Impact Project

Impact Research Centre, The University of Melbourne,
153 Barry Street, Carlton, Victoria 3053 Australia.

Telephone: (03) 344 7417 (*from overseas:* 61 3 344 7417)
Telex: AA 35185 UNIMEL Telegrams: UNIMELB, Parkville
Facsimile: (03) 344 5104 (*from overseas:* 61 3 344 5104)

ESTIMATION OF THE EXTENDED LINEAR EXPENDITURE SYSTEM WITH ASSETS

by

Philip D. Adams
University of Melbourne

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ABSTRACT

In this paper we report the results of estimating a model of consumption and savings choice called the Extended Linear Expenditure System with Assets (ELESA). The ELESA generalises the well known Extended Linear Expenditure System to accommodate multiple investment opportunities and uncertainty of perceived future asset returns. Quarterly Australian time-series data are used to fit the model to a seven commodity classification of consumption and a five asset disaggregation of household sector non-human wealth. In respect to the model's ability to explain investment behaviour, initial maximum likelihood estimates prove to be somewhat unsatisfactory. A significant improvement in performance is achieved, however, if the supply of physical assets is regarded as inelastic in the short-run. This involves treating the quantity of these assets exogenously and their rates of return endogenously.

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1 INTRODUCTION

Adams (1986) has recently proposed a model, called the 'Extended Linear Expenditure System with Assets' (hereinafter ELESA), which jointly explains the consumption and savings behaviour of a representative household[1]. The ELESA is a development of Lluch's (1973) 'Extended Linear Expenditure System' (ELES). Both models explain the allocation of a predetermined flow of income to aggregate consumption and savings, and are derived by considering similar intertemporal constrained maximization problems. Common to both systems is the individual's objective function, U, which can be written as:

$$U = \int_t^\infty e^{-\delta(h-t)} u[\mathbf{x}^h] \, dh \qquad (1),$$

where: $\mathbf{x}^h$ is an M length vector, whose typical element, x_m^h, is the expected consumption of commodity m at some instant h ($h \geq t$) of planning time;

δ is the time-preference discount rate;

and

$u[\mathbf{x}^h]$ is the instantaneous utility function at h.

The chosen form for $u[\mathbf{x}^h]$ is the Klein-Rubin; i.e.,

$$u[\mathbf{x}^h] = \sum_{m=1}^{M} \beta_m \log(x_m^h - \bar{x}_m) \qquad (2),$$

where: β_m can be shown to represent the marginal budget share of m;

and

$\bar{x}_m$ may be thought of as the 'subsistence minimum' quantity of commodity m, instantaneous utility at h being defined only when $x_m^h > \bar{x}_m$ for all m.

Whilst the ELESA and ELES are based on the same instantaneous utility function, only the former makes explicit allowance for multiple investment opportunities. Its predecessor, on the other hand, recognises just a single composite asset (i.e., non-human wealth), which is held solely to optimize the life-time consumption pattern of the individual. The ELESA explains the allocation of a predetermined net wealth total over a menu of available assets and liabilities. Savings in general have the same function as in the ELES, however because perceived future asset returns are allowed to evolve stochastically, portfolios are composed to conform to a preferred trade-off between return and risk. Under the assumption that expectations of asset prices are generated by stationary log-normal processes, the optimal consumption and portfolio rules of the ELESA are derived in Adams (1986) using the stochastic programming technique outlined in Merton (1971).

The purpose of this paper is to shed some light on the ELESA's ability to explain real-world behaviour. The model is estimated econometrically using quarterly data for the Australian economy disaggregated to a level of seven broadly-defined commodity groupings and five asset categories. Our findings will be of particular value to builders of macroeconomic and more micro-oriented models interested in the relationship between consumption and household wealth, and in the extent of substitutability between various asset and commodity categories.

The remainder of the paper is organised as follows. In section 2, we briefly recapitulate the initial part of the original derivation of the ELESA and, in so doing, extend the results of Adams (1986) to accommodate a more general definition of rates of return. It is shown, under some fairly general assumptions, that the extensions to the system lead to no additional computational difficulties nor, as a consequence, to any loss in tractability. Section 3 provides details of the econometric specification of the model derived in section 2 and relevant data considerations. In section 4 we report initial estimates of the system. The initial results, however, are found to be quite disappointing, especially with respect to the model's ability to explain investment behaviour. In section 5, the ELESA is respecified to reflect more closely the realities of the data, and estimates of the revised system are presented. Concluding remarks are offered, and an agenda for further work is discussed, in section 6.

2 THEORETICAL SPECIFICATION OF THE MODEL

As will become clear to the reader, the following derivation omits, for the sake of brevity, many of the finer points of detail contained in Adams (1986). Therefore, those readers who are unfamiliar with the model are directed to that study and to the articles referenced therein for further information about the basic properties of the system and the implications of various underlying assumptions.

Consider a representative agent who, at the current instant of time t, must decide on how much to save in aggregate, on how to allocate his total consumption amongst commodities, and on how to spread his savings over a portfolio of K 'distinct' assets and liabilities (hereinafter, for simplicity, referred to as assets)[2]. It is assumed that the agent is uncertain about the future returns on each asset in his portfolio. This uncertainty is expressed formally by writing the subjectively perceived real post-tax rate of return on asset k as the stochastic process:

$$\left[\frac{dq_k^h + s_k^h dh \; (1 - i^h)}{q_k^h} - \rho_k^h \, dh \right] = \alpha_k^t \, dh + \sigma_k^t \, dz_k^h \qquad (k = 1, \ldots, K) \quad (3),$$

where: q_k^h is the (deflated) unit market value of asset k at planning-point h in terms of a given basket of consumption goods;

s_k^h is the (deflated) instantaneous, constant return-to-scale dividend (or interest) paid on a unit of k during the interval of time dh in terms of a given basket of consumption goods;

i^h is the average rate of taxation imposed upon the individual's income at h $(0 \leq i^h \leq 1)$;

ρ_k^h is the instantaneous rate at which asset k depreciates physically at h, per unit time;

α_k^t is the instantaneous expected 'permanent' real rate of return on asset k, per unit time;

σ_k^t is the instantaneous standard deviation of the real rate of return on asset k;

and

z_k^h is a standard Wiener process, with dz_k^h being the associated white noise.

The process on the right of (3) is known as a 'geometric Brownian motion'.

Equation (3) explains the expected real post-tax rate of return on a unit of asset k over a short time interval as a random normal variable, with a stationary (through planning-time) mean $\alpha_k^t dh$ and variance $(\sigma_k^t)^2 dh$. On the left hand side, provision is made for cash income from investments on the one hand, and for capital gains on the other; as well as for taxation, and for physical depreciation. This is in contrast to Adams (1986), which dealt explicitly only with untaxed income from capital gains generated by non-depreciating assets. For simplicity, we have assumed differential tax rates on dividends (interest) and capital gains, with the rate on the latter set to zero. To assume otherwise would mean having to model the individual's optimal selling strategy for assets, since capital gains and losses are only taxed when an asset is sold[3]. This task is well beyond the scope of the present paper.

In principle, equation (3) allows us to incorporate several explicit sources of investment uncertainty into the system. For instance, we could recognise the possibility of random fluctuations in the rate of general price inflation, or in the flows of pecuniary income from investments. However, in the present context, we find it convenient to remove one such source of uncertainty by assuming that future rates of inflation are perceived with certainty by the individual and are stationary throughout the plan. Thus, the nominal return on a unit of asset k at h can be simply written as:

$$\left[\frac{d(q_k^h P^h) + s_k^h P^h dh \; (1 - i^h)}{q_k^h P^h} - \rho_k^h \, dh \right] = (\alpha_k^t + \pi^t) \, dh + \sigma_k^t \, dz_k^h$$

$$= b_k^t \, dh + \sigma_k^t \, dz_k^h \qquad (k = 1, \ldots, K) \qquad (4),$$

where P^h is a suitably defined index of (nominal) consumer goods prices at h, and π^t is taken to be the static expected rate of consumer price inflation from the perspective of the consumer at t. Notice that b_k^t dh, the expected permanent nominal rate of return on asset k, is just the expectation of the term on the left of (4).

In Appendix A of this paper the continuous time budget constraint for the individual is derived from an underlying discrete-time model as:

$$dW^h = \sum_{k=1}^{K-1} (b_k^t - b_K^t) w_k^h W^h dh + b_K^t W^h dh + (y^h - \mathbf{x}^{h\prime} \mathbf{p}^h) dh$$

$$+ \sum_{k=1}^{K} \sigma_k^t w_k^h W^h \, dz_k^h + \mathbf{o}(\Delta h) \qquad (5),$$

in which: y^h is the exogenously given contribution to net savings at h from sources other than capital gains and cash income derived from non-human wealth, per unit time; W^h is the planned level of wealth held in non-human form at h; w_k^h is the fraction of W^h held as asset k; p_m^h is the m'th element of $\mathbf{p}^h$; and $\mathbf{o}(\Delta h)$ encompasses all terms, which after division by Δh approach zero as Δh approaches zero (a prime (′) is used to denote the transpose of a column vector). Note that, apart from summing to one, there are no other restrictions on the individual portfolio shares because borrowing and short-selling are allowed. For example, a negative value for w_k^h indicates that at point h in the plan, asset k is a liability of the household.

Henceforth, we shall for simplicity assume: first, the existence of an instantaneously riskless asset (for ease of notation, asset K; thus $\sigma_K^t = 0$ and $r^t = b_K^t$); second, that y^h is constant in real terms; and third, that all relative commodity prices are expected to remain unchanged. Under these assumptions, equation (5) can be rearranged so that:

$$dW^h = \sum_{k=1}^{K-1} \{[(b_k^t - r^t)w_k^h W^h] + r^t W^h + (y^t - \mathbf{x}^{h\prime}\mathbf{p}^t)e^{(\pi^t(h-t))}\}dh$$

$$+ \sum_{k=1}^{K-1} \sigma_k^t w_k^h W^h \, dz_k^h + o(\Delta h) \qquad (6).$$

A comparison of equation (6) with the corresponding expression in Adams (1986), reveals that apart from the terms contained within $o(\Delta h)$, which disappear as part of the solution technique, and a slight change in notation (i.e., the use of the b's here instead of the a's in Adams, 1986), the two equations are the same. Therefore, since the objective functionals (i.e., equations (1) and (2)) are also identical, by analogy to the derivation of the ELESA from the problem set up in Adams (1986), we find that equation (6) implies the initial instant (i.e., h = t) solution:

$$x_m^t p_m^t = \bar{x}_m p_m^t + \beta_m^* [W^t + (\frac{y^t - \bar{\mathbf{x}}'\mathbf{p}^t}{r^t})] \qquad (m = 1, \ldots, M) \qquad (7),$$

and

$$w_k^t W^t = \sum_{i=1}^{K-1} \sigma_t^{(i,k)} (b_i^t - r^t) [W^t + (\frac{y^t - \bar{\mathbf{x}}'\mathbf{p}^t}{r^t})] \qquad (k = 1, \ldots, K-1),$$

$$w_K^t W^t = (W^t - \sum_{i=1}^{K-1} w_i^t W^t) \qquad (8),$$

in which, by definition, $\beta_m^* = \delta\beta_m$; and $\sigma_t^{(i,k)}$ is the (i,k)'th element of $\mathbf{\Omega}^{-1}$, the (K-1 x K-1) inverse of the instantaneous covariance matrix of asset rates of return from the perspective of the individual at t.

Equations (7) and (8) have the following interpretations. Equation (7) equates current expenditure on commodity m to the sum of subsistence expenditure on that commodity and a fraction of current income from non-human wealth (evaluated at the safe rate of return, r) and other sources in excess of the total cost of subsistence expenditure. At the same time, equation (8) explains the stock demand for risky asset k as the product of a weighted sum of expected permanent rates of return on risky assets relative to the return on the safe asset and income in excess of subsistence expenditure. The weight $\sigma^{(i,k)}$ is itself a function of the perceived underlying riskiness of each asset in the portfolio. This representation of portfolio behaviour is clearly consistent with the standard mean-variance model in a single period setting.

Three further things may be noted about (7) and (8). First, if r^t is held constant at, say, $\tilde{r}$, then $\tilde{\mu} = \delta / \tilde{r}$ is the marginal propensity to consume out of safe income $(r^tW^t + y^t)$ for $r^t = \tilde{r}$. Second (as noted in Cooper and McLaren, 1981), if $\bar{\mathbf{x}}'\mathbf{p}^t$ has the interpretation of subsistence expenditure, then $\bar{\mathbf{x}}'\mathbf{p}^t / r^t$ is subsistence wealth. Finally, if W^t is treated as predetermined from the perspective of the individual at t, then the endogenous portfolio shares, w_k^t, are shares in a predetermined aggregate, and there is no feedback from aggregate consumption at t (endogenised by (7)) to W^t. The final point is an assumption maintained throughout the econometric analysis which follows.

At this stage, equations (7) and (8) only describe the planned behaviour of the representative household at the initial point of the plan $h = t$. To derive equations which describe actual behaviour we need to introduce the notion of 'continual replanning', so that realisations of x_m^t and w_k^t are seen as a succession of initial solutions of continuously revised optimal plans. Moreover, for each moment t, it is assumed that the household adjusts both the level and composition of its aggregate stock of non-human wealth instantaneously to their currently optimal settings, and that perceptions of future asset returns, commodity prices, and other income change smoothly through real time. The assumption of instantaneous adjustment contrasts with that of gradual adjustment often made in the portfolio allocation literature[4]. In the latter approach, changes in asset holdings in any period are dependent on deviations between desired and actual stocks.

3 THE EMPIRICAL MODEL

3.1 Econometric Specification

The system of equations (7) and (8) represents the theoretically true continuous time form of the ELESA estimated in this paper. To make the model operational from an econometric viewpoint, it is necessary to translate the continuous-time system into its discrete-time analogue. This is done, following the approach of Bergstrom and Wymer (1976), by averaging each variable over an arbitrary observed interval t, which is assumed to be of unit length (in this paper, a unit of time equals one quarter). The following notation is used. Let t, as before, indicate instants of (continuous) time when written as a superscript (or subscript if appended to $\sigma^{(i,j)}$). When written as a subscript (superscript), t will be an integer corresponding to a discrete-time interval of unit length. Thus, if f is the rate of flow of any flow variable at time t, then we denote its discrete-time (or 'measurable') analogue to be:

$$f_t = -\int_0^1 f^{t-v}\,dv \qquad ,$$

which is the flow observed over the period (t − 1, t). Similarly, if s^t is the observed value of any stock variable at t, then

$$s_t = 0.5\ (s^{t-1} + s^t) \qquad (9),$$

is its discrete-time analogue, which notionally is timed at the mid-point of the interval (t-1, t). The averaging of variables in this manner 'can be viewed as a necessary timing adjustment to ensure that stock and flow variables are "measured" at a common point in each period' (Bacon and Johnston, 1977, p. 120). Notice that in (7) and (8), the flow variables are x_m^t (m = 1,..., M), r^t, y^t and b_k^t (k = 1,.., K-1); whilst the stock variables are p_m^t $(m = 1,\ldots, M)$, W^t, w_k^t $(k = 1,\ldots, K-1)$, and $\sigma_t^{(i,k)}$ $(i,k = 1,\ldots, K-1)$. However, because the consumption price variables are defined as the ratio of the flow of nominal expenditures to the real flow of those expenditures, they are conceptually measured at the mid-point of each period and so need not be averaged using (9).

After conversion to discrete-time analogues, and the addition of the stochastic element $e_{j,t}$, we write the econometric specification of the ELESA as:

$$x_{m,t}p_{m,t} = \bar{x}_m p_{m,t} + \beta_m^* [W_t + (\frac{y_t - \bar{x}'p_t}{r_t})] + e_{m,t} \qquad (m = 1,\ldots, M) \qquad (10),$$

and

$$w_{k,t}W_t = \sum_{i=1}^{K-1} \sigma_t^{(i,k)} (b_{i,t} - r_t) [W_t + (\frac{y_t - \bar{x}'p_t}{r_t})] + e_{M+k,t} \qquad (k = 1,\ldots, K-1) \qquad (11),$$

where $e_{j,t}$ is assumed to be joint normally distributed with the following classical properties:

$E(e_{j,t}) = 0$ (for all j and t)

$E(e_{i,t}e_{j,\tau}) = 0$ (for all i and j, and $t \neq \tau$) ,

and

$E(e_{i,t}e_{j,t}) = \delta_{i,j}$.

Notice that the fraction of the portfolio invested in the safe asset at t is obtained from:

$$w_{K,t} = 1 - \sum_{k=1}^{K-1} w_{k,t} \quad .$$

At this point, we have a system of (M + K - 1) equations and one identity involving at least (2M + 2) variables, namely $\{x_{1,t}, .., x_{M,t}, p_{1,t}, ..., p_{M,t}\}$, and 2M parameters, namely $\{\bar{x}_1, ..., \bar{x}_M, \beta_1^*, ..., \beta_M^*\}$, with δ recoverable from the normalisation condition $\Sigma\beta_m^* = \delta$. As always, some choice must now be made about which of the remaining entities in (10) and (11) should be treated as variables, and which as parameters. To remind the reader, $b_{k,t}$ is the conditional expected permanent (or long-run) post-tax rate of nominal return on risky asset k (conditional on developments observed by the agent up to and including t); r_t is the expected rate of return on the riskless asset; while $\mathbf{\Omega}^{-1}$ is the inverse of $\mathbf{\Omega}$, whose typical element $\sigma^t_{(i,k)}$ is the conditional co-variance of expected returns (nominal or real) on assets i and k. Most portfolio models estimated in the literature implicitly assume that, even though expected returns on assets change over time, the underlying riskiness of those assets does not. Thus, the matrix of interest rate responses is a constant. In this paper, the same strategy is adopted, so that the b's and r continue to be specified as variables, but the elements of $\mathbf{\Omega}^{-1}$ are treated as parameters to be estimated econometrically along with the other parameters of the system[5].

In the final system, then, there are (M + K - 1) equations and one identity, involving (2M + (K - 1)(K - 1)) parameters (before imposing symmetry on $\mathbf{\Omega}^{-1}$). The vexing questions of how to derive data for the essentially unobservable b variables and for r are addressed next.

In his 1957 book, A Theory of the Consumption Function, Milton Friedman introduced the concept of 'permanent income'. In the most common version of the 'permanent income hypothesis', permanent income is taken to be generated by an infinite distributed lag in observed income. A similar approach is used here to generate values for the permanent rates of return on risky assets. In other words, if $n_{k,t}$ is the observed rate of return on asset k at t, then we suppose for each of the risky assets that:

$$b_{k,t} = \sum_{s=0}^{\infty} \lambda_{k,s} \, n_{k,t-s} \qquad (k = 1, \ldots, K-1) \qquad (12),$$

where $\lambda_{i,s}$ is a typical coefficient of the lag structure, with $\sum_{s=0}^{\infty} \lambda_{k,s} = 1$ for all k.

In its present form, equation (12) contains an infinite number of parameters and hence is a nonfeasible estimation scheme. To obtain an operational version, we assume, on intuitive grounds, that:

$$\lambda_{k,t-\tau_k-s} = 0 \qquad (s = 0, 1, 2, \ldots ; k = 1, \ldots, K-1) \quad ,$$

and so,

$$b_{k,t} = \sum_{s=0}^{\tau_k-1} \lambda_{k,s} \, n_{k,t-s} \qquad (k = 1, \ldots, K-1) \quad ,$$

where, in the nomenclature of Powell (1973a), τ_k is the 'effective length of the memory process' for asset k. Under this assumption, history of (τ_k + 1) periods ago play no role in forming the agent's perception of future returns on asset k.

To estimate the expectations model implied by equation (12) and the subsequent restriction on lag coefficients, we adopt the so-called 'moving expectations sub-case of the linear extrapolative hypothesis' proposed in Powell (1973b, pp. 340-42)[6]. This method involves fitting a trend line to the last τ_k periods of data on $n_{k,t}$, and extrapolating forwards one period along the trend. Thus, in algaebraic terms:

$$b_{k,t} = \omega_{k,t} + (t + 1)\, n_{k,t} \qquad (k = 1,\ldots, K-1) \qquad (13),$$

where $\omega_{k,t}$ and $\eta_{k,t}$ are coefficients to be estimated.

The use of ordinary least squares to estimate the coefficients in equation (13) from a regression of data for $n_{k,t}$ over the τ_k periods terminating at t on time and on a constant, implies the following scheme:

$$\lambda_{k,s} = \frac{(4\tau_k - 6s - 4)}{\tau_k(\tau_k - 1)} \qquad (s = \tau_k-1,\ldots, 0;\ k = 1,\ldots, K-1) \qquad (14).$$

From (14) we can observe the nature of the distributed lag structure being proposed. The lag scheme contains only one parameter, τ_k, and the associated lag coefficients form a monotone decreasing sequence which sums to unity. However, unlike the more common adaptive expectations models, negative weights are allowed for the more remote observations[7].

To make the treatment complete, a procedure must be found for choosing values of τ_k $(k = 1,\ldots, K-1)$. Unfortunately, as with other lag schemes, in the absence of strong a priori information there does not seem to be any clear-cut method for deriving 'best' values for these parameters[8]. Thus, in section 4, an entirely ad hoc experimental procedure is employed, which uses the value of the system's log likelihood function as the sole criterion for comparing alternative settings of $\{\tau_k\}$.

A different methodology to that proposed in (14) appears called for to derive values for r_t (i.e., from the viewpoint of the individual at t, the permanent rate of return on the safe asset). The safe asset is one for which the individual is certain that he can earn a market rate of return, r_t, during t by holding the asset. Note that we do not assume that **future** values, $n_{K,t}$ (s > t), are necessarily known with certainty. Thus, provided such an asset exists,

$$r_t = n_{K,t} \tag{15}$$

will be our generating function for r_t.

3.2 Data Considerations

The following are the basic data requirements for estimating the ELESA:

- the market value of household sector wealth for K assets (W_t, $w_{k,t}$; k = 1,.., K);
- the post-tax market yield on each asset ($n_{k,t}$; k = 1,.., K);
- seasonally adjusted private consumption expenditure at current prices on M commodities ($p_{m,t}x_{m,t}$; m = 1,.., M);
- M commodity price indexes ($p_{m,t}$; m = 1,.., M);
- the aggregate contribution to household net savings from sources other than capital gains and cash income from non-human wealth (y_t);

and

- the total number of persons in the economy (used to express all national-level data in per capita terms).

Data sources and descriptions of manipulation techniques used to construct series not available directly from official sources are presented in Appendix B. The remainder of this sub-section is devoted to a short discussion of the classifications adopted for commodities and assets, as detailed in Tables 3.1 and 3.2, respectively.

Because the Klein-Rubin is a directly additive utility function, at t, the marginal utility received by the individual from commodity i is independent of his consumption of commodity j ($j \neq i$). Thus, to prevent violations of this additivity assumption from being too frequent, the commodities have been grouped into fairly broad categories.

Of the seven categories identified in Table 3.1, two (i.e., household durables and motor vehicles) include items that are of a durable nature. For these commodities, purchases are treated as part of <u>current</u> consumption, and as such, their durability is ignored; in particular, no explicit account is taken of future service flows generated by current purchases. The ideal treatment of course would be to estimate the value of services yielded by the respective stocks and to include those flows (rather than the purchases) as arguments of the instantaneous utility function. However, the measurement of service yields for these two durable items is extremely difficult, and so in this paper we accept the National Accounts definition of household durable expenditure as consumption[9].

The categorisation of assets depicted in Table 3.2 was chosen in order to cover the important components of household sector non-human wealth, whilst, at the same time, minimizing possible problems of multicollinearity that may occur at finer degrees of disaggregation. To remain consistent with the theory, wherever possible an item was treated as an asset if, in the author's opinion, the most important determinants of it's demand were the portfolio considerations of the individual. Thus, dwellings were not included here as part of non-human wealth, because it is assumed that the demand for these assets is largely determined by the direct utility which stems from the service flows yielded by the stock currently held. As a test of this treatment of dwellings, the ELESA was

Table 3.1: Classification of Seven Commodities Used in Time-Series Estimation

Commodity number	Name	Composition[a]
m = 1	Food	Food
2	Tobacco, cigarettes and alcohol	Cigarettes and tobacco, and alcoholic drinks
3	Clothing, footwear and drapery	Footwear, and clothing and drapery
4	Rent	Imputed rent of owner-occupiers and other rent
5	Household durables	Household appliances and other household durables
6	Motor vehicles	Purchases of motor vehicles and operation of motor vehicles
7	Other	Health, gas, electricity, fuel, entertainment and recreation, financial services, fares, postal and telephone services, and 'other' goods and services

(a) In terms of the classification adopted by the Australian Bureau of Statistics for its 'Quarterly Estimates of National Income and Expenditure' bulletin (Catalogue No. 5206.0).

Table 3.2: Classification of Five Assets Used in Time-Series Estimation

Asset number	Name	Composition[(a)]
1	Unincorporated business fixed assets	Farm and non-farm unicorporated business fixed assets
2	Equity	Ordinary and preference shares in the issued capital of private non-financial corporations
3	Financial assets	Trading bank fixed deposits; notes and coin; shares, deposits and units in permanent building societies, etc.; debentures, notes and deposits; Commonwealth Government securities; local and semi-government securities; and net contributions to life insurance offices and pension funds (private and public)
4	Household liabilities	Trading and savings bank advances; advances from building societies, the Commonwealth Development bank, life offices, finance companies and credit co-operativies; and concessionary housing finance from governments
5	Safe assets	Trading bank current deposits and savings bank deposits

(a) In terms of the classification adopted in Adams (1987).

applied in Adams (1988) to a classification of consumption which excluded rent, and a five asset disaggregation of non-human wealth, which included the stock of structural dwellings owned by households. The results from that application proved unsatisfactory, thus lending empirical support to the treatment of dwellings adopted in this paper.

Apart from dwellings, there are several other household assets excluded from Table 3.2 because of the lack of reasonably accurate data on their stocks and rates of return. These excluded assets include: natural and/or non-reproducible resources (other than land), trade credit (net), financial assets formed abroad, and working capital. The last-mentioned consist of inventories held by unincorporated businesses of raw materials, semi-finished and finished products which, though tangible, are turned over fairly quickly.

Asset category one (i.e., unincorporated business fixed assets) comprises items that are direct claims to the ownership of the physical capital of unincorporated businesses. On the other hand, categories two, three and four contain assets which are essentially financial in nature. Thus, unlike most studies of portfolio behaviour, in this paper substitiution is permitted between the physical and financial components of household wealth.

Included in the financial assets category, for the sake of completeness, are two assets the demands for which do not depend strictly on portfolio considerations. The first of these, 'net contributions to life offices and pension funds', are 'contractual' by nature rather than 'marketable'. For these, significant penalties are incurred if the assets are sold before maturity. Thus, there are likely to be considerable lags involved with the adjustment of desired to actual stocks. Similarly, the demand for notes and coin does not depend strictly upon considerations of risk and return. Instead, as has been long recognised, especially by Keynes, 'transactions' motives and 'precautionary' motives along with 'speculative' motives are all, in principle, joint determinants of demand. The treatment adopted in this paper follows the portfolio approach of Tobin (1958), which visualises notes and coin (money) as a potential investment asset, the demand for which is not related to current transactions needs.

Our choice for the risk-free asset is a composite of savings bank deposits and trading bank current deposits. These liabilities of the banking system seem to best fit the criteria for risklessness set out above; i.e., they provide an almost perfectly safe nominal income stream over any given three-month period and their nominal capital values at maturity are known with certainty.

4 INITIAL RESULTS

4.1 Estimating Forms

In principle, equations (10) and (11) should be estimated as a single system of simultaneous equations to take account of the cross-equation constraints on the parameters. However, this strategy did not prove feasible because the size of the system was so large that it exceeded the bounds of the TSP econometrics software package used for the estimation. Thus, the less efficient approach was adopted of estimating the two sub-systems (or modules) separately with values for the utility function parameters in the portfolio module fixed at levels derived from the estimation of the consumption module.

To check whether the estimates of the model obtained in this manner are significantly less efficient than those which could be derived if the model were estimated as a single system, the following Likelihood Ratio test was devised. Remember, that the two sub-systems are connected only because the $\bar{x}$ parameters of the instantaneous utility function appear in both. Thus, what we want to know is whether the value of the likelihood function for the portfolio module can be improved significantly if the values of the $\bar{x}$ parameters differ from their values as estimated from the consumption module alone (which, hereafter, are denoted $\bar{x}_m^*$ (m = 1,.., 7)). For simplicity, we assume that the likelihood function of the portfolio module responds to changes in aggregate subsistence expenditure, but not to changes in the ratios among the individual $\bar{x}_m$ parameters. With this in mind, let $L(H_0)$ be the value of the conditional likelihood function for the portfolio module given $\bar{x}_m = \bar{x}_m^*$ (m = 1,.., 7). Further, let $L(H_1)$ be the value of the corresponding likelihood with $\bar{x}_m$ unconstrained to the extent that $\bar{x}_m = \xi\bar{x}_m^*$, where ξ is an unknown scalar. Under the null hypothesis (and conditional on the values of $\bar{x}_m$),

$$-2(\log(L(H_0)) - \log(L(H_1)))$$

is distributed asymptotically as a Chi-squared statistic with one degree of freedom. The result of this test for the model estimated in this section was 106.6. Therefore, given a 1 per cent critical value of 6.64, we would reject the null hypothesis. However, the conditional maximum likelihood value of ξ (namely, $\tilde{\xi} = 10.4$, asymptotic standard error (s.e.) = 7.99), makes little sense in terms of the consumption module. Indeed, under the null hypothesis, $[(\tilde{\xi} - 1)/\text{s.e.}]$ is asymptotically normally distributed and the realised value of this statistic does not lead to rejection of H_0 at the one per cent level. Thus, the evidence on this matter is mixed.

The theory provided strong predictions concerning the coefficients of the investment module. In particular, Ω^{-1}, the matrix of rates of return coefficients, was predicted to be positive definite. However, imposing this restriction during estimation proved to be an extremely complex task, and so only the weaker requirement of symmetry was actually imposed[10]. This reduced the number of σ paramaters to be estimated from sixteen to ten.

Finally, an additive dummy variable, QINF, was introduced into the investment module equations to take account of possible variation in the interest rate coefficients caused by the highly volatile behaviour of consumer price inflation between 1973 and 1978. During the first two years of this period, inflation accelerated sharply to rates unprecedented except for the Korean war episode in the early 1950's. Inflation then declined, until by the end of 1977, it was back to a rate which was more consistent with those observed before 1973[11]. A volatile rate of inflation is likely to lead to increased household uncertainty about future nominal asset returns, and thus to a systematic variation in the interest rate coefficients which reflect the underlying levels of riskiness attached to each asset. QINF is designed to capture these variations by taking on non-zero values between 1974:3 and 1978:1. During this interval QINF equals the expected rate of consumer price inflation in each quarter. A series for inflationary expectations was obtained from data for observed rates of inflation in exactly the same way as series for permanent rates of return were derived from data for observed rates of return. The length of the memory length parameter, τ_{inf}, was determined by experimentation along with the values of the other four lag parameters.

4.2 Results

The estimates of the ELESA reported in this section were obtained by Full Information Maximum Likelihood (hereinafter FIML) estimation. The sample covered the period 1971:2 to 1986:4, giving a total of 63 quarterly observations. Parameter estimates and summary statistics for the consumption module are given in Table 4.1; while Table 4.2 contains corresponding items for the investment module. Figures in parentheses are the asymptotic standard errors; 'asymptotic t-values' can thus be obtained as the ratio of the parameter estimates to their standard errors. For each equation we report values for: the sum of squared residuals (SSR); a goodness-of-fit statistic; and the Durbin-Watson single equation test statistic for first-order serial correlation. The measure of goodness-of-fit was calculated as the R-squared from a simple regression of the left-hand variable on its predicted values, and is therefore to be interpreted solely as a descriptive indicator of predictive ability within the sample.

As mentioned above, settings for the 'effective memory length' parameters, τ_k $(k = 1, \ldots, 4)$ and τ_{inf} were determined by experimentation

Table 4.1: Initial Maximum Likelihood Estimates of the Consumption Module of ELESA, Derived from Australian Data Spanning 1971:2 through 1986:4

Equation for the consumption of	MLE of β^*_m	Implied estimate of β_m	MLE of $\bar{x}_m$	SSR	Goodness-of-fit(a)	DW
1 Food	0.001159 (0.0008)	0.1262 (0.0019)	102.270 (3.3417)	66338	0.9548	1.8516
2 Tobacco, cigarettes and alcohol	0.000600 (0.0004)	0.0653 (0.0025)	35.404 (1.7077)	19920	0.8865	1.6254
3 Clothing, footwear and drapery	0.000557 (0.0004)	0.0606 (0.0027)	35.793 (2.3195)	17225	0.8564	1.6245
4 Rent	0.000805 (0.0001)	0.0875 (0.0106)	158.930 (6.5391)	101013	0.8489	0.5950
5 Household durables	0.001362 (0.0009)	0.1482 (0.0061)	-34.722 (4.3830)	104013	0.5937	1.6070
6 Motor vehicles	0.001123 (0.0009)	0.1222 (0.0045)	21.057 (4.1471)	65545	0.7515	1.7357
7 Other	0.003584 (0.0002)	0.3899 (0.0028)	94.490 (9.8785)	633931	0.7870	1.8170

Implied estimate of $\delta = \sum_{m=1}^{m}$ (MLE of β^*_m) = 0.0092 (0.0006).

Implied estimate of the mean marginal propensity to consume out of safe income $\tilde{\mu}$ = (implied estimate of δ)/$\bar{r}$ = 0.9508 (0.0589), where $\bar{r}$ is the sample mean value of r_t.

(a) Derived as the R-squared from simple regresions of the left hand variables on their respective predicted values.

Table 4.2: Initial Maximum Likelihood Estimates of the Investment Module of ELESA, Derived from Australian Data Spanning 1971:2 through 1986:4

Equation for the posession of:	MLE of[(a)]:		SRR	Goodness-of-fit[(b)]	DW
	$\sigma^{(k,k)}$	C^k			
1 Unincorporated business fixed assets	1.1941 (0.1834)	-697.21 (829.32)	2.49×10^8	0.4594	1.0418
2 Equity	0.0830 (0.0285)	-170.13 (345.99)	2.79×10^7	0.1913	0.6355
3 Financial assets	1.1271 (0.2451)	-736.28 (1007.58)	3.24×10^8	0.5213	0.9874
4 Household liabilities	0.4251 (0.1308)	483.82 (607.87)	1.24×10^8	0.4737	1.0987
5 Safe assets	Nc.	Nc.	Nc.	0.2975	Nc.

MLE of:

$\sigma^{(2,1)}$	$\sigma^{(3,1)}$	$\sigma^{(4,1)}$	$\sigma^{(3,2)}$	$\sigma^{(4,2)}$	$\sigma^{(4,3)}$
0.3014 (0.0557)	1.1828 (0.2400)	-0.8503 (0.1384)	0.2639 (0.0654)	-0.2101 (0.0355)	-0.4674 (0.1594)

(a) C^k is the coefficient on the dummy variable QINF.

(b) Derived as the R-squared from simple regressions of left hand variables on their respective predicted values.

Nc. Not calculated or not applicable.

using the value of the log likelihood function for the investment module as the test criterion. Throughout the exercise, the sample was held constant at 63 observations. This was done to avoid the potential trap, noted in Powell (1973a, pp. 21 and 22), of making τ_k (k = 1,..., 4) and τ_{inf} larger and larger at the expense of the number of usable data points until the situation is reached where there are no degrees of freedom and the likelihood function is a spike. Therefore, the experimentation was restricted to values for each of the lag parameters in the range of 4 to 28 quarters; the upper-end being the number of quarters for which data were available on rates of return prior to the start of the sample, 1971:2. Values for τ_k (k = 1,..., 4) and τ_{inf} which yielded the highest likelihood were:

$\tau_1 = 25$ [unincorporated business fixed assets]
$\tau_2 = 28$ [equity]
$\tau_3 = 8$ [financial assets]
$\tau_4 = 9$ [equity]
$\tau_{inf} = 16$ [general inflation].

Because variations in observed rates of return on unincorporated business fixed assets (k = 1) and equity (k = 2) are far greater than those observed for financial assets (k = 3) and household liabilities (k = 4), it makes sense that the values for τ_1 and τ_2 should exceed those for τ_3 and τ_4. Note, that because of the high computational costs associated with searching for the 'maximum likelihood' estimates of τ_k (k = 1,..., 4) and τ_{inf}, their values are held constant at the above settings during the remainder of the analysis.

The rest of this section is devoted to a discussion of the results: first those reported for the consumption module in Table 4.1, then those for the investment module in Table 4.2.

Table 4.1 gives Maximum Likelihood Estimates (hereinafter MLE's) for β_m^* and $\bar{x}_m$ ($m = 1,\ldots, 7$), and implied estimates for the marginal budget shares, β_m ($m = 1,\ldots, 7$), and for the time-preference discount rate, δ. The last-mentioned was derived from the normalisation constraint on the β^*'s; while values for β_m ($m = 1,\ldots, 7$) were computed from $\beta_m = (\beta_m^* / \delta)$. Also provided is an implicit estimate of the Marginal Propensity to Consume (MPC) out of safe income, valued at the sample mean of the risk free rate of return, $\tilde{r} = 0.97$ per cent per quarter.

A glance at the indicator of goodness-of-fit shows that the descriptive ability of each equation over the sample is quite satisfactory, as would be expected from using time-series data. Values of the Durbin-Watson statistic suggest that positive first-order serial correlation exists in the residuals of only one equation, that for rent. This is supported by a visual inspection of the residual plots. Because quarterly data were used, each equation was also checked for fourth-order serial correlation using the Wallis (1972) test[12]. The Wallis test (which, like the Durbin-Watson, strictly applies to single equations, rather than equation systems) rejected the existence of fourth-order serial correlation at the 1 per cent level in all equations except that for rent.

Our estimate of the mean MPC out of safe income is 0.95, based on an estimate for δ of 0.92 per cent per quarter. This value for δ is somewhat higher than the 0.76 per cent per quarter obtained in Cooper and McLaren (1981) from a derivative of the ELES applied to Australian data over the period 1959:3 to 1978:2. Thus, our results point to an increase in the time-preference discount rate of the representative household during the past decade. To derive a figure for the MPC out of actual income (i.e., Household Disposable Income; hereinafter, HDI) implied by our estimate for the MPC out of safe income, we make use of the following expression, based on the chain rule of differentiation:

> MPC out of actual income = MPC out of safe income times the partial derivative of safe income with respect to actual income.

An approximate value for the right hand partial was derived as the estimate of the coefficient from a simple regression of safe income on HDI and a constant over the sample. Derived in this way, we obtain a value of the MPC out of actual income of 0.86, which is significantly higher than the generally accepted figure for Australia of around 0.75[13].

The implied marginal budget shares, β_m, are all positive as required by the theory. The largest β_m is for 'other', while the smallest is for clothing, footwear and drapery. The values for the estimated β_m^* parameters were all more than five times their respective asymptotic standard errors. Apart from the $\bar{x}$ for household durables, all of the other $\bar{x}$ parameters were positive, which gives limited support to the hypothesis that they behave as 'subsistence' quantities; but overall, their values were determined with considerably less precision than those of the β_m^* parameters. As required by the theory, in no quarter of the sample does the estimate of total subsistence expenditure, $\sum_{m=1}^{7} p_m^t \bar{x}_m$, exceed that for total consumption expenditure.

As with all consumer demand models based on the maximization of the Klein-Rubin utility function, in the ELESA the elasticity with respect to total expenditure of the marginal utility of an additional dollar's worth of consumption (i.e., the Frisch 'parameter'), ω_t, is a function of total consumption and subsistence expenditures. In particular, when evaluated at sample means,

$$\tilde{\omega} = - \frac{\tilde{V}}{\sum_{m=1}^{7} (\tilde{V}_m - \tilde{p}_m \bar{x}_m)} ,$$

where $V_{m,t} = p_{m,t} x_{m,t}$, $V_t = \sum_{m=1}^{7} V_{m,t}$, and ~ is used to indicate the sample mean value of the variable to which it is attached. The value for $\tilde{\omega}$ implied by the estimates in Table 4.1 is -1.4922 (asymptotic standard error = 0.0443), which is somewhat lower in absolute value than the generally accepted figure for Australia during the 1970's of around -1.8 (see Williams, 1978). This reduction in $-\omega$ over time is, of course, consistent with Frisch's original (1959) hypothesis that ω declines in absolute value with household income per head[14].

In Table 4.3 we report estimates of uncompensated own and cross price elasticities of demand, evaluated at sample means; while in Table 4.4 we report values for total expenditure and income elasticities, also evaluated at sample means. Formulae for the price elasticities are given in Adams (1986). Relevant formulae for the total expenditure and income elasticities of commodity m (m = 1,.., 7), are:

$$\tilde{\eta}_m = \frac{\beta_m \tilde{V}}{\tilde{V}_m}$$

and

$$\tilde{\eta}^*_m = \frac{\beta_m \tilde{\mu} (\tilde{r}\tilde{W} + \tilde{y})}{\tilde{V}_m} ,$$

respectively. Notice that there is only a simple scalar transformation which relates the total expenditure elasticity of commodity m to its income elasticity; and that the homogeneity property of the ELESA ensures that the

Table 4.3: Initial Estimates of Uncompensated Own- and Cross-Price Elasticities of Commodity Demand, Evaluated at Sample Means

Response in demand for:	Commodity whose price increases by 1 per cent:						
	Food	Tobacco, cigarettes and alcohol	Clothing, footwear and drapery	Rent	Household durables	Motor vehicles	Other
Food	-0.5686	-0.0210	-0.0200	-0.0970	0.0202	-0.0122	-0.0571
Tobacco, cigarettes and alcohol	-0.0673	-0.6369	-0.0229	-0.1105	0.0232	-0.0140	-0.0652
Clothing, footwear and drapery	-0.0663	-0.0237	-0.6304	-0.1091	0.0228	-0.0138	-0.0644
Rent	-0.0372	-0.0133	-0.0127	-0.3260	0.0128	-0.0078	-0.0361
Household durables	-0.1490	-0.0532	-0.0508	-0.2450	-1.3129	-0.0310	-0.1446
Motor vehicles	-0.0937	-0.0335	-0.0319	-0.1541	0.0323	-0.8516	-0.0909
Other	-0.0891	-0.0318	-0.0304	-0.1465	0.0307	-0.0185	-0.8533

Table 4.4: Initial Estimates of Total Expenditure and Income Elasticities, Evaluated at Sample Means

Commodity m	Total expenditure elasticity	Income elasticity
1. Food	0.7679	0.7297
2. Tobacco, cigarettes and alcohol	0.8780	0.8343
3. Clothing, footwear and drapery	0.8664	0.8233
4. Rent	0.4857	0.4615
5. Household durables	1.9457	1.8488
6. Motor vehicles	1.2239	1.1629
7. Other	1.1635	1.1055

sum of ordinary price and income elasticities for any commodity is zero.

The price elasticities reported in Table 4.3 are all negative, except for the cross-price elasticities with respect to household durables. These positive values are due to the negative sign on the estimated $\bar{x}_5$ parameter, which also forces the corresponding own-price elasticity to be greater than one in absolute value. By contrast, the estimated own-price elasticities for the other six commodities are all less than one. How well do these results compare with previous studies for Australia based on the ELES (see, for example, Lluch, Powell, and Williams, 1977, p. 54), and 'Working's model' under additive preferences (Chung and Powell, 1987)? In short, they do not compare very well at all. For example, Chung and Powell (p. 53) report a value for the own-price elasticity of food in 1985-86 of -0.18; while Lluch, Powell and Williams give a sample mean (1955-1968) estimate of -0.27. For rent, Chung and Powell's estimate for 1985-86 (from Table 8.3) is -0.84; while Lluch, Powell and Williams report a value of -0.69.

With respect to the total expenditure and income elasticities, there is also general disagreement between our estimates and those reported in the other studies cited (to which can be added, Tulpule and Powell's 1977 application of a variant of ELES). This disagreement is especially serious for the most important items in the budget; namely, food and rent. In particular, our estimate of 0.73 for the expenditure elasticity of food is higher than those reported in the other studies, which lie in the range 0.35 to 0.50; while our estimate of 0.46 for rent is much less than the commonly reported figure of between 1.0 and 2.0. Indeed, for most developed countries, the finding that the expenditure elasticity for rent is less than one is extremely unusual (see Theil and Clements, 1987, ch. 2).

We turn now to Table 4.2, which contains parameter estimates and summary statistics for the portfolio module. Unlike their consumption counterparts, the portfolio equations appear to perform quite poorly with respect to their predictive ability within the sample. This is especially true for the equations explaining the holdings of equity and safe assets, although the poorness of fit of the equity equation could be due to the way data were constructed - annual incomes from dividends were capitalised to obtain the value of equities at the end of each financial year (linear interpolation was then used to derive quarterly observations). The Durbin-Watson test suggests that there is significant first-order serial correlation in the residuals of each equation; while the Wallis test, perhaps surprisingly, indicates the existence of fourth-order serial correlation at the 1 per cent level in the residuals of equation 3 only[15]. It should be noted, however, that serial correlation in the residuals of the portfolio equations does not necessarily indicate mis-specification, since the extensive use of interpolation in the construction of the wealth data (see Adams, 1987) may be responsible for the observed unsatisfactory serial properties of the residuals.

Each of the own-interest rate coefficients are positive, as required by the theory, and are statistically significant at the one per cent level. The cross-interest rate coefficients are also significant. Negative values on $\sigma^{(4,1)}$, $\sigma^{(4,2)}$ and $\sigma^{(4,3)}$ imply that an increase in the expected permanent interest rate charged on household liabilities, ceteris paribus, will induce an immediate reduction in the respective portfolio shares of the remaining risky assets in the portfolio. Thus, household liabilities can be regared as (net) substitutes for the other three risky assets. This intuitively appealing result is consistent with our a priori expectations on the matter.

The signs on the dummy variable coefficients in the first three equations are negative, while in the fourth equation the sign is positive. This suggests that the household sector sought to isolate itself from the increased uncertainty generated by high and volatile rates of inflation between 1974 and 1978, by substituting away from risky assets (i.e., by reducing its overall level of indebtedness and its investment in assets 1, 2 and 3) into safe assets. An asymptotic Likelihood Ratio test of the joint restrictions that the dummy variable coefficients are all zero yields a Chi-square value of 66.1 with four degrees of freedom. Therefore, we conclude that even though the t-statistics on each of the individual coefficients are less than one, the dummy variables together are playing an important role.

As a final check of the results, in Table 4.5 we present the inverse of the matrix of interest rate coefficients. If the estimated coefficients are fully consistent with the theory, then Table 4.5 will represent a positive definite covariance matrix. The diagonal elements (i.e.,the variances) will be indicators of the underlying riskiness attached to each asset by the representative household. The smaller is the absolute value of an element the smaller is the associated degree of risk. Similarly, the off-diagonal elements will be measures of the degree to which rates of return on any two assets are perceived to be linearly related. A positve value indicates a direct relationship, while a negative value implies an inverse relationship.

Table 4.5: The Matrix of Rates of Return Coefficients Implied by the Estimates in Table 4.2

The permanent rate of return on:	The permanent rate of return on:			
	Unincorporated business fixed assets	Equity	Financial assets	Household liabilities
Unincorporated business fixed assets	6.0472	-35.3238	-0.5524	-5.9732
Equity	-35.3238	162.1545	5.5938	15.6528
Financial assets	-0.5524	5.5938	1.5546	3.3695
Household liabilities	-5.9732	15.6528	3.3695	1.8472

Clearly, the matrix in Table 4.5 fails the test of positive definiteness. Thus, we must reject the theory-based hypothesis that the estimated interest-rate coefficients in Table 4.2 are elements of the inverse of a covariance matrix.

5 REVISED ESTIMATES

5.1 Estimating Forms

Our approach to the estimation of the ELESA has so far yielded an estimated system of equations whose residuals are, in some cases, highly serially correlated. The residuals for the rent equation and for each of the portfolio equations are by far the worst behaved. There are two alternative responses to this problem. We can either model the serially correlated errors in a mechanical fashion as outlined in Berndt and Savin (1975), or explore the possibility that the serial correlation is caused by mis-specification and make appropriate corrections. In this section, the latter strategy is taken up.

The estimation of the ELESA has, up to this point, proceeded on the implicit assumption that all markets operate in such a way as to allow the representative individual to buy and sell as much of any item at the going market price or rate of return. This assumption of exogenous market prices conveniently allows us to identify market equilibrium data as tracing out a set of observations on household demand in each market. However, as Chung and Powell (1987, p. 31) point out, the above assumption is not the only one which allows demand equations to be identified. For example, if the quantity available of a particular commodity in any period is predetermined, and the price adjusts endogenously, then movements of the supply schedule will also trace out a demand curve. Indeed, this is often the case in the markets for unincorporated business fixed assets and dwellings where the stocks existing in any quarter are largely predetermined because of technological constraints which make it extremely difficult to build and install items from scratch in any given three month period. It therefore seems that a more sensible way to handle our data is by departing from the 'endogenous quantity, exogenous price' paradigm for dwellings and unincorporated business fixed assets, in favour of one in which the rate of return on the latter and the price of rental services are treated as endogenous variables, and the corresponding quantities as exogenous variables. Thus, in this section, instead of (10) and (11), we have fitted the re-normalised system:

$$x_{m,t}p_{m,t} = \bar{x}_m p_{m,t} + \beta_m^* [W_t + (\frac{y_t - \bar{x}' p_t}{r_t})] + e^*_{m,t}$$

$$(m = 1, 2, 3, 5, 6, 7),$$

$$p_{4,t} = \frac{\beta_4^* (r_t W_t + y_t - \sum_{j \neq 4}^{7} p_{j,t}\bar{x}_j)}{x_{4,t} - \bar{x}_4(\beta_4^* - 1)} + e^*_{4,t} \qquad (16),$$

and

$$b_{1,t} = r_t + \frac{1}{\sigma^{(1,1)}} \left[\frac{r_t W_t w_{1,t}}{(r_t W_t + y_t - \bar{x}' p_t)} - \sum_{i \neq k}^{K-1} \sigma^{(i,1)}(b_{i,t} - r_t) \right] + e^*_{M+1,t}$$

$$w_{k,t}W_t = \sum_{i=1}^{K-1} \sigma_t^{(i,k)}(b_{i,t} - r_t)\, [W_t + (\frac{y_t - \bar{x}' p_t}{r_t})] + e^*_{M+k,t}$$

$$(k = 2, 3, 4) \qquad (17),$$

with appropriate provisions for the additive dummy variable, QINF, and its coefficients. Note that, like the $e_{i,t}$, the $e^*_{i,t}$ are assumed to be joint normally distributed random elements with classical serial properties.

5.2 Results

FIML estimates for the revised system of equations (16) and (17) are given in Tables 5.1 and 5.2. Key features of the results are as follows:

(1) The serial properties of the residuals for those consumption equations that have not been re-normalised remain about the same, while that for equation 4 is a substantial improvement on what was previously obtained -- the Durbin-Watson and Wallis tests now both reject the presence of serial correlation in the residuals of each equation[16];

(2) On the other hand, the serial properties of the residuals in the portfolio module have not been improved, with evidence still of significant first-order serial correlation in the residuals of all four equations[17];

(3) The goodness-of-fit of each of the seven consumption equations (including that now explaining the shadow price of rent) remain satisfactory, while the goodness-of-fit of the four portfolio equations (including that for the rate of return on unincorporated business fixed assets) has improved significantly;

(4) The estimated coefficients in the consumption module are fairly sensitive to the new specification -- for instance, the Frisch parameter (at sample mean values of variables) has fallen in absolute value to -0.9723 (asymptotic standard error = 0.0382), and there are now three 'subsistence parameters' with estimated negative values, whereas before there was only one;

Table 5.1: Maximum Likelihood Estimates of the Consumption Module of ELESA with the Equation for Rent Inverted, Derived from Australian Data Spanning 1971:2 through 1986:4

	Equation for:	MLE of β^*_m	Implied estimate of β_m	MLE of $\bar{x}_m$	SSR	Goodness-of-fit(a)	DW
1	Consumption of food	0.000929 (0.0004)	0.0952 (0.0027)	78.598 (5.0951)	42745	0.9254	1.8438
2	Consumption of tobacco, cigarettes and alcohol	0.000462 (0.0002)	0.0473 (0.0028)	26.726 (1.7966)	12099	0.9085	1.5857
3	Consumption of clothing, footwear and drapery	0.000503 (0.0002)	0.0516 (0.0038)	15.980 (2.5406)	14889	0.8761	1.5305
4	Price of rent	0.003337 (0.0004)	0.3418 (0.0268)	-164.180 (37.2290)	4.855	0.7016	1.5402
5	Consumption of household durables	0.000991 (0.0005)	0.1015 (0.0075)	-47.231 (4.0747)	60229	0.7348	1.4636
6	Consumption of motor vehicles	0.000372 (0.0004)	0.0382 (0.0045)	73.667 (6.0101)	9229	0.9529	1.3704
7	Consumption of other	0.003167 (0.0008)	0.3244 (0.0110)	-15.403 (10.908)	484581	0.8317	1.8474

Implied estimate of $\delta = \sum_{m=1}^{m}$ (MLE of β^*_m) = 0.0098 (0.0005).

Implied estimate of the mean marginal propensity to consume out of safe income $\bar{\mu}$ = (implied estimate of δ)/$\bar{r}$ = 1.0097 (0.0502), where $\bar{r}$ is the sample mean value of r_t.

(a) Derived as the R-squared from simple regresions of the left hand variables on their respective predicted values.

Table 5.2: Maximum Likelihood Estimates of the Investment Module of ELESA with the Equation for the Holdings of Unincorporated Business Fixed Assets Inverted, Derived from Australian Data Spanning 1971:2 through 1986:4

Equation for:	MLE of$^{(a)}$:		SRR	Goodness-of-fit$^{(b)}$	DW
	$\sigma(k,k)$	c^k			
1 The permanent rate of return on unincorporated business fixed assets	1.0789 (0.1745)	-1065.00 (443.10)	0.0184	0.6694	1.1772
2 The holdings of equity	0.0388 (0.0182)	-288.75 (402.94)	2.23×10^7	0.8382	0.5636
3 The holdings of financial assets	0.0784 (0.5630)	-1246.79 (796.31)	2.45×10^8	0.9685	0.9263
4 The holdings of household liabilities	0.2410 (0.1773)	778.44 (515.42)	9.56×10^7	0.9444	0.9717
5 The holdings of safe assets	Nc.	Nc.	Nc.	0.8986	Nc.

MLE of:					
$\sigma(2,1)$	$\sigma(3,1)$	$\sigma(4,1)$	$\sigma(3,2)$	$\sigma(4,2)$	$\sigma(4,3)$
0.2714 (0.0339)	1.1079 (0.1399)	-0.7512 (0.0911)	0.1232 (0.0457)	-0.1150 (0.0263)	-0.1112 (0.2999)

(a) C^k is the coefficient on the dummy variable QINF.

(b) Derived as the R-squared from simple regressions of left hand variables on their respective predicted values.

Nc. Not calculated or not applicable.

and

(5) In contrast to (4), the estimated coefficients in the portfolio module are quite insensitive to the new specification -- household liabilities remain substitutes for the other three risky assets in the portfolio.

In Tables 5.3 and 5.4, revised estimates of price, income and expenditure elasticities, all calculated at sample means, are presented. Turning first to the price elasticities, we find that as a direct consequence of the negative values for $\bar{x}_4$, $\bar{x}_5$ and $\bar{x}_7$, the corresponding own-price elasticities are each now greater than one in absolute value. The own-price elasticity for food has increased in absolute terms from -0.57 to -0.66; while that for rent is now -1.50 compared with -0.32 previously. Overall, the revised estimates of price elasticities still remain in conflict with those reported in Lluch, Powell and Williams (1977, pp. 55 and 56) and Chung and Powell (1987, pp. 51-53). On the other hand, the expenditure and income elasticities now compare more favourably with the other studies, at least with respect to food and rent. However, the very low expenditure elasticity for an apparent luxury item like motor vehicles is cause for some alarm.

Finally, in Table 5.5 we present the inverse of the matrix of rate of return coefficients implied by the revised estimates in Table 5.1. As with the matrix presented earlier, the matrix depicted in Table 5.5 fails the test for consistency with the underlying theory. Thus, once again, we must reject the hypothesis that the estimated interest-rate coefficients are elements of the inverse of a covariance matrix.

Table 5.3: Revised Estimates of Uncompensated Own- and Cross-Price Elasticities of Commodity Demand, Evaluated at Sample Means

Response in demand for:	Commodity whose price increases by 1 per cent:						
	Food	Tobacco, cigarettes and alcohol	Clothing, footwear and drapery	Rent	Household durables	Motor vehicles	Other
Food	-0.6591	-0.0127	-0.0072	0.0797	0.0222	-0.0342	0.0078
Tobacco, cigarettes and alcohol	-0.0398	-0.0139	-0.0079	0.0875	0.0243	-0.0375	0.0086
Clothing, footwear and drapery	-0.0460	-0.0162	-0.8322	0.1012	0.0281	-0.0434	0.0092
Rent	-0.1185	-0.0416	-0.0237	-1.4962	0.0726	-0.1118	0.0256
Household durables	-0.0836	-0.0294	-0.0167	0.1838	-1.4469	-0.0789	0.0180
Motor vehicles	-0.0240	-0.0084	-0.0048	0.0528	0.0147	-0.4373	0.0052
Other	-0.0607	-0.0213	-0.0121	0.1334	0.0371	-0.0572	-1.0268

Table 5.4: Revised Estimates of Total Expenditure and Income Elasticities, Evaluated at Sample Means

Commodity m	Total expenditure elasticity	Income elasticity
1. Food	0.5789	0.5839
2. Tobacco, cigarettes and alcohol	0.6358	0.6415
3. Clothing, footwear and drapery	0.7351	0.7416
4. Rent	1.8941	1.9109
5. Household durables	1.3353	1.3472
6. Motor vehicles	0.3838	0.3872
7. Other	0.9695	0.9781

Table 5.5: The Matrix of Rates of Return Coefficients Implied by the Estimates in Table 5.2

The permanent rate of return on:	The permanent rate of return on:			
	Unincorporated business fixed assets	Equity	Financial assets	Household liabilities
Unincorporated business fixed assets	-0.8128	8.4579	0.9337	1.9338
Equity	8.4579	-93.0610	3.3749	-16.4929
Financial assets	0.9337	3.3749	1.9342	5.4129
Household liabilities	1.9338	-16.4929	5.4129	4.8028

6 CONCLUDING REMARKS AND AGENDA FOR FURTHER WORK

In this paper we have estimated a model of household behaviour which generalises Lluch's (1973) ELES to a setting of multiple investment opportunities. Estimates were based on a sample of quarterly Australian data disaggregated to the level of seven commodities and five assets. The analysis began with a brief presentation of the theoretical development of the model, called the Extended Linear Expenditure System with Assets; followed by a description of the necessary modifications required for econometric estimation. Initial results were then reported. These results proved somewhat disappointing, especially in respect to the overall performance of the equations explaining asset accumulation and the demand for rental services. It was found, however, that by treating the rental price and the expected rate of return on unincorporated business fixed assets endogenously, and the corresponding quantities exogenously, improved the interpretability of the estimates and mitigated the symptoms of the initial mis-specification problem. Certainly, the serial properties of the rental equation improved significantly.

Despite the early estimation difficulties, the results obtained in this paper suggest that the ELESA can quite satisfactorilly explain household consumption and portfolio behaviour in Australia's recent past. Further empirical work probably will have to await the availability of richer data sources. In the meantime, some further attention to dynamics seems called for. Presently, the model assumes that households re-assess expected rates of return on assets over time, but not the underlying riskiness of those assets (i.e., the elements of $\mathbf{\Omega}$). A more plausible specification would incorporate some sort of learning process that would allow households to re-assess risk as well as expected returns.

APPENDIX A: DERIVATION OF THE CONTINUOUS TIME BUDGET CONSTRAINT

In this Appendix we derive the true continuous time form of the intertemporal budget constraint for the individual. Following Merton (1971, pp. 377-78), we do so by firstly examining the underlying discrete-time specification of the model, and then taking limits.

Consider an individual who has planning periods of Δh time units in length and whose consumption and investment decisions for period $[h, h+\Delta h)$ are made at the beginning of the period. We assume that all prices for $[h, h+\Delta h)$ are known at h, as is the agent's initial stock of non-human wealth, W^h. At h, the agent chooses an optimal flow of consumption per unit time, $c^h\Delta h$, and an optimal distribution of wealth, such that:

$$W^h = (c^h - y^h)\Delta h + \sum_{k=1}^{K} w_k^h(W^h - c^h\Delta h + y^h\Delta h) \qquad \text{(A1)},$$

in which

$$\sum_{k=1}^{K} w_k^h = 1 \qquad \text{(A2)},$$

where $w_k^h = q_k^h P^h D_k^h / W^h$ is the proportion of total wealth at h invested in asset k, D_k^h is the quantity of asset k held at h; and y^h is the exogenously given flow of disposable labour income which accrues during h.

At the end of the planning period, the individual's wealth will differ from that at the beginning of the period according to: (i) the value of savings from wage income; (ii) the amount of net capital gain (or loss) accruing at the time new prices are called for $[h+\Delta h, h+2\Delta h)$; (iii) the flow of dividends and interest received (or paid) during the period; and (iv) the proportion of each asset used up through depreciation. In algebraic terms:

$$W^{h+\Delta h} = \sum_{k=1}^{K} w_k^h \, (W^h - c^h\Delta h + y^h\Delta h)$$

$$+ \sum_{k=1}^{K} [w_k^h(W^h - c^h\Delta h + y^h\Delta h)(\frac{q_k^{h+\Delta h}P^{h+\Delta h} - q_k^h P^h + s_k^h P^h \Delta h(1-i^h)}{q_k^h P^h} - \rho_k^h \Delta h)]$$

$$= \sum_{k=1}^{K} [w_k^h(W^h - c^h\Delta h + y^h\Delta h)(\frac{q_k^{h+\Delta h}P^{h+\Delta h} + s_k^h P^h \Delta h(1-i^h)}{q_k^h P^h} - \rho_k^h \Delta h)] \qquad (A3).$$

Equation (A3) implies:

$$W^h = \sum_{k=1}^{K} [w_k^{h-\Delta h}(W^{h-\Delta h} - c^{h-\Delta h} \, \Delta h + y^{h-\Delta h} \, \Delta h)(\frac{q_k^h P^h + s_k^{h-\Delta h}P^{h-\Delta h}\Delta h(1-i^{h-\Delta h})}{q_k^{h-\Delta h}P^{h-\Delta h}})]$$

$$- \sum_{k=1}^{K} w_k^{h-\Delta h}(W^{h-\Delta h} - c^{h-\Delta h}\Delta h + y^{h-\Delta h}\Delta h) \, \rho_k^{h-\Delta h}\Delta h \qquad (A4).$$

May (1970) has shown that the single budget constraint in discrete-time, derived here by equating (A4) with (A1), implies two continuous time constraints: one a stock constraint, the other a flow constraint. The continuous time flow constraint is obtained by first deducting (A1) from (A3),

$$W^{h+\Delta h} - W^h = \sum_{k=1}^{K} [w_k^h(W^h - c^h\Delta h + y^h\Delta h)(\frac{q_k^{h+\Delta h}P^{h+\Delta h} - q_k^h P^h + s_k^h P^h \Delta h(1-i^h)}{q_k^h P^h})]$$

$$- \sum_{k=1}^{K} [w_k^h(W^h - c^h\Delta h + y^h\Delta h)\rho_k^h\Delta h] \; + \; (y^h - c^h)\Delta h \qquad (A5),$$

and then taking limits (from the right-hand side) as Δh goes to zero to yield:

$$dW^h = \sum_{k=1}^{K} [w_k^h(W^h - c^h dh + y^h dh)\left(\frac{d(q_k^h P^h) + s_k^h P^h dh(1-i^h)}{q_k^h P^h} - \rho_k^h dh\right)]$$

$$+ (y^h - c^h)dh \qquad (A6).$$

It is assumed that all variables in (A5) are right-continuous functions of time, and, throughout this paper, the choice of functions is restricted to this class.

By applying the same limit process to equation (A2), we obtain the continuous time stock constraint, i.e.:

$$W^h = \sum_{k=1}^{K} w_k^h W^h \qquad (A7).$$

The final objective of this Appendix, the sole continuous time budget constraint, is obtained by substituting equation (4) from the main body of the text and (A7) into (A6), to give:

$$dW^h = \sum_{k=1}^{K-1} (b_k^t - b_K^t) w_k^h W^h dh + b_K^t W^h dh + (y^h - c^h)\, dh$$

$$+ \sum_{k=1}^{K} \sigma_k^t w_k^h W^h \, dz_k^h + o(\Delta h) \qquad (A8),$$

where $o(\Delta h)$ is the asymptotic order symbol encompassing all terms which, after division by Δh, approach zero as Δh approaches zero. In equation (A8), the w_k^h are now unconstrained because the relation $w_K^h = (1 - \sum_{k=1}^{K-1} w_k^h)$ will ensure that the stock constraint is at all times satisfied.

APPENDIX B: THE TIME-SERIES DATA BASE

In this Appendix we give details of the collation and editing of data used for the estimation of the ELESA. We do so by proceeding step by step through the list of the basic data requirements of the model given in section 3.2 of the text.

B.1 The market value of household sector net wealth and its components

Data for the market value of household sector net wealth and for each of its components over the period 1969:4 to 1986:4 were taken directly from Adams (1987). To ensure that the portfolio shares sum to one, the total net wealth of households was defined as a net aggregate of those assets identified in Table 3.2 alone.

Data for W_t and $w_{k,t}$ $(k = 1,\ldots, 5)$ used in the econometric analysis are presented in Tables B.1.1 and B.1.2. These estimates were obtained from the data taken from Adams (1987) by firstly dividing through by the population estimates in Table B.6, and then averaging to ensure that the stocks are (notionally) valued at the mid-point of each quarter (see section 3.1 of the text).

B.2 Post-tax rates of return on assets

The data for post-tax market rates of return on assets for 1969:4 to 1986:4 were derived from pre-tax figures given in Adams (1987). Data on an **after-tax** basis were calculated by applying figures for the average personal income tax rate calculated from data for 'Gross PAYE Instalments - Persons' and 'Household Income and Outlay Account - Wages, Salaries and Supplements' taken directly from the Australian Bureau of Statistics' 1987 publication Time Series Data on Magnetic Tape and Microfiche, June Quarter, Catalogue number 1311.0 (hereinafter ABS, Cat. No. 1311.0). Throughout these calculations the assumption was maintained that only dividends and interest receipts were taxable; capital gains accrue tax-free.

To derive estimates of rates of estimates that are consolidated groupings of individual assets (and/or liabilities), an assumption about the timing of interest payments was required. The assumption made was that during any period income accrues (or is paid) from an asset (or liability) according to the market value of that asset held by households at the end of the preceeding period. For example, if $n_{j,t}$ and $n_{k,t}$ are the after-tax rates of return during t on assets j and k respectively, then the pre-tax yield on the composite of these two assets was calculated as:

$$n_{jk,t} = \frac{(V_{j,t-1}n_{j,t} + V_{k,t-1}n_{k,t})}{(V_{j,t-1} + V_{k,t-1})} ,$$

where, $V_{i,t-1}$ is the market value of asset i at the end of period (t - 1).

The post-tax market yield series for equity was found to oscillate violently due to significant variability in its capital-gains component. Thus, to remove some of this volatility for the purpose of calculating the appropriate permanent rate of return series, a lagging four-quarter moving average was taken through the capital-gains component.

Data for post-tax rates of return used to derive estimates of $b_{k,t}$ $(k = 1,..., 4)$ and r_t are given in Table B.2.1; while the actual permanent rate of return data used in the econometric analysis are presented in Table B.2.2. Note that, the estimates of $b_{k,t}$ $(k = 1,..., 4)$ were compiled from equation (13), with $\tau_1 = 25$, $\tau_2 = 28$, $\tau_3 = 8$ and $\tau_4 = 9$.

B.3 Private consumption expenditure

Disaggregated seasonally adjusted data on private final consumption expenditure in current prices for 1969:3 to 1986:4 were taken from ABS (Cat. No. 1311.0). Aggregation from the given sixteen commodity disaggregation to the seven commodity aggregation used here was obtained by simple summation of the component items given in Table 3.1 of the text. Table B.3 contains the per-capita data used in the econometric analysis.

B.4 Commodity price indexes

ABS (Cat. No. 1311.0) also provides disaggregated seasonally unadjusted estimates of private consumption expenditure in average 1979-80 prices. Imputation of price indexes for each of the sixteen commodity groupings was obtained by dividing the expenditure data in current prices through by the corresponding items of expenditure in constant prices. Average 1979-80 budget shares were used as weights to aggregate these individual price indexes from sixteen to seven commodities. Data for each of the price indexes are given in Table B.4.1. Note, that because the consumption price variables are defined as the ratio of the flow of nominal expenditures to the real flow of those expenditures, they are conceptually measured at the mid-point of each period and so need not be averaged using equation (9) in the text as do the other stock variables in the analysis.

In Table B.4.2 we give values for the general level of consumer price inflation, and for the dummy variable QINF. The level of inflation was derived as a fixed (1979-80) weighted sum of the proportional changes in the seven individual price indexes. Non-zero values for QINF were computed from the inflation data using equation (13), with τ_{inf} set to 16.

B.5 The aggregate contribution to household net savings from sources other than capital gains and cash income from non-human wealth

To derive a series for the aggregate after-tax contribution to household net savings from sources other than capital gains and cash income from non-human wealth, explicit allowance must be made for the fact that dwellings are excluded from non-human wealth in the econometric analysis. To understand why, consider the following. By definition, at any point in time what is not consumed in the form of goods and services from the current flow of income must be saved, i.e., (in a notation separate from that used in the text)

$$d\left(\sum_{k=1}^{K} P_k N_k\right) = \text{Income} - (\text{Expenditure on goods and services}) \qquad \text{(B1)},$$

where P_k is the price per unit of asset k, N_k is the number of units of asset k held by the household, and K is the total number of household assets (including dwellings). For simplicity, we assume that there are no taxes or depreciation of physical assets, nor are there differences between secondhand and new prices of assets.

Income can be generated from either assets or non-asset (i.e. other) sources. In other words:

$$\text{Income} = \sum_{k=1}^{K} (dP_k + M_k)N_k + \bar{Y} \qquad \text{(B2)},$$

where M_k is the non-capital gain component of the unit return on asset k, and $\bar{Y}$ is defined as the sum of wages, salaries and supplements, personal benefit payments, capital and current grants to non-profit institutions, third party insurance transfers, and net unrequited transfers from overseas. Substitution of (B2) into (B1) yields:

$$d\left(\sum_{k=1}^{K} P_k N_k\right) = \sum_{k=1}^{K} (dP_k + M_k)N_k + \bar{Y} \ - \text{(Expenditure on goods and services)} \qquad \text{(B3)}.$$

To make the budget constraint (equation (B3)) consistent with the underlying constraint facing the individual in the analysis, it is necessary to remove dwellings (i.e., for notational convenience, asset K) from the left-hand side of (B3) so that:

$$d\left(\sum_{k=1}^{K-1} P_k N_k\right) = \sum_{k=1}^{K-1} (dP_k + M_k)N_k + (\bar{Y} - d(P_K N_K) + (dP_K + M_K)N_K) \ - \text{(Expenditure on goods and services)} \qquad \text{(B4)}.$$

Equation (B4) tells us, that to preserve the budget constraint identity at all points in the sample, y_t should be calculated as the sum of incomes from 'other' sources and from dwellings (including the capital gains component) less the change in market value of the stock of dwellings held by households in the current period. Seasonally adjusted data for $\bar{Y}$ were taken directly from the 'National Accounts - Household Income and Outlay Account' section of ABS (Cat. No. 1311.0). Data for the market value of dwellings owned by households and the pre-tax return on those dwelling (including imputed rent) were taken from Adams (1987). Figures for y_t on an after-tax basis were obtained by applying the average rates of income tax given in Table B.2.1 to the wages, salaries and supplements component and that part of the return from dwellings consisting of actual cash income from rent. Incomes from the remaining sources were assumed to accrue tax-free. Per-capita data for y_t are given in Table B.5.

B.6 Population

To complete our data base we must obtain a series for the population of Australia during the period 1969:3 to 1986:4. Our population data, reported in Table B.6 were taken directly from the Australian Treasury's data base for the NIF-10 model, documented by the Australian Bureau of Statistics in its 1987 publication NIF-10S Model - Data on Magnetic Tape and Microfiche, Catalogue number 1313.0.

Table B.1.1: Household Net Wealth ($ per person) Data Used Used for Econometric Analysis[a]

Date	W_t	Date	W_t
1971:1	3628.13940	1982:1	11228.33789
1971:2	3692.06543	1982:2	11418.93262
1971:3	3804.87842	1982:3	11770.36328
1971:4	3965.87598	1982:4	12408.51270
1972:1	4107.21387	1983:1	13069.65137
1972:2	4235.14014	1983:2	13595.84375
1972:3	4317.18457	1983:3	14205.63184
1972:4	4370.95557	1983:4	14811.32520
1973:1	4424.89502	1984:1	15121.20703
1973:2	4441.76904	1984:2	15250.48535
1973:3	4445.60645	1984:3	15567.80664
1973:4	4441.70703	1984:4	16121.77930
1974:1	4398.71631	1985:1	16621.24414
1974:2	4297.11719	1985:2	16974.58398
1974:3	4361.15039	1985:3	17314.73828
1974:4	4617.30273	1985:4	17773.95508
1975:1	4839.91113	1986:1	18191.88672
1975:2	5039.36963	1986:2	18577.64453
1975:3	5282.36572	1986:3	19017.28711
1975:4	5548.58887	1986:4	19544.59180
1976:1	5781.89160		
1976:2	5980.37158		
1976:3	6159.69141		
1976:4	6325.78027		
1977:1	6446.47705		
1977:2	6560.56250		
1977:3	6725.09863		
1977:4	6935.80469		
1978:1	7141.25049		
1978:2	7315.66699		
1978:3	7539.53516		
1978:4	7835.28125		
1979:1	8105.66016		
1979:2	8304.44629		
1979:3	8545.35352		
1979:4	8882.72070		
1980:1	9147.45410		
1980:2	9371.26660		
1980:3	9661.45410		
1980:4	9961.98828		
1981:1	10162.96387		
1981:2	10306.37891		
1981:3	10578.04785		
1981:4	10944.18164		

(a) Data in this table were computed from the raw data by firstly dividing through by the population estimates in Table B.6, and then averaging to ensure that all stocks are (notionally) measured at the mid-point of each quarter.

Table B.2.1: Data for Post-Tax Rates of Return and the Average Rate of Personal Income Tax

Date	Post-tax rate of return per quarter on:					Average rate of income tax
	Unincorporated business fixed assets	Equity	Financial assets	Household liabilities	Safe assets	
1971:1	0.0363	-0.0109	0.0138	0.0167	0.0083	0.15
1971:2	0.0408	-0.0037	0.0135	0.0169	0.0083	0.16
1971:3	0.0380	-0.0182	0.0143	0.0173	0.0083	0.16
1971:4	0.0553	-0.0269	0.0138	0.0173	0.0082	0.17
1972:1	0.0444	0.0523	0.0132	0.0171	0.0081	0.17
1972:2	0.0409	0.0519	0.0129	0.0174	0.0080	0.17
1972:3	0.0535	0.0358	0.0133	0.0180	0.0080	0.17
1972:4	0.0439	0.0401	0.0131	0.0183	0.0083	0.16
1973:1	0.0419	0.0292	0.0129	0.0186	0.0082	0.16
1973:2	0.0523	0.0026	0.0128	0.0188	0.0082	0.16
1973:3	0.1369	0.0036	0.0149	0.0203	0.0080	0.17
1973:4	0.0703	-0.0356	0.0153	0.0222	0.0103	0.17
1974:1	0.0357	0.0255	0.0156	0.0226	0.0102	0.18
1974:2	0.1064	-0.0252	0.0169	0.0230	0.0101	0.18
1974:3	0.1802	-0.0881	0.0178	0.0257	0.0110	0.19
1974:4	0.0961	-0.0565	0.0174	0.0268	0.0108	0.20
1975:1	0.0803	0.0086	0.0174	0.0272	0.0111	0.18
1975:2	0.0887	-0.0003	0.0172	0.0273	0.0105	0.17
1975:3	0.0883	0.0155	0.0178	0.0276	0.0102	0.19
1975:4	0.0727	0.0544	0.0172	0.0280	0.0097	0.20
1976:1	0.0612	0.0618	0.0170	0.0274	0.0095	0.20
1976:2	0.0604	0.0465	0.0169	0.0268	0.0093	0.21
1976:3	0.0515	0.0658	0.0180	0.0271	0.0098	0.19
1976:4	0.0656	-0.0003	0.0178	0.0279	0.0095	0.21
1977:1	0.0563	0.0293	0.0176	0.0275	0.0095	0.21
1977:2	0.0647	0.0247	0.0173	0.0274	0.0093	0.22
1977:3	0.0473	0.0170	0.0186	0.0277	0.0095	0.21
1977:4	0.0515	0.0231	0.0182	0.0279	0.0096	0.20
1978:1	0.0544	0.0269	0.0175	0.0273	0.0094	0.20
1978:2	0.0495	0.0390	0.0175	0.0272	0.0095	0.20
1978:3	0.0523	0.0581	0.0185	0.0272	0.0096	0.19
1978:4	0.0479	0.0413	0.0130	0.0274	0.0094	0.20
1979:1	0.0415	0.0619	0.0173	0.0276	0.0090	0.21
1979:2	0.0654	0.0398	0.0173	0.0279	0.0088	0.22
1979:3	0.0573	0.0638	0.0126	0.0282	0.0089	0.21
1979:4	0.0547	0.0724	0.0129	0.0288	0.0089	0.21
1980:1	0.0645	0.1193	0.0132	0.0298	0.0090	0.22
1980:2	0.0553	0.0671	0.0135	0.0305	0.0094	0.21
1980:3	0.0560	0.1209	0.0207	0.0311	0.0099	0.20
1980:4	0.0511	0.0929	0.0211	0.0324	0.0099	0.21
1981:1	0.0886	0.0594	0.0214	0.0334	0.0103	0.22
1981:2	0.0549	0.0733	0.0225	0.0338	0.0103	0.21
1981:3	0.0619	0.0143	0.0230	0.0353	0.0104	0.22
1981:4	0.0641	0.0052	0.0246	0.0364	0.0111	0.21

Table B.2.1 (con't)

Date	Post-tax rate of return per quarter on:					Average rate of income tax
	Unincorporated business fixed assets	Equity	Financial assets	Household liabilities	Safe assets	
1982:1	0.0651	-0.0257	0.0236	0.0372	0.0107	0.23
1982:2	0.0483	-0.0196	0.0252	0.0385	0.0111	0.23
1982:3	0.0879	-0.0188	0.0255	0.0401	0.0112	0.23
1982:4	0.0416	0.0041	0.0238	0.0403	0.0111	0.23
1983:1	0.0381	0.0090	0.0223	0.0393	0.0108	0.22
1983:2	0.0368	0.0555	0.0222	0.0391	0.0107	0.22
1983:3	0.0378	0.0743	0.0222	0.0381	0.0107	0.22
1983:4	0.0489	0.0648	0.0219	0.0377	0.0102	0.22
1984:1	0.0183	0.0643	0.0207	0.0375	0.0098	0.23
1984:2	0.0232	0.0284	0.0216	0.0373	0.0096	0.23
1984:3	0.0571	0.0325	0.0220	0.0373	0.0094	0.24
1984:4	0.0418	0.0402	0.0223	0.0370	0.0093	0.24
1985:1	0.0329	0.0387	0.0219	0.0374	0.0095	0.23
1985:2	0.0717	0.0673	0.0226	0.0384	0.0096	0.23
1985:3	0.0565	0.0653	0.0238	0.0424	0.0096	0.23
1985:4	0.0652	0.0713	0.0245	0.0440	0.0101	0.24
1986:1	0.0571	0.0682	0.0234	0.0453	0.0102	0.24
1986:2	0.0327	0.0905	0.0235	0.0466	0.0105	0.24
1986:3	0.0382	0.0469	0.0236	0.0463	0.0103	0.25
1986:4	0.0417	0.1066	0.0238	0.0475	0.0100	0.26

Table B.2.2: Permanent Rate of Return Data Used for Econometric Analysis[a]

Date	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$
1971:1	0.0433	0.0433	0.0142	0.0173
1971:2	0.0422	0.0330	0.0140	0.0174
1971:3	0.0412	0.0214	0.0143	0.0176
1971:4	0.0433	0.0085	0.0142	0.0177
1972:1	0.0437	0.0101	0.0136	0.0175
1972:2	0.0437	0.0106	0.0131	0.0175
1972:3	0.0446	0.0088	0.0130	0.0179
1972:4	0.0446	0.0064	0.0129	0.0183
1973:1	0.0431	0.0035	0.0128	0.0186
1973:2	0.0437	-0.0036	0.0125	0.0190
1973:3	0.0570	-0.0059	0.0137	0.0200
1973:4	0.0616	-0.0135	0.0148	0.0217
1974:1	0.0661	-0.0105	0.0157	0.0229
1974:2	0.0733	-0.0110	0.0169	0.0239
1974:3	0.0905	-0.0228	0.0182	0.0258
1974:4	0.0956	-0.0315	0.0188	0.0276
1975:1	0.0965	-0.0242	0.0189	0.0288
1975:2	0.0992	-0.0222	0.0183	0.0292
1975:3	0.1032	-0.0184	0.0183	0.0293
1975:4	0.1031	-0.0061	0.0179	0.0296
1976:1	0.1011	0.0070	0.0173	0.0291
1976:2	0.0993	0.0128	0.0169	0.0278
1976:3	0.0966	0.0249	0.0174	0.0273
1976:4	0.0942	0.0217	0.0176	0.0274
1977:1	0.0912	0.0230	0.0177	0.0274
1977:2	0.0880	0.0239	0.0176	0.0273
1977:3	0.0827	0.0218	0.0182	0.0275
1977:4	0.0786	0.0195	0.0184	0.0279
1978:1	0.0740	0.0248	0.0181	0.0278
1978:2	0.0690	0.0323	0.0177	0.0274
1978:3	0.0642	0.0416	0.0182	0.0272
1978:4	0.0599	0.0483	0.0182	0.0272
1979:1	0.0539	0.0574	0.0177	0.0273
1979:2	0.0511	0.0599	0.0173	0.0275
1979:3	0.0479	0.0661	0.0149	0.0279
1979:4	0.0436	0.0694	0.0132	0.0287
1980:1	0.0404	0.0852	0.0119	0.0296
1980:2	0.0365	0.0864	0.0113	0.0305
1980:3	0.0395	0.0896	0.0150	0.0314
1980:4	0.0371	0.0889	0.0186	0.0326
1981:1	0.0414	0.0871	0.0217	0.0339
1981:2	0.0425	0.0863	0.0249	0.0348
1981:3	0.0509	0.0763	0.0261	0.0360
1981:4	0.0539	0.0684	0.0272	0.0371

(a) Data in this table were derived from data in Table B.2.1, using equation (13), with $\tau_1 = 25$, $\tau_2 = 28$, $\tau_3 = 8$ and $\tau_4 = 9$.

Table B.2.2 (con't)

Date	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$
1982:1	0.0560	0.0567	0.0265	0.0382
1982:2	0.0565	0.0454	0.0257	0.0393
1982:3	0.0630	0.0369	0.0263	0.0407
1982:4	0.0616	0.0268	0.0257	0.0417
1983:1	0.0590	0.0203	0.0242	0.0417
1983:2	0.0564	0.0213	0.0229	0.0411
1983:3	0.0534	0.0244	0.0218	0.0399
1983:4	0.0532	0.0259	0.0213	0.0387
1984:1	0.0480	0.0272	0.0200	0.0374
1984:2	0.0445	0.0233	0.0201	0.0365
1984:3	0.0448	0.0219	0.0209	0.0363
1984:4	0.0432	0.0206	0.0217	0.0364
1985:1	0.0406	0.0209	0.0218	0.0367
1985:2	0.0433	0.0242	0.0223	0.0375
1985:3	0.0439	0.0291	0.0234	0.0401
1985:4	0.0453	0.0358	0.0245	0.0429
1986:1	0.0450	0.0464	0.0244	0.0456
1986:2	0.0429	0.0563	0.0243	0.0480
1986:3	0.0413	0.0638	0.0243	0.0493
1986:4	0.0401	0.0795	0.0243	0.0501

Table B.3: Current-Price Expenditure ($ per person) Data Used for Econometric Analysis[a]

Date	$p_{1,t}x_{1,t}$	$p_{2,t}x_{2,t}$	$p_{3,t}x_{3,t}$	$p_{4,t}x_{4,t}$	$p_{5,t}x_{5,t}$	$p_{6,t}x_{6,t}$	$p_{7,t}x_{7,t}$
1971:2	75.23	37.42	36.66	53.80	31.76	45.08	54.64
1971:3	76.84	37.77	36.86	55.37	33.36	49.27	55.67
1971:4	77.06	38.04	36.82	57.28	33.34	47.89	56.90
1972:1	78.39	38.40	36.97	59.00	33.50	47.76	57.72
1972:2	79.75	39.46	38.03	60.58	35.18	48.48	58.70
1972:3	82.00	40.06	39.91	62.38	36.39	48.15	61.03
1972:4	84.64	40.87	40.87	64.14	37.51	50.41	63.24
1973:1	84.92	42.05	41.98	66.05	39.38	53.20	64.34
1973:2	88.93	43.84	44.21	68.12	41.91	54.35	67.31
1973:3	92.30	45.82	45.82	70.24	45.16	56.45	69.50
1973:4	96.22	46.54	48.26	73.89	48.55	58.18	72.87
1974:1	101.76	47.70	50.69	75.93	51.65	56.99	77.03
1974:2	106.24	49.33	51.37	78.99	54.36	60.92	81.61
1974:3	109.50	52.64	54.82	83.79	58.02	66.22	86.33
1974:4	109.96	52.34	54.87	88.35	59.64	68.97	92.18
1975:1	113.40	56.99	58.28	92.84	64.20	72.57	96.37
1975:2	116.46	59.17	60.10	97.46	68.09	75.36	99.76
1975:3	120.56	62.11	60.67	103.68	71.95	76.83	106.05
1975:4	124.99	64.95	62.78	108.96	73.73	78.60	110.75
1976:1	128.67	67.69	63.33	113.60	75.76	85.26	115.10
1976:2	132.97	70.05	65.84	118.15	79.60	88.51	117.01
1976:3	137.85	70.52	68.11	125.41	81.69	87.23	121.43
1976:4	142.52	72.08	69.24	131.33	82.92	94.12	125.23
1977:1	147.16	73.12	70.65	136.35	82.30	92.26	127.38
1977:2	153.11	75.68	72.08	141.14	82.72	92.80	131.13
1977:3	154.30	74.91	73.85	148.06	82.07	93.60	135.62
1977:4	161.61	76.61	75.76	154.33	82.77	95.37	137.32
1978:1	164.83	77.95	76.97	158.48	82.00	96.79	138.17
1978:2	170.83	79.88	80.09	163.10	86.15	101.82	145.76
1978:3	175.38	83.63	80.23	170.87	85.57	105.37	149.89
1978:4	181.35	86.20	81.56	176.49	86.83	106.92	153.07
1979:1	184.69	87.86	83.02	182.07	86.89	112.24	155.41
1979:2	191.59	88.25	84.19	187.32	86.67	116.57	158.46
1979:3	197.61	91.80	85.20	196.10	89.67	127.39	161.12
1979:4	201.82	91.97	85.06	200.79	91.15	127.04	164.84
1980:1	210.57	94.70	88.76	204.97	94.57	135.12	172.27
1980:2	216.47	97.58	90.71	211.02	98.26	134.74	177.61
1980:3	224.06	99.62	94.60	221.42	100.77	133.39	181.47
1980:4	230.50	101.24	95.23	226.38	106.44	139.39	186.67
1981:1	236.32	103.67	96.14	232.96	108.18	144.01	190.74
1981:2	240.97	106.59	100.15	240.10	110.00	148.39	196.36
1981:3	249.12	109.48	102.27	251.45	115.88	150.78	200.55
1981:4	253.62	110.73	104.42	262.72	115.38	151.79	207.72

(a) Data in this table were derived from the raw data by dividing through by population numbers in Table B.6.

Table B.3 (con't)

Date	$p_{1,t}x_{1,t}$	$p_{2,t}x_{2,t}$	$p_{3,t}x_{3,t}$	$p_{4,t}x_{4,t}$	$p_{5,t}x_{5,t}$	$p_{6,t}x_{6,t}$	$p_{7,t}x_{7,t}$
1982:1	257.64	113.64	104.58	273.12	117.67	154.19	213.98
1982:2	266.57	117.27	107.66	286.93	119.45	162.54	221.90
1982:3	268.51	118.37	108.59	302.91	119.81	173.25	226.04
1982:4	279.05	123.00	110.82	314.92	120.18	176.54	226.09
1983:1	290.09	121.57	114.85	325.25	125.81	175.44	231.40
1983:2	284.02	122.18	115.22	335.91	125.30	175.30	235.78
1983:3	293.31	126.81	117.73	347.47	127.26	179.02	239.09
1983:4	297.24	130.40	117.20	359.24	132.67	183.41	251.62
1984:1	298.81	128.84	119.81	370.78	134.51	186.87	256.84
1984:2	304.45	133.39	118.35	384.61	131.46	187.84	261.64
1984:3	307.52	135.78	120.65	396.69	135.65	187.83	269.25
1984:4	312.67	136.11	124.74	410.89	137.52	191.96	275.03
1985:1	321.32	139.39	126.59	424.41	139.39	201.60	281.71
1985:2	328.40	142.01	130.33	440.39	144.93	212.04	291.65
1985:3	340.40	145.42	133.07	458.98	147.51	207.77	301.09
1985:4	349.48	148.06	140.87	476.60	149.44	210.07	303.68
1986:1	358.01	153.65	133.48	494.06	149.06	202.54	307.17
1986:2	364.19	155.60	139.38	515.16	153.16	193.25	314.52
1986:3	372.41	155.98	142.56	536.56	155.73	199.65	320.56
1986:4	377.49	158.40	142.85	556.78	152.99	206.28	315.42

Table F.4.1: Consumption Price Index Data Used for Econometric Analysis

Date	$p_{1,t}$	$p_{2,t}$	$p_{3,t}$	$p_{4,t}$	$p_{5,t}$	$p_{6,t}$	$p_{7,t}$
1971:1	0.4120	0.4064	0.3830	0.3547	0.5301	0.4024	0.4117
1971:2	0.4153	0.4184	0.3894	0.3825	0.5349	0.4068	0.4157
1971:3	0.4166	0.4230	0.3922	0.3705	0.5389	0.4142	0.4236
1971:4	0.4245	0.4233	0.3997	0.3801	0.5492	0.4197	0.4367
1972:1	0.4236	0.4302	0.4060	0.3887	0.5482	0.4229	0.4414
1972:2	0.4308	0.4346	0.4104	0.3959	0.5527	0.4280	0.4517
1972:3	0.4382	0.4437	0.4145	0.4044	0.5540	0.4383	0.4563
1972:4	0.4444	0.4431	0.4190	0.4119	0.5610	0.4391	0.4595
1973:1	0.4563	0.4517	0.4280	0.4207	0.5674	0.4385	0.4623
1973:2	0.4800	0.4588	0.4435	0.4305	0.5742	0.4419	0.4718
1973:3	0.5020	0.4663	0.4600	0.4397	0.5868	0.4637	0.4846
1973:4	0.5232	0.4843	0.4689	0.4587	0.5979	0.4783	0.4974
1974:1	0.5463	0.4888	0.4880	0.4680	0.6112	0.4927	0.5075
1974:2	0.5634	0.5181	0.5180	0.4826	0.6294	0.5031	0.5296
1974:3	0.5753	0.5461	0.5483	0.5079	0.6716	0.5508	0.5649
1974:4	0.5803	0.5558	0.5759	0.5308	0.6961	0.5775	0.5926
1975:1	0.5859	0.5822	0.5972	0.5528	0.7144	0.5872	0.6131
1975:2	0.5993	0.5948	0.6158	0.5759	0.7273	0.6090	0.6379
1975:3	0.6125	0.6346	0.6372	0.6072	0.7397	0.6432	0.6631
1975:4	0.6290	0.7068	0.6674	0.6334	0.7630	0.6875	0.7044
1976:1	0.6548	0.7220	0.6908	0.6555	0.7731	0.7026	0.7332
1976:2	0.6645	0.7475	0.7224	0.6756	0.7892	0.7150	0.7546
1976:3	0.6849	0.7416	0.7426	0.7096	0.8019	0.7286	0.7596
1976:4	0.7124	0.7460	0.7742	0.7344	0.8174	0.7415	0.7750
1977:1	0.7276	0.7524	0.8026	0.7557	0.8373	0.7657	0.8002
1977:2	0.7457	0.7680	0.8250	0.7754	0.8575	0.7815	0.8198
1977:3	0.7754	0.7718	0.8489	0.8030	0.8649	0.7957	0.8299
1977:4	0.7918	0.7813	0.8628	0.8298	0.8806	0.8163	0.8445
1978:1	0.7980	0.7912	0.8859	0.8445	0.8900	0.8285	0.8632
1978:2	0.8229	0.7986	0.8998	0.8594	0.9041	0.8427	0.8797
1978:3	0.8465	0.8217	0.9203	0.8916	0.9160	0.8555	0.8880
1978:4	0.8726	0.9486	0.9260	0.9113	0.9278	0.8768	0.9019
1979:1	0.8928	0.9504	0.9465	0.9305	0.9340	0.9057	0.9171
1979:2	0.9289	0.9624	0.9630	0.9467	0.9455	0.9293	0.9293
1979:3	0.9587	0.9780	0.9802	0.9794	0.9599	0.9669	0.9526
1979:4	0.9765	0.9867	0.9873	0.9932	0.9895	0.9830	0.9873
1980:1	1.0182	1.0079	1.0062	1.0033	1.0228	1.0167	1.0194
1980:2	1.0481	1.0282	1.0254	1.0231	1.0308	1.0337	1.0434
1980:3	1.0641	1.0429	1.0465	1.0632	1.0625	1.0324	1.0748
1980:4	1.0953	1.0522	1.0633	1.0764	1.0723	1.0586	1.1057
1981:1	1.1184	1.0791	1.0825	1.0976	1.0803	1.1060	1.1302
1981:2	1.1386	1.0992	1.1033	1.1207	1.0976	1.1213	1.1623
1981:3	1.1693	1.1208	1.1198	1.1633	1.1095	1.1386	1.1870
1981:4	1.1890	1.1439	1.1383	1.2036	1.1424	1.1649	1.2283

Table B.4.1 (con't)

Date	$p_{1,t}$	$p_{2,t}$	$p_{3,t}$	$p_{4,t}$	$p_{5,t}$	$p_{6,t}$	$p_{7,t}$
1982:1	1.1999	1.1780	1.1549	1.2392	1.1587	1.1857	1.2704
1982:2	1.2231	1.2022	1.1688	1.2904	1.1852	1.2168	1.2966
1982:3	1.2546	1.2526	1.1899	1.3491	1.2063	1.2865	1.3318
1982:4	1.2891	1.3181	1.2041	1.3880	1.2345	1.2949	1.3848
1983:1	1.3133	1.3462	1.2212	1.4216	1.2452	1.3350	1.4186
1983:2	1.3831	1.3663	1.2558	1.4560	1.2708	1.3390	1.4408
1983:3	1.3846	1.3915	1.2701	1.4909	1.2880	1.3840	1.4538
1983:4	1.4033	1.4795	1.2844	1.5258	1.3065	1.4143	1.4997
1984:1	1.4285	1.5151	1.3048	1.5608	1.3128	1.4227	1.5216
1984:2	1.4265	1.5443	1.3292	1.6044	1.3188	1.4568	1.5372
1984:3	1.4528	1.5650	1.3443	1.6375	1.3268	1.4732	1.5556
1984:4	1.4720	1.5941	1.3603	1.6797	1.3338	1.4858	1.5608
1985:1	1.4859	1.6261	1.3795	1.7178	1.3378	1.5148	1.5957
1985:2	1.5174	1.6491	1.4052	1.7647	1.3519	1.5933	1.6289
1985:3	1.5332	1.6833	1.4337	1.8211	1.3863	1.6226	1.6617
1985:4	1.5628	1.7250	1.4729	1.8724	1.4235	1.6378	1.6851
1986:1	1.5962	1.7903	1.5021	1.9213	1.4565	1.6887	1.7121
1986:2	1.6424	1.8332	1.5538	1.9865	1.4908	1.6393	1.7360
1986:3	1.6805	1.8888	1.5835	2.0510	1.5318	1.7197	1.7689
1986:4	1.7036	1.9425	1.6256	2.1081	1.5760	1.8228	1.8612

Table F.4.2: Data for the General Level of Inflation and QINF Used for Econometric Analysis[a]

Date	Inflation	QINF (% per quarter)	Date	Inflation	QINF (% per quarter)
1971:1	0.0183	0.0000	1982:1	0.0244	0.0000
1971:2	0.0215	0.0000	1982:2	0.0242	0.0000
1971:3	0.0058	0.0000	1982:3	0.0329	0.0000
1971:4	0.0222	0.0000	1982:4	0.0301	0.0000
1972:1	0.0104	0.0000	1983:1	0.0222	0.0000
1972:2	0.0162	0.0000	1983:2	0.0230	0.0000
1972:3	0.0145	0.0000	1983:3	0.0140	0.0000
1972:4	0.0096	0.0000	1983:4	0.0257	0.0000
1973:1	0.0137	0.0000	1984:1	0.0161	0.0000
1973:2	0.0247	0.0000	1984:2	0.0138	0.0000
1973:3	0.0312	0.0000	1984:3	0.0141	0.0000
1973:4	0.0330	0.0000	1984:4	0.0134	0.0000
1974:1	0.0363	0.0000	1985:1	0.0160	0.0000
1974:2	0.0383	0.0000	1985:2	0.0238	0.0000
1974:3	0.0377	3.7910	1985:3	0.0216	0.0000
1974:4	0.0380	4.1534	1985:4	0.0184	0.0000
1975:1	0.0299	4.2253	1986:1	0.0234	0.0000
1975:2	0.0333	4.3997	1986:2	0.0182	0.0000
1975:3	0.0404	4.4871	1986:3	0.0268	0.0000
1975:4	0.0344	5.0836	1986:4	0.0258	0.0000
1976:1	0.0336	4.9038			
1976:2	0.0269	4.5845			
1976:3	0.0197	4.0327			
1976:4	0.0259	3.5512			
1977:1	0.0279	3.1392			
1977:2	0.0244	2.7698			
1977:3	0.0215	2.4248			
1977:4	0.0212	2.1089			
1978:1	0.0171	1.6321			
1978:2	0.0194	0.0000			
1978:3	0.0205	0.0000			
1978:4	0.0293	0.0000			
1979:1	0.0198	0.0000			
1979:2	0.0188	0.0000			
1979:3	0.0275	0.0000			
1979:4	0.0228	0.0000			
1980:1	0.0289	0.0000			
1980:2	0.0214	0.0000			
1980:3	0.0238	0.0000			
1980:4	0.0219	0.0000			
1981:1	0.0228	0.0000			
1981:2	0.0216	0.0000			
1981:3	0.0231	0.0000			
1981:4	0.0277	0.0000			

(a) Non-zero values of QINF were derived from the data for consumer price inflation using equation (13), with $\tau_{inf} = 16$.

Table B.5: Data for the Aggregate Contribution to Household Net Savings from Sources Other Than Capital Gains and Cash Income from Non-Human Wealth ($ per person) Used for Econometric Analysis[a]

Date	y_t	Date	y_t
1971:2	457.52	1982:1	1952.51
1971:3	353.39	1982:2	2270.68
1971:4	331.76	1982:3	1937.85
1972:1	287.80	1982:4	1962.95
1972:2	434.41	1983:1	1159.62
1972:3	325.35	1983:2	1394.24
1972:4	280.45	1983:3	1987.53
1973:1	273.15	1983:4	1453.86
1973:2	559.45	1984:1	1683.80
1973:3	202.29	1984:2	1413.25
1973:4	391.69	1984:3	1702.66
1974:1	545.00	1984:4	1570.74
1974:2	740.99	1985:1	1891.80
1974:3	976.70	1985:2	1899.03
1974:4	782.67	1985:3	2304.70
1975:1	780.66	1985:4	2223.90
1975:2	757.05	1986:1	2121.03
1975:3	707.56	1986:2	2300.47
1975:4	849.74	1986:3	2035.15
1976:1	496.83	1986:4	2035.73
1976:2	882.23		
1976:3	1069.78		
1976:4	759.22		
1977:1	815.17		
1977:2	827.60		
1977:3	988.63		
1977:4	867.22		
1978:1	774.83		
1978:2	1109.17		
1978:3	931.62		
1978:4	729.05		
1979:1	914.28		
1979:2	1072.70		
1979:3	505.66		
1979:4	625.09		
1980:1	1472.76		
1980:2	947.44		
1980:3	296.49		
1980:4	1938.57		
1981:1	1290.05		
1981:2	1456.43		
1981:3	1258.26		
1981:4	1333.82		

(a) Data in this table were computed from the raw data by dividing through by the population estimates in Table B.6.

Table F.6: Population Data Used for Econometric Analysis

Date	Population (thousands)	Date	Population (thousands)
1969:4	12664	1982:2	15178
1970:1	12735	1982:3	15232
1970:2	12802	1982:4	15277
1970:3	12861	1983:1	15333
1970:4	12935	1983:2	15379
1971:1	13008	1983:3	15417
1971:2	13067	1983:4	15452
1971:3	13131	1984:1	15508
1971:4	13198	1984:2	15556
1972:1	13254	1984:3	15599
1972:2	13304	1984:4	15649
1972:3	13354	1985:1	15704
1972:4	13409	1985:2	15752
1973:1	13459	1985:3	15796
1973:2	13505	1985:4	15852
1973:3	13553	1986:1	15913
1973:4	13614	1986:2	15964
1974:1	13670	1986:3	16028
1974:2	13723	1986:4	16080
1974:3	13772		
1974:4	13832		
1975:1	13863		
1975:2	13893		
1975:3	13927		
1975:4	13969		
1976:1	14005		
1976:2	14033		
1976:3	14066		
1976:4	14110		
1977:1	14155		
1977:2	14192		
1977:3	14231		
1977:4	14281		
1978:1	14330		
1978:2	14359		
1978:3	14397		
1978:4	14431		
1979:1	14478		
1979:2	14515		
1979:3	14554		
1979:4	14602		
1980:1	14646		
1980:2	14695		
1980:3	14746		
1980:4	14807		
1981:1	14874		
1981:2	14927		
1981:3	14989		
1981:4	15054		

ENDNOTES

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1. In this paper, as in most empirical microeconomic studies of consumption choice, we deal with the behaviour of one 'representative household' on which is imposed the demand-theoretic constraints associated with the behaviour of an individual unit. Arguments in favour of this approach can be found in Barnett (1979). The reader should note that households, as a group in the economy, include all resident persons and their unincorporated enterprises; and all domestic non-profit organisations serving these persons and enterprises other than those financial organisations, such as Friendly and Building Societies, which form part of the financial enterprises sector.

2. An asset is 'distinct' if it cannot be written as a linear combination of other assets in the portfolio.

3. Optimal selling strategies for risky depreciable assets with proportional taxes are examined in Williams (1985).

4. For some early examples of the 'stock adjustment approach' to portfolio modelling, see Feige (1967), Brainard and Tobin (1968), and Sharpe (1973).

5. In a recent paper, Bollerslev, Engle and Wooldridge (1988), a porfolio model with time-varying variances and co-variances is presented. This model can be viewed as a generalisation of earlier work in this area by Friedman (1985). Of course, values for the elements of Ω^{-1} could also be obtained by inverting an estimate of Ω derived extraneously be extrapolating from past changes in observed rates of return. However, with this approach there are serious problems relating to aggregation across households and across assets. Indeed, in view of the limited number of different assets held by most households, and widespread financial intermediation not modelled here, the notion of a 'representative

asset holder' presents formidable difficulties in the the context of these aggregate data.

6. Powell's hypothesis was proposed in the context of measuring permanent income for a scalar consumption function in the United States.

7. Further details of the lag structure are given in Powell (1974, p. 133).

8. For a detailed discussion on this point, see Judge, Griffiths, Hill and Lee (1980, part 6).

9. A generalised form of the ELES to cover the case of durables is described in Dixon and Lluch (1977). In that model, purchases of durables depend both on changes in stocks as well as on levels of expectations.

10. One method of imposing positive definiteness on Ω^{-1} during estimation is to make use of the 'Cholesky factorisation' of the matrix. Lau (1978) has shown that every positive definite matrix has a Cholesky factorisation with Cholesky values that are strictly greater than zero. Therefore, to impose the condition that Ω^{-1} be positive definite, we need only to constrain the Cholesky values to be > 0.

11. Uncontrolled interest rates and rates of return on unincorporated business fixed assets followed a similar pattern. However, the rate of return on equity generally declined during 1975, before turning around and gradually increasing over the remainder of the period.

12. Values of the Wallis (1972) test for each one of the consumption module equations were:
    equation 1 test = 1.69,
    equation 2 test = 0.57,
    equation 3 test = 1.58,
    equation 4 test = 0.69,
    equation 5 test = 1.58,
    equation 6 test = 1.57,

equation 7 test = 1.69.

13. The value of the coefficient on HDI was 0.91. A similar regression, this time with HDI as the dependent variable, was carried out because of doubts about the nature of the causality between HDI and safe income. This yielded the similar coefficient value (after reciprocation) of 0.89. The value adopted in the calculation in the text was 0.90.

14. The Frisch conjecture, which seems to work well in the context of fitted systems based on the Klein-Rubin utility function, does not seem to be supported by some other approaches; see, for example, Theil and Clements (1987, ch. 2).

15. Significant first-order serial correlation is a problem encountered in the estimation of many portfolio models; see, for example, Clements (1976), Berndt, McCurdy and Rose (1980) and Upcher and McLaren (1986). Values of the Wallis test for each of the investment module equations were:
equation 1 test = 1.42,
equation 2 test = 1.06,
equation 3 test = 1.38,
equation 4 test = 1.37.

16. Values of the Wallis test for each of the equations in the revised consumption module were:
equation 1 test = 1.64,
equation 2 test = 1.49,
equation 3 test = 1.45,
equation 4 test = 1.52,
equation 5 test = 1.41,
equation 6 test = 1.37,
equation 7 test = 1.69.

17. The Wallis test, however, now rejects the existence of fourth-order serial correlation in all four equations. Values for the test were:
equation 1 test = 1.55,
equation 2 test = 1.31,
equation 3 test = 1.45,
equation 4 test = 1.42.

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