IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Department of Labor and Immigration and the Industries Assistance Commission
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THE THEORETICAL STRUCTURE OF THE ORANI MODULE

by

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1. INTRODUCTION

The Industries Assistance Commission and the Department of Labor and Immigration are currently engaged in a joint study directed towards the development of an econometric model of the Australian economy. The model (named IMPACT) is to consist of three interacting modules.[1] These will be:

(i) a macroeconomic module (MACRO) ;

(ii) a general equilibrium module (ORANI) ;

(iii) a demographic module (BACHUROO) .

The purpose of this note is to outline the theoretical structure of the ORANI module. It has been prepared with the idea of :

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* Many of the ideas which have gone into the development of the ORANI module arose during the numerous extended coffee breaks which I had with Subhash Thakur during my term at the International Monetary Fund. Alan Powell and George Ryland contributed substantial clarifying comments, and Vern Caddy and Matthew Butlin have given editorial assistance.

1. An outline of the aims of the project and the analytical framework to be used can be found in : Alan A. Powell and Tony Lawson, "IMPACT : An Economic-Demographic Model of Australian Industry Structure. Preliminary Outline," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper I-01, Industries Assistance Commission, Melbourne, September 1975.

(a) informing people who are currently making inputs to the project as to its nature; and

(b) soliciting comments from participants at this early and crucial stage.

The notes are concerned only with the theory and are intended to give quick answers to questions such as "What's being assumed about production?"; "What is the nature of investment behaviour?"; or "How are consumer decisions modelled?"

The organization of this paper is as follows. Section 2 outlines the basic methodology. Sections 3 - 6 describe in detail the production functions for current goods and capital, the household sector, investment behaviour, the price system and the balance equations. On the basis of assumed optimizing behaviour by agents, demand functions for capacity creation, consumer goods, intermediate inputs, and primary factors, are derived. Then Section 7 brings all the parts of the module together in a summary form.

2. BASIC APPROACH

The module is developed from the general form

$$F\begin{pmatrix} Y_t \\ Y_{t-1} \end{pmatrix} = 0 , \qquad t = 0, 1 \ldots \tag{2.1}$$

where Y_t is the vector of variable values at time t, and F is a vector function.

The first noteworthy feature of the module is that the lag structure is of a very simple form and may be modified at later stages of the project. According to (2.1), the current values of variables are independent of all but the immediate past history. Computations will be recursive. Given Y_{-1}, we will use (2.1) to find Y_0, then knowing Y_0 we will find Y_1, etc..

The second possibly unfamiliar aspect of (2.1) is that no variables are prior-restricted to being exogenous. The system is rectangular - the number of equations, n, (i.e., the dimension of F) being less than the number of current variables, m, (i.e., the dimension of Y_t). In each experiment with the module, m - n components of Y_t will be set exogenously, and then, given the values of the predetermined vector Y_{t-1}, (2.1) will be solved to determine the values of the remaining n components of Y_t. Maximum flexibility to switch variables between being exogenous and endogenous will be maintained. For example, in some experiments, tariff levels and the exchange rate may be exogenous and the model will determine the balance of trade. In other experiments, the model may generate the tariff levels or the exchange rate necessary to achieve an exogenously specified level for the balance of trade. Another interesting possibility is to let the model determine the wage levels in various occupations consistent with exogenously given levels of employment, or alternatively we could treat the wage rates as exogenous, and determine the employment levels.

A third point about which the reader may be concerned is the lack of a time subscript on F. Changes in production functions and consumer tastes (for example) can be handled by allowing the functional form F to change over time. Alternatively, we can include variables which describe technological change etc. in the vector Y_t. For the purposes of this section, we are adopting the second approach.[2]

Computations will be carried out in a linearized version of system (2.1). We assume that F is differentiable. Then Taylor's theorem implies that there exist numbers $\Theta_{1t}, \ldots \Theta_{nt}$, each in the interval [0, 1], such that

$$F\begin{pmatrix} Y_t \\ Y_{t-1} \end{pmatrix} = \begin{bmatrix} f_1(Z_{1t}) \\ \cdot \\ \cdot \\ \cdot \\ f_n(Z_{nt}) \end{bmatrix} \begin{pmatrix} Y_t - Y_{t-1} \\ Y_{t-1} - Y_{t-2} \end{pmatrix} = 0 , \quad t = 0, 1 \ldots. \tag{2.2}$$

where $f_j(Z_{jt})$ is the row vector of first order partial derivatives of F_j, the j^{th} component of F, evaluated at Z_{jt}

and $Z'_{jt} = (Y'_{t-1}, Y'_{t-2}) + \Theta_{jt}(Y'_t - Y'_{t-1}, Y'_{t-1} - Y'_{t-2})$.

It will be convenient[3] to express (2.2) as

$$\begin{bmatrix} f_1(Z_{1t}) \\ \cdot \\ \cdot \\ \cdot \\ f_n(Z_{nt}) \end{bmatrix} \widehat{\begin{bmatrix} Y_{t-1} \\ Y_{t-2} \end{bmatrix}} \begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} = 0 , \quad t = 0, 1 \ldots. \tag{2.3}$$

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2. Technological change and changes in tastes are not discussed in the rest of the paper. Their treatment in the module has not been finalized.

3. The proportionate change formulation is conventient because equation parameters are easily interpreted as elasticities and shares. This point will become clear in the detailed description of the model, sections 3 to 6. Other nonlinear models solved in terms of proportionate changes include :

Footnote continued at bottom of page 5 ...

where the $\wedge$ changes the column vector into a diagonal matrix, and y_t, y_{t-1} are vectors of proportionate changes, (e.g., y_{ti}, the i^{th} component of y_t, is $(Y_{ti} - Y_{(t-1)i})/ Y_{(t-1)i}$.

Finally, we rewrite (2.3) in easier notation as

$$\left[Q(Z(t)) \ \widehat{\begin{bmatrix} Y_{t-1} \\ Y_{t-2} \end{bmatrix}} \right] \begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix} = 0 , \qquad t = 0, 1 \ldots. \qquad (2.4)$$

where $Z(t)$ is the matrix formed by the column vectors $Z_{1t}, \ldots, Z_{nt}$, i.e., the j^{th} column of $Z(t)$ shows the point at which f_j should be evaluated in linearizing the system (2.1).

In each experiment with the module, we will reorder the elements in y_t and the columns of Q so that we can write

$$Q_1(Z(t)) \left[\hat{\bar{Y}}_{t-1} \right] \bar{y}_t + Q_2(Z(t)) \left[\hat{K}_{t-1} \right] k_t = 0 , \qquad t = 0, 1 \ldots. \quad (2.5)$$

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continued ...

L. Johansen, A Multisectoral Study of Economic Growth, 2nd ed., Amsterdam : North-Holland Publishing Co., 1964

L. Taylor and S. L. Black, "Practical General Equilibrium Estimation of Resource Pulls under Trade Liberalization," Journal of International Economics, 4 (1974), pp. 35-58

Nico Klijn, "Revaluation and Changes in Tariff Protection - The Short Term Effects with Special Reference to Agriculture," Paper presented to the 18th Annual Conference of the Australian Agricultural Economics Society, Perth, February, 1974. (This paper uses a theoretical structure similar to that used by Taylor and Black, but incorporates important modifications to accommodate the Australian situation.)

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where $\bar{y}_t$ is the n-component vector of proportionate changes in variables chosen as endogenous for the experiment and k_t is the ((m-n) + m)-component vector of proportionate changes in the remaining current and predetermined variables (i.e., the exogenous variables).[4] We assume that Q_1 is nonsingular and that (2.5) can be solved as

$$\bar{y}_t = -\left(Q_1(Z(t)) \left[\hat{\bar{Y}}_{t-1}\right]\right)^{-1} Q_2(Z(t)) \left[\hat{K}_{t-1}\right] k_t , \qquad t = 0, 1 \ldots. \tag{2.6}$$

$$\bar{Y}_t = \left[I + \hat{\bar{y}}_t\right] \bar{Y}_{t-1} , \qquad t = 0, 1 \ldots. \tag{2.7}$$

System (2.6) shows how each of the endogenous variables is affected by changes in the exogenous variables and system (2.7) translates the changes, $\bar{y}_t$, into levels $\bar{Y}_t$.

Unfortunately, Taylor's theorem does not tell us the value for Z(t), only that each column is on the line segment connecting (Y_t', Y_{t-1}') and (Y_{t-1}', Y_{t-2}') . Hence, we will normally approximate the system (2.6) by

$$\bar{y}_t = -\left(Q_1(\psi_0(t)) \left[\hat{\bar{Y}}_{t-1}\right]\right)^{-1} Q_2(\psi_0(t)) \left[\hat{K}_{t-1}\right] k_t , \qquad t = 0, 1, \ldots. \tag{2.8}$$

where $\psi_0(t)$ is the $2m \times n$ matrix, each column of which is $\begin{pmatrix} Y_{t-1} \\ Y_{t-2} \end{pmatrix}$.

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4. $\begin{pmatrix} \bar{Y}_{t-1} \\ K_{t-1} \end{pmatrix}$ is the vector $\begin{pmatrix} Y_{t-1} \\ Y_{t-2} \end{pmatrix}$, reordered so that the endogenous variables occur in the first n rows.

Rather than evaluating the rows of Q at the appropriate set of points, Z_{1t}, Z_{2t} ... Z_{nt}, (2.8) implies that we evaluate each row at the "base" point $\begin{pmatrix} Y_{t-1} \\ Y_{t-2} \end{pmatrix}$. If necessary, accuracy may be increased by making a two or more stage approximation. For example, we could use

$$\bar{Y}_t = \left[I + \hat{\bar{y}}_t(2)\right] \left[I + \hat{\bar{y}}_t(1)\right] \bar{Y}_{t-1} \qquad t = 0, 1 \ldots. \qquad (2.9)$$

where

$$\bar{y}_t(1) = -\left(Q_1(\psi_0(t)) \left[\hat{\bar{Y}}_{t-1}\right]\right)^{-1} Q_2(\psi_0(t)) \left[\hat{K}_{t-1}\right] (\tfrac{1}{2}k_t) ,$$

$$\bar{y}_t(2) = -\left(Q_1(\psi_1(t)) \left[\hat{\bar{Y}}_{t-1}(1)\right]\right)^{-1} Q_2(\psi_1(t)) \left[\hat{K}_{t-1}(1)\right] (\tfrac{1}{2}k_t) ,$$

$$\bar{Y}_{t-1}(1) = [\, I + \hat{\bar{y}}_t(1) \,] \; \bar{Y}_{t-1} ,$$

$$\bar{K}_{t-1}(1) = [\, I + \tfrac{1}{2}\hat{k}_t \,] \; K_{t-1} ,$$

and $\psi_1(t)$ is the $2m \times n$ matrix with each column made up of the vectors $Y_{t-1}(1)$, $K_{t-1}(1)$ written with appropriate ordering. For each time, t, the approximation (2.9) involves (i) moving the exogenous variables through half their total change, (ii) evaluating the effect on the endogenous variables, (iii) updating the evaluation of the derivative matrices Q_1 and Q_2, (iv) moving the exogenous variables through the remaining half of their change, and (v) completing the computation of the effect on the endogenous variables.

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At early stages of the project, we will not have a complete list and modelling of all the "relevant" variables. Hence, initially, we will report impact multipliers (i.e., matrices of the form $B = -(Q_1 \hat{\hat{Y}}_{t-1})^{-1} (Q_2 \hat{K}_{t-1})$ rather than complete time-path solutions such as (2.7). The elements of B show the "one period" elasticities of variables chosen to be endogenous with respect to changes in exogenous variables.

These results can be useful even in an incompletely specified model. For example, if the elasticity of output in industry i with respect to exchange rate changes is largely independent of assumptions concerning technological change, it is legitimate to compute it in a model which has no convincing description of technological change.

The equations which constitute the ORANI module (this gives a particular form to the general differential system (2.4)) and the assumptions underlying their derivation are outlined below.

3. DEMAND EQUATIONS[5]

3.1 Demand for Inputs for the Production of Current Goods.

The production function available to the industry which produces good j is specified as being of the form

$$X_j = \min \left\{ \frac{X_{1j}}{A^{(1)}_{1j}}, \frac{X_{2j}}{A^{(1)}_{2j}}, \ldots, \frac{X_{gj}}{A^{(1)}_{gj}}; \frac{X_{(g+1)j}}{A^{(1)}_{(g+1)j}}, \ldots, \frac{X_{(g+m)j}}{A^{(1)}_{(g+m)j}} \right\} \tag{3.1}$$

$j = ;. \ldots, g$

5. A more detailed derivation of the demand equations can be found in: Vern Caddy, "Demand Equations in the ORANI Module : An Exposition," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper OP-02, Industries Assistance Commission, Melbourne, July 1975.

where X_j is the output of industry j, X_{ij} is the "effective input" of good i into the production of j, and $A_{ij}^{(1)}$ is a parameter representing the minimum amount of i required to produce a unit of j.

The concept of "effective inputs" is introduced to allow for the possibility that units of a particular input drawn from one source may not constitute perfect substitutes for units of the same input drawn from an alternative source.[6] The manner in which units of a given input, differentiated according to source, are combined to provide a measure of "effective input" is defined by the function

$$X_{ij} = \left\{ \sum_s (X_{isj})^{-\rho_{ij}^{(1)}} b_{isj}^{(1)} \right\}^{-1/\rho_{ij}^{(1)}} \qquad (3.2)$$

where X_{isj} is the input of i from source s into the production of good j and $b_{isj}^{(1)}$ and $\rho_{ij}^{(1)}$ are parameters (with $b_{isj}^{(1)} > 0$; $\rho_{ij}^{(1)} > -1$) . The form of this function implies that the elasticity of substitution between quantities of input i from its various sources of supply, when they are being used to satisfy industry j's requirements for that input is

$$\sigma_{ij}^{(1)} = \frac{1}{1 + \rho_{ij}^{(1)}} \qquad (3.3)$$

6. The idea of imperfect substitution between domestic and foreign sources is emphasized in Paul Armington, "Adjustment of Trade Balances : Some Experiments with a Model of Trade Among Many Countries," IMF Staff Papers 17 (1970), pp. 488-526

In equation (3.1) it will be noticed that there are $(g + m)$ inputs. The first g of these are interpreted as being <u>manufactured inputs</u>, e.g., steel, petroleum, etc. For these, two sources of supply are identified, namely, domestic production and imports. In the notation used $s = 1$ refers to domestic production and $s = 2$ to imports.

The subscript $g + 1$ identifies the input of <u>primary factors</u>. The module distinguishes between the three "traditional" sources of supply for primary inputs, i.e., labor, land and capital. These three elements are combined in the manner specified by (3.2) to yield a particular "effective" level of input. Thus, $X_{(g+1)j}$ is the "effective" input of primary factors to industry j, $X_{(g+1)1j}$ is the input of labor, $X_{(g+1)2j}$ is the input of capital, and $X_{(g+1)3j}$ is the input of land. It should be noted that equation (3.2) allows for substitution between the various primary inputs but <u>not</u> between primary and manufactured inputs. Similarly, while substitution between domestically produced and imported units of a manufactured input is allowable, the possibility of substitution between different types of manufactured inputs is excluded.

"Goods" $(g + 2)$ to $(g + m)$ can be used flexibly to introduce other costs of production, e.g., taxes, costs of holding liquidity, and costs of holding inventories. For illustrative purposes we will assume that $m = 2$ and that $X_{(g+2)j}$ is the input of tax tickets, i.e., in order to produce a unit of good j a certain number of tax tickets must be purchased. Changes in the price of these tickets simulate changes in the tax rate. Alternative specifications would be necessary to capture the role of profit taxes rather than production taxes.

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The production specification can be summarized as

(a) Leontief technology - this implies constant returns to scale and no substitution between different goods or between intermediate inputs and primary factors and

(b) substitution between alternative sources of the same good, with the substitution possibilities described by CES functions.[7]

The assumption made about the behaviour of producers is that they are efficient. This means that for any given level of output X_j, the producers in industry j will select the combination of inputs which minimizes costs and is compatible with that output level, i.e., they choose X_{isj} for all i and s so as to minimize

$$\sum_{i=1}^{g} \sum_{s} P_{is} X_{isj} + \sum_{i=g+1}^{g+m} \sum_{s} P_{isj} X_{isj} \qquad (3.4)$$

(Total costs)

subject to (3.1) & (3.2) where X_j is treated as fixed and P_{is} is the price of good i from source s. For inputs g + 1 to g + m the additional subscript j is attached to the price term to allow for variations across industries in the primary factor prices and tax rates.

On the basis of the behavioural model (3.4) it is possible to derive demand functions of the form

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7. It is worth pointing out that because imports are not necessarily perfect substitutes for domestic products, the model accommodates crosshauling.

$$X_{isj} = f_{isj}(P_j, X_j) \qquad (3.5)$$

where P_j is the vector of prices appearing in (3.4). In fact, with the particular specifications (3.1) & (3.2) it can be shown[8] that a differential form of (3.5) is

$$\left.\begin{aligned} x_{isj} &= x_j - \sigma_{ij}^{(1)}\Big(p_{is} - \sum_{s=1}^{2} p_{is}S_{isj}^{(1)}\Big) && \text{(i)} \\ & \qquad i = 1, \ldots, g; \\ x_{isj} &= x_j - \sigma_{ij}^{(1)}\Big(p_{isj} - \sum_{s} p_{isj}S_{isj}^{(1)} && \text{(ii)} \\ & \qquad i = g+1, \ldots, g+m \end{aligned}\right\} \qquad (3.6)$$

where the lower case x's and p's refer to proportionate changes in the corresponding upper case variables (i.e., $x_{isj} = \frac{dX_{isj}}{X_{isj}}$) and $S_{isj}^{(1)}$ is the share of product (is) in the total costs of input i used by industry j (i.e., $S_{isj}^{(1)} = \frac{P_{is}X_{isj}}{\sum_s P_{is}X_{isj}}$).

The interpretation of (3.6) is straightforward. If there are no changes in the prices of good i, then a 10% increase in X_j leads to a 10% increase in X_{isj}. However, if P_{is} rises relative to the prices of good i from alternative sources, then X_{isj} increases less rapidly than X_j.

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8. See Appendix 6, and also Caddy, op. cit.

The model's demand side for intermediate inputs and primary factors is specified by (3.6). Its implementation requires knowledge of the $S_{isj}^{(1)}$ and the $\sigma_{ij}^{(1)}$. The prices and outputs will be determined by other parts of the model.

3.2 Demand for Inputs for the Production of Capital Goods.

The technical specification and behavioural assumptions associated with the production of capital equipment are similar to those adopted for the production of current goods.

The technology for the creation of capital in industry j is of the form

$$Y_j = \min_{i=i,\ldots,g} \left\{ \frac{Y_{ij}}{A_{ij}^{(2)}} \right\} \tag{3.7}$$

where $A_{ij}^{(2)}$ is a parameter, Y_{ij} is the "effective" input of good i into creating capital for industry j, and Y_j is the capital creation in industry j.

Y_{ij} is defined by

$$Y_{ij} = \left[\sum_s (Y_{isj})^{-\rho_{ij}^{(2)}} b_{isj}^{(2)} \right]^{-\frac{1}{\rho_{ij}^{(1)}}} \tag{3.8}$$

where the definitions are analogous to those made following (3.2).

A point of contrast between the technologies described by (3.1), (3.2) and (3.7), (3.8) is that capital creation requires no primary factors. The use of labor, capital and land will be recongized via the inputs of "construction," i.e., the construction industry uses labor and capital, and the creation of capital requires heavy inputs of construction.

Our behavioural assumption is that capacity expansions are achieved at minimum cost, i.e., the Y_{isj} are chosen so that they minimize

$$\sum_{i=1}^{g} \sum_{s} P_{is} Y_{isj} \tag{3.9}$$

subject to (3.7) and (3.8), where Y_j is treated as a parameter.

The efficiency assumption leads to the demand function

$$y_{isj} = y_j - \sigma_{ij}^{(2)} (p_{is} - \sum_{s} S_{isj}^{(2)} p_{is}) \tag{3.10}$$

where the lower case letters denote proportionate changes in the quantities defined by the corresponding upper case letters, $\sigma_{ij}^{(2)} \equiv \frac{1}{1 + \rho_{ij}^{(2)}}$ is the elasticity of substitution between alternative sources of good i in the construction of capital for industry j, and $S_{isj}^{(2)}$ is the share of product (is) in the total costs of good i used in capital creation for industry j (i.e., $S_{isj}^{(2)} = \frac{P_{is} Y_{isj}}{\sum_{s} P_{is} Y_{isj}}$) .

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The module's demand for inputs into capital creation is specified by (3.10). Its implementation requires a knowledge of $S_{isj}^{(2)}$ and $\sigma_{ij}^{(2)}$. The y_j and price changes are endogenous.

3.3 Demand for Consumption Goods.

In the module, household demands are explained by a utility maximizing model. It is assumed that the household sector chooses the consumption bundle of "effective inputs" (C_i) which maximizes the utility function

$$U\ (C_1, C_2, \ldots , C_g) \tag{3.11}$$

subject to the restrictions imposed by the budget constraint

$$\sum_{i=1}^{g} \sum_{s} P_{is}\ C_{is} = C \quad . \tag{3.12}$$

C refers to total household expenditure, C_{is} to final consumption of good i from source s and P_{is} is as previously defined. The relationship between the level of "effective input" and the quantities of that good purchased from the alternative sources is defined by

$$C_i = \left[\sum_{s} (C_{is})^{-\rho_i^{(3)}} b_{is}^{(3)} \right]^{-\frac{1}{\rho_i^{(3)}}} \quad . \tag{3.13}$$

The parameters $b_{is}^{(3)}$ and $\rho_i^{(3)}$ will have values greater than zero and minus one respectively.

The model (3.11) - (3.13) implies various useful relationships concerning "cross" and "own price" elasticities. It can be shown that[9]

$$\text{(a)} \qquad \eta_{isjt} = \eta_{ij} S_{jt}^{(3)} \qquad (3.14)$$

(for all i, s, j and t, $j \neq i$)

$$\text{(b)} \qquad \eta_{isit} = (\eta_{ii} + \sigma_i^{(3)}) \, S_{it}^{(3)} \qquad (3.15)$$

(for all i, s and t with $s \neq t$)

$$\text{(c)} \qquad \eta_{isis} = (\eta_{ii} + \sigma_i^{(3)}) \, S_{is}^{(3)} - \sigma_i^{(3)} \qquad (3.16)$$

(for all i and s)

where η_{isjt} is the elasticity of demand for good i from source s with respect to the price of good j from source t; $\sigma_i^{(3)}$ is the elasticity of substitution between alternative sources of good i ($\sigma_i^{(3)} = \frac{1}{1 + \rho_i^{(3)}}$) ; $S_{js}^{(3)}$ is the share of total consumption expenditure on good j that goes in purchases of good j from source s (i.e., $S_{js}^{(3)} = \frac{P_{js}C_{js}}{\sum_s P_{js}C_{js}}$) ; and η_{ij} are the "outside" elasticities. The "outside" elasticities specify the elasticity of demand for good i with respect to changes in the price of j (i.e., they reflect the percentage change in C_j which will result when each of the P_{is}, for all s, are raised by 1%.)[10] These "outside" elasticities

9. See Appendix 1.

10. The concept of the "outside" elasticity is clarified in Appendix 1.

are the output from the usual studies of demand systems where products are not distinguished by source.[11]

The equations expressing the demand for consumption goods are of the form

$$c_{is} = \sum_j \sum_t \eta_{isjt} p_{jt} + \varepsilon_{is} c \qquad (3.17)$$

where the lower case letters represent proportionate changes in consumption, prices and total household expenditure, and the η_{isjt} are measured by combining the system (3.14)-(3.16) with a set of measurements of the "outside" elasticities. ε_{is} is the expenditure elasticity of demand for product i from source s and from (3.13) it can be seen that

$$\varepsilon_{is} = \varepsilon_i \text{ for all } s.$$ [12]

Finally, if the additivity assumption is adopted for the utility function (3.11) then the Frisch method[13] can be used to estimate the "outside" elasticities on the basis of measurements of the expenditure elasticities.

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11. For example, Alan A. Powell, Empirical Analytics of Demand Systems, (Lexington, Mass. : D.C. Heath/Lexington Books, 1974), pp. 46; 52-54

12. This follows from the fact that the function (3.13) (which relates inputs from the alternative sources to the "effective" input level) has constant returns to scale. This means that the least cost method of achieving any expansion in a particular "effective" input, which results from a change in income, is by making equiproportionate changes in the quantity of inputs drawn from each of the alternative sources.

13. Ragnar Frisch, "A Complete Scheme for Computing all Direct and Cross Demand Elasticities in a Model with Many Sectors," Econometrica, Vol. 27 (1959), pp. 177-196

4. INVESTMENT

The assumptions concerning investment that have been adopted are as follows:

(a) Capital takes one year to install. (This assumption could be varied if information becomes available.)

(b) Investors behave as though they believe that an additional unit to the existing capital in industry j will have an earning profile of

$$\Xi_j = (-\Pi_j, P_{(g+1)2j}, P_{(g+1)2j}(1-\delta_j), P_{(g+1)2j}(1-\delta_j)^2 \ldots) . \qquad (4.1)$$

That is, in the first year investors must make an outlay of Π_j (the cost of a unit of capital in industry j), in the next year they will earn $P_{(g+1)2j}$ (the current annual rental value of capital in industry j), and in subsequent years earnings are expected to reduce at the rate δ_j (reflecting capital depreciation).

The profile Ξ_j implies a rate of return or marginal efficiency of capital (MEC) of [14]

$$MEC_j = \frac{P_{(g+1)2j}}{\Pi_j} - \delta_j . \qquad (4.2)$$

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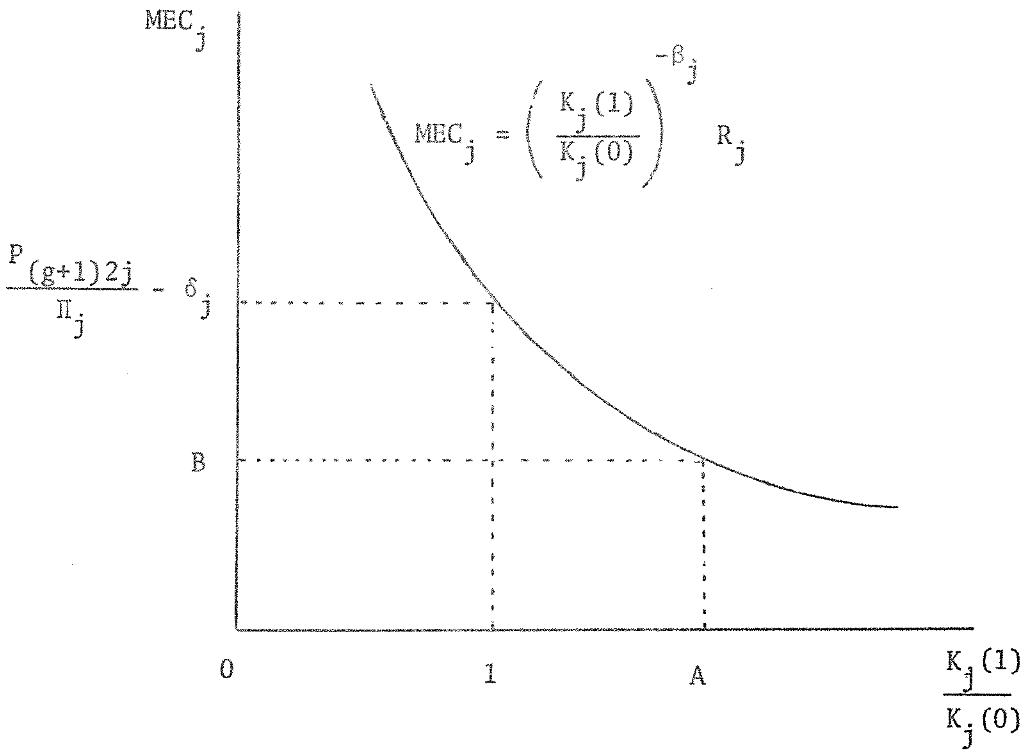
14. See Appendix 2.

(c) Investors are cautious about expanding the capital stock too fast in industry j. While they perceive (4.2) as representing the MEC at the existing level of capital stock, they behave as if

$$\frac{\partial MEC_j}{\partial K_j(1)} \cdot \frac{K_j(1)}{MEC_j} = -\beta_j \qquad (4.3)$$

where $K_j(1)$ is the capital stock planned for one year's time and β_j is a positive parameter, i.e., the elasticity of the MEC with respect to increases in capital stock is $-\beta_j$.

This situation is represented in the figure below.



The horizontal axis measures the ratio of future capital stock, (i.e., capital stock existing in one year's time), to current capital stock, and the vertical axis shows the MEC. For example, with the existing capital stock, the MEC is

$$R_j \equiv \frac{P_{(G+1,2)j}}{\Pi_j} - \delta_j \quad . \tag{4.4}$$

If investment plans were set so that $K_j(1)/K_j(0)$ would reach the level A, then the rate of return on an additional unit of capital would be expected to be B.

(d) The total annual investment expenditure (Y) is allocated across industries so as to equate the expected MEC's. This means that there exists some rate of return Λ such that

$$\left(\frac{K_j(1)}{K_j(0)}\right)^{-\beta_j} R_j \leqslant \Lambda \tag{4.5}$$

$$K_j(1) = K_j(0)(1 - \delta_j) + Y_j \tag{4.6}$$

$$\sum_j \Pi_j Y_j = Y , \qquad Y_j > 0 \text{ for all } j \tag{4.7}$$

and the strict inequality in (4.5) implies that $Y_j = 0$. Y_j is the number of new units of capital stock to be created for industry j and Π_j is the price per unit of that capital.

It is initially assumed that (4.5) holds as an equality for all j. In this case a differential form of the relationships (4.5) - (4.7) is

$$-\beta_j(k_j(1) - k_j(0)) + Q_j(p_{(g+1)2j} - \pi_j) = \lambda \tag{4.8}$$

$$k_j(1) = k_j(0)(1 - g_j) + y_j g_j \tag{4.9}$$

$$\sum_j (\pi_j + y_j) z_j = y \tag{4.10}$$

where the lower case letters denote proportionate changes in the variables defined by the upper case letters; z_j is the share of industry j in the aggregate investment expenditure,

(i.e., $z_j = \frac{Y_j \Pi_j}{\sum_j Y_j \Pi_j}$) ; $g_j = \frac{Y_j}{K_j(1)}$; and $Q_j = \frac{(R_j + \delta_j)}{R_j}$.

If an experiment (i.e., a solution of the model for a particular set of exogenous variables) reveals large negative values for $k_j(1)$, it may be necessary to introduce the inequality (4.5). One simple procedure would be to set $k_j(1)$ at its minimum and recompute. The minimum value that $k_j(1)$ can take is the negative of the rate of depreciation.

5. PURE PROFITS AND THE PRICE SYSTEM

It is assumed that pure profits are zero in both current production and the creation of capacity. Hence we have

$$\sum_{i=1}^{g} \sum_{s} P_{is} X_{isj} + \sum_{i=g+1}^{g+m} \sum_{s} P_{isj} X_{isj} = P_{j1} X_j \qquad (5.1)$$

and

$$\sum_{i=1}^{g} \sum_{s} P_{is} Y_{isj} = \Pi_j Y_j \qquad (5.2)$$

(for $j = 1, \ldots, g$) .

(Note that P_{j1} is the price of domestically produced good j.)

Differential forms of (5.1) and (5.2) are [15]

$$p_{j1} = \sum_{i=1}^{g} \sum_{s} a_{isj}^{(1)} + \sum_{i=g+1}^{k} \sum_{s} a_{isj}^{(1)} p_{isj} \qquad (5.3)$$

and

15. The derivation of (5.3) & (5.4) from (5.1) & (5.2) is given in Appendix 3.

$$\pi_j = \sum_{i=1} \sum_s a^{(2)}_{isj} \, p_{is} \tag{5.4}$$

where the lower case p's and π's represent proportionate changes, $a^{(1)}_{isj}$ is the share of the cost of current good j which is accounted for by inputs of product i from source s and $a^{(2)}_{isj}$ is (is)'s share of the cost of creating a unit of capital for industry j.

6. MARKET CLEARING

The equations introduced into the model to ensure balance between the various supplies and demands have the form

$$X_i = \sum_j X_{i1j} + \sum_j Y_{i1j} + C_{i1} + E_{i1} + F_{i1} \tag{6.1}$$

$$i = 1, \ldots, g$$

$$L = \sum_j X_{(g+1)1j} \tag{6.2}$$

$$K_j(0) = X_{(g+1)2j} \tag{6.3}$$

$$N = \sum_j X_{(g+1)3j} \tag{6.4}$$

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Equation (6.1) equates supply and demand for each of the domestically produced goods. Total demand is made up of

(i) demand for intermediate inputs into the production of current goods (X_{i1j}) ;

(ii) demand for intermediate inputs into the production of capital equipment (Y_{i1j}) ;

(iii) demand for consumption goods (C_{i1}) ;

(iv) export demand (E_{i1}) ;

(v) exogenous demands, e.g., government purchases, accumulation of stocks (F_{i1}) .

The only noteworthy point about (6.1) is the absence of imports. This results from the fact that in the ORANI module imports of good i are recognized as a distinct product and consequently are not subtracted from the right hand side of the balance equation. Non-exportables are handled simply by setting E_{i1} equal to zero.

Equation (6.2) equates the supply of labor (L) to the demand for it. It also implies that labor is homogeneous and justifies the assumption that there is only one wage rate, i.e.,

$$P_{(g+1)1j} = P_{(g+1)1} \qquad \text{for all } j. \tag{6.3}$$

The specification of the labor market given by (6.2) is the crudest possible. More interesting specifications, such as providing for the recognition of various types of labor, may be included once the problems of getting the basic model to work have been overcome. Another possibility is to allow for "unemployment" by replacing (6.2) with a wage equation.

Equation (6.3) equates supply and demand for capital in each industry. Unlike labor, capital is not regarded as being homogeneous, but is assumed to be industry specific.

The last of the balance equations (6.4) ensures equality between the demand for, and supply of, land. As for labor it is assumed that there is a single rental price for all land, i.e.,

$$P_{(g+1)3j} = P_{(g+1)3} \qquad \text{for all } j \, . \tag{6.5}$$

The percentage change forms of the equations (6.1) - (6.4) are:

$$x_i = \sum_j x_{i1j} B^{(1)}_{i1j} + \sum_j y_{i1j} B^{(2)}_{i1j} + c_{i1} B^{(3)}_{i1j} + e_{i1} B^{(4)}_{i1} + f_{i1} B^{(5)}_{i1} \tag{6.6}$$

$$\ell = \sum_j \ell_j \, x_{(g+1)1j} \tag{6.7}$$

$$k_j(0) = x_{(g+1)2j} \tag{6.8}$$

$$n = \sum_j n_j \, x_{(g+1)3j} \tag{6.9}$$

where ℓ_j and n_j are the shares of the total labor force and land used by industry j, and the B's are the shares of the sales of good i being absorbed by the various types of demand identified on the right hand side of (6.1). Again, the lower case letters have been used to represent proportional changes in the variable identified by the corresponding upper case letter.

7. THE COMPLETE MODEL

The complete model is presented in Appendix 4, the equations being transcribed from earlier sections of the notes. It will be clear to the reader that further equations could be added for the determination of the balance of trade and the relationship between import prices and tariff changes. Appendix 5 lists the variables.

To conclude these notes, some of the influences of a tariff change on the domestic economy which are captured by the model will be outlined. Assume that there is an increase in the tariff on commodity i, resulting in an increase in P_{i2}. Equations (3.6); (3.10) and (3.17) allow for substitution of intermediate input requirements (both for current and capital production) and household demands. The output effects are then captured by equation (6.6). The cost effects are reflected through equations (5.3) and (5.4) and shifts in the relative rental value of capital in different industries are allowed to influence investment decisions via (4.8).

APPENDIX 1

Derivation of the Household Demand Elasticities [1]

By comparing the necessary conditions for a maximum, it can be shown that the solution to the problem

$$\left.\begin{aligned} &\text{maximize} && U(C_1, \ldots, C_g) \\ &\text{subject to} && C_i = \left(\sum_s (C_{is})^{-\rho_i^{(3)}} b_{is}^{(3)}\right) \\ &\text{and} && \sum_i \sum_s P_{is} C_{is} = C \end{aligned}\right\} \quad (A1.1)$$

is the same as the solution to the problems

$$\left.\begin{aligned} &\text{minimize} && \sum_s P_{is} C_{is} \\ &\text{subject to} && C_i = \left(\sum_s (C_{is})^{-\rho_i^{(3)}} b_{is}^{(3)}\right)^{-\frac{1}{\rho_i^{(3)}}} \end{aligned}\right\} \quad (A1.2)$$

where C_i is determined as the solution to the problem,

$$\left.\begin{aligned} &\text{maximize} && U(C_i, \ldots, C_g) \\ &\text{subject to} && \sum_i P_{i*} C_i = C \end{aligned}\right\} \quad (A1.3)$$

and P_{i*} is the price index defined by (A1.8) below. This justifies the following two-stage procedure.

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1. The household demand elasticities are derived from a nested-utility model. For a similar model see M. Brown and D. Heien, "The S-Branch Utility Tree: A Generalization of the Linear Expenditure System," Econometrica 40, No. 4 (July 1972).

Problem (A1.2) leads to (from manipulating the necessary first order conditions)

$$\frac{P_{ij}}{P_{iq}} = \left(\frac{C_{iq}}{C_{ij}}\right)^{1+\rho_i^{(3)}} \frac{b_{ij}^{(3)}}{b_{iq}^{(3)}} \quad . \tag{A1.4}$$

Substituting into the constraint function leads to

$$C_{ij} = C_i \left(\sum_q \left(\frac{P_{ij}}{P_{iq}} \cdot \frac{b_{iq}^{(3)}}{b_{ij}^{(3)}} \right)^{\frac{-\rho_i^{(3)}}{1+\rho_i^{(3)}}} b_{iq}^{(3)} \right)^{\frac{1}{\rho_i^{(3)}}} \quad . \tag{A1.5}$$

Then to derive the price index, we note that first order conditions in problem (A1.3) require

$$\frac{\partial U}{\partial C_i} = \lambda_1 P_{i*} \quad . \tag{A1.6}$$

But problem (A1.1) implies

$$\frac{\partial U}{\partial C_i} \cdot \frac{\partial C_i}{\partial C_{ij}} = \lambda_2 P_{ij} \quad . \tag{A1.7}$$ [1]

Multiplying both sides of (A1.7) by C_{ij}, summing over j and using the linear homogeneity of C_i (from equation (A1.1), gives

---

1. $\lambda_1 = \lambda_2 =$ marginal utility of expenditure.

$$P_{i*} = \sum_j P_{ij} \left(\frac{C_{ij}}{C_i} \right)$$

$$= \sum_j P_{ij} \left[\sum_q \left(\frac{P_{ij}}{P_{iq}} \cdot \frac{b_{iq}^{(3)}}{b_{ij}^{(3)}} \right)^{-\frac{\rho_i^{(3)}}{1+\rho_i^{(3)}}} b_{iq}^{(3)} \right]^{\frac{1}{\rho_i^{(3)}}} . \qquad \text{(A1.8)}$$

From problem (A1.3) we are able to derive demand functions of the form

$$C_i = f_i \, (P_{i*}, \ldots, P_{g*}, C) \quad . \qquad \text{(A1.9)}$$

Equation (A1.9) gives the notion of the "outside" elasticity referred to in section 3.3 in the text. The "outside" elasticity is defined as

$$\eta_{ij} = \frac{\partial f_i}{\partial P_j} \cdot \frac{P_{j*}}{C_i} \quad .$$

The elasticity of demand for good ij with respect to the price good ℓm is given by

$$\eta_{ij\ell m} = \left\{ \left(\frac{\partial C_{ij}}{\partial P_{\ell m}} \right)_{C_i \text{ const.}} + \left(\frac{\partial C_{ij}}{\partial C_i} \right) \left(\frac{\partial C_i}{\partial P_{\ell *}} \right) \left(\frac{\partial P_{\ell *}}{\partial P_{\ell m}} \right) \right\} \frac{P_{\ell m}}{C_{ij}}$$

$$= \frac{\partial C_i}{\partial P_{\ell m}} \cdot \frac{P_{\ell m}}{C_{ij}} + \left(\frac{\partial C_{ij}}{\partial C_i} \cdot \frac{C_i}{C_{ij}} \right) \left(\frac{\partial C_i}{\partial P_{\ell *}} \cdot \frac{P_{\ell *}}{C_i} \right) \left(\frac{\partial P_{\ell *}}{\partial P_{\ell m}} \cdot \frac{P_{\ell m}}{P_{\ell *}} \right) . \qquad \text{(A1.10)}$$

Note that

$$\frac{\partial C_{ij}}{\partial C_i} \cdot \frac{C_i}{C_{ij}} = 1 \qquad \text{(A1.11)}$$

and that for $i \neq \ell$

$$\frac{\partial C_{ij}}{\partial P_{\ell m}} = 0 \ . \qquad \text{(A1.11a)}$$

For $i = \ell$, $j \neq m$, it follows from equation (A1.5) that[2]

$$\frac{\partial C_{ij}}{\partial P_{\ell m}} \cdot \frac{P_{\ell m}}{C_{ij}} = \frac{\sigma_i^{(3)} \, (P_{im})^{\rho_i^{(3)}\sigma_i^{(3)}} \, (b_{im}^{(3)})^{\sigma_i^{(3)}}}{\sum_q (P_{iq})^{\rho_i^{(3)}\sigma_i^{(3)}} \, (b_{iq}^{(3)})^{\sigma_i^{(3)}}} \ . \qquad \text{(A1.12)}$$

Now

$$S_{im}^{(3)} = \frac{P_{im} C_{im}}{\sum_j P_{ij}^{\rho_i^{(3)}\sigma_i^{(3)}} \, (b_{ij}^{(3)})^{\sigma_i^{(3)}}}$$

$$\text{(A1.13)}$$

$$= \frac{P_{im}^{\rho_i^{(3)}\sigma_i^{(3)}} \, (b_{im}^{(3)})^{\sigma_i}}{\sum_j P_{ij}^{\rho_i^{(3)}\sigma_i^{(3)}} \, (b_{ij}^{(3)})^{\sigma_i^{(3)}}} \ .$$

---

2. Note $\sigma_i^{(3)} = \dfrac{1}{1+\rho_i^{(3)}}$.

Substituting (A1.13) into (A1.12) gives

$$\frac{\partial C_{ij}}{\partial P_{\ell m}} \cdot \frac{P_{\ell m}}{C_{ij}} = \sigma_i^{(3)} S_{im}^{(3)} \qquad \text{(A1.14)}$$

$$i = \ell,\ j \neq m .$$

Since (A1.5) is homogeneous of degree zero in prices, it follows immediately from (A1.14) that

$$\frac{\partial C_{ij}}{\partial P_{\ell m}} \cdot \frac{P_{\ell m}}{C_{ij}} = -\sigma_i \left(1 - S_m^{(3)}\right) \qquad \text{(A1.15)}$$

$$i = \ell,\ j = m .$$

We also note that

$$P_{i*} = \left\{ \sum_q \left(P_{iq}\right)^{\rho_i^{(3)} \sigma_i^{(3)}} \left(b_{iq}^{(3)}\right)^{\sigma_i^{(3)}} \right\}^{1 + \frac{1}{\rho_i^{(3)}}} ,$$

and if we substitute this into

$$\frac{\partial P_{\ell *}}{\partial P_{\ell m}} \cdot \frac{P_{\ell m}}{P_{\ell *}} \quad \text{we obtain}$$

$$\frac{\partial P_{\ell *}}{\partial P_{\ell m}} \cdot \frac{P_{\ell m}}{P_{\ell *}} = S_{\ell m}^{(3)} . \qquad \text{(A1.16)}$$

Finally, if we substitute from (A1.11), (A1.11a), (A1.14), (A1.15) and (A1.16) into (A1.10), we obtain the final formulae shown in the text as (3.14), (3.15) and (3.16).

APPENDIX 2

Derivation of the Marginal Efficiency of Capital

The problem is to find r such that

$$\Pi_j = \frac{1}{1+r} \sum_{t=0}^{\infty} P_{(g+1)2j} \left(\frac{1-\delta_j}{1+r} \right)^t \qquad \text{(A2.1)}$$

$$= P_{(g+1)2j} \frac{1}{\delta_j + r} \quad .$$

Thus,

$$r = \frac{P_{(g+1)2j}}{\Pi_j} - \delta_j \quad . \qquad \text{(A2.2)}$$

APPENDIX 3

Derivation of Equations (5.3) and (5.4)

From (5.1) we have

$$\sum_{i=1}^{g+m} \sum_{s} p_{isj} a_{isj}^{(1)} = p_{j1} + x_j - \sum_{i=1}^{g+m} \sum_{s} x_{isj} a_{isj}^{(1)} \qquad \text{(A3.1)}$$

where (A3.1) is a percentage change form of (5.1). Cost minimizing in the context of the constant returns to scale production function (3.1) - (3.2) implies that

$$x_j = \sum_{i=1}^{g+m} x_{isj} a_{isj}^{(1)} \quad .$$

Hence (A3.1) reduces to (5.3). The derivation of (5.4) is along similar lines.

APPENDIX 4

The Complete Model

	Equation Identification No.	Description
$x_{isj} = x_j - \sigma_{ij}^{(1)}(p_{is} - \sum_{t=1}^{2} p_{it}S_{itj}^{(1)})$ $i, j = 1, \ldots, g;$ $s = 1, 2$.	(3.6)(i)	Demands for intermediate inputs for production of current goods.
$x_{(g+1)sj} = x_j - \sigma_{(g+1)j}^{(1)}\{p_{(g+1)sj} - \sum_{t=1}^{3} p_{(g+1)tj}S_{(g+1)tj}^{(1)}\}$ $j = 1, \ldots, g;$ $s = 1, 2, 3$.	(3.6)(ii)	Demands for primary inputs for production of current goods.
$y_{isj} = y_j - \sigma_{ij}^{(2)}(p_{is} - \sum_{t=1}^{2} p_{it}S_{itj}^{(2)})$ $i, j = 1, \ldots, g;$ $s = 1, 2$.	(3.10)	Demands for intermediate inputs for creation of capital.
$c_{is} = \varepsilon_{is}c + \sum_{u=1}^{g}\sum_{v=1}^{2} \eta_{isuv}p_{uv}$ $i = 1, \ldots, g;$ $s = 1, 2$.	(3.17)	Household demands for consumption goods.
$-\beta_j(k_j(t) - k_j(t-1)) + Q_j(p_{(g+1)2j} - \pi_j) = \lambda$ $j = 1, \ldots, g.$	(4.8)	MEC's equated to long run "safe" interest rate.
$k_j(t) = k_j(t - 1)(1 - g_j) + y_jg_j$ $j = 1, \ldots, g.$	(4.9)	Capital accumulation equation.

$$\sum_{j=1}^{g} (\pi_j + y_j)\, z_j = y. \qquad (4.10)$$

Investment budget.

$$p_{j1} = \sum_{i=1}^{g} \sum_{s=1}^{2} a_{isj}^{(1)} p_{is} + \sum_{i=g+1}^{g+m} \sum_{s} a_{isj}^{(1)} p_{isj} \qquad (5.3)$$

$$j = 1, \ldots, g .$$

Zero profits in production of current goods.

$$\pi_j = \sum_{i=1}^{g} \sum_{s=1}^{2} a_{isj}^{(2)} p_{is} \quad . \qquad (5.4)$$

Zero profits in creation of capital.

$$p_{(g+1)1j} = p_{(g+1)1} \; . \qquad (6.3)$$

Homogeneous labor supply.

$$p_{(g+1)3j} = p_{(g+1)3} \; . \qquad (6.5)$$

Land is homogeneous.

$$x_i = \sum_{j=1}^{g} x_{i1j} B_{i1j}^{(1)} + \sum_{j=1}^{g} y_{i1j} B_{i1j}^{(2)} + c_{i1} B_{i1}^{(3)} + e_{i1} B_{i1}^{(4)} + f_{i1} B_{i1}^{(5)} \qquad (6.6)$$

$$i = 1, \ldots, g .$$

Demand for domestically produced goods equals supply.

$$\ell = \sum_{j=1}^{g} \ell_j x_{(g+1)1j} \; . \qquad (6.7)$$

Demand equals supply for labor.

$$k_j(0) = x_{(g+1)2j} \qquad (6.8)$$

$$j = 1, \ldots, g .$$

Demand equals supply for capital in each industry.

$$n = \sum_{j=1}^{g} n_j \, x_{(g+1)3j} \; . \qquad (6.9)$$

Demand equals supply for land.

APPENDIX 5

The Variables

(N.B. All variables are expressed in proportional change terms)

Variable	Definition	Subscript Range
x_{isj}	Demand for intermediate inputs of good i from source s for the production of current good j.	$i,j = 1,\ldots,g$; $s = 1, 2$.
$x_{(g+1)sj}$	Demand for primary factor s to be used in the manufacture of current good j.	$j = 1,\ldots,g$; $s = 1, 2, 3$.
y_{isj}	Demand for intermediate inputs of good i from source s for the creation of capital in industry j.	$i,j = 1,\ldots,g$; $s = 1, 2$.
c_{is}	Demand by consumers for good i from source s.	$i = 1,\ldots,g$; $s = 1, 2$.
$k_j(t)$	Capital available for use by industry j in the next year.	$j = 1,\ldots,g$.
y_j	Investment in industry j.	$j = 1,\ldots,g$.
λ	The marginal efficiency of capital.	
p_{j1}	Price of domestically produced goods.	$j = 1,\ldots,g$.
e_{j1}	Exports of good j.	$j = 1,\ldots,g$.
π_j	Price of new capital good in industry j.	$j = 1,\ldots,g$.
$p_{(g+1)1j}$	Price of labor in industry j.	$j = 1,\ldots,g$.
$p_{(g+1)3j}$	Price of land in industry j.	$j = 1,\ldots,g$.
x_i	Output of industry i.	$i = 1,\ldots,g$.
$p_{(g+1)1}$	Price of labor (when labor is considered to be homogeneous.)	

continued ...

Variable	Definition	Subscript Range
$p_{(g+1)3}$	Price of land (when land when land is considered to be homegeneous.)	
$p_{(g+1)2j}$	Rental value of capital in industry j.	$j = 1,\dots,g.$
p_{i2}	Price of imported product i.	$i = 1,\dots,g.$
y	Aggregate investment expenditure.	
c	Aggregate household expenditure.	
$p_{(g+r)sj}$	Taxes and other costs of production incurred by industry j.	$r > 1$, $s = 1$; $j = 1,\dots,g.$
f_{i1}	"Other" demands for product i (e.g., government purchases, accumulation of stock.)	$i = 1,\dots,g.$
ℓ	Supply of labor.	
n	Supply of land.	
$k_j(t-1)$	Supply of capital currently available in industry j.	$j = 1,\dots,g.$

APPENDIX 6

Derivation of the Demand for Inputs for Current Production.

The problem faced by producers in industry j is to minimize[1]

$$\left.\begin{aligned} &\sum_{i=1}^{g} \sum_{s} P_{is} X_{isj} + \sum_{i=g+1}^{k} \sum_{s} P_{is} X_{isj} \\ &\text{subject to} \\ &X_j = \min_{i=1,\ldots,g+m} \left\{ \frac{\left[\sum_{s} (X_{isj})^{-\rho_{ij}^{(1)}} b_{isj}^{(1)} \right]^{-\frac{1}{\rho_{ij}^{(1)}}}}{A_{ij}^{(1)}} \right\} . \end{aligned}\right\} \quad (A6.1)$$

Efficiency implies that the "min." can be disregarded and the problem splits over i, reducing to

$$\left.\begin{aligned} &\text{minimize} \quad \sum_{s} P_{is} X_{isj} \\ &\text{subject to} \\ &X_j = \frac{\left[\sum_{s} (X_{isj})^{-\rho_{ij}^{(1)}} b_{isj}^{(1)} \right]^{-\frac{1}{\rho_{ij}^{(1)}}}}{A_{ij}^{(1)}} . \end{aligned}\right\} \quad (A6.2)$$

The first order conditions require that

$$P_{is} - \psi \frac{\partial X_j}{\partial X_{isj}} = 0 \qquad (A6.3)$$

where ψ is a Lagrange multiplier.

---

1. It is convenient to drop the j subscript from the factor prices.

Thus

$$P_{is} = \frac{\psi}{A_{ij}^{(1)}} \left(\frac{A_{ij}^{(1)} X_j}{X_{isj}} \right)^{1+\rho_{ij}^{(1)}} b_{isj}$$

and so

$$\frac{b_{itj}^{(1)}}{b_{isj}^{(1)}} \cdot \frac{P_{is}}{P_{it}} = \left(\frac{X_{itj}}{X_{isj}} \right)^{1+\rho_{ij}^{(1)}} . \qquad (A6.4)$$

On making X_{isj} the subject of (A6.4)and substituting the results into the constraint function in (A6.2) the input demand function is

$$X_{itj} = A_{ij}^{(1)} X_j \left\{ \sum_s \left(\frac{P_{is}}{P_{it}} \cdot \frac{b_{itj}^{(1)}}{b_{isj}^{(1)}} \right)^{\frac{\rho_{ij}^{(1)}}{1+\rho_{ij}^{(1)}}} b_{isj} \right\}^{\frac{1}{\rho_{ij}^{(1)}}} . \qquad (A6.5)$$

Totally differentiating (A6.5) and expressing in percentage change form gives

$$x_{itj} = x_j - \frac{1}{1+\rho_{ij}^{(1)}} \left\{ \sum_s S_{isj}^{(1)} p_{it} - \sum_s S_{isj}^{(1)} p_{is} \right\}$$

$$= x_j - \sigma_{ij}^{(1)} \left\{ p_{it} - \sum_s S_{isj}^{(1)} p_{is} \right\} \qquad (A6.6)$$

where

$$\sigma_{ij}^{(1)} = \frac{1}{1+\rho_{ij}^{(1)}}$$

and

$$S_{isj}^{(1)} = \frac{(b_{isj}^{(1)})^{\sigma_{ij}^{(1)}} (P_{is})^{\rho_{ij}^{(1)}\sigma_{ij}^{(1)}}}{\sum_r (b_{irj}^{(1)})^{\sigma_{ij}^{(1)}} (P_{ir})^{\rho_{ij}^{(1)}\sigma_{ij}^{(1)}}} = \frac{P_{is}X_{isj}}{\sum_r P_{ir}X_{irj}} \quad .$$