \frac{(k_j-1)}{(k_i-1)}\, y_j + \frac{(k_i-k_j)}{(k_i-1)}\, z + \frac{p_i-p_j}{(k_i-1)} \qquad (i,j = 1,\ldots,N) \quad .$$

Of course, making such a transformation adds no new structural information to the system; (12) is fully equivalent to (1). The particular form (12) was chosen, however, because in this form parametric restrictions

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1. For a discussion of the iterative procedure involving the S^* see Dixon, Vincent and Powell, <u>op. cit</u>. p. 37.

within and across equations are particularly easy to handle. In fitting (12), naturally, we must be careful to select a set of euqations which are equivalent to (1). We do this by choosing equations of the form (12) for any (n-1) pairs (i,j) such that the coefficient matrix for the endogenous variables y_i has full rank. The reader may verify that this can only be achieved if the selection of pairs includes every i (i = 1,...,N) at least once. It follows that our full rank choice of equations of the form (12) results in (N-1) linearly independent equations in N endogenous variables, $y_1,\ldots,y_N$. To close the model and restore equivalence with (1) we add the identity

$$z \equiv \sum_{i=1}^{N} y_i S_i \quad . \tag{13}$$

To estimate the system (12) - (13), we append zero mean random errors μ_i to (1) (i.e., to (11)). The errors in equation (12) consequently take the form :

$$v_{ij} = \mu_i - \frac{(k_j-1)}{(k_i-1)} \mu_j \quad . \tag{14}$$

Given that we are treating the product shares S_j in (13) as non-stochastic,[1]

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1. Implicit in equation (12) are product shares which are variable and endogenous. In (13) these shares are treated as fixed (the S_i do not vary from observation to observation, being set at their sample mean values) and exogenous. Since the sample deviations between a z series based on fixed S_i and the corresponding series based on variable S_i are extremely small, this inconsistency seems likely to have, at worst, minor consequences.

the linear constraint (13) on the y_i (and thereby on the μ_i) implies that the rank of the (contemporaneous) variance-covariance matrix of the errors μ_i cannot exceed (n-1) . Here we assume that the rank of this matrix is exactly (N-1) . The (N-1) errors v_{ij} belonging to any full-rank selection of (12) are a full rank linear combination of the elements of the complete n-order μ vector; the rank of the (contemporaneous) variance-covariance matrix of the v_{ij} variables in the selected sub-set is, therefore, full (and equal to N-1) . Hence we can apply full information maximum likelihood (FIML) methods to (12) and (13).[1] As is well known from the demand systems literature, the FIML estimates do not depend on which particular full-rank representation of the equation system (12) - (13) is chosen.[2] FIML estimates of the system (12) and (13) were obtained from a straightforward application of Wymer's RESIMUL package.[3] In the structural form estimations, the product shares were allowed to vary from one data point to the next. Our approximate reduced form, fitted

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1. Notice that this is equivalent to making z exogenous. In the reduced form treatment (4), on the other hand, z remains endogenous. (In fact, the elimination of z from the right of (1) results in the additional exogenous variables appearing on the right of (4)).

2. A.P. Barten, "Maximum Likelihood Estimation of a Complete System of Demand Equations", European Economic Review, Vol. 1, No. 1 (Fall 1969), pp. 7-73.
A.A. Powell, "Aitken Estimators as a Tool in Allocating Pre-determined Aggregates", Journal of the American Statistical Association, 64, No. 327 (September 1969), pp. 913-922.

3. C.R. Wymer, "Computer Programs: RESIMUL Manual", London School of Economics, July 1973 (mimeo). This work could not even have been attempted without the access generously made available by Dr Wymer to his extremely advanced programs.