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# A FORWARD LOOKING APPROACH TO PORTFOLIO ANALYSIS USING A COMPUTABLE GENERAL EQUILIBRIUM MODEL

by

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Preliminary Working Paper No. IP-37 Melbourne June 1988



ISSN 0813 7986 ISBN 0 642 10272 4

## ABSTRACT

The historical approach to portfolio analysis is to use past data to estimate the expected rates of return of financial assets and the correlations between the assets (i.e. the variance-covariance matrix). A potential problem with this approach, however, is that even if the returns on two assets have been uncorrelated in the past, they may nevertheless be correlated in the future. This could be the case if the returns on the two assets were closely linked with, for example, protection policy. This relationship would not be revealed by historical time-series data if there had been no significant change in protection policy. Thus, even if returns on these assets have not been closely correlated in the past, we might nevertheless conclude that a portfolio containing both assets is not well diversified if we expect a change in protection policy. In this paper a forward looking approach to portfolio analysis is developed. The first step involves specifying future economic scenarios in terms of the exogenous variables of a computable general equilibrium (hereafter CGE) model. These models represent a rapidly emerging field in applied economic analysis. The CGE model is then solved for the effects of the economic scenarios on industry rates of return. These projections are then mapped from industries to corporations according to their base-period holdings across industries. Finally, the expected return and risk is projected for any given portfolio of corporate stocks.

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# A FORWARD LOOKING APPROACH TO PORTFOLIO ANALYSIS USING A COMPUTABLE GENERAL EQUILIBRIUM MODEL*

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## 1. INTRODUCTION

The historical approach to portfolio analysis is to use a time-series analysis of past trends to estimate the expected rates of return of financial assets and the correlations between the assets (i.e. the variance-covariance matrix). A potential problem with this approach, however, is that even if the return on two assets have been uncorrelated in the past, they may nevertheless be correlated in the future. For example, this could be the case if the returns on the two assets were closely linked with protection policy. This relationship would not be revealed by historical time-series data if there had been no significant change in protection policy. Thus, even if returns on these assets have not been closely correlated in the past, a portfolio in which both assets loom large is not well diversified if we have reason to expect a major change in protection.

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* This research was supported in part by the Superannuation Fund Investment Trust, Canberra. The author is particularly indebted to Alan Powell for extensive editoral feedback and comments. Thanks are also due to Philip Adams, George Codsi, Peter Dixon, Richard Douglas, Wolfgang Ernst, Mark Horridge, Mike Kenderes, Bob Officer, Ken Pearson, and Norm Sinclair for comments and assistance. Peter Dixon should in addition be acknowledged for some unpublished notes on the topic of this paper that were very helpful. Thanks are also due to the participants of seminars at Griffith University, the Reserve Bank of Australia, the University of British Columbia, and the University of Melbourne.

In this paper a forward looking approach to portfolio analysis is developed. This approach combines a multi-index model of asset returns with factors derived from a computable general equilibrium (hereafter CGE) model. CGE models represent a rapidly emerging field in applied economic analysis.[1] The first step of this approach is to specify future economic scenarios in terms of the exogenous variables of a CGE model. The CGE model is then solved for the effects of the economic scenarios on industry rates of return. These projections are then mapped from industries to corporations according to their base-period holdings across industries. Finally, the expected return and risk are projected for any given portfolio of corporate stocks.

The remainder of the paper is organized as follows. Section 2 contains a review of the background literature. The theoretical foundations are presented in section 3. A brief description of a CGE model of the Australian economy is given in section 4. Note that as this paper is largely concerned with developing and illustrating new techniques, the CGE model is highly stylized. This allows attention to be focused on mechanisms which are key to the purposes at hand without the distraction of excessive dimensionality. Also contained in section 4 is a discussion of some elasticities computed from the CGE model. In section 5 the rate of return on capital is defined. In section 6 it is shown how the CGE model can be used for forecasting rates of return by industries. This first requires scenarios to be specified, from outside of the CGE model, about future developments in the variables exogenous to it. These scenarios together with the appropriate elasticities are then used to produce forecasts of some key macroeconomic and industry variables. A mapping between the CGE model's industries and some illustrative corporations is described in section 7. The forecast rates of return by corporations and associated variance-covariance matrix are calculated in section 8. In section 9 some portfolios consisting of the corporations are defined. Then in section 10 the expected rates of return by portfolio and associated variances are calculated. In section 11 the frontier of efficient portfolios is derived. Some concluding remarks are offered in section 12. Finally, some technical notes are contained in an appendix.

## 2. LITERATURE REVIEW

Modern portfolio theory (hereafter MPT), pioneered by Markowitz (1952), was the first framework explicitly to capture risk in a portfolio sense. MPT defines risk as the variability of portfolio returns rather than those of individual assets. It is assumed that an investor can calculate the risk and return of portfolios consisting of all possible combinations of investment alternatives. Given this information the rational investor will prefer portfolios that provide the highest expected return for a given level of risk or those that offer the least risk for a given expected return. Investors will then choose from this set of efficient portfolios according to their appetites for risk. A risk-averse investor may prefer a portfolio with lower variance and commensurately lower returns. The problem with MPT, however, is that an investor must forecast the expected return of every asset, the variance of every asset's expected returns, and the correlation between the expected returns for every pair of assets (i.e., the variance-covariance matrix). The complexity of this task has largely prevented MPT's extensive use by practitioners.

The capital asset pricing model (hereafter CAPM), developed by Sharpe (1964), Lintner (1965), and Mossin (1966), represents an extension of MPT. The CAPM dramatically reduced the complexity of the forecasting task by assuming that stock prices move relative to a market-value-weighted portfolio of all possible risky investments. The investor forecasts the relationship (which is assumed to be linear) between the return on each of the assets and the market portfolio. The investor must also forecast the variance of the market portfolio and of each of the assets. However, by assuming that the error terms of the estimated linear relationships are uncorrelated, the covariances among individual assets can be deduced from the estimated coefficients and the variance of the market portfolio.[2]

The CAPM also assumes the existence of a risk-free asset. The addition of a risk-free asset significantly changes the set of investment opportunities. In fact the rational investor will only purchase a combination of the market portfolio and the risk-free security (see Tobin (1965) and Merton (1971), section 5). The relative portions of each purchased will again depend on the investor's tolerance for risk. Note that to forecast the composition of the market portfolio requires estimates of each security's risk. The CAPM's risk index is referred to as beta. For any security, beta is calculated as the covariance of the security's returns with the market's return divided by the variance of the market's returns. Thus the beta of the market is one. Securities with less risk than the market have a beta less than one, more risky assets have betas in excess of one. The beta for a portfolio is the weighted average of the betas for each asset contained in the portfolio.

There exists a large literature which examines the CAPM from both theoretical and empirical perspectives. All models are to some extent an abstraction from reality. However, many of the assumptions of the CAPM have come under critical review. For example, in CAPM the investor is assumed to maximize the utility of wealth; however, wealth can be defined in many ways (e.g., pre-tax versus post-tax or dividends versus capital gains); -- see Miller and Scholes (1982) and Vandell and Stevens (1982). Beta as a measure of risk has also provoked controversy; see Friend and Blume (1970), Cooley, Roenfeldt, and Modani

(1977) and Arnott (1983). The CAPM assumes homogenous expectations, but in reality each investor has their own expectations; see the state preference model of Hirshleifer (1965). In the standard treatment all investors have a common single-period planning horizon, and make transactions at the same instant. (See however, Merton (1973) for an intertemporal, continuous-time, CAPM.) The existence of a risk-free asset and the assumption that investors can borrow or lend unlimited amounts at the risk-free rate is also questioned; see Black (1972), Lintner (1975) and Lewellen and Ang (1982). The linear risk-return relationship has been questioned in empirical studies; see Fama and MacBeth (1973). Finally, Roll (1977) has even questioned the validity of the empirical tests of CAPM. The CAPM is based on expectations; as Roll points out, however, the empirical tests have been tests of what actually occurred rather than of expectations.

The criticism of the CAPM that is of most interest here is that it omits factors which are important in determining rates of return. In particular, the CAPM makes no distinction among industry groups. However, many authors have found that industry-related factors do effect rates of return by corporations. For example, industries are observed to have different betas. Harrington (1987) reports betas for 44 industry groups that range from a high of 1.69 for the brokerage industry to a low of 0.65 for electric utilities. Harrington also observed that the betas of companies within most industries are quite similiar. For example, the range for companies in the electric utilities industry was 0.83 to 0.43.

The first study to show the importance of industry factors was King (1966). King studied the returns between 1927 and 1960 of a sample of 63 firms on the New York stock Exchange chosen from six industry groups. These industries were tobacco products, petroleum products, metals, railroads, utilities, and retail stores. The 63 stocks were first paired in all possible ways. The pair with the highest correlation was made into a composite stock. Then the remaining 62 "stocks" were again paired in all possible ways, etc. On each round a stock combined with another stock or with a composite stock, or two composite stocks combined. It was found that the composite stocks so identified tended to be defined along industry lines. King found that after allowing for the general effect of the market, industry membership appeared to be the most important single influence in determining stock price.

Another study to use factor analysis to study the influence of market and industry factors on stock price behaviour was Meyers (1973). Meyers' objective was to demonstrate that King (1966) had overstated the role of industry factors due partly to the empirical methods and partly to the sample King used. However, Livingston (1977) has identified problems with both King's and Meyers' papers. Specifically, Livingston shows how variants of factor analysis performed on the same body of data can lead to different results. Furthermore, market factors extracted by factor analysis vary considerably with the sample of data used. Livingston prefers the estimation of residual covariances by regression against a broad market index. Using this approach Livingston concludes that industry comovement of securities is of considerable importance. The results support the view that "industry movement of securities must be accounted for in diversifying one's portfolio" (Livingston (1977, p. 873)).

The multi-index portfolio selection models also estimate residual covariances by regression against a broad market index. In general these models have found large residual effects which frequently follow industry lines. The first of these studies was by Cohen and Pogue (1967). They specified a multi-index MPT model using industry indexes. Subsequent studies specifying industry indexes include Campanella (1972), Aber (1973), Elton and Gruber (1973), and Fertuck (1975). However, a basic problem with these studies was that some industry indexes turn out to be highly collinear.

The next step was to attempt to capture more explicitly the fundamental factors that were generating the industry effects. This approach was foreshadowed by Treynor (1972) who observed that stocks are primarily vehicles for participating in factors and argued that the principal task of the security analyst is to describe how each stock is likely to be affected by these factors. Farrell (1974 and 1975) specified a multi-index MPT model using broader-than-industry indexes. Farrell wanted to define indexes of stocks that were significantly correlated within their own grouping but largely independent of other groups. Using cluster analysis Farrell found that significant independent groupings could be found in terms of what he called growth, cyclical, stable and oil stocks. Farrell then showed that a portfolio would not be well diversified if it was concentrated in only one of these groupings. Furthermore, Farrell found that a multi-index MPT model using the above groupings did outperform a single-index MPT model. This result was also found in a follow-up study to Farrell by Martin and Klemkosky (1976).

Arnott (1980) employed a method similar to Farrell's except that he used a larger sample, eliminated data lying beyond three standard deviations from the mean, and only created clusters up to the point where he saw unusual groups joining the cluster (Farrell had combined groups until no positive correlations remained). Arnott identified five groups which he labelled quality growth, utilities, oil and related, basic industries, and consumer cyclical. Arnott conjectured that these groups embodied important fundamental characteristics. For example Arnott (1980, p.59) claimed that "the utility factor is dominated by interest-rate sensitivity, while the cyclical factor is strongly influenced by the economic outlook." The key result of Arnott's study was that these groups represented a 30 per cent improvement in explanatory power over the single-index model.

The effect of a range of factors on asset prices was also studied by Sharpe (1982). Sharpe specified 16 factors of which eight represented industry sectors. He found that many of these factors were significant during the period 1931-79. However, many of the factors used in Sharpe's study may be proxies for other more fundamental factors.

The arbitrage pricing theory (hereafter APT), developed by Ross (1976), is in some ways an extension of the multifactor models. APT relates the expected return of an asset to the return of the risk-free asset and a series of other common factors. If investors know the sensitivities of assets' expected returns with respect to a factor, then according to APT they will trade until arbitrage profits are eliminated.

The APT does not identify what the common factors are. However, it is possible to use a purely statistical means, similar to factor analysis, to estimate the number of factors that affect the market. Roll and Ross (1980) found that there were perhaps as many as four factors important in pricing assets. Similar results have also been obtained by Reinganum (1981), Fogler (1982), and Brown and Weinstein (1983). It should be noted that the Roll and Ross (1980) study has been criticized by Dhrymes (1984) who claims that the number of factors observed is sample size dependent. For example, there may exist factors that are industry-specific that will only be identified when the sample includes stocks from all industries.

Chen, Roll, and Ross (1983) have attempted to identify the common factors by first estimating their number and then testing for correlation between them and some macroeconomic variables, such as inflation, oil prices, and industrial production. Note that only the unanticipated changes in these macroeconomic variables are considered to influence the pricing of assets. This study found that four factors were important in determining returns: industrial production; changes in the risk premium; changes in the term structure of interest rates; and unanticipated changes in inflation.

Estep, Hanson, and Johnson (1983) have shown how the relative performance of portfolios constructed so as to be sensitive to particular factors, such as those identified by Chen, Roll, and Ross (1983), do indeed vary quite considerably. Estep, Hanson, and Johnson also examine the performance of portfolios sensitive to additional factors; namely, defence spending and real oil prices.

Finally, the Salomon Brothers' STOCKFACTS system, which was developed by Data Resources Incorporated, may use techniques which are sympathetic to those developed in this paper. The STOCKFACTS system apparently makes use of an input-output table to estimate the historic factor sensitivity of stocks to a number of macroeconomic variables.

The approach developed in this paper can be related to the above literature in the following ways. It is similar to the APT approach of Chen, Roll, and Ross (1983) in that asset returns are assumed to be influenced by developments in some key macroeconomic variables. However, rather than statistically test for these factors and then attempt to identify them via a correlation exercise, here we use an economic model to trace explicitly the effects of forecast changes in macroeconomic variables on asset returns. The approach taken here is also related to the literature initiated by King (1966) on industry factors, in that the economic model first traces the effects to the industry level. The effects on corporations of changes in the macroeconomy are then estimated according to their holdings across industries.

# 3. THEORETICAL FOUNDATIONS

In this section we will develop the theoretical foundations of a multi-index model of asset returns with factors derived from a CGE economic model. Next a synthesis is presented of the multi-index model, the CAPM, and the APT. Finally, the statistical properties of the return generating process are specified.

## 3.1 A Multi-Index Model of Asset Returns with Factors Derived from a CGE Economic Model

In this section we will develop the theoretical foundations of a multi-index model of asset returns with factors derived from a CGE economic model. To avoid any potential confusion with respect to timing, the notation will include time superscripts. We will start with the assumption that returns on the $i^{th}$ asset are generated by a multi-index model of the form:

$$R_i^t = a_i + b_{i1}F_1^t + \ldots + b_{ik}F_k^t + e_i^t \qquad i=1,\ldots,m; \qquad (3.1)$$

where $R_i{}^t$ is the return on asset i in period t[3]; $a_i$ is a constant which captures the nonfactor-related aspects of the return on asset i; $b_{iv}$ is the sensitivity of asset i's returns (as estimated from the CGE model) to movements in the common factor $F_v$; $F_v{}^t$ is the value of the common factor at time t; and $e_i{}^t$ is the realized error at time t.

Next we note that the rate of return of asset i, the value of factor v, and the error term associated with asset i in period t+1 can be written:

$$R_i^{t+1} = R_i^t + \Delta R_i^t \qquad i=1,\ldots,m; \qquad (3.2)$$

$$F_v^{t+1} = F_v^t + \Delta F_v^t \qquad v=1,\ldots,k; \qquad (3.3)$$

and

$$e_i^{t+1} = e_i^t + \Delta e_i^t \qquad i=1,\ldots,m. \qquad (3.4)$$

Furthermore, we can calculate the change in the rate of return on asset i over the period t to t+1, $\Delta R_i{}^t$, from equation (3.1):

$$\Delta R_i^t = b_{i1}\Delta F_1^t + \ldots + b_{ik}\Delta F_k^t + \Delta e_i^t \qquad i=1,\ldots,m. \qquad (3.5)$$

Now substitute equation (3.5) into equation (3.2):

$$R_i^{t+1} = R_i^t + b_{i1}\Delta F_1^t + \ldots + b_{ik}\Delta F_k^t + \Delta e_i^t \qquad i=1,\ldots,m. \qquad (3.6)$$

Finally, substitute equations (3.3) and (3.4) into equation (3.6):

$$R_i^{t+1} = R_i^t + b_{i1}(F_1^{t+1} - F_1^t) + \ldots$$

$$+ b_{ik}(F_k^{t+1} - F_k^t) + (e_i^{t+1} - e_i^t) \qquad i=1,\ldots,m. \qquad (3.7)$$

Equation (3.7) can also be written:

$$R_i^{t+1} = R_i^t + b_{i1}(F_1^{t+1} - F_1^t) + \ldots$$

$$+ b_{ik}(F_k^{t+1} - F_k^t) + u_i^{t+1} \qquad i=1,\ldots,m; \qquad (3.8)$$

where

$$u_i^{t+1} = e_i^{t+1} - e_i^t \qquad i=1,\ldots,m. \qquad (3.9)$$

Equation (3.8) is the central equation of our approach as it describes how the rates of return on assets evolve over time.

The expected return for asset i can be obtained by taking the expectation of equation (3.8):

$$E^t(R_i^{t+1}) = R_i^t + b_{i1}(E^t(F_1^{t+1}) - F_1^t) + \dots$$

$$+ b_{ik}(E^t(F_k^{t+1}) - F_k^t) \qquad i=1,\dots,m; \quad (3.10)$$

where $E^t(R_i{}^{t+1})$ is the expectation at time t of the rate of return on asset i in period t+1; and $E^t(F_v{}^{t+1})$ is the expectation at time t of the value of the common factor $F_v$ at time t+1. Note that it is assumed that $E^t(u_i{}^{t+1})$ is zero; see section 3.5. Equation (3.10) can also be written:

$$R_i^t = E^t(R_i^{t+1}) - b_{i1}(E^t(F_1^{t+1}) - F_1^t) - \dots$$

$$- b_{ik}(E^t(F_k^{t+1}) - F_k^t) \qquad i=1,\dots,m. \quad (3.11)$$

Substitute equation (3.11) for $R_i{}^t$ in equation (3.8):

$$R_i^{t+1} = E^t(R_i^{t+1}) + b_{i1}(F_1^{t+1} - E^t(F_1^{t+1})) + \dots$$

$$+ b_{ik}(F_k^{t+1} - E^t(F_k^{t+1})) + u_i^{t+1} \qquad i=1,\dots,m. \quad (3.12)$$

Equation (3.12) says that investors have an ex ante expected return for asset i, but that unexpected changes in the factors will lead to results that are different from what investors had expected.

Why Use a CGE Model?

The above multi-index model of asset returns could be adopted for use with economic models other than those of the CGE class. However, CGE models have the following advantages. A CGE model goes beyond the historical correlations among aggregate variables which form the basis of many traditional macro-econometric models, thus enabling it to examine the effects of structural changes. This facility is especially valuable when dealing with unprecedented changes in the relations among macro-variables (e.g., the oil shocks of the 1970s). Furthermore, a CGE model is transparent in the sense that all mechanisms responsible for results are explicit. This enables users to both check for errors in the model and to obtain insights that would otherwise have been difficult to deduce.

## 3.2 Capital Asset Pricing Model

The development of the CAPM has been attributed to Sharpe (1964), Lintner (1965), and Mossin (1966). Here we follow Sharpe's derivation of the CAPM. The CAPM assumes that investors only care about portfolio risk and expected return:

$$U = g(E^t(R_p^{t+1}), Sd^t(R_p^{t+1})) \quad ; \qquad (3.13)$$

where U is the level of utility; $E^t(R_p{}^{t+1})$ is the expectation at time t of the return on portfolio p in period t+1; and $Sd^t(R_p{}^{t+1})$ is the standard deviation at time t of the return on portfolio p in period t+1. It is assumed that investors agree on the prospects of various assets (including their expected values, standard deviations, and correlation coefficients). Furthermore, investors are assumed to prefer an efficient portfolio to an inefficient one. A portfolio is said to be efficient if (and only if) there is no alternative portfolio with either (1) the same $E^t(R_p{}^{t+1})$ and a lower $Sd^t(R_p{}^{t+1})$, (2) the same $Sd^t(R_p{}^{t+1})$ and a higher $E^t(R_p{}^{t+1})$, or (3) a higher $E^t(R_p{}^{t+1})$ and a lower $Sd^t(R_p{}^{t+1})$. A frontier of efficient portfolios constructed from the set of available risky assets is depicted in Figure 3.1 by the curved line AA.

The CAPM then assumes the existence of a risk-free asset (i.e., an asset with zero standard deviation and zero covariance with all other assets) with a rate of return in period t+1 of $R_F{}^{t+1}$. It is also assumed that all investors are able to borrow or lend funds at rate $R_F{}^{t+1}$ on equal terms. These two assumptions allow the creation of a new efficient frontier of portfolios, depicted by the line $R_F{}^{t+1}Z$ in Figure 3.1. (Note that the line $R_F{}^{t+1}Z$ is sometimes called the capital market line.) Point M on the line $R_F{}^{t+1}Z$ corresponds to the efficient portfolio consisting of just risky assets. Consider for the moment that the only risky assets are equities and the risk-free asset is bonds. Given this, then the line $R_F{}^{t+1}Z$ sets out all the alternative combinations of a pure equity portfolio M with bonds (mixed with bonds up to the point M and levered by loans beyond point M). The combination of a pure equity portfolio M with bonds that an investor chooses will depend on their particular taste with respect to risk. Since $R_F{}^{t+1}Z$ defines the set of efficient portfolios, the mutual funds theroem (Tobin (1965), Merton (1971)) is established: any portfolio derived by a rational investor can be obtained as a linear combination of the risk-free asset and the market portfolio M of risky assets.

The central result of the CAPM is an equilibrium condition expressing the expected return of an asset as a function of the

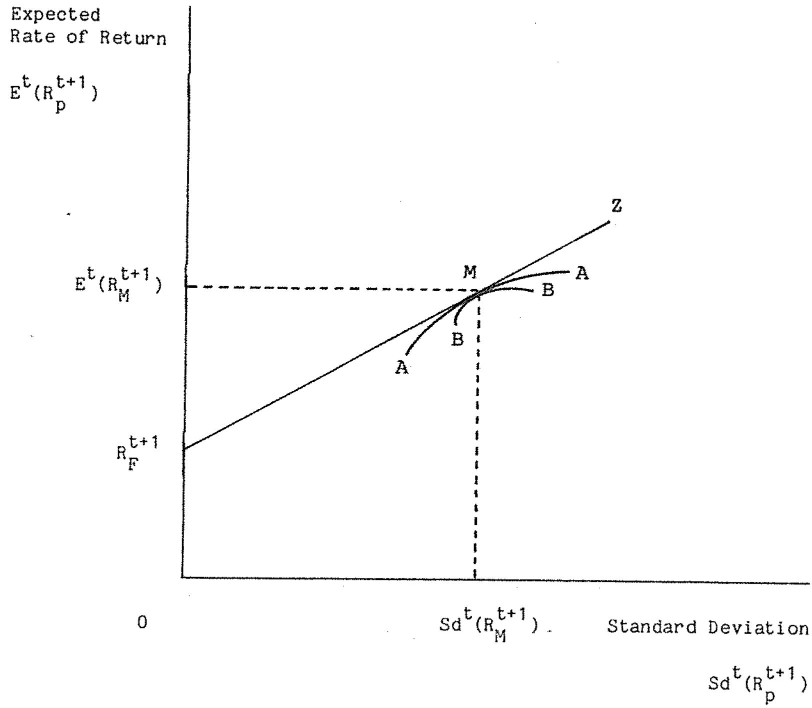


FIGURE 3.1: DERIVATION OF THE CAPITAL ASSET PRICING MODEL

After Levy and Sarnat (1984, p. 409).

risk-free rate of return, the expected return of the market portfolio, and the covariability of the asset's returns with the market's returns. This condition is derived as follows. (Note that the subsequent exposition draws on Levy and Sarnat's (1984) treatment of Sharpe's derivation.) Suppose there existed a portfolio N (different from M) which is a mix of some risky asset i and the optimal market portfolio M. The expected rate of return of portfolio N in period t+1, $E^t(R_N{}^{t+1})$, given by:

$$E^t(R_N^{t+1}) = S_i^{t+1}\ E^t(R_i^{t+1}) + (1 - S_i^{t+1})\ E^t(R_M^{t+1}) \quad ; \qquad (3.14)$$

where $S_i{}^{t+1}$ is the proportion of portfolio N invested in asset i in period t+1; $E^t(R_i{}^{t+1})$ is the expected rate of return on asset i in period t+1; and $E^t(R_M{}^{t+1})$ is the expected rate of return of the market portfolio in period t+1. By changing the proportion $S_i{}^{t+1}$, we trace the curve BB which describes all portfolios consisting of a mix of these two assets; see Figure 3.1.

The key point in the subsequent derivation is that at point M the curves AA and BB have the same slope as the line $R_F{}^{t+1}Z$. This is due to the fact that the curve BB must touch the point M (i.e., at M the investor holds 100 per cent M and a zero per cent (separate) holding of asset i), but BB cannot cross AA. If BB did cross AA, then the curve AA would not be the efficient frontier of <u>all</u> risky assets as was previously assumed.

The slope of the line $R_F{}^{t+1}Z$ is given by:

$$\text{Slope at point M} = (E^t(R_M^{t+1}) - R_F^{t+1})/Sd^t(R_M^{t+1}) \quad . \qquad (3.15)$$

The expected rate of return and the variance of portfolio N are given by:

$$E^t(R_N^{t+1}) = S_i^{t+1} E^t(R_i^{t+1}) + (1 - S_i^{t+1})\ E^t(R_M^{t+1}) \quad ; \qquad (3.16)$$

and

$$Var^t(R_N^{t+1}) = [S_i^{t+1}]^2\ Var^t(R_i^{t+1}) + [1 - S_i^{t+1}]^2\ Var^t(R_M^{t+1})$$

$$+ 2S_i^{t+1}(1 - S_i^{t+1})\ Cov^t(R_i^{t+1}, R_M^{t+1}) \quad . \qquad (3.17)$$

Next we calculate the following derivatives:

$$\partial E^t(R_N^{t+1})/\partial S_i^{t+1} = E^t(R_i^{t+1}) - E^t(R_M^{t+1}) \quad ; \qquad (3.18)$$

and

$$\partial Sd^t(R_N^{t+1})/\partial S_i^{t+1} = [2S_i^{t+1}\ Var^t(R_i^{t+1}) - 2(1 - S_i^{t+1})\ Var^t(R_M^{t+1}) +$$

$$2(1 - 2S_i^{t+1})\ Cov^t(R_i^{t+1}, R_M^{t+1})]/[2Sd^t(R_N^{t+1})] \quad . \qquad (3.19)$$

At point M we know that $S_i{}^{t+1}$ equals zero and that $Sd^t(R_M{}^{t+1})$ equals $Sd^t(R_M{}^{t+1})$. Thus, if the above derivative is evaluated at point M we find:

$$\partial Sd^t(R_N^{t+1})/\partial S_i^{t+1} = [Cov^t(R_i^{t+1}, R_M^{t+1}) - Var^t(R_M^{t+1})]/Sd^t(R_M^{t+1}) \quad . \qquad (3.20)$$

Next we note that by the chain rule of differentiation the following holds:

$$\partial E^t(R_N^{t+1})/\partial Sd^t(R_N^{t+1}) = [\partial E^t(R_N^{t+1})/\partial S_i^{t+1}]/[\partial Sd^t(R_N^{t+1})/\partial S_i^{t+1}] \quad . \qquad (3.21)$$

The left-hand side of equation (3.21) is in fact the slope of the curve BB at point M. Thus, using equations (3.15), (3.18), and (3.20) we can rewrite equation (3.21):

$$(E^t(R_M^{t+1}) - R_F^{t+1})/Sd^t(R_M^{t+1}) = [E^t(R_i^{t+1}) - E^t(R_M^{t+1})]\ Sd^t(R_M^{t+1})/$$

$$[Cov^t(R_i^{t+1}, R_M^{t+1}) - Var^t(R_M^{t+1})] \quad . \qquad (3.22)$$

Clearing fractions in equation (3.22), we obtain the CAPM risk-return relationship:

$$E^t(R_i^{t+1}) = R_F^{t+1} + (E^t(R_M^{t+1}) - R_F^{t+1})\beta_i \quad ; \qquad (3.23)$$

where

$$\beta_i = Cov^t(R_i^{t+1}, R_M^{t+1})/Var^t(R_M^{t+1}) \quad . \qquad (3.24)$$

The $\beta_i$ measures the risk of asset i in the portfolio context. Note that since the same derivation is true for each asset i, the risk-return relationship (3.23) holds for all risky assets.

## 3.3 Arbitrage Pricing Theory

Following Roll and Ross (1980) and Fogler (1982), the derivation of the APT begins with the assumption that investors believe that the returns on the $i^{th}$ asset are generated by a multi-index model of the form (cf. equation (3.12)):

$$R_i^{t+1} = E^t(R_i^{t+1}) + b_{i1}(F_1^{t+1} - E^t(F_1^{t+1})) + \ldots$$

$$+ b_{ik}(F_k^{t+1} - E^t(F_k^{t+1})) + u_i^{t+1} \qquad i=1,\ldots,m; \qquad (3.25)$$

where $R_i^{t+1}$ is the actual return on asset i in period t+1; $E^t(R_i^{t+1})$ is the expectation at time t of the rate of return on asset i in period t+1; $b_{iv}$ is the sensitivity of asset i's returns to movements in the common factor $F_v$; $F_v^{t+1}$ is the actual value of the common factor at time t+1; $E^t(F_v^{t+1})$ is the expectation at time t of the value of the common factor $F_v$ at time t+1; and $u_i^{t+1}$ is the realized error at time t+1. It is assumed for all i that $E^t(u_i^{t+1})$ equals zero, and that $u_i^{t+1}$ is uncorrelated with $u_j^{t+1}$, for all i and j. Equation (3.25) says that investors have an ex ante expected return for asset i, but that unexpected changes in certain factors may lead to results that are different from what investors had expected.

Now consider an investor who is currently holding a portfolio and is considering an alteration to it. Any new portfolio will differ from the old portfolio by proportions $x \equiv (x_1,\ldots,x_m)$ such that:

$$\text{(Share of asset i held in prospective portfolio)} = x_i + \text{(share of asset i held in current portfolio)}\ ; \tag{3.26}$$

where

$$\sum_{i=1}^{m} x_i = 0 \quad . \tag{3.27}$$

Condition (3.27) says that additional purchases of assets must be financed by sales of other assets. Portfolios that use no wealth such as x are called arbitrage portfolios. The additional return obtainable from altering the current portfolio by x is given by:

$$x \cdot R^{t+1} \equiv \sum_{i=1}^{m} x_i \, R_i^{t+1} \quad . \tag{3.28}$$

Substitute equation (3.25) into equation (3.28):

$$x \cdot R^{t+1} = \sum_{i=1}^{m} x_i \; E^t(R_i^{t+1}) + (F_1^{t+1} - E^t(F_1^{t+1})) \sum_{i=1}^{m} x_i \; b_{i1} \quad + \ldots$$

$$+ \; (F_k^{t+1} - E^t(F_k^{t+1})) \sum_{i=1}^{m} x_i \; b_{ik} + \sum_{i=1}^{m} x_i u_i^{t+1} \quad . \tag{3.29}$$

The next step is to consider an arbitrage portfolio that has no systematic risk.[4] In other words, **x** is chosen in such a way that the return on the arbitrage portfolio is unaffected by unexpected changes in any common factor:

$$\sum_{i=1}^{m} x_i b_{iv} = 0 \qquad \text{for all } v, \quad v=1,\ldots,k. \tag{3.30}$$

Furthermore, it is assumed that this particular arbitrage portfolio is well diversified (and so, the number of assets m is relatively large). From the law of large numbers, the following condition then holds approximately:

$$\sum_{i=1}^{m} x_i u_i^{t+1} = 0 \quad . \tag{3.31}$$

That is, x is chosen such that the unsystematic risk has been diversified away.[5] Thus it is possible to choose arbitrage portfolios that are free of both systematic and unsystematic risk. Now, substitute equations (3.30) and (3.31) into equation (3.29):

$$x \cdot R^{t+1} = \sum_{i=1}^{m} x_i \; E^t(R_i^{t+1}) \quad . \tag{3.32}$$

Finally, no portfolio is in equilibrium if it can be improved upon without incurring additional risk or using additional resources. In other words, in equilibrium all portfolios of these m assets which satisfy the conditions of using no wealth and having no risk must also earn no return on average:

$$\sum_{i=1}^{m} x_i E^t(R_i^{t+1}) = 0 \quad . \tag{3.33}$$

APT also makes a further statement. Consider the set of values of x which simultaneously satisfy (3.27) and (3.28); i.e., consider the set of all arbitrage portfolios with zero systematic risk. According to APT, such portfolios necessarily are displacements by proportions x from an optimal portfolio, which can be seen as follows. Let E be the vector with typical element $E^t(R_i{}^{t+1})$ $(i=1,\ldots,m)$. Clearly, if $x \cdot E$ were greater than zero, then the original portfolio could not have been optimal because the displacement x resulted in an expected return higher by $x \cdot E$ without any increase in systematic risk. Alternatively, if $x \cdot E$ were negative, then a displacement of $-x$ would lead to an improvement relative to the initial portfolio (again, contrary to hypothesis). Accordingly, $x \cdot E$ must be zero. In other words, equations (3.27) and (3.28) imply (3.33).

Conditions (3.27) and (3.28) say that x lies in the orthogonal complement of the space spanned by the set of vectors $B^\circ = \{1, b_1, \ldots, b_k\}$, where 1 is a vector containing m units, and the $b_v$ are the vectors with typical elements $b_{iv}$ $(i=1,\ldots, m;\ v=1,\ldots, k)$. Condition (3.33) says that the vector E is orthogonal to any x under discussion. Then it follows from the argument in the previous paragraph that E lies in the linear space which is the orthogonal complement of the orthogonal complement of $B^\circ$; i.e., in the space spanned by $B^\circ$ itself. In other words,

$$E^{t}(R_{i}^{t+1}) = \lambda_{0}^{t+1} + \lambda_{1}^{t+1} b_{i1} + \ldots + \lambda_{k}^{t+1} b_{ik} \qquad i=1,\ldots,m. \qquad (3.34)$$

According to Roll and Ross (1980, p.1079), equation (3.34) "is the central conclusion of APT." If k were equal to 1, equation (3.34) would be similar to the statement in the CAPM that expected returns are linear with respect to a security's beta (i.e., higher beta means higher expected return). The $\lambda_v^{t+1}$ can be thought of as the market prices for risk related to the $v^{th}$ factor.[6]

Finally, if there exists a riskless asset with a return $R_0$, then by, assuming $R_0^{t+1} = \lambda_0^{t+1}$ equation (3.34) becomes a statement about excess returns:

$$E^{t}(R_{i}^{t+1}) - R_{0}^{t+1} = \lambda_{1}^{t+1} b_{i1} + \ldots + \lambda_{k}^{t+1} b_{ik} \qquad i=1,\ldots,m. \qquad (3.35)$$

### 3.4 A Comparison Between the Multi-Index Model of Asset Returns with Factors Derived from a CGE Economic Model, CAPM, and APT

Here we first comment about the use of the multi-index model developed in this paper as compared with the CAPM and the APT. A synthesis of the three approaches is then presented.

The principal advantage of the approach developed here over the CAPM is that our approach captures factors, which are omitted by the CAPM, that are important in determining the returns on assets. Furthermore, all of the CGE model's behavioural relationships and accounting identities are encapsulated in the estimates of the factor loadings. This is a more satisfactory way of estimation, as compared with just relying on correlations observed in time series. One problem with the time-series approach of the APT, for example, is that the forecast changes in some of the macroeconomic variables may be unprecedented. (This would have been the case, for example, when forecasting the price of oil in the early 1970s.)

Sharpe (1984) has developed a synthesis of factor (or multi-index) models, CAPMs, and the APT. In this section we will draw upon Sharpe's work to explain the conditions under which the multi-index model developed in this paper, the CAPM, and the APT are consistent systems.

The first step is to specify the relationship between an asset's beta and its sensitivities to the factors:

$$\beta_i = b_{i1}\,\beta_{M1} + \ldots + b_{ik}\,\beta_{Mk} \qquad i=1,\ldots,m; \qquad (3.36)$$

where the $\beta_i$'s and $b_{iv}$'s are as defined above; and the $\beta_{Mv}$'s are the beta values of the factors to changes in the return on the market portfolio M. The $\beta_{Mv}$'s can be calculated:

$$\beta_{Mv} = Cov^t(F_v^{t+1},\ R_M^{t+1}) \ /\ Var^t(R_M^{t+1}) \qquad v=1,\ldots,k. \qquad (3.37)$$

If the assumptions of the CAPM hold then expected returns must conform to equation (3.23). Furthermore, if returns are generated by a k-factor model, beta values must conform to equation (3.36). Using equation (3.36) we can rewrite equation (3.23):

$$E^t(R_i^{t+1}) = R_F^{t+1} + (E^t(R_M^{t+1}) - R_F^{t+1})(b_{i1}\beta_{M1} + \ldots + b_{ik}\beta_{Mk})$$

$$i=1,\ldots,m. \qquad (3.38)$$

Equation (3.38) can also be written:

$$E^t(R_i^{t+1}) = R_F^{t+1} + b_{i1}\ [(E^t(R_M^{t+1}) - R_F^{t+1})\beta_{M1}] + \ldots$$

$$+ b_{ik}[(E^t(R_M^{t+1}) - R_F^{t+1})\beta_{Mk}] \qquad i=1,\ldots,m. \qquad (3.39)$$

If the conditions required for the APT hold, then equation (3.34) must also apply. Comparing equations (3.34) and (3.39) leads to the conclusion that:

$$\lambda_0^{t+1} = R_F^{t+1} \qquad (3.40)$$

and

$$\lambda_v^{t+1} = (E^t(R_M^{t+1}) - R_F^{t+1})\beta_{Mv} \qquad v=1,\ldots,k. \qquad (3.41)$$

Thus, if equations (3.40) and (3.41) hold, the multi-index model, CAPM, and APT are consistent.

Finally, note that multi-index equations such as (3.8) are sometimes referred to as reduced-form equations. The structural-form equations from which these are derived are the subject matter of the re.ainder of this paper. However, first we will study some of the statistical properties of the return-generating process.

## 3.5 Statistical Properties of the Multi-Index Model of Asset Returns with Factors Derived from a CGE Economic Model

In this section we will examine some of the statistical properties of the return generating process. Recall from equation (3.8) that:

$$R_i^{t+1} = R_i^t + b_{i1}\ (F_1^{t+1} - F_1^t) + \ldots$$

$$+ b_{ik}(F_k^{t+1} - F_k^t) + u_i^{t+1} \qquad i=1,\ldots,m. \qquad (3.42)$$

It is assumed that the $u_i{}^t$ have expectations and own and cross serial covariances all equal to zero:

$$E^t(u_i^t) = 0 \qquad i=1,\ldots,m; \text{ for all } t; \quad (3.43)$$

$$E^t(u_i^t\, u_i^{t+\tau}) = 0 \qquad i=1,\ldots,m \ \&\ \tau \neq 0 \text{ for all } t; \quad (3.44)$$

and

$$E^t(u_i^t\, u_j^{t+\tau}) = 0 \qquad i,\ j=1,\ldots,m;\ i \neq j \ \&\ \tau \neq 0 \text{ for all } t. \quad (3.45)$$

It is further assumed that the $u_i{}^t$ are uncorrelated with the factors; i.e., that:

$$Cov^t(u_i^t,\ F_v^t) = 0 \qquad i=1,\ldots,m;\ v=1,\ldots,k \text{ for all } t. \quad (3.46)$$

Equation (3.42) can also be written:

$$R^{t+1} = R^t + B\cdot F^{t+1} - B\cdot F^t + U^{t+1} \quad ; \qquad (3.47)$$

where $R^{t+1}$ is an $(m\times1)$ vector of the rates of return of the (m) assets in period t+1; $R^t$ is an $(m\times1)$ vector of the rates of return of the (m) assets in period t; B is an $(m\times k)$ matrix of the elasticities of the (m) rates of return with respect to each of the (k) exogenous variables; $F^{t+1}$ and $F^t$ are $(k\times1)$ vectors of the values of the (k) exogenous variables in period t+1 and t, respectively; and $U^{t+1}$ is a $(m\times1)$ vector of error terms.

The $(m\times1)$ vector of the expectations at time t of the rates of return, $E^t(R^{t+1})$, can be obtained by taking the expectation of equation (3.47):

$$E^t(R^{t+1}) = R^t + B\cdot E^t(F^{t+1}) - B\cdot F^t \quad ; \quad (3.48)$$

where $R^t$, B, and $F^t$ are as defined above; and $E^t(F^{t+1})$ is an $(m\times1)$ vector of the expectations at time t of the values of the factors in period t+1.

The $(m\times m)$ variance-covariance matrix of the rates of return of the (m) assets, $Cov^t(R^{t+1})$, can also be calculated from equation (3.47):

$$Cov^t(R^{t+1}) = B\cdot Cov^t(F^{t+1})\cdot B' + Cov^t(U^{t+1}) \quad ; \quad (3.49)$$

where B is an $(m\times k)$ matrix as defined above;B' is the $(k\times m)$ transpose of B; and $Cov^t(U^{t+1})$ is an $(m\times m)$ matrix of covariances of residuals.

The matrices on the right-hand-side of equation (3.48) are estimated as follows. The B matrix is calculated by solving the CGE economic model for the elasticities of the rates of return on assets with respect to the exogenous factors. These calculations are explained in the remainder of this paper. The $Cov^t(F^{t+1})$ matrix in (3.49) must be estimated independently of the CGE economic model. In this paper we will simply assume a given $Cov^t(F^{t+1})$ matrix. However, in future work this matrix will be estimated. Finally, for the illustrative application below, we will assume that $Cov^t(U^{t+1})$ is a null matrix; i.e., that the $u_i^{t+1}$ in equation (3.42) all are zero. Note that this last assumption implies that the only uncertainty captured by equation (3.49) is the component due to uncertainty with respect to the forecasts of the exogenous factors. However, in practice, there are other sources of uncertainty such as errors in the specification of the CGE model, errors in the estimated parameters of the CGE model, inherent randomness in the model, etc.[7]

Before we leave this section it is worth noting that the CGE model could in future work be used as part of the estimation of the $Cov^t(U^{t+1})$ matrix. For example, given an historical $Cov^t(F^t)$ matrix and the B matrix calculated from the CGE model, it may be possible to separate out the systematic component of an historical covariance matrix $Cov^t(R^t)$ as follows. First, the systematic component, $B \cdot Cov^t(F^t) \cdot B'$, would be calculated (doubtless under some specific stationarity assumptions). Then following equation (3.49), the systematic component would be subtracted from $Cov^t(R^t)$ to produce an estimate of the matrix $Cov^t(U^t)$.

# 4. A STYLIZED CGE MODEL OF THE AUSTRALIAN ECONOMY

In this section we will briefly describe a CGE model of the Australian economy. As this paper is largely concerned with developing and illustrating new techniques, the CGE model is highly stylized. This allows attention to be focused on mechanisms which are key to the purposes at hand without the distraction of excessive dimensionality. Also contained in this section is a discussion of the economic environment assumed for the simulations with the model. Finally, an analysis is presented of some elasticities computed with the stylized CGE model.

## 4.1 MO87: A Three-Sector Miniature ORANI Model

The ORANI model, developed by Dixon, Parmenter, Sutton, and Vincent (1982), is a very detailed CGE model of the Australian economy. For this paper a miniature version of the ORANI model, MO87, has been developed as a vehicle for generating portfolio-analytic decision rules within a CGE framework. MO87 is capable of providing estimates of the elasticities of rates of return on capital for industries with fundamentally differing commercial exposures (exporting, import-competing, non-traded) with respect to exogenous variables of interest (e.g., terms of trade, tariffs, aggregate demand). The MO87 model is fully documented in Higgs (1987a); here we will briefly describe some of its features.

The theoretical structure of MO87 consists of a set of equations. These fall into groups: (a) representing household and other final demands for commodities; (b) describing demands for intermediate and primary-factor inputs; (c) relating commodity prices to costs; (d) specifying market clearing conditions for primary factors and commodities; and (e) defining miscellaneous aggregate variables and indexes. MO87 distinguishes six commodities, of which three are produced domestically and three are imported, plus two primary factors, labour and capital. The domestic commodities are assumed to be produced by three single-product industries representative of the export, import-competing, and non-traded areas of the economy. There are three sources of final demands modelled here: investment, household consumption, and exports.

The MO87 model is calibrated using an aggregated version of the 1978-79 Australian input-output table; see Australian Bureau of Statistics (1984). Finally, the model is solved in a linearized form (following Johansen (1960)) using the GEMPACK suite of general purpose software for CGE models; see Pearson (forthcoming).

## 4.2 Assumed Economic Environment

Certain features of the economy are not projected endogenously by MO87. For these, the user of the model must specify an environment (i.e., closure of the model) before computing a solution. In other words, there are more variables than equations in the model; therefore, the user must set values for some of the variables exogenously so that the number of unknown variables equals the number of equations.

The economic environment assumed here has the following key aspects.[8] The industry capital stocks are set exogenously. This choice can serve to define the time-period of the model's projections. For example, setting industry capital stocks exogenously at zero change defines a short-run simulation, the length of which has been estimated for the ORANI model by Cooper (1983) as about 2 years.

The second key exogenous variable is the nominal exchange rate. This variable acts as the numeraire. Note that a typical CGE model will have mechanisms which determine changes in the real exchange rate, but will lack mechanisms suitable for partitioning such variation into changes in the domestic inflation rate (relative to the overseas rate) on the one hand, and changes in the nominal exchange rate on the other. With the nominal exchange rate as the numeraire, changes in domestic price indices can be interpreted as changes in domestic relative to world prices.

The third key group of exogenous variables are the tariffs or tariff equivalents of quantitative restrictions on imports. By making tariffs exogenous we can compute the effects of exogenously projected changes in the government's policy on protection.

The fourth key exogenous variable is real absorption (i.e., aggregate consumption plus aggregate investment deflated by the appropriate price indices). In this case the CGE model indicates, among other things, the change in the balance of trade that would be required to maintain a projected target level for real absorption.

The final exogenous variables of interest are world prices. Here we allow for the calculation of the effects of exogenously projected movements in the terms of trade.

## 4.3 Projections

In this section we will use MO87 to study the effects of increases in the world price of Australia's exports, tariffs, real absorption, and the nominal exchange rate on some macroeconomic variables, industry outputs, rental rates and creation costs of capital.

### 4.3.1 Macroeconomic and Industry Output Projections

The short-run (two-year) elasticities of some macroeconomic variables and industry outputs with respect to a one per cent increase in the world price of Australia's exports, tariffs, real absorption, and the nominal exchange rate are given in Table 4.1. Each of the exogenous shocks will be discussed in turn.

The first column shows the effects of a one per cent increase in the world price of Australia's exports (at initial export levels). This price rise causes an increase in both Australia's exports and the output of the export industry. The world price rise is inflationary and results in an appreciation of the real exchange rate.[9] The international competitiveness of the import-competing sector deteriorates. As a result, aggregate imports are projected to increase and the output of the import-competing industry contracts. The improvement in export prices causes a net improvement in the balance of trade. Finally, aggregate employment and the output of the non-traded industry are projected to increase slightly.

TABLE 4.1: MACROECONOMIC AND INDUSTRY OUTPUT PROJECTIONS*

| Variable | One per cent increase in: | | | |
|---|---|---|---|---|
| | World Price of Exports | Ad Valorem Tariffs | Real Absorption | Nominal Exchange Rate |
| | $P_W$ | $t$ | $a_R$ | $\phi$ |
| Macroeconomic | | | | |
| Real exchange rate | 0.2526 | 0.1366 | 0.8253 | 0.0000 |
| Aggregate exports (foreign currency value) | 1.0507 | -0.0665 | -0.4905 | 0.0000 |
| Aggregate imports (foreign currency value) | 0.2529 | -0.0265 | 1.4083 | 0.0000 |
| Balance of trade | 0.0011 | -0.0000 | -0.0038 | 0.0000 |
| Aggregate employment | 0.1042 | -0.0201 | 0.8034 | 0.0000 |
| Industry Outputs | | | | |
| 1. Export | 0.5192 | -0.0936 | -0.2781 | 0.0000 |
| 2. Import-competing | -0.0419 | 0.0168 | 0.5647 | 0.0000 |
| 3. Non-traded | 0.0187 | -0.0072 | 0.7717 | 0.0000 |

* All projections, with the exception of the balance of trade, are percentage deviations from the value the variable would have taken in the absence of the shock at the head of each column. The balance of trade, while also a deviation from control, is the change in the balance of trade as a share of base-period GDP.

A one per cent increase in tariffs causes an increase in the price of imported goods, which in turn is inflationary and results in an appreciation of the real exchange rate. The international competitiveness of the traded industries deteriorates. Aggregate exports decline as does the output of the export industry. However, as protection from import competition has increased, aggregate imports decline and the output of the import-competing industry increases. The net effect is for a very small decline in the balance of trade. This in turn results in small decreases in both aggregate employment and the output of the non-traded industry.

The third column of Table 4.1 shows the effects of a one per cent increase in real absorption (i.e., in real aggregate demand). An increase in real absorption is inflationary, causing an appreciation of the real exchange rate. The decline in the international competitiveness of the trading industries results in a fall in aggregate exports, a contraction in the output of the export industry, and an increase in aggregate imports. However, the output of the import-competing industry is projected to increase. This is because the expansionary effects of the increase in real demand outstrip the contractionary consequences of the loss in competitiveness.

The last column of Table 4.1 shows the effects of a one per cent increase in the nominal exchange rate. If wages are assumed to be 100 per cent indexed to the consumer price index, as they are here, then a one per cent increase in the nominal exchange rate (i.e., the numeraire) causes a one per cent increase in all prices. However, as there are no changes in relative prices, a one per cent increase in the nominal exchange rate causes zero change in all real variables. For example, although the selling price of each industry rises by one per cent, this is exactly matched by the increase in its unit cost, thus leaving outputs unchanged.

Finally, note that the elasticities in Table 4.1 are qualitatively similar to those computed by the ORANI model; see, for example, Dixon, Powell, and Parmenter (1979) and Higgs (1986).

### 4.3.2 Projections of Rental Prices and Creation Costs of Capital

The short-run (two-year) elasticities of rental values and creation costs of capital with respect to a one per cent increase in the world price of Australia's exports, tariffs, real absorption, and the nominal exchange rate are given in Table 4.2. These elasticities can be explained as follows. First we note that since the sizes of the capital stocks in use in each industry are exogenous in the assumed economic environment, none of the shocks under discussion have any effect on them. Recall from Table 4.1 that an increase in tariffs, for example, causes a decline in the output of the export industry. As capital does not respond to this shock, the decline in output occurs via a reduction in the use of the variable primary factor (labour). This causes an increase in the capital-labour ratio in the export industry. As capital is now relatively abundant, the marginal physical product of capital in the export industry falls. This causes a decline in the marginal product of capital and thus the rental rate on capital in the export industry falls (see Table 4.2). The increase in tariffs causes an increase in the output of the import-competing industry, and hence an increase in its rental rate on capital. Finally, the increase in tariffs causes a fall in the output of the non-traded industry. This

TABLE 4.2: PROJECTIONS OF RENTAL PRICES AND CREATION COSTS OF CAPITAL BY INDUSTRIES*

| Variable | One per cent increase in: | | | |
|---|---|---|---|---|
| | World Price of Exports | Ad Valorem Tariffs | Real Absorption | Nominal Exchange Rate |
| | $p_W$ | $t$ | $a_R$ | $\phi$ |
| Rental Rates on Capital, $p*_j$ | | | | |
| 1. Export | 1.1651 | -0.0279 | 0.3365 | 1.0000 |
| 2. Import-competing | 0.2018 | 0.1570 | 1.5094 | 1.0000 |
| 3. Non-traded | 0.2777 | 0.1270 | 1.8583 | 1.0000 |
| Creation Costs of Capital, $\pi*_j$ | | | | |
| 1. Export | 0.2262 | 0.1497 | 0.6921 | 1.0000 |
| 2. Import-competing | 0.1867 | 0.1605 | 0.6141 | 1.0000 |
| 3. Non-traded | 0.2261 | 0.1452 | 0.8362 | 1.0000 |

* All projections are percentage deviations from what the variable in question would have been in the absence of the shock at the head of the column.

results in an increase in the capital-labour ratio in this industry, and a fall in the marginal physical product of capital. However, the price of the output of the non-traded industry increases by more than the fall in the marginal physical product of capital, and the net effect is an increase in the marginal value product of capital. Thus the rental rate on capital in the non-traded industry increases (see Table 4.2).

An increase in tariffs causes an increase in the price of imported goods. This is inflationary and results in an increase in the creation costs of capital in each of the industries (see Table 4.2). As a unit of capital in the import-competing sector uses slightly more imports (relative to domestically produced inputs) than does capital in the other industries, there is a slightly greater projected increase in the creation cost of capital in the import-competing industry.

The elasticities with respect to an increase in the world price of Australia's exports, real absorption and the nominal exchange rate, listed in Table 4.2, can also be explained by appealing to arguments similar to those above. It is shown in section 5 how these elasticities can be used to produce forecasts of industry rates of return on capital.

# 5. INDUSTRY RATES OF RETURN ON CAPITAL

This section contains two parts. The first defines industry rates of return on capital. The second calculates the base-period rates of return for the industries in the CGE model.

## 5.1 Definition of Rates of Return by Industry

The rate of return on capital is defined in this paper by the following thought experiment. At instant t a unit of capital stock in industry j is purchased which is held for $\ell$ years. During this period, a rental is earned; however the physical quantity of capital left from the initial purchase has declined due to depreciation. The remainder is sold at the market price of capital goods at instant $t+\ell$. Note that it is assumed (i) that there are no margins separating the sale and purchase prices of second-hand assets; and (ii) that there is no difference between the price of a unit of second-hand capital and the price of a unit of new capital with the same productive capacity.[10] The average annual rate of return is then defined as the sum of the average annual rental received plus the average capital gain per annum all divided by the initial purchase price.

Let $R_{*j}^{t}$ denote the average annual rate of return on a unit of capital in industry j over the period beginning at instant $t-\ell$ and ending at instant t:

$$\begin{aligned} R_{*j}^{t} = {} & \bar{P}_{*j}^{t}(1 - \tau_{*j}^{t})/\Pi_{*j}^{t-\ell} \\ & + (\Pi_{*j}^{t}(1 - \ell d_{*j}^{t}) - \Pi_{*j}^{t-\ell})/(\ell\Pi_{*j}^{t-\ell}) \qquad j=1,\ldots,n; \qquad (5.1) \end{aligned}$$

where $\bar{P}*_j{}^t$ is the average annual flow of rentals on a unit of capital in industry j over the period t-ℓ to t; $\tau*_j{}^t$ is the ratio of taxes paid over the period t-ℓ to t to the rentals earned over that period -- hereafter referred to as the average annual tax rate on the rental earned by a unit of capital in industry j over the period t-ℓ to t; $\Pi*_j{}^{t-\ell}$ and $\Pi*_j{}^t$ are the creation costs of a unit of capital in industry j at t-ℓ and t, respectively; $d*_j{}^t$ is the depreciation rate on a unit of capital in industry j over the period t-ℓ to t; and ℓ is the number of years between t-ℓ and t. According to equation (5.1) the rate of return consists of a rental component plus a capital gains component.

Consider the following numerical example. Assume for the moment that ℓ equals one year and that at instant t-1 it costs \$100 to buy a unit of capital in industry j (i.e., $\Pi*_j{}^{t-1}$ = \$100). Over the period t-1 to t this unit of capital earned \$20 in rentals (i.e., $\bar{P}*_j{}^t$ = \$20); however this was taxed at a rate of 20 per cent (i.e., $\tau*_j{}^t$ = 0.20). Thus, the after-tax rental component contributes 16 percentage points to the rate of return (i.e., 100 × \$20 (1 - 0.20)/\$100). Furthermore, the value of a new unit of capital increased to \$120 (i.e., $\Pi*_j{}^t$ = \$120); however the unit of capital bought at t-1 has depreciated by 5 per cent (i.e., $d*_j{}^t$ = 0.05). Thus the capital gain contributes 14 percentage points to the rate of return (i.e., 100 × (\$120(1 - 0.05) - \$100)/\$100).[11] Therefore the rate of return on a unit of capital in industry j over the period t-1 to t is 30 per cent.

Next we note that the rate of return on capital in industry j over the period t to t+ℓ, $R*_j{}^{t+\ell}$, can be calculated as:

$$R_{*j}^{t+\ell} = R_{*j}^{t} + \Delta R_{*j}^{t} \qquad j=1,\ldots,n; \qquad (5.2)$$

where $\Delta R_{*j}^t$ is the change in the rate of return on capital in industry j between the period t-ℓ to t and the period t to t+ℓ. It is possible to approximate $\Delta R_{*j}^t$ by totally differentiating equation (5.1):[12]

$$100\Delta R_{*j}^{t} = \bar{p}_{*j}^{t}\,\alpha_{*j}^{t} + \pi_{*j}^{t}\,\beta_{*j}^{t} + \pi_{*j}^{t-\ell}\,\gamma_{*j}^{t}$$

$$+ 100\Delta\tau_{*j}^{t}\,\lambda_{*j}^{t} + 100\Delta d_{*j}^{t}\psi_{*j}^{t} \qquad j=1,\ldots,n; \qquad (5.3)$$

where

$$\alpha_{*j}^{t} = \bar{P}_{*j}^{t}\,(1 - \tau_{*j}^{t}) \,/\, \Pi_{*j}^{t-\ell} \qquad j=1,\ldots,n; \qquad (5.4)$$

$$\beta_{*j}^{t} = (1 - \ell d_{*j}^{t})\,\Pi_{*j}^{t} \,/\, (\ell\Pi_{*j}^{t-\ell}) \qquad j=1,\ldots,n; \qquad (5.5)$$

$$\gamma_{*j}^{t} = -\alpha_{*j}^{t} - \beta_{*j}^{t} \qquad j=1,\ldots,n; \qquad (5.6)$$

$$\lambda_{*j}^{t} = -\bar{P}_{*j}^{t} \,/\, \Pi_{*j}^{t-\ell} \qquad j=1,\ldots,n; \qquad (5.7)$$

$$\psi_{*j}^{t} = -\Pi_{*j}^{t} \,/\, \Pi_{*j}^{t-\ell} \qquad j=1,\ldots,n. \qquad (5.8)$$

The above notation can be explained as follows. The lower-case variables are the percentage changes between the period t-ℓ to t and the period t to t+ℓ in the respective upper-case variables; and $100\Delta R_{*j}^t$, $100\Delta\tau_{*j}^t$, and $100\Delta d_{*j}^t$ are the percentage-point changes in the rate of return on capital, the tax rate, and the depreciation rate, respectively.

It should be noted that the percentage change in the average annual flow of rentals between the period t-ℓ to t and the period t to t+ℓ, $\bar{p}_{*j}^t$, is not the same as the percentage change in the annual flow of rentals as computed by the CGE model, $p_{*j}^t$. The latter is strictly interpreted as the percentage change in the value of the annual rental rate, $P_{*j}^\tau$, which has accrued up to the instant $t+\ell$.[13] To calculate the rate of return over the period $t$ to $t+\ell$, however, we require the percentage change in the average value of the variable $P_{*j}^\tau$ over the interval (rather than its terminal value). In this paper it is assumed that $P_{*j}^\tau$ adjusts at a constant rate over the period $t$ to $t+\ell$; see Figure 5.1. Thus the percentage change in the average annual flow of rentals over the period $t$ to $t+\ell$ can be calculated as:

$$\bar{p}_{*j}^t = p_{*j}^t \,/\, L \qquad j=1,\ldots,n; \qquad (5.9)$$

where L=2. This is only one of a whole range of assumptions that could be made. For example, $P_{*j}^\tau$ may adjust very quickly such that $\bar{p}_{*j}^t$ is approximately equal to $p_{*j}^t$(L=1). To take the other extreme, $P_{*j}^\tau$ may adjust very slowly such that $\bar{p}_{*j}^t$ is approximately equal to zero ($L\to\infty$). In future work it may be possible to empirically determine the correct assumption to make concerning the adjustment path (i.e., to estimate L); see Cooper, McLaren, and Powell (1985) for related work on this issue.

In the next step (5.9) is used to express equation (5.3) wholly in terms of the variables projected by the CGE model:

$$\begin{aligned} 100\Delta R_{*j}^t &= p_{*j}^t \alpha_{*j}^t / L + \pi_{*j}^t \beta_{*j}^t + \pi_{*j}^{t-\ell} \gamma_{*j}^t \\ &\quad + 100\Delta\tau_{*j}^t \lambda_{*j}^t + 100\Delta d_{*j}^t \,\psi_{*j}^t \qquad j=1,\ldots,n. \qquad (5.10) \end{aligned}$$

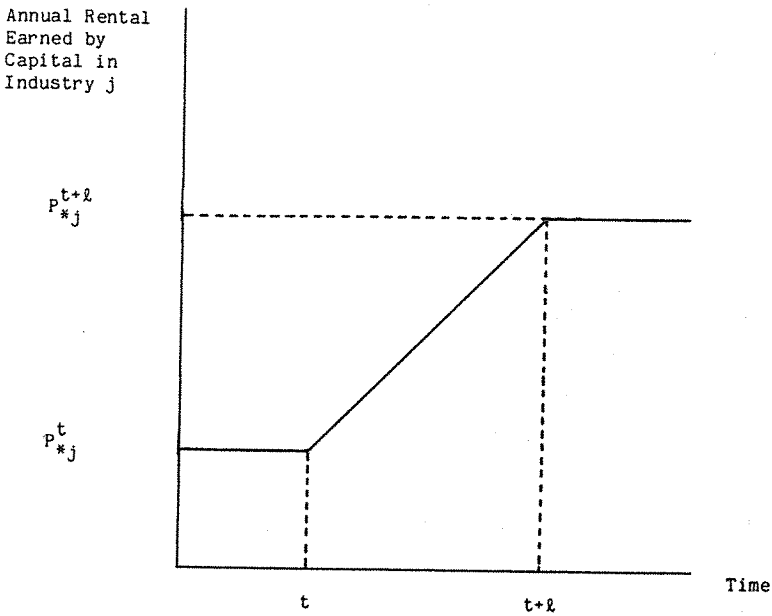


FIGURE 5.1: THE ASSUMED DYNAMICS OF THE PROJECTED CHANGE IN RENTALS IN THE CGE MODEL

As we usually only observe a price times a quantity in the data, to implement equation (5.10) the coefficients $\alpha_{*j}{}^{t}$, $\beta_{*j}{}^{t}$, $\gamma_{*j}{}^{t}$, $\lambda_{*j}{}^{t}$, and $\psi_{*j}{}^{t}$ can be calculated as follows:

$$\alpha^{t}_{*j} = \bar{P}^{t}_{*j}K^{t-\ell}_{*j}(1 - \tau^{t}_{*j}) \;/\; (\Pi^{t-\ell}_{*j}\; K^{t-\ell}_{*j}) \qquad j=1,\ldots,n; \qquad (5.11)$$

$$\beta^{t}_{*j} = (1 - \ell d^{t}_{*j})\Pi^{t}_{*j}\; K^{t-\ell}_{*j} \;/\; (\ell\Pi^{t-\ell}_{*j}\; K^{t-\ell}_{*j}) \qquad j=1,\ldots,n; \qquad (5.12)$$

$$\gamma^{t}_{*j} = -\,\alpha^{t}_{*j} - \beta^{t}_{*j} \qquad j=1,\ldots,n; \qquad (5.13)$$

$$\lambda^{t}_{*j} = -\,\bar{P}^{t}_{*j}\; K^{t-\ell}_{*j} \;/\; (\Pi^{t-\ell}_{*j}\; K^{t-\ell}_{*j}) \qquad j=1,\ldots,n; \qquad (5.14)$$

$$\psi^{t}_{*j} = -\,\Pi^{t}_{*j}\; K^{t-\ell}_{*j} / \;(\Pi^{t-\ell}_{*j}\; K^{t-\ell}_{*j}) \qquad j=1,\ldots,n. \qquad (5.15)$$

In the above equations, $K_{*j}{}^{\tau}$ is the size of the capital stock in use in industry j during the period beginning at instant $\tau$ ($\tau = t-\ell$, t). Equations (5.11) to (5.15) are based on the following two assumptions. The first is that at t we can observe the replacement value of the initial capital stock, $\Pi_{*j}\; K_{*j}{}^{t-\ell}$. The second assumption concerns the timing of physical depreciation of the capital stock. Here we assume that all of the depreciation $\ell d_{*j}{}^{t}\; K_{*j}{}^{t-\ell}$ occurs just before the instant t; see Figure 5.2. As a result of this last assumption, the average annual revenue that is earned over the period $t-\ell$ to t is equal to the product of the initial capital stock and the average annual rental earned over the period $t-\ell$ to t, $\bar{P}_{*j}{}^{t}K_{*j}{}^{t-\ell}$.

Equation (5.10) can be illustrated by considering two consecutive intervals of time -- see Table 5.1. Again we will assume

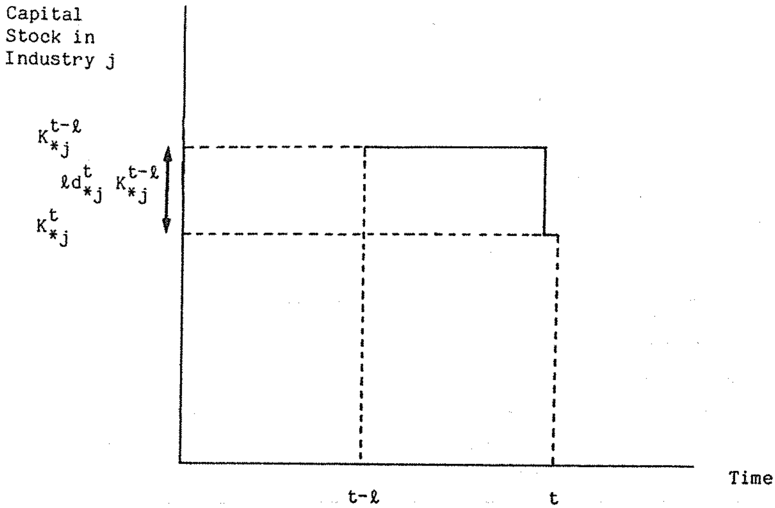


FIGURE 5.2: THE ASSUMED DYNAMICS OF DEPRECIATION IN THE CAPITAL STOCK IN THE CGE MODEL

TABLE 5.1: NUMERICAL EXAMPLE TO ILLUSTRATE HOW INDUSTRY RATES OF RETURN ARE CALCULATED

| 1. Variables defined as stocks at a given point of time | Instants | | |
|---|---|---|---|
| | t-1 | t | t+1 |
| Creation cost of capital, $\Pi_{*j}K_j$ | \$100 | \$120 | \$160 |

| 2. Variable defined as average flows over periods | Period | |
|---|---|---|
| | (t-1 to t) | (t to t+1) |
| Rental earned on capital, $P_{*j}K_{*j}$ | \$20 | \$30 |
| Tax rate on rentals, $\tau_{*j}$ | 0.20 | 0.15 |
| Depreciation rate, $d_{*j}$ | 0.05 | 0.05 |
| Rate of return, $R_{*j}$ | 0.30 | 0.48 |

| 3. Variables defined as percentage changes in stocks | Between Instants | |
|---|---|---|
| | (t-1 and t) | (t and t+1) |
| Percentage changes in creation costs, $\pi_{*j}$ | 20 | 33.33 |

4. Variables defined as differences between periods (t-1 to t) and (t to t+1) of flow variables defined in 2 above

| | |
|---|---|
| Percentage change in rental rates, $p_{*j}$ | 50 |
| Percentage-point change in tax rate, $100\Delta\tau_{*j}$ | -5 |
| Percentage-point change in depreciation rate, $100\Delta d_{*j}$ | 0 |

5. Coefficients

| | |
|---|---|
| $\alpha_{*j}$ (a) | 0.16 |
| $\beta_{*j}$ (b) | 1.14 |
| $\gamma_{*j}$ (c) | -1.30 |
| $\lambda_{*j}$ (d) | -0.20 |
| $\psi_{*j}$ (e) | -1.20 |

| | |
|---|---|
| Percentage-point change in the rate of return, $100\Delta R_{*j}$ | |
| - actual | 18.00 |
| - estimated using equation (5.2) | 21.00 |

(a) See equation (5.11).
(b) See equation (5.12).
(c) See equation (5.13).
(d) See equation (5.14).
(e) See equation (5.15).

that $\ell$ equals one year. Recall from the example above that the rate of return over the period t-1 to t was 30 per cent (i.e., $R*_j{}^t = 0.30$). Now assume that at instant t a producer in industry j buys $120 worth of capital (i.e., $\Pi*_j{}^t K*_j{}^t$ = $120). Over the period t to t+1 this capital earns $30 in rentals (i.e., $P*_j{}^{t+1} K*_j{}^t$ = $30); however this is taxed at a rate of 15 per cent (i.e., $\tau*_j{}^{t+1} = 0.15$). Next we assume that at instant t+1 the replacement value of the capital increases to $160 (i.e., $\Pi*_j{}^{t+1} K*_j{}^t$ = $160); however the capital bought at time t has depreciated by 5 per cent (i.e., $d*_j{}^{t+1} = 0.05$).

According to equation (5.1), the above producer over the period t to t+1 earns a rate of return of 48 per cent. This rate of return can also be calculated, subject to linearization errors, using equations (5.2) and (5.10). The percentage changes $p*_j{}^t$, $\pi*_j{}^{t-1}$, $\pi*_j{}^t$, the percentage-point changes $100\Delta\tau*_j{}^t$, $100\Delta d*_j{}^t$, and the coefficients $\alpha*_j{}^t$, $\beta*_j{}^t$, $\gamma*_j{}^t$, $\lambda*_j{}^t$, and $\psi*_j{}^t$ can all be calculated from the above data and they are listed in Table 5.1. If these values are substituted into equation (5.10) the change in the rate of return over the period t to t+1 is equal to 0.21. Thus according to equation (5.2) the rate of return on capital industry j over the period t to t+1 is equal to 51 per cent (i.e., 0.30 + 0.21), which is fairly close to the actual rate of return of 48 per cent.

Next we note that for industry j the percentage change in the rental between the periods t-$\ell$ to t and t to t+$\ell$ and the percentage change in the creation costs of capital between instants t and t+$\ell$ are estimated by the CGE model as:

$$p_{*j}^t = \eta_{P_{*j}F_1} f_1^t + \dots + \eta_{P_{*j}F_k} f_k^t \qquad j=1,\dots,n; \qquad (5.16)$$

$$\pi^t_{*j} = \eta_{\Pi_{*j}F_1} f^t_1 + \dots + \eta_{\Pi_{*j}F_k} f^t_k \qquad j=1,\dots,n; \qquad (5.17)$$

where $\eta_{P_{*j}F_v}$ and $\eta_{\Pi_{*j}F_v}$ are the elasticites of the rental and creation cost, respectively, in industry j with respect to the $v^{th}$ exogenous variable (i.e., factor); and $f_v{}^t$ is the percentage change in the $v^{th}$ exogenous variable over the period t to t+ℓ. Equations (5.16) and (5.17) can be substituted into equation (5.10):

$$100\Delta R^t_{*j} = (\alpha^t_{*j}/L) \sum_{v=1}^{k} \eta_{P_{*j}F_v} f^t_v$$

$$+ \beta^t_{*j} \sum_{v=1}^{k} \eta_{\Pi_{*j}F_v} f^t_v + C^t_{*j} \qquad j=1,\dots,n; \qquad (5.18)$$

where:

$$C^t_{*j} = \pi^{t-\ell}_{*j} \gamma^t_{*j} + 100\Delta\tau^t_{*j} \lambda^t_{*j} + 100\Delta d^t_{*j} \psi^t_{*j} \qquad j=1,\dots,n. \qquad (5.19)$$

Note that we assume that $\pi^{t-\ell}_{*j}$, $\Delta\tau^t_{*j}$, and $\Delta d^t_{*j}$ are known from the data base.

Finally, we can substitute equation (5.18) into equation (5.2):

$$100R^{t+\ell}_{*j} = 100R^t_{*j} + C^t_{*j} + B^t_{*j1} f^t_1 + \dots + B^t_{*jk} f^t_k \qquad j=1,\dots,n; \qquad (5.20)$$

where

$$B^t_{*jv} = (\alpha^t_{*j}/L)\eta_{P_{*j}F_v} + \beta^t_{*j} \eta_{\Pi_{*j}F_v} \qquad j=1,\dots,n \quad v=1,\dots,k. \qquad (5.21)$$

The similarity between (5.20) and the return generating process as expressed in section 3, equation (3.8), can be clearly seen if we rewrite equation (5.20) as:

$$100R_{*j}^{t+\ell} = 100R_{*j}^{t} + C_{*j}^{t} + b_{*j1}^{t}\ (F_1^{t+\ell} - F_1^{t}) + \ldots$$
$$+ b_{*jk}^{t}(F_k^{t+\ell} - F_k^{t}) + u_{*j}^{t+\ell} \qquad j=1,\ldots,n; \qquad (5.22)$$

where

$$b_{*jv}^{t} = B_{*jv}^{t} f_v^{t}\ F_v^{t}/100 \qquad j=1,..,n \quad v=1,\ldots,k. \qquad (5.23)$$

Two final points can be made concerning equation (5.22). The first is that it contains the term $u_{*j}{}^{t+\ell}$ which is the realized error at time $t+\ell$. The second concerns the role of the term $C_{*j}{}^{t}$. This term is implicitly contained in equation (3.8), where the factors contributing to it are changes in: the creation costs of capital over the period $t-\ell$ to $t$, the tax rate over the period $t$ to $t+\ell$, and the depreciation rate over the period $t$ to $t+\ell$.[14]

## 5.2 Calculation of the Rates of Return by Industry in the CGE Model

In this section we calculate the rate of return and the values of the $C_{*j}{}^{t}$'s and the $B_{*jv}{}^{t}$'s for the industries in the CGE model. To do this particular care must be taken of the length of the solution period of the CGE model.

Recall from section 4.2 that the economic environment chosen for the simulations presented in this paper was one where industry capital stocks are exogenous. Cooper (1983) has estimated that this closure produces projections which pertain to the values of the endogenous variables about two years after the introduction of shocks in the exogenous variables (i.e., $\ell=2$). Following equation (5.1), the average annual rate of return on capital in industry j over the period t-2 to t, $R_{*j}^t$, is given by:

$$R_{*j}^t = \bar{P}_{*j}^t(1 - \tau_{*j}^t) / \Pi_{*j}^{t-2}$$

$$+ (\Pi_{*j}^t(1 - 2d_{*j}^t) - \Pi_{*j}^{t-2}) / (2\Pi_{*j}^{t-2}) \qquad j=1,\ldots,n; \qquad (5.24)$$

where $\bar{P}_{*j}^t$ is the annual average flow of rentals on a unit of capital in industry j over the period t-2 to t; $\tau_{*j}^t$ is the annual average tax rate on the rental earned by a unit of capital in industry j over the period t-2 to t; $\Pi_{*j}^{t-2}$ and $\Pi_{*j}^t$ are the creation costs of a unit of capital in industry j at time t-2 and t, respectively; and $d_{*j}^t$ is the annual average depreciation rate on capital in industry j over the period t-2 and t. Note that we have assumed that the depreciation $2d_{*j}^t$ occurs just before the point in time t (see Figure 5.2). Futhermore, as we usually only observe a price times a quantity in the data, equation (5.24) can also be written:

$$R_{*j}^t = \bar{P}_{*j}^t K_{*j}^{t-2}(1 - \tau_{*j}^t) / (\Pi_{*j}^{t-2} K_{*j}^{t-2})$$

$$+ (\Pi_{*j}^t K_{*j}^{t-2}(1 - 2d_{*j}^t) - \Pi_{*j}^{t-2} K_{*j}^{t-2}) / (2\Pi_{*j}^{t-2} K_{*j}^{t-2}) \qquad j=1,\ldots,n; \qquad (5.25)$$

where $K_{*j}^{t-2}$ is the capital stock in industry j at instant t-2. Similar to equations (5.11) to (5.15) above, we are assuming that at time t it is possible to observe the replacement value of the initial capital stock, $\Pi_{*j}^t K_{*j}^{t-2}$.

The base-period data concerning the capital stocks of the industries in the CGE model are given in Table 5.2. It can be seen from

TABLE 5.2: RATES OF RETURN BY INDUSTRY IN THE CGE MODEL

| Variable | Industry | | |
|---|---|---|---|
| | 1. Export | 2. Import-Competing | 3. Non-Traded |
| Average annual rental earned on capital,[(a)] $\bar{P}^{t}_{*j}K^{t-2}_{*j}$ | \$ 6,159m | \$ 3,187m | \$ 18,144m |
| Creation cost of capital,[(b)] $\Pi^{t-2}_{*j}\ K^{t-2}_{*j}$ | \$18,420m | \$11,159m | \$ 92,016m |
| $\Pi^{t}_{*j}K^{t-2}_{*j}$ | \$22,811m | \$13,857m | \$113,400m |
| Average tax rate on rentals,[(c)] $\tau_{*j}$ | 0.20 | 0.20 | 0.20 |
| Average depreciation rate,[(d)] $d^{t}_{*j}$ | 0.07 | 0.08 | 0.06 |
| Rate of return,[(e)] $R^{t}_{*j}$ | 0.30 | 0.25 | 0.20 |

(a) Measured at 1978-79 prices; source Higgs (1987a).

(b) Measured at 1978-79 prices. Note that the values of the capital stock at time t-2 were calibrated to achieve the rates of return shown in this table.

(c) This rate has been chosen for illustrative purposes. However, note that the effective capital income tax rate, defined as the ratio of taxes on profits and self-employment to pre-tax gross operating surplus, for 1984-85 was 16 per cent; see Meagher and Parmenter (1987).

(d) Based on estimates contained in Bruce and Horridge (1986).

(e) See equation (5.25).

the table that the rates of return in the export, import-competing, and non-traded industries are assumed to be 30 per cent, 25 per cent, and 20 per cent, respectively.[15] Given the data in Table 5.2, it is possible to calculate the coefficients, $\alpha*_j{}^t$, $\beta*_j{}^t$, $\gamma*_j{}^t$, $\lambda*_j{}^t$, and $\psi*_j{}^t$, and the observed percentage change in the creation cost of capital, $\pi*_j{}^{t-2}$; see Table 5.3. Table 5.3 also contains the projected percentage-point changes in tax rates and depreciation rates. Given the above coefficients and projections we can substitute these into equation (5.19) to calculate the values of the $C*_j{}^t$'s; see the bottom row of Table 5.3.

Finally, the values of the industry factor sensitivity coefficients (i.e., the $B*_{jv}{}^t$'s) are calculated by substituting the values of the coefficients $\alpha*_j{}^t$ and $\beta*_j{}^t$ and the appropriate elasticities into equation (5.21); see Table 5.4. For example, the coefficient for the export industry with respect to the world price of exports is calculated as follows. From Table 5.3 we know that for the export industry $\alpha*_j{}^t$ and $\beta*_j{}^t$ are equal to 0.2675 and 0.4300, respectively. Furthermore, recall from Table 4.2 that the elasticities of the rental rate and the creation costs of capital in the export industry with respect to an increase in the world price of exports are 1.1651 and 0.2262, respectively. Finally, the adjustment coefficient in equation (5.9) is taken to be two (i.e., L = 2). If these values are substituted into equation (5.21) we find that the coefficient for the export industry with respect to the world price of exports is equal to 0.2530 (i.e., $(0.2675/2) \times 1.1651 + 0.4300 \times 0.2262$).

TABLE 5.3: COEFFICIENTS IMPLIED BY THE BASE-PERIOD INDUSTRY DATA AND ASSUMED DEVELOPMENTS IN INDUSTRY TAX AND DEPRECIATION RATES

| Coefficient/Variable | Industry | | |
| --- | --- | --- | --- |
| | 1. Export | 2. Import-Competing | 3. Non-Traded |
| $\alpha_{*j}^{t}$ (a) | 0.2675 | 0.2285 | 0.1577 |
| $\beta_{*j}^{t}$ (b) | 0.4300 | 0.4200 | 0.4400 |
| $\gamma_{*j}^{t}$ (c) | -0.6975 | -0.6485 | -0.5977 |
| $\lambda_{*j}^{t}$ (d) | -0.3343 | -0.2855 | -0.1971 |
| $\psi_{*j}^{t}$ (e) | -1.2383 | -1.2417 | -1.2323 |
| $\pi_{*j}^{t-2}$ | 23.8382 | 24.1778 | 23.2394 |
| $\Delta\tau_{*j}^{t}$ | 0.0000 | 0.0000 | 0.0000 |
| $\Delta d_{*j}^{t}$ | 0.0000 | 0.0000 | 0.0000 |
| $C_{*j}^{t}$ (f) | -16.6271 | -15.6793 | -13.8902 |

(a) See equation (5.11).
(b) See equation (5.12).
(c) See equation (5.13).
(d) See equation (5.14).
(e) See equation (5.15).
(f) See equation (5.19).

TABLE 5.4: INDUSTRY FACTOR SENSITIVITY COEFFICIENTS*

| Factor | Industry | | |
|---|---|---|---|
| | 1. Export | 2. Import-Competing | 3. Non-Traded |
| World price of exports, $p_W$ | 0.2530 | 0.1014 | 0.1213 |
| Ad valorem tariffs, t | 0.0606 | 0.0853 | 0.0739 |
| Real absorption, $a_R$ | 0.3426 | 0.4303 | 0.5144 |
| Nominal exchange rate, $\varphi$ | 0.5637 | 0.5342 | 0.5188 |

* That is, the $B*_{jv}{}^{t}$ coefficients; see equation (5.21).

in equation (5.9) is taken to be two (i.e., $L = 2$). If these values are substituted into equation (5.21) we find that the coefficient for the export industry with respect to the world price of exports is equal to 0.2530 (i.e., $(0.2675/2) \times 1.1651 + 0.4300 \times 0.2262$).

# 6. INDUSTRY FORECASTS

In this section it is shown how a CGE model can be used for forecasting industry rates of return on capital. To forecast with a CGE model first requires scenarios to be specified, from outside of the CGE model, about future developments in the exogenous variables. These scenarios together with the appropriate elasticities are then used to produce forecasts of rates of return on capital in each industry. Note that the only uncertainty captured by the forecasts is with respect to future developments in the exogenous variables. However, in practice there are other sources of uncertainty such as errors in the specification of the CGE model, errors in the estimated parameters of the CGE model, inherent randomness in the model, etc; see Higgs and Powell (forthcoming).

## 6.1 Future Developments in the Exogenous Variables

To forecast with a CGE model, the future developments in all the exogenous variables must be specified. For illustrative purposes, however, we will specify changes in just a few of the exogenous variables. First, we will assume that there will be a decline in Australia's terms of trade over the next two years. In particular we expect the foreign currency price of Australia's exports to decline by 40 per cent with a probability of 0.2, by 30 per cent with a probability of 0.6, and by 20 per cent with a probability of 0.2. Next, we assume that tariffs will be reduced by 80 per cent with a probability of 0.3, by 60 per cent with a probability of 0.4, and by 40 per cent with a probability of 0.3. These probabilities indicate that we are not very confident about the exact size of the tariff cut. However, it is assumed that we have to be able to deduce from some source (say government announcements) that there will be significant tariff cuts somewhere between 40 and 80 per cent. We assume that real absorption will increase over two years by 6.0 per cent with a probability of 0.1, by 7.0 per cent with a probability of 0.8, and by 8.0 per cent with a probability of 0.1. Note that our relative confidence about the forecasts for real absorption is only assumed for illustrative purposes. In practice we would probably be more uncertain about our real absorption forecast. To simplify the analysis we will assume that, with the exception of the nominal exchange rate, all other exogenous variables are forecast to remain unchanged with a probability of one. It is assumed that the nominal exchange rate will depreciate by 40 per cent with a probability of one. This last assumption is made to generate sufficient inflation to ensure that projected nominal rates of return lie in a range comparable to that of their initial values.[16] This admitedly artificial procedure is adopted for presentational reasons only; in a realistic simulation the introduction of changes in all or most of the exogenous variables would ensure that the initial and projected values of rates of return remained in broadly comparable ranges without the very high rates of inflation implied above. Finally, it is assumed that the future developments in the exogenous variables are uncorrelated.[17] The above future developments in the exogenous variables are summarized in Table 6.1.

Note that the above percentage changes in the exogenous variables are assumed to occur instantaneously just before the start of the two-year projection period, and to be sustained throughout the full two year period. If we did not expect the future developments in exogenous variables to take this form, then the changes in them can be

TABLE 6.1: FUTURE DEVELOPMENTS IN THE EXOGENOUS VARIABLES*

| World Price of Exports $p_W$ | | Ad Valorem Tariffs $t$ | | Real Absorption $a_R$ | |
|---|---|---|---|---|---|
| Percentage change | Probability | Percentage change | Probability | Percentage change | Probability |
| -40.0 | 0.2 | -80.0 | 0.3 | 6.0 | 0.1 |
| -30.0 | 0.6 | -60.0 | 0.4 | 7.0 | 0.8 |
| -20.0 | 0.2 | -40.0 | 0.3 | 8.0 | 0.1 |
| | 1.0 | | 1.0 | | 1.0 |

* With the exception of the nominal exchange rate, all other exogenous variables are assumed to remain unchanged with a probability of one. It is assumed that the nominal exchange rate depreciates by 40 per cent with a probability of one.

converted (i.e., temporally aggregated) into equivalent two-year sustained shocks. A method for making this conversion has been developed by Cooper, McLaren, and Powell (1985).

The future developments in the exogenous variables listed in Table 6.1 give rise to 27 scenarios (i.e., 3×3×3); see Table 6.2. For example, the first scenario is when the low world price change occurs with the low tariff cut and the low real absorption increase, etc. Note that since the future developments in the exogenous variables are assumed here to be uncorrelated, the probability of a scenario occurring is simply given by the multiplication of the probabilities for the individual settings of the exogenous variables.

## 6.2 Industry Forecasts

It is now possible to forecast industry rates of return on capital. Following from equations (3.8) and (5.20), the annual average rate of return on capital in industry j over the period t to t+2 given the future developments of scenario s, $R_{*j}^{t+2,s}$, can be written:

$$100R_{*j}^{t+2,s} = 100R_{*j}^{t} + C_{*j}^{t} + B_{*j1}^{t}\, f_{1}^{t,s} + \ldots$$
$$+ B_{*jk}^{t}\, f_{k}^{t,s} + u_{*j}^{t+2,s} \qquad j=1,\ldots,n \quad s=1,\ldots,q; \qquad (6.1)$$

where $f_h^{t,s}$ is the sustained percentage change in the $h^{th}$ exogenous factor occurring at instant t and persisting at least until t+2 given scenario s; $u_{*j}^{t+2,s}$ is the error term associated with the $j^{th}$ industry in period t+2 given scenario s; and $R_{*j}^{t}$, $C_{*j}^{t}$, and the $B_{*jv}^{t}$'s are as defined above by equations (5.1), (5.19) and (5.21), respectively.

TABLE 6.2 : ECONOMIC SCENARIOS AND THEIR ASSOCIATED PROBABILITIES

| Scenario | Forecast percentage change in the World Price of Exports $p_W$ | Probability of the World Price of Exports Forecast | Forecast percentage change in Ad Valorem Tariffs $t$ | Probability of Tariff change Forecast | Forecast percentage change in Real Absorption $a_R$ | Probability of Real Absorption Forecast | Probability of the Scenario Occurring |
|---|---|---|---|---|---|---|---|
| [I] | [II] | [III] | [IV] | [V] | [VI] | [VII] | [III] × [V] × [VII] |
| L:L:L[a] | -40.0 | 0.2 | -80.0 | 0.3 | 6.0 | 0.1 | 0.006 |
| L:L:M | -40.0 | 0.2 | -80.0 | 0.3 | 7.0 | 0.8 | 0.048 |
| L:L:H | -40.0 | 0.2 | -80.0 | 0.3 | 8.0 | 0.1 | 0.006 |
| L:M:L | -40.0 | 0.2 | -60.0 | 0.4 | 6.0 | 0.1 | 0.008 |
| L:M:M | -40.0 | 0.2 | -60.0 | 0.4 | 7.0 | 0.8 | 0.064 |
| L:M:H | -40.0 | 0.2 | -60.0 | 0.4 | 8.0 | 0.1 | 0.008 |
| L:H:L | -40.0 | 0.2 | -40.0 | 0.3 | 6.0 | 0.1 | 0.006 |
| L:H:M | -40.0 | 0.2 | -40.0 | 0.3 | 7.0 | 0.8 | 0.048 |
| L:H:H | -40.0 | 0.2 | -40.0 | 0.3 | 8.0 | 0.1 | 0.006 |
| M:L:L | -30.0 | 0.6 | -80.0 | 0.3 | 6.0 | 0.1 | 0.018 |
| M:L:M | -30.0 | 0.6 | -80.0 | 0.3 | 7.0 | 0.8 | 0.144 |
| M:L:H | -30.0 | 0.6 | -80.0 | 0.3 | 8.0 | 0.1 | 0.018 |
| M:M:L | -30.0 | 0.6 | -60.0 | 0.4 | 6.0 | 0.1 | 0.024 |
| M:M:M | -30.0 | 0.6 | -60.0 | 0.4 | 7.0 | 0.8 | 0.192 |
| M:M:H | -30.0 | 0.6 | -60.0 | 0.4 | 8.0 | 0.1 | 0.024 |
| M:H:L | -30.0 | 0.6 | -40.0 | 0.3 | 6.0 | 0.1 | 0.018 |
| M:H:M | -30.0 | 0.6 | -40.0 | 0.3 | 7.0 | 0.8 | 0.144 |
| M:H:H | -30.0 | 0.6 | -40.0 | 0.3 | 8.0 | 0.1 | 0.018 |
| H:L:L | -20.0 | 0.2 | -80.0 | 0.3 | 6.0 | 0.1 | 0.006 |
| H:L:M | -20.0 | 0.2 | -80.0 | 0.3 | 7.0 | 0.8 | 0.048 |
| H:L:H | -20.0 | 0.2 | -80.0 | 0.3 | 8.0 | 0.1 | 0.006 |
| H:M:L | -20.0 | 0.2 | -60.0 | 0.4 | 6.0 | 0.1 | 0.008 |
| H:M:M | -20.0 | 0.2 | -60.0 | 0.4 | 7.0 | 0.8 | 0.064 |
| H:M:H | -20.0 | 0.2 | -60.0 | 0.4 | 8.0 | 0.1 | 0.008 |
| H:H:L | -20.0 | 0.2 | -40.0 | 0.3 | 6.0 | 0.1 | 0.006 |
| H:H:M | -20.0 | 0.2 | -40.0 | 0.3 | 7.0 | 0.8 | 0.048 |
| H:H:H | -20.0 | 0.2 | -40.0 | 0.3 | 8.0 | 0.1 | 0.006 |
| | | | | | | | 1.000 |

* In all of the scenarios it is assumed with a probability of one that the nominal exchange rate depreciates by 40 per cent and all other exogenous variables (except for those mentioned in this table) remain unchanged.

a L = Low, M = Medium, H = High.

To illustrate equation (6.1), we will compute the rate of return on capital in the export industry if the first scenario were to occur. Note that the $C*_j{}^t$ and the $B*_{jv}{}^t$'s are listed in Tables 5.3 and 5.4, respectively. Recall from Table 6.2 that the first scenario consists of a 40 per cent decline in the world price of Australia's exports, an 80 per cent across-the-board tariff cut, a 6 per cent increase in real absorption, a 40 per cent depreciation in the nominal exchange rate and no change in the remaining exogenous variables. A 40 per cent decline in the world price of Australia's exports would cause a 10.12 percentage-point decline in the rate of return in the export industry (i.e., 0.2530 × -40). An 80 per cent across-the-board tariff cut would cause a 4.85 percentage-point decrease in the rate of return in the export industry (i.e., 0.0606 × -80). A six per cent increase in real absorption would cause a 2.06 percentage-point increase in the rate of return in the export industry (i.e., 0.3426 × 6). Finally, a 40 per cent depreciation in the nominal exchange rate would cause the rate of return in the export industry to increase by 22.55 percentage points (i.e., 0.5637 × 40). Thus the net effect due to changes in the exogenous variables, for the first scenario, is for a 9.64 percentage-point increase in the rate of return on capital in the export industry (i.e., -10.12 - 4.85 + 2.06 + 22.55). If this percentage-point change is substituted into equation (6.1) together with $C*_j{}^t$ and $R*_j{}^t$ for the export industry (i.e., -16.6271 and 0.30; see Tables 5.3 and 5.2, respectively) and it is assumed that the error term is zero, then the average annual rate of return on capital in the export industry over the next two years if the first scenario were to occur is 23.01 per cent (i.e., 100 × 0.30 - 16.6271 + 9.64 + 0.0). The forecasts of the rate of return on capital in the export industry for the remaining 26 scenarios can be calculated in a similar fashion. These forecasts, along with those for the import-competing and non-traded industries, are depicted in Figure 6.1.

It can be seen from Figure 6.1 that the export industry's forecasts have the highest variance, followed by the import-competing industry and then, finally, by the non-traded industry (although it is a borderline case between these last two industries). Furthermore, the export industry has the highest expected return on capital, followed by the import-competing industry and then by the non-traded industry. The exact vector of expected returns and variance-covariance matrix of industry rates of return can be calculated as follows.

Following from equation (3.48), the expected rate of return on industry j can be obtained by taking the expectation of equation (6.1):

$$100E^{t}(R_{*j}^{t+2,s}) = 100R_{*j}^{t} + C_{*j}^{t} + B_{*j1}^{t}\, E^{t}(f_{1}^{t}) + \dots$$
$$+ B_{*jk}^{t}\, E^{t}(f_{k}^{t,s}) \qquad j=1,\dots,n. \qquad (6.2)$$

Equation (6.2) can also be written:

$$100E^{t}(R_{*.}^{t+2,s}) = 100R_{*.}^{t} + C_{*.}^{t} + B_{*..}^{t}\, E^{t}(f_{.}^{t,s}) \quad ; \qquad (6.3)$$

where $E^{t}(R_{*.}^{t+2,s})$ is an $(n\times 1)$ vector of the expectations at time t of the average annual rates of return of the (n) industries over the period t to t+2; $R_{*.}^{t}$ is an $(n\times 1)$ vector of the average annual rates of return of the (n) industries over the period t-2 to t; $C_{*.}^{t}$ is an $(n \times 1)$ vector of the $C_{*.}^{t}$'s for the (n) industries; $B_{*..}^{t}$ is an $(n \times k)$ matrix of the sensitivity coefficients of the rates of return of the (n) industries with respect to each of the (k)

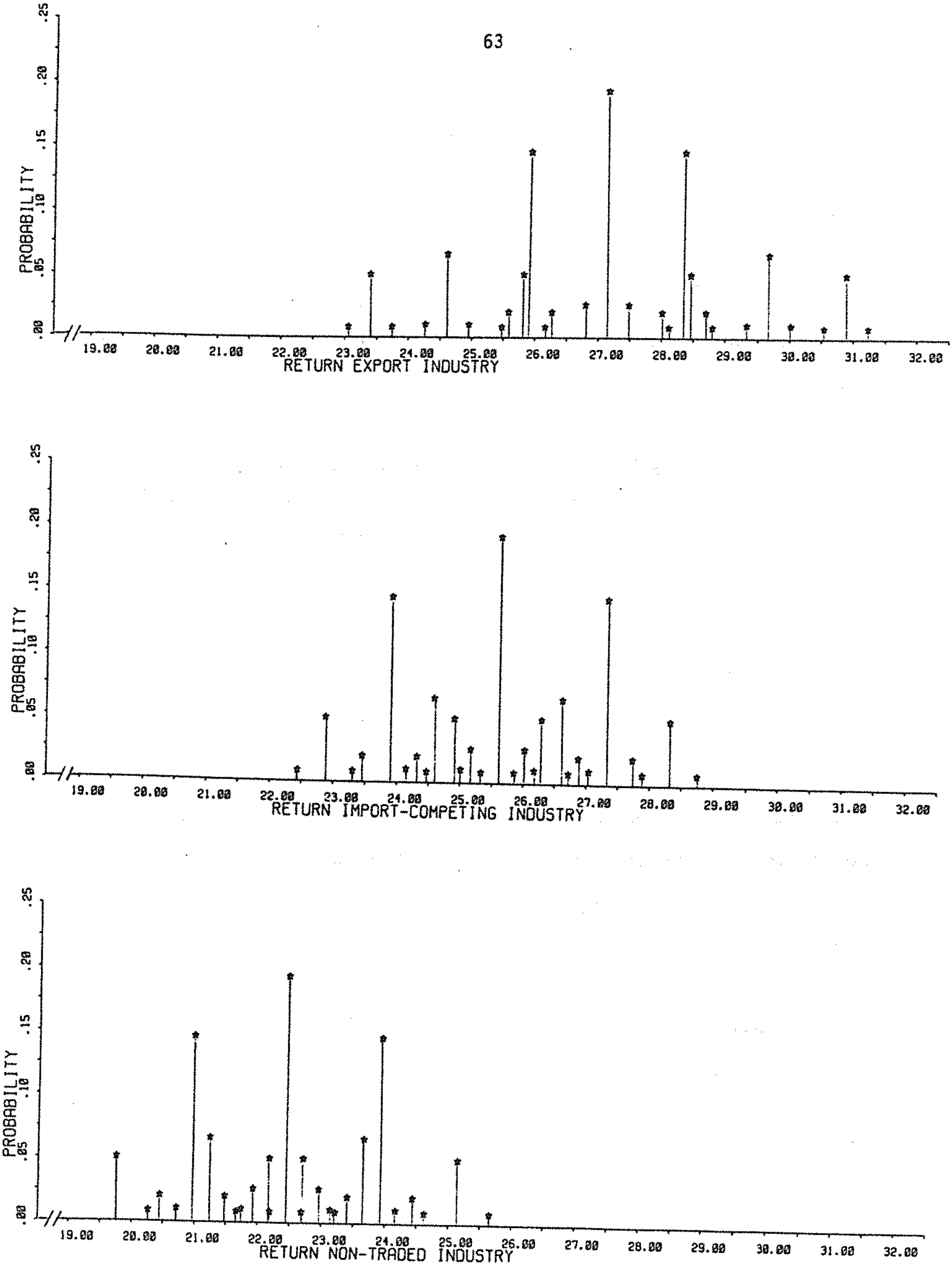


FIGURE 6.1: FORECAST AVERAGE ANNUAL PERCENTAGE-POINT RATES OF RETURN BY INDUSTRY

exogenous variables; and $E^t(f._{}^{t,s})$ is a $(k \times 1)$ vector of the expectations at time t of the percentage changes in the (k) exogenous variables. The $R_{*.}^{t}$ vector is given in Table 5.2, the $C_{*.}^{t}$ vector can be obtained from Table 5.3, the $B_{..}^{t}$ matrix is contained in Table 5.4, and the $E^t(f._{}^{t})$ vector can be calculated from the data given in Table 6.1. The vector of expected annual average rates of return can be estimated by substituting these vectors and matrix into equation (6.3):

$$100E^t(R_{*.}^{t+2,s}) = 100\begin{bmatrix} 0.30 \\ 0.25 \\ 0.20 \end{bmatrix} + \begin{bmatrix} -16.6271 \\ -15.6793 \\ -13.8902 \end{bmatrix} + \begin{bmatrix} 0.2530 & 0.0606 & 0.3426 & 0.5637 \\ 0.1014 & 0.0853 & 0.4303 & 0.5342 \\ 0.1213 & 0.0739 & 0.5144 & 0.5188 \end{bmatrix} \begin{bmatrix} -30.0 \\ -60.0 \\ 7.0 \\ 40.0 \end{bmatrix}$$

$$= \begin{bmatrix} 27.0931 \\ 25.5408 \\ 22.3896 \end{bmatrix}$$

Thus the average annual expected rates of return on capital in the export, import-competing, and non-traded industries over the period t to t+2 are 27.09 per cent, 25.54 per cent, and 22.39 per cent, respectively.

Following from equation (3.49), the variance-covariance of the rates of return on industries can be estimated as:

$$10^4 Cov^t(R_{*.}^{t+2,s}) = [B_{*..}^{t}]\, Cov^t(f_{.}^{\,t,s})\, [B_{*..}^{t}]' + Cov^t(u_{*.}^{t+2,s}) \quad ; \qquad (6.4)$$

where $Cov^t(R_{*.}^{t+2,s})$ is an $(n \times n)$ variance-covariance matrix at time t of the average annual rates of return of the (n) industries over the period t to t+2; $B_{*..}^{t}$ is an (n × k) matrix as defined above; $Cov^t(f_{.}^{t,s})$ is a (k × k) variance-covariance matrix at time t of the percentage changes in the (k) exogenous variables; and $Cov^t(u_{*.}^{t+2,s})$ in an (n × n) variance-covariance matrix at time t of the errors associated with industry rates of return forecasts. The variance-covariance matrix $Cov^t(f_{.}^{t})$ can be calculated from the data given in Table 6.1, and it is assumed for illustrative purposes that $Cov^t(u_{*.}^{t+2,s})$ is a null matrix. The variance-covariance matrix of industry rates of return can be estimated by substituting these matrices into equation (6.4):

$$10^4 Cov^t(R_{*.}^{t+2,s}) = \begin{bmatrix} 0.2530 & 0.0606 & 0.3426 & 0.5637 \\ 0.1014 & 0.0853 & 0.4303 & 0.5342 \\ 0.1213 & 0.0739 & 0.5144 & 0.5188 \end{bmatrix} \begin{bmatrix} 40.0 & 0 & 0 & 0 \\ 0 & 240.0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.0 \end{bmatrix} \begin{bmatrix} 0.2530 & 0.1014 & 0.1213 \\ 0.0606 & 0.0853 & 0.0739 \\ 0.3426 & 0.4303 & 0.5144 \\ 0.5637 & 0.5342 & 0.5188 \end{bmatrix}$$

$$= \begin{bmatrix} 3.4652 & 2.2962 & 2.3376 \\ 2.2962 & 2.1946 & 2.0492 \\ 2.3376 & 2.0492 & 1.9522 \end{bmatrix} .$$

Thus the variances of the average annual percentage-point returns on capital in the export, import-competing, and non-traded industries are 3.4652, 2.1946, and 1.9522, respectively.

As only a few of the exogenous variables were forecast to change, the above results are only illustrative. However it is conjectured that because the export industry is vulnerable to fluctuations on the world market, then in a more realistic example it would still exhibit a relatively high variance. Similarly, the import-competing industry is subject to some degree of uncertainty due to fluctuations in international markets. In a more realistic example it may continue to exhibit the second highest variance. The non-traded industry is not directly exposed to fluctuations on world markets. As a result, we would expect that in practice it would still have a relatively low variance.

Three final points should be made concerning the illustrative forecasts. The first is that they have been constructed such that industries with relatively higher expected returns also exhibit relatively higher variances (i.e., risk). There is nothing in our approach that either requires or ensures that this always occurs. The second point is that the forecasts have been constructed such that the distributions of the rates of return depicted in Figure 6.1 are overlapping. Again there is nothing in our approach that either requires or ensures that this will always occur. Finally, it is not necessary to work with a collection of discrete values for the possible developments in the exogenous variables; allowing continuous variations in them is straightforward (see Higgs (1987b)).

# 7. CORPORATE RATES OF RETURN

Here we first define the rate of return for corporations as observed by investors. We then explain the relationship assumed between this rate of return and the rate of return on capital, as discussed in section 5.1. Finally, we define a mapping from industries (the traditional subject matter of CGE models) to corporations. This mapping is based on the distribution of the corporations' capital across the areas of activity defined by the CGE model's industries.

## 7.1 Definition of Rates of Return for Corporations

The rate of return for corporations as observed on the stockmarket is defined as follows. At instant t the investor purchases one share of corporation i which the investor holds for $\ell$ years. During this period dividends are paid, and the share price may change from the initial share price. The rate of return is then defined as the sum of dividends received plus the capital gain (or loss as applicable) on the change in the share price, all divided by the initial price paid for the share.

Let $R_{i*}{}^{t}$ denote the annual average rate of return for corporation i over the period $t-\ell$ to t:

$$R_{i*}^{t} = \bar{D}_{i*}^{t}/\P_{i*}^{t-\ell} + (\P_{i*}^{t} - \P_{i*}^{t-\ell})/\P_{i*}^{t-\ell} \qquad i=1,\ldots,m;(7.1)$$

where $\bar{D}_{i*}^{t}$ is the average annual flow of dividends on one share in corporation i over the period $t-\ell$ to t; and $\P_{i*}{}^{t-\ell}$ and $\P_{i*}{}^{t}$ are the market prices of a share of corporation i at instants $t-\ell$ and t, respectively. Note that it is possible to modify equation (7.1) to allow for the effects of reinvesting dividends, double taxation (if applicable), stock splits, etc. However, equation (7.1) will suffice for our purposes.

Next we note that the rate of return on shares in corporation i over the period t to t+ℓ, $R_{i*}{}^{t+\ell}$, can be calculated:

$$R_{i*}^{t+\ell} = R_{i*}^{t} + \Delta R_{i*}^{t} \qquad i=1,\ldots,m; \quad (7.2)$$

where $\Delta R_{i*}{}^{t}$ is the change in the rate of return in corporation i over the period t to t+ℓ.

## 7.2 Relationship Between the Rates of Return for Corporations and the Rates of Return for Industries

In this section we discuss the relationship between the rates of return for corporations as observed on stockmarkets and the rates of return on capital earned by industries. Recall from equation (5.1) that the rate of return on capital in an industry is defined as the after-tax sum of the rentals earned plus the capital gain on the replacement value of the depreciated capital stock, all divided by the initial purchase price of the capital. Here we are indifferent between how rentals and capital gains earned on the replacement value of the depreciated capital stock are allocated between dividends and capital gains in terms of the share price. Rather we just assume that the <u>change</u> in the rate of return on corporations is a weighted average of the changes in the rates of return on capital in industries:

$$100\Delta R^t_{i*} = \sum_{j=1}^{n} 100\Delta R^t_{ij}\ S^t_{ij} + 100\Delta R^t_{in+1}\ S^t_{in+1} \qquad i=1,\ldots,m; \quad (7.3)$$

where $100\Delta R_{ij}{}^t$ is the percentage-point change in the annual average rate of return on the component of corporation i located in domestic industry j over the period t-$\ell$ to t; $100\Delta R_{in+1}{}^t$ is the percentage-point change in the annual average rate of return on the component of corporation i located overseas over the period t-$\ell$ to t; $S_{ij}{}^t$ is the share of corporation i's total capital stock located in domestic industry j at time t; and $S_{in+1}{}^t$ is the share of corporation i's total capital stock located overseas at time t. The $S_{ij}{}^t$ can be calculated as:

$$S^t_{ij} = \Pi^t_{ij}\ K^t_{ij} \Big/ \sum_{k=1}^{n+1} (\Pi^t_{ik}\ K^t_{ik}) \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n+1; \end{array} \quad (7.4)$$

where $\Pi_{ij}{}^t$ is the creation cost of a unit of capital of corporation i located in industry j at time t; and $K_{ij}{}^t$ is the quantity of corporation i's capital stock located in industry j at time t. Finally, note that:

$$\sum_{j=1}^{n+1} S^t_{ij} = 1 \qquad i=1,\ldots,m. \quad (7.5)$$

Our next task is to determine the link between the $\Delta R_{ij}{}^t$'s in equation (7.3) and the $\Delta R_{*j}{}^t$'s discussed in section 5. It is desirable to allow for the possibility of differential changes in the rates of return on capital between corporations located in the same industry. For example, it may be that due to, say, better management techniques, a corporation earns a higher rate of return than the average for the industry it operates in. As the CGE model will only project the average response for the industry, some additional shift variables must be included to explicitly account for the factors that distinguish this corporation from the others in the industry. Two mappings are derived in this paper. The first mapping allows for differential changes in just the rates of return projections between corporations located in the same industry. The second mapping not only allows for differential changes in rates of return projections but also specific differential changes in rentals, creation costs of capital, tax rates, and depreciation rates between corporations located in the same industry. This second mapping is more general; however it is significantly more complex and data-intensive relative to the first mapping. As a result, the derivation of the second mapping has been relegated to Appendix A.2. Below we derive the first mapping.

The annual average percentage-point change in the rate of return on the component of corporation i located in domestic industry j

$$100\Delta R_{ij}^{t} = 100\Delta R_{*j}^{t} + h_{ij}^{(100\Delta R)t} \qquad \begin{matrix} i=1,\dots,m \\ j=1,\dots,n \end{matrix} \quad ; \qquad (7.6)$$

where $h_{ij}^{(100\Delta R)t}$ is the annual average percentage-point differential between the rate of return on the capital of corporation i located in industry j and the overall rate of return on capital in industry j. For example, if it is known that the component of corporation i located in industry j always earns a rate of return which is, say, 5 percentage-points higher than the average for industry j, then this can be incorporated by setting $h_{ij}^{(100\Delta R)t}$ equal to 5.0. Note that for consistency between an industry projection and the projections of the corporations located within that industry the following condition must hold:

$$\sum_{i=1}^{m} h_{ij}^{(100\Delta R)t} \; S_{ij}^{t} = 0 \qquad j=1,\ldots,n. \qquad (7.7)$$

The next step is to substitute equation (7.6) into equation (7.3):

$$100\Delta R_{i*}^{t} = \sum_{j=1}^{n} \left[100\Delta R_{*j}^{t} + h_{ij}^{(100\Delta R)t}\right] S_{ij}^{t}$$

$$+ 100\Delta R_{in+1}^{t} \; S_{in+1}^{t} \qquad i=1,\ldots,m. \qquad (7.8)$$

We can now substitute equation (7.8) into equation (7.2):

$$100R_{i*}^{t+\ell} = 100R_{i*}^{t} + \sum_{j=1}^{n} \left[100\Delta R_{*j}^{t} + h_{ij}^{(100\Delta R)t}\right] S_{ij}^{t}$$

$$+ 100\Delta R_{in+1}^{t} \; S_{in+1}^{t} \qquad i=1,\ldots,m. \qquad (7.9)$$

Thus given estimates of the distribution of a corporation's capital stocks, the projected changes in rates of return by industries, and the relative performance of the corporation within each industry, it is possible to forecast the rate of return by corporation.

Now recall from equations (5.18) to (5.21) that:

$$100\Delta R_{*j}^{t} = C_{*j}^{t} + B_{*j1}^{t} f_{1}^{t} + \ldots + B_{*jk}^{t} f_{k}^{t} \qquad j=1,\ldots,n; \qquad (7.10)$$

where

$$C_{*j}^{t} = \pi_{*j}^{t-\ell} \gamma_{*j}^{t} + 100\Delta\tau_{*j}^{t} \lambda_{*j}^{t} + 100\Delta d_{*j}^{t} \psi_{*j}^{t} \qquad j=1,\ldots,n; \qquad (7.11)$$

and

$$B^t_{*jv} = (\alpha^t_{*j}/L)\eta_{P_{*j}F_v} + \beta^t_{*j}\ \eta_{\Pi_{*j}F_v} \qquad j=1,\ldots,n;\ v=1,\ldots,k. \qquad (7.12)$$

Thus equation (7.9) can be written:

$$100R^{t+\ell}_{i*} = 100R^t_{i*} + \sum_{j=1}^{n}[C^t_{*j} + B^t_{*j1}f^t_1 + \ldots + B^t_{*jk}f^t_k + h^{(100\Delta R)t}_{ij}]S_{ij}$$

$$+\ 100\Delta R^t_{in+1}\ S^t_{in+1} \qquad i=1,\ldots,m. \qquad (7.13)$$

The similarity between equation (7.13) and the return generating process as expressed in section 3, equation (3.8), can be more clearly seen if we rewrite equation (7.13):

$$100R^{t+\ell}_{i*} = 100R^t_{i*} + \sum_{j=1}^{n}[C^t_{*j} + h^{(100\Delta R)t}_{ij} + b^t_{*j1}(F^{t+\ell}_1 - F^t_1) + \ldots$$

$$+\ b^t_{*jk}(F^{t+\ell}_k - F^t_k)]S^t_{ij} + 100\Delta R^t_{in+1}\ S^t_{in+1} + u^{t+\ell}_{i*}$$

$$i=1,\ldots,m; \qquad (7.14)$$

where

$$b^t_{*jv} = B^t_{*jv}f^t_v\ F^t_v/100 \qquad j=1,\ldots,n;\ v=1,\ldots,k. \qquad (7.15)$$

Again note that the term $C_{*j}{}^t$ is implicitly contained in equation (3.8) where the factors are changes in the creation costs of capital over the period $t-\ell$ to $t$, the tax rate over the period $t$ to $t+\ell$, and the depreciation rate over the period $t$ to $t+\ell$. The term $h_{ij}{}^{(100\Delta R)t}$ is also implicitly contained in equation (3.8); however in this case the factors are events or effects not captured by the CGE model.

## 7.3 A Mapping of Some Illustrative Corporations Across Industries

In this section we define a mapping of some illustrative corporations across industries and specify rates of return for these corporations. As in section 5, we assume that the forecast period is two years ahead (i.e., $\ell=2$). In practice the historical annual average rates of return on corporations would be calculated directly from stockmarket data. However, for this illustrative example we used the following equation:

$$100R_{i*}^{t} = \sum_{j=1}^{n} (100R_{*j}^{t} + h_{ij}^{(100\Delta R)t-2})S_{ij}^{t}$$

$$+ 100R_{in+1}^{t} S_{in+1}^{t} \qquad i=1,\ldots,m. \qquad (7.16)$$

A mapping of the distribution of six hypothetical corporations' capital stocks across industries (i.e., the $S_{ij}^{t}$'s) is given in Table 7.1. (Note that these six do not provide an exhaustive list.) Also contained in Table 7.1 are estimates of the performance of these corporations relative to the industry averages (i.e., the $h_{ij}^{(100\Delta R)t-2}$'s). Equation (7.16) also requires the industry rates of return and the foreign rate of return. Recall from Table 5.2 that the average annual rates of return over the period t-2 to t for the export, import-competing, and non-traded industries are 30 per cent, 25 per cent, and 20 per cent, respectively. Finally, it is assumed that the foreign rate of return is 25 per cent for all corporations.

It can be seen from Table 7.1 that the first three corporations AG, TCF, and SERVICES are located solely in the export, import-competing, and non-traded industries, respectively. Furthermore

TABLE 7.1: A MAPPING OF SOME ILLUSTRATIVE CORPORATIONS ACROSS INDUSTRIES

| Corporation | Domestic Industry | | | Foreign Industries | Total |
|---|---|---|---|---|---|
| | 1. Export | 2. Import-Competing | 3. Non-Traded | | |
| 1. AG | | | | | |
| Capital Stocks, $\Pi_{ij}{}^t K_{ij}{}^t$ | \$100m | 0 | 0 | 0 | \$100m |
| Shares, $S_{ij}{}^t$ | 1.0 | 0 | 0 | 0 | 1.0 |
| Shift variable, $h_{ij}^{(100\Delta R)t-2}$ | 0 | 0 | 0 | 0 | 0 |
| 2 TCF | | | | | |
| Capital Stocks, $\Pi_{ij}{}^t K_{ij}{}^t$ | 0 | \$100m | 0 | 0 | \$100m |
| Shares, $S_{ij}{}^t$ | 0 | 1.0 | 0 | 0 | 1.0 |
| Shift variable, $h_{ij}^{(100\Delta R)t-2}$ | 0 | 0 | 0 | 0 | 0 |
| 3. SERVICES | | | | | |
| Capital Stocks, $\Pi_{ij}{}^t K_{ij}{}^t$ | 0 | 0 | \$100m | 0 | \$100m |
| Shares, $S_{ij}{}^t$ | 0 | 0 | 1.0 | 0 | 1.0 |
| Shift variable, $h_{ij}^{(100\Delta R)t-2}$ | 0 | 0 | 0 | 0 | 0 |
| 4. MIX | | | | | |
| Capital Stocks, $\Pi_{ij}{}^t K_{ij}{}^t$ | 0 | \$50m | \$50m | 0 | \$100m |
| Shares, $S_{ij}{}^t$ | 0 | 0.5 | 0.5 | 0 | 1.0 |
| Shift variable, $h_{ij}^{(100\Delta R)t-2}$ | 0 | 0 | 0 | 0 | 0 |
| 5. OS | | | | | |
| Capital Stocks, $\Pi_{ij}{}^t K_{ij}{}^t$ | 0 | 0 | \$50m | \$50m | \$100m |
| Shares, $S_{ij}{}^t$ | 0 | 0 | 0.5 | 0.5 | 1.0 |
| Shift variable, $h_{ij}^{(100\Delta R)t-2}$ | 0 | 0 | 0 | 0 | 0 |
| 6. STAR | | | | | |
| Capital Stocks, $\Pi_{ij}{}^t K_{ij}{}^t$ | 0 | 0 | \$100m | 0 | \$100m |
| Shares, $S_{ij}{}^t$ | 0 | 0 | 1.0 | 0 | 1.0 |
| Shift variable, $h_{ij}^{(100\Delta R)t-2}$ | 0 | 0 | 1.0 | 0 | 1.0 |

the rates of return for these corporations are assumed not to deviate from the industry rates of return (i.e., their $h_{ij}^{(100\Delta R)t-2}$'s are equal to zero). Thus the rates of return for the AG, TCF, and SERVICES corporations are 30 per cent, 25 per cent, and 20 per cent, respectively.

The operations of the MIX corporation are half located in the import-competing industry and half in the non-traded industry. Furthermore, the components of the MIX corporation located in these industries are assumed not to deviate from the respective industry rates of return. Thus the rate of return for the MIX corporation is 22.5 per cent (i.e., $0.5 \times 25 + 0.5 \times 20$).

The operations of the OS corporation are also located across more than one industry. The OS corporation is half located in the domestic non-traded industry and half in foreign industries. As above, the rates of return for components of the OS corporation located in these industries are assumed not to deviate from the respective industry rates of return. Furthermore, as it assumed that the foreign industries earn a 25 per cent rate of return, then the rate of return for the OS corporation is also 22.5 per cent (i.e., $0.5 \times 20 + 0.5 \times 25$).

Finally, the STAR corporation is assumed to be located solely in the non-traded industry. Due to, say, better than average management techniques, the STAR corporation earns a rate of return that is one percentage-point higher than the rate of return for the non-traded industry as a whole. Thus the rate of return for the STAR corporation is 21 per cent (i.e., $20 + 1$).

# 8. CORPORATE FORECASTS

In this section it is shown how a CGE model can be used for forecasting rates of return for corporations. First the theory for forecasting the rates of return and for calculating the variance-covariance matrix of forecasting errors is developed. Then stochastic scenarios for corporation-specific exogenous variables are detailed for the six illustrative corporations discussed in section 7. Finally, these developments together with those for the other exogenous variables, as specified in section 6.1, are used to produce forecasts of the rates of return by corporations and to estimate the variance-covariance matrix of these forecasts.

## 8.1 Forecasting the Rates of Return for Corporations

Following from equations (3.8) and (7.13), the average annual rate of return for corporation i over the period t to t+2 given the future developments of scenario s, $R_{i*}^{t+2,s}$, can be written:

$$
\begin{aligned}
100R_{i*}^{t+2,s} = 100R_{i*}^{t} + \sum_{j=1}^{n} [C_{*j}^{t} + h_{ij}^{(100\Delta R)t,s} + \\
B_{*j1}^{t} f_{1}^{t,s} + \ldots + B_{*jk}^{t} f_{k}^{t,s}] S_{ij}^{t} + \\
100\Delta R_{in+1}^{t,s} \; S_{in+1}^{t} + u_{i*}^{t+2,s} \qquad i=1,\ldots,m \\
s=1,\ldots,q; \qquad (8.1)
\end{aligned}
$$

where $R_{i*}^{t}$ is the average annual rate of return for corporation i prior to time t; $C_{*j}^{t}$ is defined in equation (5.19); $h_{ij}^{(100\Delta R)t,s}$ is the value assigned to the average annual percentage-point differential between the rate of return on the component of corporation i located in industry j and the rate of return of industry j; the $B_{*jv}^{t}$'s are defined in equation (5.21); $f_v^{t,s}$ is the percentage change in the $v^{th}$ exogenous variable given scenario s; $S_{ij}^{t}$ is the share of corporation i's total capital stock located in domestic industry j at t; $100\Delta R_{in+1}^{t,s}$ is the percentage-point change in the rate of return of the foreign component of corporation i given scenario s; $S_{in+1}^{t}$ is the share of corporation i's total capital stock located overseas at t; and $u_{i*}^{t+2,s}$ is the error term associated with the $i^{th}$ corporation given scenario s.

Following from equation (3.48), the expected rate of return for corporation i can be obtained by taking the expectation of equation (8.1):

$$100E^t(R_{i*}^{t+2,s}) = 100R_{i*}^{t} + \sum_{j=1}^{n}[C_{*j}^{t} + E^t(h_{ij}^{(100\Delta R)t,s}) + B_{*j1}^{t}E^t(f_1^{t,s}) + \ldots + B_{*jk}^{t}E^t(f_k^{t,s})]S_{ij}^{t} + 100E^t(\Delta R_{in+1}^{t,s})S_{in+1}^{t} \qquad i=1,\ldots,m. \tag{8.2}$$

Equation (8.2) can also be written:

$$100E^t(R_{.*}^{t+2,s}) = 100\,R_{.*}^{t} + S_{..}^{t}\,C_{*.}^{t} + S_{..}^{t} \circ E^t(h_{..}^{(100\Delta R)t,s})\underline{1} + S_{..}^{t}\,B_{*..}^{t}\,E^t(f_{.}^{t,s}) + 100\,S_{.n+1}^{t}\,E^t(\Delta R_{.n+1}^{t,s}) \quad ; \tag{8.3}$$

where $E^t(R_{.*}^{t+2,s})$ is an $(m\times 1)$ vector of the expectations at time t of the average annual rates of return of the (m) corporations over the period t to t+2; $R_{.*}^{t}$ is an $(m\times 1)$ vector of the average annual rates of return of the (m) corporations prior to time t; $S_{..}^{t}$ is an $(m\times n)$ matrix of the shares of the capital stocks of the (m) corporations located across the (n) domestic industries; $C*_{.}^{t}$ is an (n×1) vector of the $C*_{j}^{t}$'s for the (n) industries; $E^{t}(h_{..}^{(100\Delta E)t,s})$ is an (m×n) matrix of the expectations at time t of the rate-of-return shift variables for the components of the (m) corporations located in the (n) industries; $\underline{1}$ is an (n×1) unit vector; $B*_{..}^{t}$ is an (n×k) matrix of the factor sensitivities for each of the (n) industries with respect to the (k) exogenous variables; $E^{t}(f_{.}^{t,s})$ is a (k×1) vector of the expectations at time t of the percentage changes in the (k) exogenous variables; $S_{.n+1}^{t}$ is an (m×m) diagonal matrix of the shares of foreign capital owned by the (m) corporations in their respective total capital stocks; and $E^{t}(\Delta R_{.n+1}^{t,s})$ is an (m×1) vector of the expectations at time t of the changes in the rates of return of the foreign capital stock owned by the (m) corporations. Note that the operator "∘" in equation (8.3) denotes a Hadamard product.[18]

The next task is to compute the variance-covariance matrix of rates of return by corporations. To do this we first express equation (8.1) in matrix notation:

$$100R_{.*}^{t+2,s} = Ab + c \quad ; \tag{8.4}$$

where $R_{.*}^{t+2,s}$ is an (m×1) vector of the average annual rates of return of the (m) corporations over the period t to t+2 given scenario s; A is an (m ×(m×n + n×k + 2m)) matrix of shares and coefficients; b is a ((m×n + n×k + 2m)× 1) vector of the stochastic exogenous variables; and c is an (m×1) vector of the non-stochastic elements of equation (8.1) for each of the (m) corporations. Equation (8.4) is depicted in Figure 8.1.

The variance-covariance matrix of the rates of return by corporations, $Cov^t(R_{.*}^{t+2,s})$, can be estimated from equation (8.4):

$$10^4 Cov^t(R_{.*}^{t+2,s}) = A\ Cov^t(b)A' \quad ; \qquad (8.5)$$

where $Cov^t(b)$ is the variance-covariance matrix of the exogenous variables. A decomposition of the $Cov^t(b)$ matrix is depicted in Figure 8.2.

The number of calculations involved when illustrating equation (8.5) can be dramatically reduced if some assumptions are made concerning the structure of the $Cov^t(b)$ matrix. Here we will assume that the error terms are uncorrelated with the exogenous variables. This means that $H_1U, \ldots, H_mU$, FU and RU are null matrices. We will also assume that the variance-covariance matrix of error terms (i.e., U) is a null matrix. The third assumption we make is that the performance of a corporation relative to the average performance of the industry it operates in is uncorrelated with developments in the economy (i.e., with the shocks to the CGE model). Thus $H_1F, \ldots, H_mF$ are null matrices. Next we assume that the performance of a corporation relative to the average performances of the domestic industries it operates in is uncorrelated with the index of the changes in the rate of return on its foreign investments. Hence $H_1R, \ldots, H_mR$ are null matrices. Perhaps more contentiously, for illustrative purposes we also assume that changes in the rates of return on the foreign investments of corporations are uncorrelated with the exogenous variables affecting the domestic economy; that is, the matrix FR is null. Finally, it is assumed that the relative performance of a corporation with respect to industry averages is not correlated with

$$
100\begin{bmatrix} R^{t+2,s}_{1*} \\ \vdots \\ R^{t+2,s}_{m*} \end{bmatrix}
=
\left[\begin{array}{ccccccccc|ccccc}
S^t_{11} & \cdots & S^t_{1n}, & 0 & \cdots & 0, & 0 \cdots 0, & 0 & \cdots\ 0, & B^t_{*11}\,S^t_{11}, & \cdots, & B^t_{*1k}\,S^t_{11}, & \cdots, & B^t_{*n1} \\
0 & \ddots & & & & & & & \vdots & \vdots & & \vdots & & \\
\vdots & & & & & & & \ddots & 0 & \vdots & & \vdots & & \\
0 & \cdots & 0, & 0 & \cdots & 0, & 0 \cdots 0, & S^t_{m1} & \cdots\ S^t_{mn}, & B^t_{*11}\,S^t_{m1}, & \cdots, & B^t_{*1k}\,S^t_{m1}, & \cdots, & B^t_{*n1}
\end{array}\right]
$$

$$
\underbrace{\overbrace{\qquad}^{n}\cdots\cdots\overbrace{\qquad}^{n}}_{m \times n}\;\underbrace{\overbrace{\qquad\qquad}^{k}\cdots\cdots}_{n \times k}
$$

$(m \times 1)$ $(m \times (m \times n + n \times k)$

$$
+\ 100\begin{bmatrix} R^t_{1*} \\ \vdots \\ R^t_{m*} \end{bmatrix}
+
\begin{bmatrix} S^t_{11}, & \cdots, & S^t_{1n} \\ \vdots & & \vdots \\ S^t_{m1} & \cdots, & S^t_{mn} \end{bmatrix}
\begin{bmatrix} C^t_{*1} \\ \vdots \\ C^t_{*n} \end{bmatrix}
$$

$(m \times 1)$ $(m \times n)$ $(n \times 1)$

FIGURE 8.1: DIAGRAMATIC REPRESENTATION OF THE PROCESS
RATES OF RETURN FOR CORPORATIONS OVER THE

$$
\begin{bmatrix}
s^t_{1n}, \ldots, s^t_{*nk}\; s^t_{1n}, & 100s^t_{1n+1}, 0, \ldots, 0, \quad 0 & 1, 0, \ldots, 0, 0 \\
\vdots \qquad\quad \vdots & 0 \;\; \ddots \qquad\qquad \vdots & 0 \;\; \ddots \qquad \vdots \\
\vdots \qquad\quad \vdots & \vdots \qquad \ddots \qquad 0 & \vdots \qquad \ddots \quad 0 \\
s^t_{mn}, \ldots, s^t_{*nk}\; s^t_{mn}, & 0, 0, \ldots, 0, 100s^t_{mn+1} & 0, 0, \ldots, 0, 1
\end{bmatrix}
\begin{bmatrix}
h^{(100\Delta R)t,s}_{11} \\ \vdots \\ h^{(100\Delta R)t,s}_{1n} \\ \vdots \\ h^{(100\Delta R)t,s}_{m1} \\ \vdots \\ h^{(100\Delta R)t,s}_{mn} \\ f^{t,s}_{1} \\ \vdots \\ f^{t,s}_{k} \\ \vdots \\ f^{t,s}_{1} \\ \vdots \\ f^{t,s}_{k} \\ \Delta R^{t,s}_{1n+1} \\ \vdots \\ \Delta R^{t,s}_{mn+1} \\ u^{t+2,s}_{1*} \\ \vdots \\ u^{t+2,s}_{m*}
\end{bmatrix}
\begin{array}{l}
\left.\right\} n \\ \vdots \; m \times n \\ \left.\right\} n \\ \left.\right\} k \\ \vdots \; n \times k \\ \left.\right\} k \\ \left.\right\} m \\ \left.\right\} m
\end{array}
$$

$$\underbrace{\qquad\qquad}_{k} \quad \underbrace{\qquad\qquad}_{m} \quad \underbrace{\qquad\qquad}_{m}$$

+ 2m))

((m×n + n×k + 2m) × 1)

FOR GENERATING THE AVERAGE ANNUAL
PERIOD t TO t+2 GIVEN SCENARIO s

| | $h_{11}^{(100\Delta R)t,s}$ . . . | $h_{1n}^{(100\Delta R)t,s}$ . . . . . . . . | $h_{m1}^{(100\Delta R)t,s}$ . . . $h_{mn}^{(100\Delta R)t,s}$ | $f_1^{t,s}$ . . . | $f_k^{t,s}$ . . . . . . | $f_1^{t,s}$ . . . $f_k^{t,s}$ | $\Delta R_{1n+1}^{t,s}$ . . . $\Delta R_{mn+1}^{t,s}$ | $u_{1*}^{t+2,s}$ . . . $u_{m*}^{t+2,s}$ |
|---|---|---|---|---|---|---|---|---|
| $h_{11}^{(100\Delta R)t,s}$ ⋮ $h_{1n}^{(100\Delta R)t,s}$ | $H_{1,1}$ | ... | $H_{1,m}$ | $H_1F$ | ... | $H_1F$ | $H_1R$ | $H_1U$ |
| ⋮ $h_{m1}^{(100\Delta R)t,s}$ ⋮ | ⋮ | ⋱ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $h_{mn}^{(100\Delta R)t,s}$ | $H_{m,1}$ | ... | $H_{m,m}$ | $H_mF$ | ... | $H_mF$ | $H_mR$ | $H_mU$ |
| $f_1^{t,s}$ ⋮ $f_k^{t,s}$ | $H_1F$ | ... | $H_mF$ | F | ... | F | FR | FU |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $f_1^{t,s}$ ⋮ $f_k^{t,s}$ | $H_1F$ | ... | $H_mF$ | F | ... | F | FR | FU |
| $\Delta R_{1n+1}^{t,s}$ ⋮ $\Delta R_{mn+1}^{t,s}$ | $H_1R$ | ... | $H_mR$ | FR | ... | FR | R | RU |
| $u_{1*}^{t+2,s}$ ⋮ $u_{m*}^{t+2,s}$ | $H_1U$ | ... | $H_mU$ | FU | ... | FU' | RU | U |

FIGURE 8.2: THE VARIANCE-COVARIANCE MATRIX FOR THE STOCHASTIC VARIABLES

the relative performance of other corporations. Thus, of the top left-hand corner block of submatrices in Figure 8.2, only the diagonal $H_{i,i}$ matrices contain non-zero elements. Given the above assumptions, the structure of the $Cov^t(b)$ matrix reduces to a "diagonal" of non-zero submatrices; see Figure 8.3.

Finally, given the structure of the $Cov^t(b)$ matrix depicted in Figure 8.3, it is possible to rewrite equation (8.5):

$$
\begin{aligned}
10^4 Cov^t(R^{t+2,s}_{.*}) = {} & \Xi^t_{1.}\ Cov^t\ (H_{1,1})\ \Xi^{t'}_{1.} + \ldots \\
& + \Xi^t_{m.}\ Cov^t\ (H_{m,m})\ \Xi^{t'}_{m.} + \\
& + S^t_{..}\ B^t_{*..}\ Cov^t(f^{t,s}_{.})\ [B^t_{*..}]'\ [S^t_{..}]' \\
& + 10^4\ S^t_{.n+1}\ Cov^t(\Delta R^{t,s}_{.n+1})\ S^t_{.n+1} \qquad ; \qquad (8.6)
\end{aligned}
$$

where $\Xi_{i.}{}^t$ is an $(m{\times}n)$ submatrix of A, consisting of the $m^{th}$ block of n columns of the A matrix (i.e., it is a matrix of zeros with the exception of the $i^{th}$ row which consists of the shares $S_{i1}{}^t,\ldots,S_{in}{}^t$); $Cov^t(H_{i,i})$ is an $(n{\times}n)$ variance-covariance matrix at t of the shift variables describing how corporation i performs relative to the average performance of the (n) domestic industries; $S_{..}{}^t$ is an $(m{\times}n)$ matrix of the shares of the capital stocks of the (m) corporations located across the (n) domestic industries; $B*_{..}{}^t$ is an $(n{\times}k)$ matrix of the sensitivity coefficients for each of the (n) industries with respect to the (k) exogenous variables (see equation (5.21); $Cov^t(f_{.}{}^{t,s})$ is a $(k{\times}k)$ variance-covariance matrix at time t of the percentage changes in the (k) exogenous variables; $S_{.n+1}{}^t$ is an $(m{\times}m)$ diagonal matrix of the shares of foreign capital owned by the (m) corporations in their

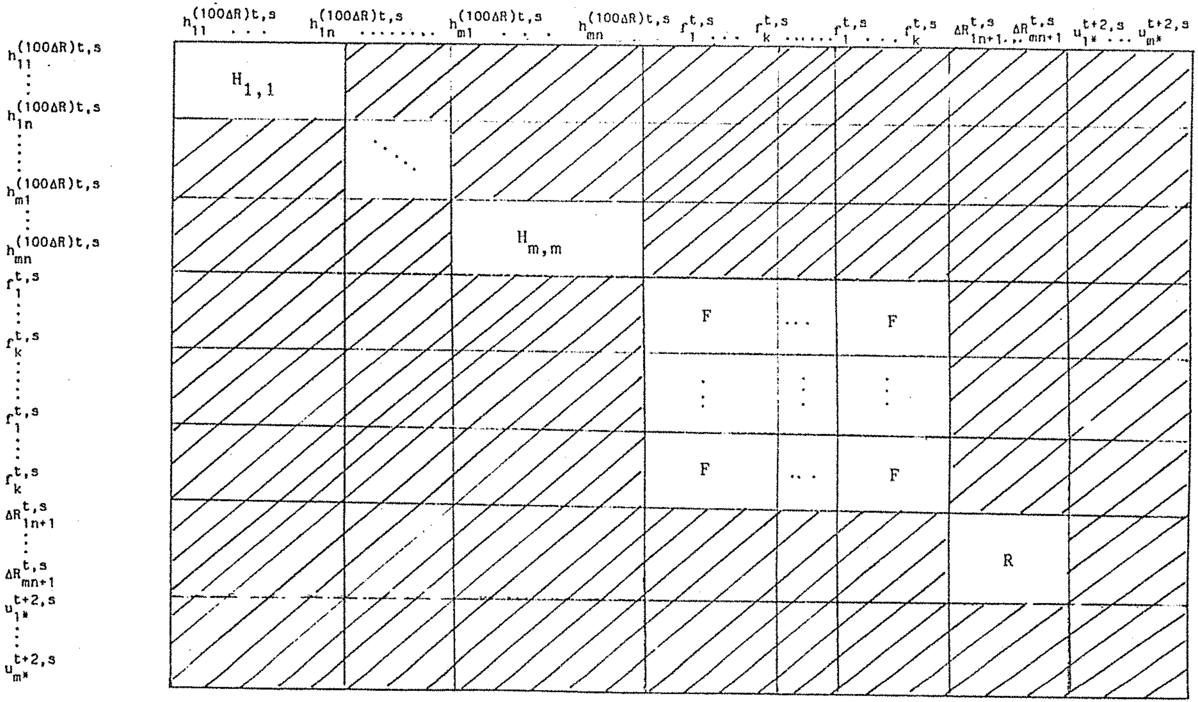


FIGURE 8.3: THE ASSUMED NON-ZERO COMPONENTS OF THE VARIANCE-COVARIANCE MATRIX FOR THE STOCHASTIC VARIABLES

respective capital stocks; and $Cov^t(\Delta R_{.n+1}{}^{t,s})$ is an $(m \times m)$ variance-covariance matrix at time t of the changes in the rates of return of the foreign capital stock owned by the (m) corporations.

## 8.2 Future Developments in the Corporation-Specific Exogenous Variables

Before equations (8.3) and (8.6) can be evaluated we have to specify the corporation-specific terms of $E^t(h_{..}{}^{(100\Delta R)t,s})$, $E^t(\Delta R_{.n+1}{}^{t,s})$, $Cov^t(H_{i,i})$, and $Cov^t(\Delta R_{.n+1}{}^{t,s})$.

Of the six illustrative corporations only one is projected to deviate from the industry average rates of return. It is assumed that the STAR corporation (i=6) is expected to earn a rate of return that is one percentage-point higher than the rate of return for the non-traded industry (j=3):

$$E^t(h_{63}^{(100\Delta R)t,s}) = 1.0 \quad ;$$

and

$$Var^t(h_{63}^{(100\Delta R)t,s}) = 0.05 \quad .$$

Futhermore, it is assumed with certainty that all other corporations earn the industry average rate of return.

Of the six illustrative corporations only one had part of its capital stock located overseas (see Table 7.1). It is assumed that the expectation and variance of the percentage-point change in the rate of return of the foreign component of the OS corporation (i=5) are:

$$10^2 E^t(\Delta R_{5n+1}^{t,s}) = 0.5 \quad ;$$

and

$$10^4 \mathrm{Var}^t(\Delta R^{t,s}_{5n+1}) = 3.0 \quad .$$

## 8.3 Calculation of the Expectations and Variance-Covariance Matrix of the Forecast Rates of Return for the Illustrative Corporations

First we calculate the vector of expected rates of return for the six illustrative corporations. Note that the $R_{.*}^t$ vector can be obtained from section 7.3. The $S_{..}^t$ matrix and the $S_{.n+1}^t$ vector are contained in Table 7.1. The $C*_{.}^t$ vector can be obtained from Table 5.3. The $E^t(h_{..}^{(100\Delta R)t,s})$ and the $E^t(\Delta R_{.n+1}^{t,s})$ matrices can be calculated from section 8.2. The $B*_{..}^t$ matrix is contained in Table 5.4. Finally, the $E^t(f_{.}^{t,s})$ vector can be calculated from the data given in Table 6.1. The vector of expected average annual rates of return can be estimated by substituting these vectors and matrices into equation (8.3). This is shown in Figure 8.4. Thus the average annual expected rates of return over the period t to t+2 in the AG, TCF, SERVICES, MIX, OS, and STAR corporations are 27.09 per cent, 25.54 per cent, 22.39 per cent, 23.97 per cent, 23.94 per cent, and 24.39 per cent, respectively.

As a brief aside, note that when calculating the expected returns for the corporations we implicitly made use of corporate factor sensitivity coefficients. These sensitivity coefficients show the effect on the rate of return of a corporation of a one per cent increase in each of the exogenous variables (i.e., factors). Let $B_{i*v}^t$ denote the sensitivity coefficient at time t for corporation i with respect to

$$100E^{t}(R_{.*}^{t+2,s}) = 100 \begin{bmatrix} 0.300 \\ 0.250 \\ 0.200 \\ 0.225 \\ 0.225 \\ 0.210 \end{bmatrix} + \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ 0.0 & 0.5 & 0.5 \\ 0.0 & 0.0 & 0.5 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} -16.6271 \\ -15.6793 \\ -13.8902 \end{bmatrix} + \begin{bmatrix} 1.0\times0.0 & 0.0\times0.0 & 0.0\times0.0 \\ 0.0\times0.0 & 1.0\times0.0 & 0.0\times0.0 \\ 0.0\times0.0 & 0.0\times0.0 & 1.0\times0.0 \\ 0.0\times0.0 & 0.5\times0.0 & 0.5\times0.0 \\ 0.0\times0.0 & 0.0\times0.0 & 0.5\times0.0 \\ 0.0\times0.0 & 0.0\times0.0 & 1.0\times1.0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$+ \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ 0.0 & 0.5 & 0.5 \\ 0.0 & 0.0 & 0.5 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.2530 & 0.0606 & 0.3426 & 0.5637 \\ 0.1014 & 0.0853 & 0.4303 & 0.5342 \\ 0.1213 & 0.0739 & 0.5144 & 0.5188 \end{bmatrix} \begin{bmatrix} -30.0 \\ -60.0 \\ 7.0 \\ 40.0 \end{bmatrix} + 100 \begin{bmatrix} 0.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.005 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 27.0931 \\ 25.5408 \\ 22.3896 \\ 23.9652 \\ 23.9448 \\ 24.3896 \end{bmatrix}$$

FIGURE 8.4: ILLUSTRATIVE COMPUTATION OF EQUATION (8.3)

the $v^{th}$ factor. Then:

$$B^{t}_{i*v} = \sum_{j=1}^{n} B^{t}_{*jv} S^{t}_{ij} \qquad \begin{array}{l} i=1,\ldots,m \\ v=1,\ldots,k; \end{array} \qquad (8.7)$$

where the $B*_{jv}{}^{t}$'s are the industry sensitivity coefficients; see equation (5.21). The sensitivity coefficients for the six illustrative corporations are given in Table 8.1.

Finally, we calculate the variance-covariance matrix of rates of return for the six illustrative corporations. Note that the non-zero elements of the $\Xi_{i.}{}^{t}$ matrices are contained in Table 7.1. The $Cov^{t}(H_{i,i})$ matrices can be calculated from section 8.2. The $Cov^{t}(f_{.}{}^{t,s})$ can be calculated from the data given in Table 6.1. Finally, the $Cov^{t}(\Delta R_{.n+1}{}^{t,s})$ matrix can also be inferred from section 8.2; it is a null matrix apart from the fifth diagonal element. The variance-covariance matrix of rates of return by corporations can be calculated by substituting these vectors and matrices, together with $S_{..}{}^{t}$, $B*_{..}{}^{t}$, and $S_{.n+1}{}^{t}$, into equation (8.6). The resultant computation is shown in Figure 8.5. Thus the variances of the percentage-point rates of return forecasts over the period t to t+2 for the AG, TCF, SERVICES, MIX, OS, and STAR corporations are 3.4652, 2.1946, 1.9522, 2.0613, 1.2381, and 2.0022, respectively.

## 8.4 Corporate Betas

As a final note to this section we explain how the approach developed here could also be used to calculate the betas of the capital asset pricing model discussed in section 3. To calculate betas we must first define a market index:

TABLE 8.1: CORPORATE FACTOR SENSITIVITY COEFFICIENTS*

| Corporation | Factor | | | |
|---|---|---|---|---|
| | World Price of Exports $p_W$ | Ad Valorem Tariffs $t$ | Real Absorption $a_R$ | Nominal Exchange Rate $\phi$ |
| 1. AG | 0.2530 | 0.0606 | 0.3426 | 0.5637 |
| 2. TCF | 0.1014 | 0.0853 | 0.4303 | 0.5342 |
| 3. SERVICES | 0.1213 | 0.0739 | 0.5144 | 0.5188 |
| 4. MIX | 0.1114 | 0.0796 | 0.4724 | 0.5265 |
| 5. OS | 0.0607 | 0.0370 | 0.2572 | 0.2594 |
| 6. STAR | 0.1213 | 0.0739 | 0.5144 | 0.5188 |

* That is, the $B_{i*v}{}^t$ coefficients; see equation (8.7).

$$
10^4 \mathrm{Cov}^t(R_{\cdot *}^{t+2,s}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.05 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.0 \\ 0 & 0 & 0 & 0 & 0 & 0.0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}
$$

$$
+ \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ 0.0 & 0.5 & 0.5 \\ 0.0 & 0.0 & 0.5 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.2530 & 0.0606 & 0.3426 & 0.5637 \\ 0.1014 & 0.0853 & 0.4303 & 0.5342 \\ 0.1213 & 0.0739 & 0.5144 & 0.5188 \end{bmatrix} \begin{bmatrix} 40.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 240.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} 0.2530 & 0.1014 & 0.1213 \\ 0.0606 & 0.0853 & 0.0739 \\ 0.3426 & 0.4303 & 0.5144 \\ 0.5637 & 0.5342 & 0.5188 \end{bmatrix} \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.5 & 0.5 & 1.0 \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0 \end{bmatrix} \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 3.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} 0.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0 \end{bmatrix} \cdot \begin{bmatrix} 3.4652 & 2.2962 & 2.3376 & 2.3169 & 1.1688 & 2.3376 \\ 2.2962 & 2.1946 & 2.0491 & 2.1219 & 1.0246 & 2.0491 \\ 2.3376 & 2.0491 & 1.9522 & 2.0007 & 0.9761 & 1.9522 \\ 2.3169 & 2.1219 & 2.0007 & 2.0613 & 1.0004 & 2.0007 \\ 1.1688 & 1.0246 & 0.9761 & 1.0004 & 1.2381 & 0.9761 \\ 2.3376 & 2.0491 & 1.9522 & 2.0007 & 0.9761 & 2.0022 \end{bmatrix}
$$

FIGURE 8.5: ILLUSTRATIVE COMPUTATION OF EQUATION (8.6)

$$R_M^{t+2,s} = \sum_{i=1}^{m} \Omega_i \; R_{i*}^{t+2,s} \qquad ; \tag{8.8}$$

where $R_M{}^{t+2,s}$ is the average annual rate of return for the market over the period t to t+2 given the future developments of scenario s; and $\Omega_i$ is the share of corporation i in the market index. The betas can then be calculated:

$$\beta_i^t = \mathrm{Cov}^t\,(R_M^{t+2,s},\ R_{i*}^{t+2,s})/\mathrm{Var}^t(R_M^{t+2,s}) \qquad i=1,\ldots,m. \tag{8.9}$$

# 9. PORTFOLIO RATES OF RETURN

Here we define the rate of return for portfolios and specify a mapping from corporations to portfolios. This mapping is then illustrated by constructing a number of portfolios from the six corporations.

## 9.1 Definition of Rates of Return by Portfolio

Let $R_{(p)}{}^{t}$ denote the average annual rate of return for portfolio p over the period $t-\ell$ to t:

$$100R^{t}_{(p)} = 100\sum_{i=1}^{m} R^{t}_{i*}\, S^{t}_{(p)i} + 100R^{t}_{m+1*}\, S^{t}_{(p)m+1} \qquad p=1,\ldots,z; \quad (9.1)$$

where $R_{i*}{}^{t}$ is the average annual rate of return of corporation i over the period $t-\ell$ to t; $S_{(p)i}{}^{t}$ is the share of the value of portfolio p invested in corporation i at time t; $R_{m+1*}{}^{t}$ is the average annual rate of return over the period $t-\ell$ to t on assets other than domestic corporations (e.g., foreign corporations, bonds, etc.); and $S_{(p)m+1}{}^{t}$ is the share of the value of portfolio p invested in other assets at time t. Note that for simplicity, assets, other than domestic corporations, have been aggregated. However, the subsequent analysis could easily be generalized to incorporate a whole range of other assets. Finally, note that:

$$\sum_{i=1}^{m+1} S^{t}_{(p)i} = 1 \qquad p=1,\ldots,z. \quad (9.2)$$

## 9.2 Calculation of Rates of Return for Some Illustrative Portfolios

To illustrate equation (9.1) we will calculate the rate of return for some illustrative portfolios. As in sections 5.2 and 7.3, to calculate the rate of return the length of run of the CGE model must first be defined. Recall from section 4.2 that the economic environment chosen produces projections for a period commencing about two years after the exogenous shocks are injected (i.e., $\ell$=2).

A mapping of the value at t of five illustrative portfolios across corporations and other assets is given in Table 9.1. For example, the first portfolio is divided equally between the SERVICES corporation and the STAR corporation. The next three portfolios are also wholly devoted to corporations. However, the fifth portfolio is divided equally between the AG corporation and other assets.

To illustrate equation (9.1) we also require the corporate rates of return and the rate of return on other assets. Recall from section 7.3 that the average annual rates of return over the period t-2 to t for the AG, TCF, SERVICES, MIX, OS, and STAR corporations are 30 per cent, 25 per cent, 20 per cent, 22.5 per cent, 22.5 per cent and 21 per cent, respectively. Finally, it is assumed that the average annual rate of return on other assets is 15 per cent.

Given the above rates of return and the shares listed in Table 9.1 we can use equation (9.1) to calculate the rates of return over the period t-2 to t for the above portfolios as follows. The rate of return on the first portfolio is 20.5 per cent (i.e., 20.0 × 0.5 + 21.0 × 0.5). The rate of return on the second portfolio is slightly higher at 21.25

TABLE 9.1: A MAPPING OF SOME ILLUSTRATIVE PORTFOLIOS ACROSS CORPORATIONS*

| Portfolio | Corporation | | | | | | Other Assets | Total |
|---|---|---|---|---|---|---|---|---|
| | 1. AG | 2. TCF | 3. SERVICES | 4. MIX | 5. OS | 6. STAR | | |
| 1. | | | 50.00 | | | 50.00 | | 100.0 |
| 2. | | | 50.00 | | 50.00 | | | 100.0 |
| 3. | | 33.33 | 33.33 | 33.33 | | | | 100.0 |
| 4. | 16.67 | 16.67 | 16.67 | 16.67 | 16.67 | 16.67 | | 100.0 |
| 5. | 50.00 | | | | | | 50.00 | 100.0 |

* The numbers in the table are the percentages of the value of each portfolio invested in the respective corporations and other assets at time t.

per cent (i.e., 20.0 × 0.5 + 22.5 × 0.5). The third portfolio consists of investments in three corporations and earns a rate of return of 22.5 per cent (i.e., 25.0 × 0.33 + 20.0 × 0.33 + 22.5 × 0.33). The fourth portfolio is divided equally over the six corporations and it earns a rate of return of 23.50 per cent. The final portfolio consists of investments in the AG corporation and other assets and it earns a rate of return of 22.5 per cent (i.e., 30.0 × 0.5 + 15.0 × 0.5).

# 10. PORTFOLIO FORECASTS

In this section it is shown how a CGE model can be used for forecasting rates of return of portfolios. First the theory for reckoning the expected rate of return and the variance for a portfolio is developed. Then the expected return and variance thereof for assets other than the corporations, are specified. Finally, the expected rate of return and variance for the five illustrative portfolios, as defined in section 9.2, are calculated.

## 10.1 Forecasting the Rates of Return for Portfolios

The average annual rate of return on portfolio p over the period t to t+2 given scenario s, $R_{(p)}{}^{t+2,s}$, is given by the identity:

$$100R_{(p)}^{t+2,s} = 100R_{(p)}^{t} + 100\Delta R_{(p)}^{t,s} \qquad \begin{matrix} p=1,\ldots,z \\ s=1,\ldots,q; \end{matrix} \quad (10.1)$$

where $R_{(p)}{}^{t}$ is the average annual rate of return for portfolio p over the period t-2 to t and $\Delta R_{(p)}{}^{t,s}$ is change in the rate of return for portfolio p over the period t to t+2 given scenario s.

The change in the rate of return for portfolio p over the period t to t+2 given scenario s can be calculated:

$$100\Delta R_{(p)}^{t,s} = 100\sum_{i=1}^{m} \Delta R_{i*}^{t,s}\, S_{(p)i}^{t} + 100\Delta R_{m+1*}^{t,s}\, S_{(p)m+1}^{t} \qquad \begin{matrix} p=1,\ldots,z \\ s=1,\ldots,q; \end{matrix} \quad (10.2)$$

where $\Delta R_{i*}{}^{t,s}$ is the change in the rate of return of corporation i over the period t to t+2 given scenario s; and $\Delta R_{m+1*}{}^{t,s}$ is the change in the rate of return of other assets over the period t to t+2 given scenario s. Equation (10.2) can be substituted into equation (10.1):

$$100R_{(p)}^{t+2,s} = 100R_{(p)}^{t} + 100 \sum_{i=1}^{m} \Delta R_{i*}^{t,s} S_{(p)i}^{t}$$

$$+ 100\Delta R_{m+1*}^{t,s} S_{(p)m+1}^{t} \qquad \begin{array}{l} p=1,\ldots,z \\ s=1,\ldots,q. \end{array} \quad (10.3)$$

The expected rate of return for portfolio p can be obtained by taking the expectation of equation (10.3):

$$100E^{t}(R_{(p)}^{t+2,s}) = 100R_{(p)}^{t} + 100 \sum_{i=1}^{m} E^{t}(\Delta R_{i*}^{t,s}) S_{(p)i}^{t}$$

$$+ 100E^{t}(\Delta R_{m+1*}^{t,s}) S_{(p)m+1}^{t} \qquad p=1,\ldots,z. \quad (10.4)$$

Equation (10.4) can also be written:

$$100E^{t}(R_{(.)}^{t+2,s}) = 100R_{(.)}^{t} + 100S_{(.).}^{t} E^{t}(\Delta R_{.*}^{t,s})$$

$$+ 100S_{(.)m+1}^{t} E^{t}(\Delta R_{m+1*}^{t,s}) \qquad ; \qquad (10.5)$$

where $E^{t}(R_{(.)}{}^{t+2,s})$ is a $(z\times1)$ vector of the expectations at time t of the average annual rates of return of the (z) portfolios over the period t to t+2; $R_{(.)}{}^{t}$ is a $(z\times1)$ vector of the average annual rates of return of the (z) portfolios in period t-2 to t; $S_{(.).}{}^{t}$ is a $(z\times m)$ matrix of the shares of the (z) portfolios invested in the (m) corporations at time t; and $E^{t}(\Delta R_{.*}{}^{t,s})$ is an $(m\times1)$ vector of the expectations at time t of the change in the average annual rates of return of the (m) corporations over the period of t to t+2; $S_{(.)m+1}{}^{t}$ is a $(z\times1)$ vector of the shares of the (z) portfolios invested in other assets at time t; and the scalar $E^t(\Delta R_{m+1*}{}^{t,s})$ is the expected change in the average annual rate of return of other assets over the period t to t+2.

The next task is to compute the variance-covariance matrix of rates of return of portfolios. To do this we first express equation (10.3) in matrix notation:

$$100R_{(.)}^{t+2,s} = A^*b^* + c^* \quad ; \qquad (10.6)$$

where $R_{(.)}{}^{t+2,s}$ is a $(z\times1)$ vector of the average annual rates of return of the (z) portfolios over the period t to t+2 given scenario s; $A^*$ is a $(z \times (m+1+z))$ matrix of shares and coefficients; $b^*$ is a $((m+1+z) \times 1)$ vector of the changes in rates of return of the (m) corporations and the (1) other asset; and $c^*$ is a $(z\times1)$ vector of the average annual rates of return of the (z) portfolios over the period t-2 to t. Equation (10.6) is depicted in Figure 10.1.

The variance-covariance matrix of the rates of return by portfolios, $Cov^t(R_{(.)}{}^{t+2,s})$, can be deduced from equation (10.6):

$$10^4 Cov^t(R_{(.)}^{t+2,s}) = A^* \, Cov^t(b^*) \, A^{*\prime} \quad ; \qquad (10.7)$$

where $Cov^t(b^*)$ is the variance-covariance matrix of the changes in the rates of return of the corporations and other assets. A decomposition of the $Cov^t(b^*)$ matrix is depicted in Figure 10.2.

The number of calculations involved when illustrating equation (10.7) can be reduced if an assumption is made concerning the structure of the $Cov^t(b^*)$ matrix. We will assume that changes in the rates of

$$\underbrace{\begin{bmatrix} R^{t+2,s}_{(1)} \\ \vdots \\ R^{t+2,s}_{(z)} \end{bmatrix}}_{(z \times 1)} = \underbrace{\left[\begin{array}{ccc|c} S^{t}_{(1)1} & \ldots, & S^{t}_{(1)m}, & S^{t}_{(1)m+1} \\ \vdots & & \vdots & \vdots \\ S^{t}_{(z)1} & \ldots, & S^{t}_{(z)m}, & S^{t}_{(z)m+1} \end{array}\right]}_{(z \times (m+1))} \underbrace{\begin{bmatrix} \Delta R^{t,s}_{1*} \\ \vdots \\ \Delta R^{t,s}_{m*} \\ \Delta R^{t,s}_{m+1} \end{bmatrix}}_{((m+1) \times 1)} + \underbrace{\begin{bmatrix} R^{t}_{(1)} \\ \vdots \\ R^{t}_{(z)} \end{bmatrix}}_{(z \times 1)}$$

FIGURE 10.1: DIAGRAMATIC REPRESENTATION OF THE PROCESS FOR GENERATING THE AVERAGE ANNUAL RATE OF RETURN FOR PORTFOLIOS OVER THE PERIOD t TO t+2 GIVEN SCENARIO s

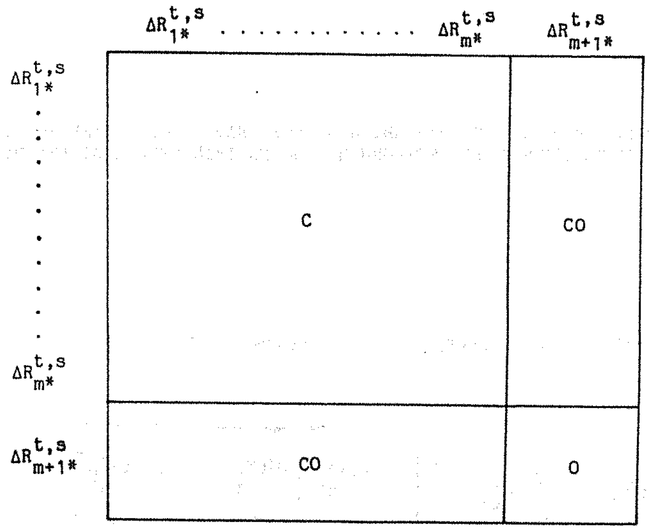


FIGURE 10.2: THE VARIANCE-COVARIANCE MATRIX OF CHANGES IN THE RATES OF RETURN OF CORPORATIONS AND OF OTHER ASSETS

return for corporations are uncorrelated with changes in the rates of return for other assets. Thus CO is a null matrix. (It should be stressed that this assumption has only been made to simplify the subsequent analysis. In practice, it would be unlikely that CO is a null matrix.) Given the above assumption, the only non-zero terms in the $Cov^t(b^*)$ matrix are the covariances among the changes in the rates of return on corporations and the variance of the changes in the rate of return on other assets; i.e., matrix C and the scalar O in Figure 10.3.

Finally, given the structure of the $Cov^t(b^*)$ matrix depicted in Figure 10.3, it is possible to rewrite equation (10.7) as:[19]

$$10^4 Cov^t(R^{t+2,s}_{(.)}) = 10^4\, S^t_{(.).}\; Cov^t(\Delta R^{t,s}_{.*})\; S^{t\,'}_{(.).}$$

$$+\ 10^4\, S^t_{(.)\,m+1}\; Cov^t(\Delta R^{t,s}_{m+1*})\; S^{t\,'}_{(.)m+1} \quad ; \qquad (10.8)$$

where $S_{(.).}{}^t$ is a (z×m) matrix as defined above; $Cov^t(\Delta R_{.*}{}^{t,s})$ is an (m×m) variance-covariance matrix of the changes in the rate of return of the (m) corporations; $S_{(.)m+1}{}^t$ is a (z×1) vector as defined above; and $Cov^t(\Delta R_{m+1*}{}^{t,s})$ is the (scalar) variance of the change in the rate of return of other assets.

## 10.2 Future Developments in Other Assets

Before equations (10.5) and (10.8) can be evaluated we have to specify the expected outcome and variance of the rate of return of other assets. It is assumed that:

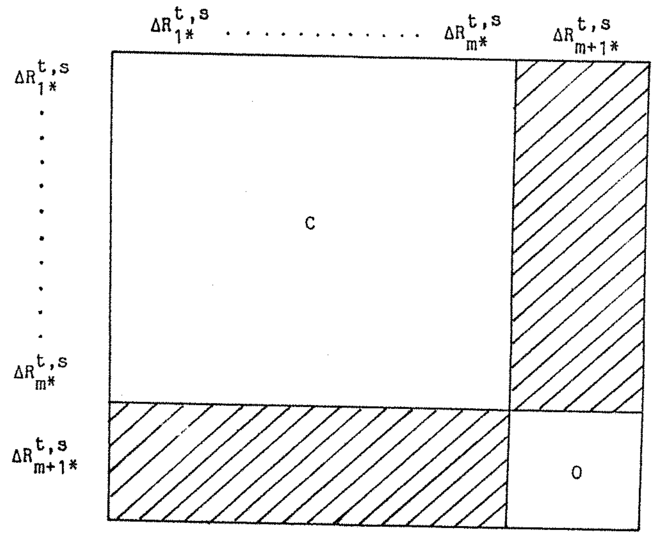


FIGURE 10.3: THE ASSUMED NON-ZERO COMPONENTS OF THE VARIANCE-COVARIANCE MATRIX OF CHANGES IN THE RATES OF RETURN OF CORPORATIONS AND OF OTHER ASSETS

$$10^2E^t(\Delta R^{t,s}_{m+1*}) = 2.0 \quad ;$$

and

$$10^4Var^t(\Delta R^{t,s}_{m+1*}) = 0.5 \quad .$$

10.3 Calculation of the Expectations and Variance-Covariance Matrix of the Forecast Rates of Return for the Illustrative Portfolios

The vector of expected rates of return for the five illustrative portfolios can be calculated as follows. Note that the $R_{(.)}{}^t$ vector can be obtained from section 9.2. The $S_{(.)\cdot}{}^t$ matrix and the $S_{(.)m+1}{}^t$ vector can be obtained from Table 9.1. The $E^t(\Delta R_{\cdot *}{}^{t,s})$ matrix is given in section 8.3. Finally, the value of $E^t(\Delta R_{m+1*}{}^{t,s})$ is given in section 10.2. The vector of expected average annual rates of return can be estimated by substituting these vectors and matrices in equation (10.5). This is shown in Figure 10.4. Thus the expected rates of return over the period t to t+2 for the five illustrative portfolios are 23.39 per cent, 23.17 per cent, 23.96 per cent, 24.56 per cent, and 22.05 per cent, respectively.

As a brief aside, note that when calculating the expected returns for portfolios we implicitly made use of portfolio factor sensitivity coefficients. These sensitivity coefficients show the effect on the rate of return of a portfolio of a one per cent increase in each of the exogenous variables (i.e., factors). Let $B_{(p)v}{}^t$ denote the sensitivity coefficient at time t for portfolio p with respect to the $v^{th}$ factor:

$$B^t_{(p)v} = \sum_{i=1}^{m} B^t_{i*v}\, S^t_{(p)i} \qquad \begin{array}{l} p=1,\ldots,z \\ v=1,\ldots,k; \end{array} \qquad (10.9)$$

$$100E^{t}(R_{(.)}^{t+2,s}) = 100\begin{bmatrix}0.2050\\0.2125\\0.2250\\0.2350\\0.2250\end{bmatrix} + 100\begin{bmatrix}0.0 & 0.0 & 0.5000 & 0.0 & 0.0 & 0.5000\\0.0 & 0.0 & 0.5000 & 0.0 & 0.5000 & 0.0\\0.0 & 0.3333 & 0.3333 & 0.3333 & 0.0 & 0.0\\0.1667 & 0.1667 & 0.1667 & 0.1667 & 0.1667 & 0.1667\\0.5000 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0\end{bmatrix}\begin{bmatrix}-0.0291\\0.0054\\0.0239\\0.0147\\0.0144\\0.0239\end{bmatrix}$$

$$+ 100\begin{bmatrix}0.0\\0.0\\0.0\\0.0\\0.5000\end{bmatrix}\begin{bmatrix}0.0200\end{bmatrix} = \begin{bmatrix}23.3896\\23.1672\\23.9628\\24.5588\\22.0466\end{bmatrix}$$

FIGURE 10.4: ILLUSTRATIVE COMPUTATION OF EQUATION (10.5)

where $B_{i*v}{}^{t}$'s are the corporate factor sensitivity coefficients, see equation (8.7). The factor sensitivity coefficients for the five illustrative portfolios are given in Table 10.1.

Next we calculate the variance-covariance matrix of rates of return for the five illustrative portfolios. Note that the $Cov^{t}(\Delta R_{.*}{}^{t,s})$ matrix can be obtained from section 8.3. The $Cov^{t}(\Delta R_{m+1*}{}^{t,s})$ is given in section 10.2. The variance-covariance matrix of rates of return by portfolios can be calculated by substituting these, together with $S_{(.).}{}^{t}$ and $S_{(.)m+1}{}^{t}$, into equation (10.8). The resultant computation is shown in Figure 10.5. Thus the variances of the percentage-point rates of return forecasts over the period t to t+2 for the five illustrative portfolios are 1.9647, 1.2856, 2.0609, 1.8377, and 0.9913, respectively.

Finally, the expected rates of return for the illustrative portfolios can be plotted against their standard deviations (i.e., risk); see Figure 10.6. It can be seen from Figure 10.6 that, for example, the fourth portfolio dominates the third portfolio in the sense that the fourth portfolio has a relatively higher expected return and a relatively lower risk. In the next section the frontier of efficient portfolios (i.e., the locus of portfolios with the minimum risk for a prescribed expected return) is derived.

TABLE 10.1: PORTFOLIO FACTOR SENSITIVITY COEFFICIENTS*

| Portfolio | Factor | | | |
|---|---|---|---|---|
| | World price of exports $p_W$ | Ad valorem tariffs $t$ | Real absorption $a_R$ | Nominal exchange rate $\phi$ |
| 1. | 0.1213 | 0.0739 | 0.5144 | 0.5188 |
| 2. | 0.0910 | 0.0555 | 0.3858 | 0.3891 |
| 3. | 0.2598 | 0.0796 | 0.4724 | 0.5265 |
| 4. | 0.1282 | 0.0561 | 0.4219 | 0.4869 |
| 5. | 0.1265 | 0.0303 | 0.1713 | 0.2819 |

* That is, the $B_{(p)}v^t$ coefficients; see equation (10.9).

$$10^4 Cov^t(R_{(.)}^{t+2,s}) = \begin{bmatrix} 0.0 & 0.0 & 0.5000 & 0.0 & 0.0 & 0.5000 \\ 0.0 & 0.0 & 0.5000 & 0.0 & 0.5000 & 0.0 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 & 0.0 & 0.0 \\ 0.1667 & 0.1667 & 0.1667 & 0.1667 & 0.1667 & 0.1667 \\ 0.5000 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} 3.4652 & 2.2962 & 2.3376 & 2.3169 & 1.1688 & 2.3376 \\ 2.2962 & 2.1946 & 2.0491 & 2.1219 & 1.0246 & 2.0491 \\ 2.3376 & 2.0491 & 1.9522 & 2.0007 & 0.9761 & 1.9522 \\ 2.3169 & 2.1219 & 2.0007 & 2.0613 & 1.0004 & 2.0007 \\ 1.1688 & 1.0246 & 0.9761 & 1.0004 & 1.2381 & 0.9761 \\ 2.3376 & 2.0491 & 1.9522 & 2.0007 & 0.9761 & 2.0022 \end{bmatrix} \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.1667 & 0.5000 \\ 0.0 & 0.0 & 0.3333 & 0.1667 & 0.0 \\ 0.5000 & 0.5000 & 0.3333 & 0.1667 & 0.0 \\ 0.0 & 0.0 & 0.3333 & 0.1667 & 0.0 \\ 0.0 & 0.5000 & 0.0 & 0.1667 & 0.0 \\ 0.5000 & 0.0 & 0.0 & 0.1667 & 0.0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.5000 \end{bmatrix} \begin{bmatrix} 0.5 \end{bmatrix} \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.5000 \end{bmatrix} = \begin{bmatrix} 1.9647 & 1.4642 & 2.0005 & 1.8825 & 1.1688 \\ 1.4642 & 1.2856 & 1.5003 & 1.4713 & 0.8766 \\ 2.0005 & 1.5003 & 2.0609 & 1.9172 & 1.1584 \\ 1.8825 & 1.4713 & 1.9172 & 1.8377 & 1.1604 \\ 1.1688 & 0.8766 & 1.1584 & 1.1604 & 0.9913 \end{bmatrix}$$

FIGURE 10.5: ILLUSTRATIVE COMPUTATION OF EQUATION (10.8)

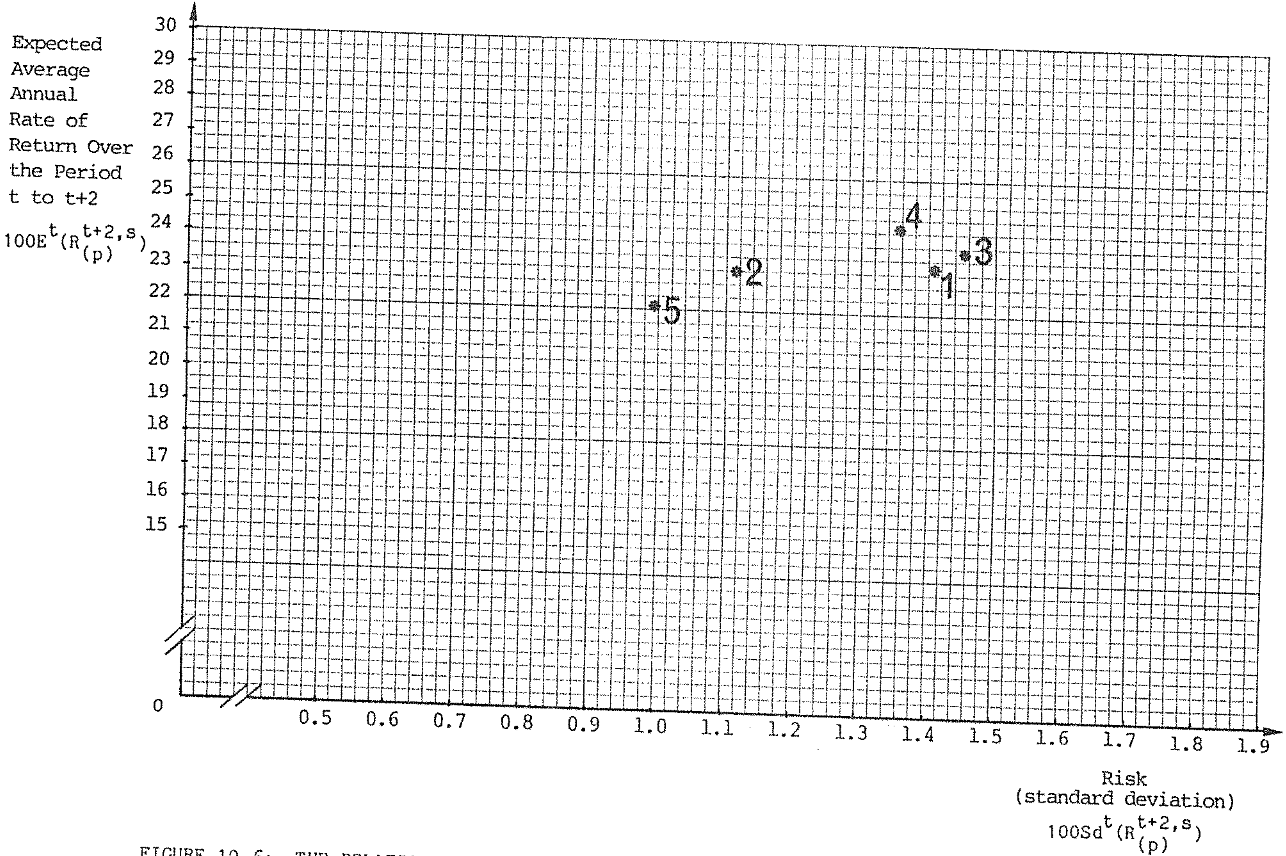


FIGURE 10.6: THE RELATIONSHIP BETWEEN EXPECTED RETURN AND RISK FOR THE ILLUSTRATIVE PORTFOLIOS

# 11. FRONTIER OF EFFICIENT PORTFOLIOS

The frontier of efficient portfolios is the locus of feasible portfolios that have the smallest variance for a prescribed expected return. In this section two frontiers of efficient portfolios are presented. The first is when agents are allowed to borrow and short-sell all assets. The second is when agents are not allowed to borrow or short-sell any assets.

## 11.1 Frontier of Efficient Portfolios When Agents are Allowed to Borrow and Short-Sell All Assets

Here we follow Merton (1972) and present the analytical derivation of the efficient portfolio frontier. Suppose that at instant t there are m+1 risky assets with the expected average annual rate of return on the $i^{th}$ asset over the period t to t+2 denoted by $E^t(R_{i*}{}^{t+2,s})$, the covariance of returns between the $i^{th}$ and the $j^{th}$ assets denoted by $Cov^t(R_{i*}{}^{t+2,s}, R_{j*}{}^{t+2,s})$, and the variance of the return on the $i^{th}$ asset denoted by $Var^t(R_{i*}{}^{t+2,s}, R_{i*}{}^{t+2,s})$. Let $S_{(p)i}{}^t$ equal the share of the value of portfolio p invested in the $i^{th}$ asset at t. Then, the frontier of efficient portfolios can be described as the set of portfolios that satisfy the following constrained minimization problem:

Choose $\quad S^t_{(p)1}, \ldots, S^t_{(p)m+1}$

to minimize $\quad \frac{1}{2} \, Var^t(R^{t+2,s}_{(p)}) \quad ; \qquad (11.1)$

subject to

$$Var^t(R_{(p)}^{t+2,s}) = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} S_{(p)i}^t \; S_{(p)j}^t \; Cov^t(R_{i*}^{t+2,s}, \; R_{j*}^{t+2,s}) \quad ; \quad (11.2)$$

$$E^t(R_{(p)}^{t+2,s}) = \sum_{i=1}^{m+1} S_{(p)i}^t \; E^t(R_{i*}^{t+2,s}) \quad ; \quad (11.3)$$

and

$$\sum_{i=1}^{m+1} S_{(p)i}^t = 1 \quad ; \quad (11.4)$$

where $Var^t(R_{(p)}{}^{t+2,s})$ is the variance (across scenarios) of the rate of return on the portfolio p over the period t to t+2. Note that the term on the left of (11.3) is a given constant -- that is, for a fixed expectation of the rate of return on the portfolio, the variance of the rate of return is minimized.

The above constrained minimization problem is analytically solved in Appendix A.3; however, the solution for the composition of portfolio p is given by:

$$S_{(p).}^t = \{(X - E^t(R_{(p)}^{t+2,s})Z)/(X^2 - YZ)\}[Cov^t(R_{.*}^{t+2,s})^{-1}]' E^t(R_{.*}^{t+2,s})$$

$$+ \{(E^t(R_{(p)}^{t+2,s})X - Y)/(X^2 - YZ)\}[Cov^t(R_{.*}^{t+2,s})^{-1}]' \underline{1} \quad ; \quad (11.5)$$

where

$$X = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} \; E^t(R_{j*}^{t+2,s}) \quad ; \quad (11.6)$$

$$Y = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} \; E^t(R_{j*}^{t+2,s}) \; E^t(R_{i*}^{t+2,s}) \quad ; \quad (11.7)$$

$$Z = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} \quad . \quad (11.8)$$

The above notation can be explained as follows: $S_{(p)}.^{t}$ is an $((m+1)\times 1)$ vector of the shares of each of the (m+1) risky assets held in the frontier portfolio p; the values in the set brackets "{}" are scalars ($v_{ij}$ is the $ij^{th}$ element of $Cov^{t}(R_{.*}{}^{t+2,s})^{-1}$); $[Cov^{t}(R_{.*}{}^{t+2,s})^{-1}]$ is the $((m+1)\times(m+1))$ inverse matrix of the variance-covariance matrix of the rates of return of the (m+1) assets; $E^{t}(R_{.*}{}^{t+2,s})$ is an $((m+1)\times 1)$ vector of the expected rates of return of the (m+1) assets; and $\underline{1}$ is an $((m+1)\times 1)$ unit vector.

The efficient frontier associated with the compositional equation (11.5) is:

$$E^{t}(R_{(p)}^{t+2,s}) = X/Z + \mathrm{Max}\{\pm\ [(Z\ Var^{t}(R_{(p)}^{t+2,s}) - 1)(YZ - X^{2})]^{1/2}/Z\} \quad . \qquad (11.9)$$

The values for X, Y, and Z can be calculated using equations (11.6), (11.7), and (11.8): X = 23.37, Y = 1,797.55, and Z = 4.30. Thus, the efficient frontier of portfolios for the numerical illustration in this paper is given by:

$$100E^{t}(R_{(p)}^{t+2,s}) = 5.4380 + \mathrm{Max}\{\pm\ [3.0863\ Var^{t}(R_{(p)}^{t+2,s}) - 0.7180]^{1/2}/0.0430\} \quad . \qquad (11.10)$$

Equation (11.10) can be illustrated as follows. Assume that we are interested in calculating the highest possible expected average annual rate of return for a portfolio with a variance of 0.5109 (in other words, a standard deviation of 0.7148). If we substitute 0.5109 for $Var^{t}(R_{(p)}{}^{t+2,s})$ in equation (11.10) we find that the highest possible expected average annual rate of return for a portfolio is 27.00 per cent. Furthermore, if we substitute 27.00 for $100E^{t}(R_{(p)}{}^{t+2,s})$ in equation (11.5) we find that the percentages of the value of the optimal portfolio with an expected return of 27.00 per cent across the seven assets are 36.70 per cent, 230.83 per cent, -213.76 per cent, -114.58 per cent, 32.79 per cent, 51.63 per cent, and 76.38 per cent, respectively. Note that, as the MIX corporation is a linear combination of the TCF and SERVICES corporations, the composition of the optimal portfolio is not unique. However, other portfolios with an expected return of 27.00 per cent will not exhibit variances of less than 0.5109. Table 11.1 contains a list of portfolios that were generated by increasing the expected rate of return in equation (11.10) from 15 per cent to 30 per cent. These portfolios are then used to generate the frontier of efficient portfolios depicted in Figure 11.1.

## 11.2 Frontier of Efficient Portfolios When Agents are Not Allowed to Borrow or Short-Sell Any Assets

Here we derive the frontier of efficient portfolios when agents are not allowed to borrow or short-sell any assets (in other words, the shares of assets in the optimal portfolios must all be greater than or equal to zero). This frontier is perhaps of more interest than the frontier described in section 11.1 as our analysis has a two-year outlook. To plot the frontier we must solve the following quadratic programming problem:[20]

Choose $\quad S^t_{(p)1}, \ldots, S^t_{(p)m+1}$

to minimize $\quad \frac{1}{2}\, Var^t(R^{t+2,s}_{(p)}) \quad ; \qquad (11.11)$

subject to

$$Var^t(R^{t+2,s}_{(p)}) = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} S^t_{(p)i}\, S^t_{(p)j}\, Cov^t(R^{t+2,s}_{i*}, R^{t+2,s}_{j*}) \quad ; \qquad (11.12)$$

TABLE 11.1: PORTFOLIOS THAT HAVE THE SMALLEST STANDARD DEVIATION FOR A PRESCRIBED EXPECTED RETURN WHEN AGENTS ARE ALLOWED TO BORROW AND SHORT-SELL ALL ASSETS

| Expected Average Annual Rate of Return Over the Period t to t+2(a) | Standard Deviation(c) | Percentage of the Value of Each Portfolio Invested in the Respective Corporations(b) | | | | | | Percentage of the Value of Each Portfolio Investment in Other Assets(b) | Total |
|---|---|---|---|---|---|---|---|---|---|
| | | 1. AG | 2. TCF | 3. SERVICES | 4. MIX | 5. OS | 6. STAR | | |
| 15.00 | 0.54 | -31.85 | -139.12 | 108.07 | 57.04 | 23.21 | 22.90 | 59.77 | 100.0 |
| 16.00 | 0.55 | -26.13 | -108.29 | 81.25 | 42.74 | 24.01 | 25.29 | 61.15 | 100.0 |
| 17.00 | 0.56 | -20.42 | -77.46 | 54.43 | 28.43 | 24.81 | 27.69 | 62.53 | 100.0 |
| 18.00 | 0.57 | -14.71 | -46.63 | 27.61 | 14.13 | 25.60 | 30.08 | 63.92 | 100.0 |
| 19.00 | 0.59 | -9.00 | -15.80 | 0.79 | -0.17 | 26.40 | 32.48 | 65.30 | 100.0 |
| 20.00 | 0.60 | -3.29 | 15.03 | -26.02 | -14.47 | 27.20 | 34.87 | 66.69 | 100.0 |
| 21.00 | 0.61 | 2.43 | 45.85 | -52.84 | -28.77 | 28.00 | 37.26 | 68.07 | 100.0 |
| 22.00 | 0.63 | 8.14 | 76.68 | -79.66 | -43.07 | 28.80 | 39.66 | 69.46 | 100.0 |
| 23.00 | 0.65 | 13.85 | 107.51 | -106.48 | -57.38 | 29.60 | 42.05 | 70.84 | 100.0 |
| 24.00 | 0.66 | 19.56 | 138.34 | -133.30 | -71.68 | 30.40 | 44.45 | 72.22 | 100.0 |
| 25.00 | 0.68 | 25.28 | 169.17 | -160.12 | -85.98 | 31.19 | 46.84 | 73.61 | 100.0 |
| 26.00 | 0.70 | 30.99 | 200.00 | -186.94 | -100.28 | 31.99 | 49.24 | 74.99 | 100.0 |
| 27.00 | 0.71 | 36.70 | 230.83 | -213.76 | -114.58 | 32.79 | 51.63 | 76.38 | 100.0 |
| 28.00 | 0.73 | 42.41 | 261.66 | -240.52 | -128.88 | 33.59 | 54.03 | 77.76 | 100.0 |
| 29.00 | 0.75 | 48.12 | 292.49 | -267.40 | -143.19 | 34.39 | 56.42 | 79.15 | 100.0 |
| 30.00 | 0.77 | 53.80 | 323.32 | -294.21 | -157.49 | 35.19 | 58.82 | 80.53 | 100.0 |

(a) That is, $100E^t(R_{(p)}^{t+2,s})$.

(b) That is, the shares $S_{(p)i}{}^t \times 100$.

(c) That is, $100Sd^t(R_{(p)}^{t+2,s})$.

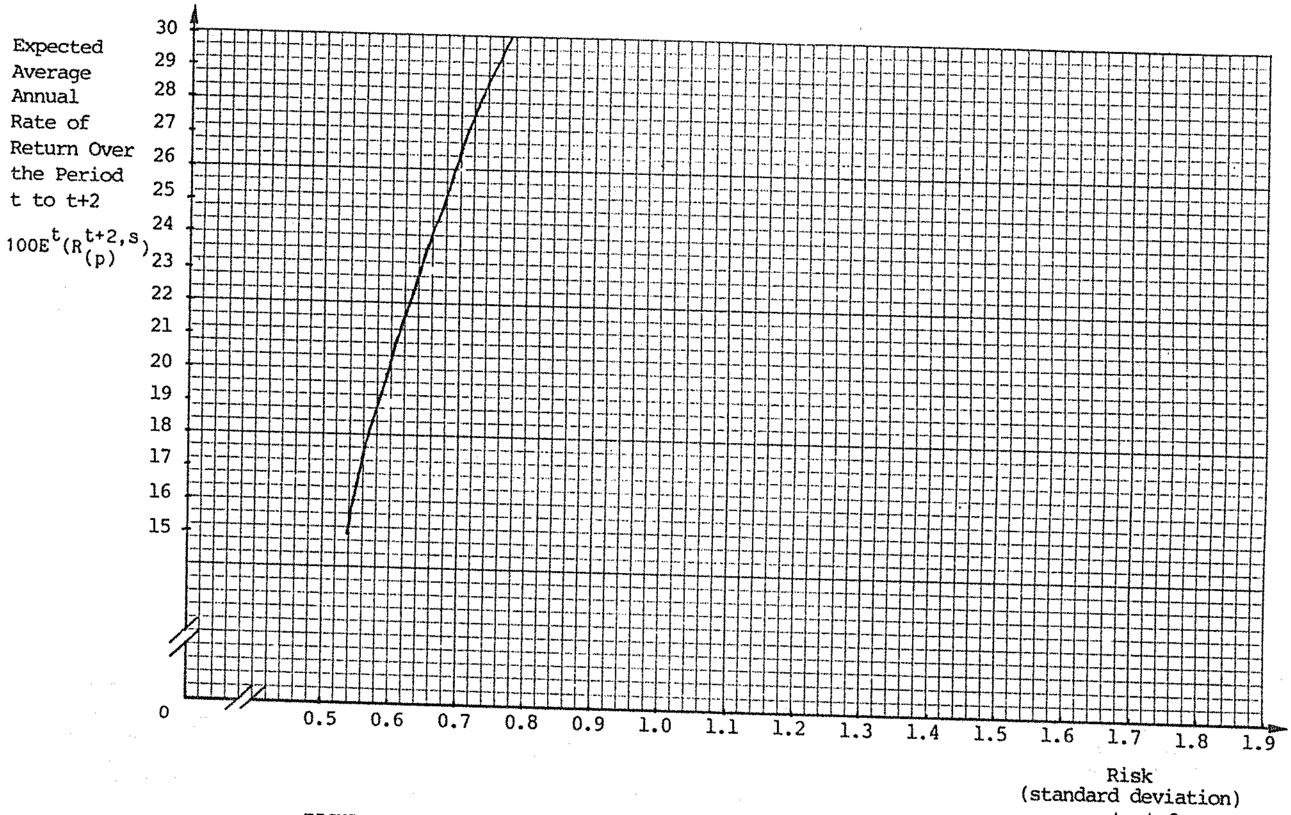


FIGURE 11.1: FRONTIER OF EFFICIENT PORTFOLIOS WHEN AGENTS ARE ALLOWED TO BORROW AND SHORT-SELL ALL ASSETS

$$\bar{E}^t(R_{(p)}^{t+2,s}) = \sum_{i=1}^{m+1} S_{(p)i}^t \; E^t(R_{i*}^{t+2,s}) \qquad ; \qquad (11.13)$$

$$\sum_{i=1}^{m+1} S_{(p)i}^t = 1 \qquad ; \qquad (11.14)$$

and

$$S_{(p)i}^t \geq 0 \qquad i=1,\ldots,m+1. \qquad (11.15)$$

The frontier of efficient portfolios can be numerically generated by changing the prescribed expected return (i.e., $\bar{E}^t(R_{(p)}{}^{t,s})$) by small amounts and resolving. Table 11.2 contains a list of portfolios that were generated by increasing the expected rate of return from 17.00 per cent to 27.09 per cent.

It can be seen from Table 11.2 that the MIX corporation does not form part of any optimal portfolio. This is because, as mentioned above, the MIX corporation is a linear combination of the TCF and SERVICES corporations. It can also be seen from the table that the standard deviations decline initially from 0.71 (with an associated expected return of 17 per cent) to 0.59 (with an associated expected return of 19 per cent), afterwhich they steadily increase. Thus, the minimum return even the most risk-adverse investor should accept is 19 per cent. Furthermore, the SERVICES and STAR corporations do not appear in any optimal portfolios with a return of greater than 19 per cent. It would appear that the OS corporation, which also has operations in the non-traded sector of domestic economy but with an overall relatively higher expected return, is dominating the SERVICES and STAR corporations. Finally, the portfolios listed in Table 11.2 are used to generate the frontier of efficient portfolios depicted in Figure 11.2.

TABLE 11.2: PORTFOLIOS THAT HAVE THE SMALLEST STANDARD DEVIATION FOR A PRESCRIBED EXPECTED RETURN WHEN AGENTS ARE NOT ALLOWED TO BORROW OR SHORT-SELL ANY ASSETS

| Expected Average Annual Rate of Return Over the Period t to t+2(a) | Standard Deviation(c) | Percentage of the Value of Each Portfolio Invested in the Respective Corporations(b) | | | | | | Percentage of the Value of Each Portfolio Investment in Other Assets(b) | Total |
|---|---|---|---|---|---|---|---|---|---|
| | | 1. AG | 2. TCF | 3. SERVICES | 4. MIX | 5. OS | 6. STAR | | |
| 17.00 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 100.00 | 100.0 |
| 18.00 | 0.62 | 0.00 | 0.00 | 7.75 | 0.00 | 8.38 | 0.00 | 83.86 | 100.0 |
| 19.00 | 0.59 | 0.00 | 0.00 | 5.02 | 0.00 | 23.57 | 1.26 | 70.16 | 100.0 |
| 20.00 | 0.61 | 0.00 | 9.52 | 0.00 | 0.00 | 31.49 | 0.00 | 58.99 | 100.0 |
| 21.00 | 0.68 | 1.81 | 13.25 | 0.00 | 0.00 | 38.67 | 0.00 | 46.27 | 100.0 |
| 22.00 | 0.78 | 4.73 | 15.83 | 0.00 | 0.00 | 45.66 | 0.00 | 33.78 | 100.0 |
| 23.00 | 0.90 | 7.65 | 18.41 | 0.00 | 0.00 | 52.65 | 0.00 | 21.30 | 100.0 |
| 24.00 | 1.04 | 10.57 | 20.98 | 0.00 | 0.00 | 59.64 | 0.00 | 8.82 | 100.0 |
| 25.00 | 1.20 | 23.02 | 20.71 | 0.00 | 0.00 | 56.27 | 0.00 | 0.00 | 100.0 |
| 26.00 | 1.46 | 58.39 | 13.59 | 0.00 | 0.00 | 28.02 | 0.00 | 0.00 | 100.0 |
| 27.00 | 1.82 | 94.00 | 6.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 100.0 |
| 27.09 | 1.86 | 100.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 100.0 |

(a) That is, $100E^t(R_{(p)}^{t+2,s})$.

(b) That is, the shares $S_{(p)i}^t \times 100$.

(c) That is, $100Sd^t(R_{(p)}^{t+2,s})$.

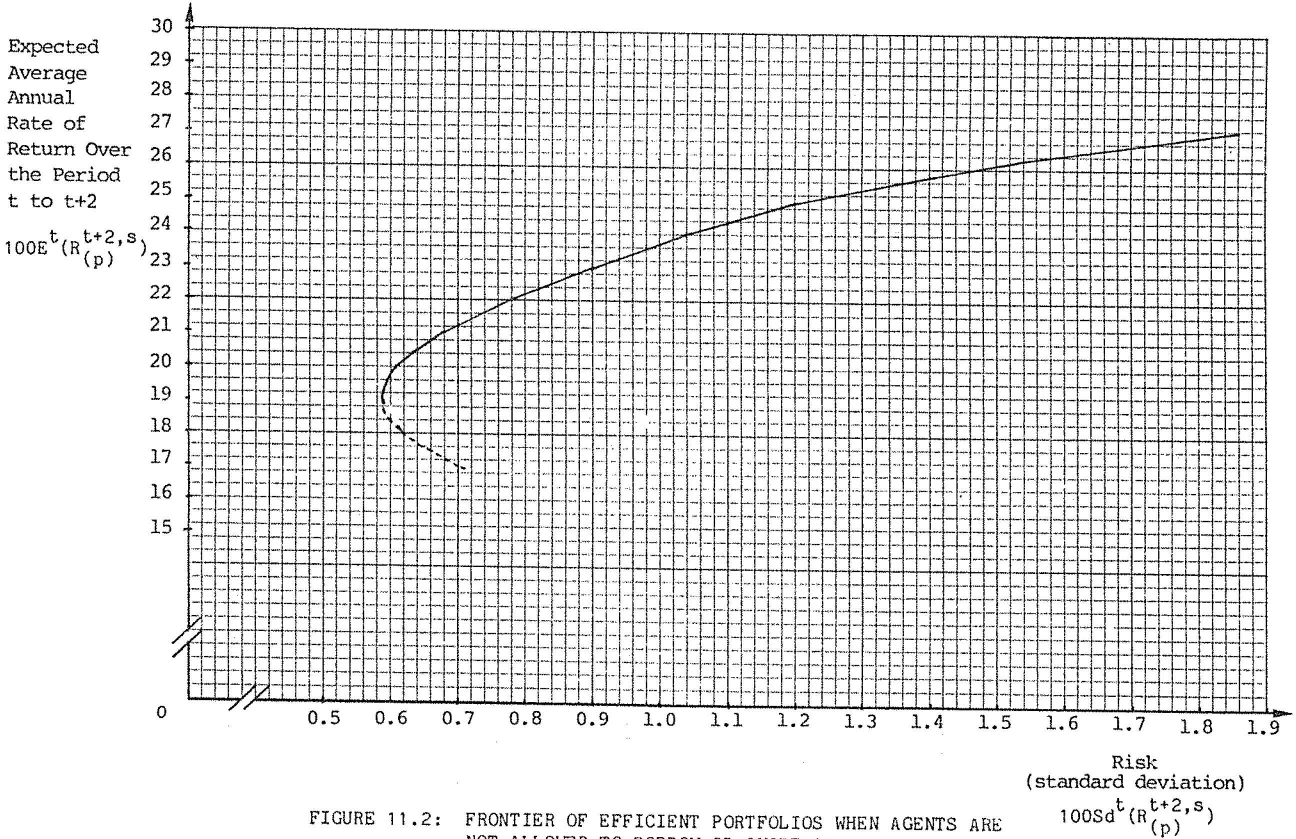


FIGURE 11.2: FRONTIER OF EFFICIENT PORTFOLIOS WHEN AGENTS ARE NOT ALLOWED TO BORROW OR SHORT-SELL ANY ASSETS

## 12. CONCLUSION

In this paper we have developed a multi-index model of asset returns with factors derived from an economic model. As our approach is forward-looking it can be used for tactical asset allocation. In fact, it has been shown how the approach can be used to derive the frontier of efficient portfolios (i.e., portfolios with the minimum risk for a given expected return.) Furthermore, although the principal focus is on the domestic equities market, our approach has potential to forecast the prospective performance of portfolios consisting of a whole range of assets (e.g., domestic and foreign equities, bonds, etc.).

In future work the procedures developed here will be used in conjunction with the ORANI model developed by Dixon, Parmenter, Sutton, and Vincent (1982). ORANI is a large general equilibrium model of the Australian economy which distinguishes 112 industries. Recently extensive research has been conducted on forecasting with the ORANI model (see, for example, Dixon (1986), Dixon and Parmenter (1986, 1987), and Dixon, Johnson and Parmenter (1988)). It is envisaged that the forecasting component of the procedures developed in this paper will draw heavily on this research. It may also be desirable to make use of regional forecasting versions of the ORANI model (see Dixon (1987), Johnson and Kee (1987), and Higgs and Powell (1988)).

Beyond the full-scale implementation of the methods developed in this paper, fruitful lines for future research would include comparative work on the performance of the CGE, CAPM and APT approaches to portfolio management. It is conjectured that if the historical period studied is one involving significant new developments in the economy, then the CGE approach developed here will be a relatively better predictor of portfolio performance.

# APPENDIX

This appendix is divided into three sections. The first derives the percentage-change form of the equation describing rates of return on capital by industry. The second describes a mapping from industry projections to corporate projections that allows for differential changes in rentals, creation costs of capital, tax rates, depreciation rates, and rates of return. Finally, the third section presents the analytical derivation of the frontier of efficient portfolios.

## A.1 DERIVATION OF THE PERCENTAGE-CHANGE FORM OF THE EQUATION DESCRIBING RATES OF RETURN ON CAPITAL BY INDUSTRY

Recall from equation (5.1) that the annual average rate of return on capital in industry j over the period $t-\ell$ to t, $R_{*j}^{t}$, is given by:

$$R_{*j}^{t} = P_{*j}^{t}\ (1-\tau_{*j}^{t})\ /\ \Pi_{*j}^{t-\ell}$$

$$+\ (\Pi_{*j}^{t}(1 - \ell d_{*j}^{t}) - \Pi_{*j}^{t-\ell})\ /(\ell\Pi_{*j}^{t-\ell}) \qquad j=1,\ldots,n; \qquad (A1.1)$$

where $P_{*j}^{t}$ is the annual average flow of rentals on a unit of capital industry j over the period $t-\ell$ to t; $\tau_{*j}^{t}$ is the tax rate on the rental earned by a unit of capital in industry j over the period $t-\ell$ to t; $\Pi_{*j}^{t-\ell}$ and $\Pi_{*j}^{t}$ are the creation costs of a unit of capital in industry j at instants $t-\ell$ and t, respectively; $d_{*j}^{t}$ is the depreciation rate on a unit of capital in industry j over the period $t-\ell$ to t; and $\ell$ is the number of years between $t-\ell$ and t. Equation (A1.1) can also be written:

$$R_{*j}^{t} = A_{*j}^{t}\ /\ (\ell\Pi_{*j}^{t-\ell}) \qquad j=1,\ldots,n; \qquad (A1.2)$$

where

$$A_{*j}^{t} = \ell P_{*j}^{t}\ (1-\tau_{*j}^{t}) + \Pi_{*j}^{t}\ (1 - \ell d_{*j}^{t}) - \Pi_{*j}^{t-\ell} \qquad j=1,\ldots,n. \qquad (A1.3)$$

Totally differentiate (A1.2):

$$d(R_{*j}^{t}) = d(A_{*j}^{t})\ /\ (\ell\Pi_{*j}^{t-\ell}) - d(\Pi_{*j}^{t-\ell})\ell A_{*j}^{t}\ /\ (\ell\Pi_{*j}^{t-\ell})^{2}$$

$$j=1,\ldots,n. \qquad (A1.4)$$

Next we totally differentiate (A1.3):

$$d(A_{*j}^{t}) = d(P_{*j}^{t})\ell(1 - \tau_{*j}^{t}) - d(\tau_{*j}^{t})\ell P_{*j}^{t}$$

$$+\ d(\Pi_{*j}^{t})(1 - \ell d_{*j}^{t}) - d(d_{*j}^{t})\ell\Pi_{*j}^{t} - d(\Pi_{*j}^{t-\ell}) \quad j=1,\ldots,n. \qquad (A1.5)$$

Substitute (A1.5) and (A1.3) into (A1.4):

$$d(R^t_{*j}) = d(P^t_{*j})\ell(1 - \tau^t_{*j}) / (\ell\Pi^{t-\ell}_{*j})$$

$$- d(\tau^t_{*j})\ell P^t_{*j} / (\ell\Pi^{t-\ell}_{*j})$$

$$+ d(\Pi^t_{*j})(1 - \ell d^t_{*j}) / \ell\Pi^{t-\ell}_{*j})$$

$$- d(d^t_{*j})\ell\Pi^t_{*j}/ (\ell\Pi^{t-\ell}_{*j})$$

$$- d(\Pi^{t-\ell}_{*j}) / (\ell\Pi^{t-\ell}_{*j})$$

$$- d(\Pi^{t-\ell}_{*j})\ell(\ell P^t_{*j} (1 - \tau^t_{*j}) + \Pi^t_{*j}(1 - \ell d^t_{*j}) - \Pi^{t-\ell}_{*j}) / (\ell\Pi^{t-\ell}_{*j})^2$$

$$j=1,\ldots,n. \qquad (A1.6)$$

Equation (A1.6) can be written:

$$100\Delta R^t_{*j} = p^t_{*j}\ \alpha^t_{*j} + \pi^t_{*j}\ \beta^t_{*j} + \pi^{t-\ell}_{*j}\ \gamma^t_{*j}$$

$$+ 100\Delta\tau^t_{*j}\ \lambda^t_{*j} + 100\Delta d^t_{*j}\psi^t_{*j} \qquad j=1,\ldots,n; \qquad (A1.7)$$

where

$$\alpha^t_{*j} = P^t_{*j}\ (1 - \tau^t_{*j}) / \Pi^{t-\ell}_{*j} \qquad j=1,\ldots,n; \qquad (A1.8)$$

$$\beta^t_{*j} = (1 - \ell d^t_{*j})\ \Pi^t_{*j} / (\ell\Pi^{t-\ell}_{*j}) \qquad j=1,\ldots,n; \qquad (A1.9)$$

$$\gamma^t_{*j} = -\alpha^t_{*j} - \beta^t_{*j} \qquad j=1,\ldots,n; \qquad (A1.10)$$

$$\lambda^t_{*j} = -P^t_{*j} / \Pi^{t-\ell}_{*j} \qquad j=1,\ldots,n; \qquad (A1.11)$$

$$\psi^t_{*j} = -\Pi^t_{*j} / \Pi^{t-\ell}_{*j} \qquad j=1,\ldots,n. \qquad (A1.12)$$

The above notation can be explained as follows: the lower-case variables are the percentage changes over the period t to $t+\ell$ in the respective upper case variables; and $100\Delta R*_j{}^t$, $100\Delta\tau*_j{}^t$, and $100\Delta d*_j{}^t$ are the percentage-point changes over the period t to $t+\ell$ in the rate of return on capital, the tax rate, and the depreciation rate, respectively.

## A.2 A MAPPING FROM INDUSTRY PROJECTIONS TO CORPORATE PROJECTIONS THAT ALLOWS FOR DIFFERENTIAL CHANGES IN RENTALS, CREATION COSTS OF CAPITAL, TAX RATES, AND DEPRECIATION RATES

Here we define a mapping from industry projections to corporate projections which allows for a greater range of variables in which differential changes are possible between corporations located in the same industry than the mapping developed in section 7.2.

This more general mapping first requires the annual average rate of return on the component of corporation i located in industry j to be defined:

$$R_{ij}^{t} = \bar{P}_{ij}^{t}\ (1 - \tau_{ij}^{t})/\Pi_{ij}^{t-\ell}$$

$$+ (\Pi_{ij}^{t}(1 - \ell d_{ij}^{t}) - \Pi_{ij}^{t-\ell})/(\ell\Pi_{ij}^{t-\ell}) \qquad i=1,\ldots,m; \quad \text{(A2.1)}$$

where $\bar{P}_{ij}{}^{t}$ is the annual average flow of rentals on a unit of capital of corporation i located in industry j over the period t-$\ell$ to t; $\tau_{ij}{}^{t}$ is the annual average tax rate on the rental earned by a unit of capital of corporation i in industry j over the period t-$\ell$ to t; $\Pi_{ij}{}^{t-\ell}$ and $\Pi_{ij}{}^{t}$ are the creation costs of a unit of capital of corporation i in industry j at t-$\ell$ and t, respectively; and $d_{ij}{}^{t}$ is the annual average depreciation rate on capital in corporation i in industry j over the period t-$\ell$ to t. As in section 5, it is assumed that the depreciation $\ell d_{ij}{}^{t}$ occurs just before the point in time t (cf. Figure 5.2).

Next we note that the rate of return on capital in corporation i over the period t to t+$\ell$, $R_{i*}{}^{t+\ell}$, can be calculated:

$$100R_{i*}^{t+\ell} = 100R_{i*}^{t} + 100\Delta R_{i*}^{t} \qquad i=1,\ldots,m; \quad (A2.2)$$

where $100\Delta R_{i*}{}^{t}$ is the percentage-point change in the rate of return on capital in corporation i over the period t to t+ℓ. Now recall from equation (7.3):

$$100\Delta R_{i*}^{t} = \sum_{j=1}^{n} 100\Delta R_{ij}^{t}\ S_{ij}^{t} + 100\Delta R_{in+1}^{t}\ S_{in+1}^{t} \qquad i=1,\ldots,m; \quad (A2.3)$$

where $100\Delta R_{ij}{}^{t}$ is the percentage-point change in the rate of return on the component of corporation i located in domestic industry j over the period t to t+ℓ, $100\Delta R_{in+1}{}^{t}$ is the percentage-point change in the rate of return on the foreign component of corporation i over the period t to t+ℓ; $S_{ij}{}^{t}$ is the share of corporation i's total capital stock located in domestic industry j over the period t-ℓ to t; and $S_{in+1}{}^{t}$ is the share of corporation i's total capital stock located overseas during the period t-ℓ to t. The percentage-point change $100\Delta R_{ij}{}^{t}$ can be obtained by totally differentiating equation (A2.1):

$$100\Delta R_{ij}^{t} = \bar{p}_{ij}^{t}\ \alpha_{ij}^{t} + \pi_{ij}^{t}\ \beta_{ij}^{t} + \pi_{ij}^{t-\ell}\ \gamma_{ij}^{t}$$

$$+\ 100\Delta\tau_{ij}^{t}\ \lambda_{ij}^{t} + 100\Delta d_{ij}^{t}\psi_{ij}^{t} \qquad i=1,\ldots,m \quad j=1,\ldots,n; \quad (A2.4)$$

where

$$\alpha_{ij}^{t} = \bar{P}_{ij}^{t}\ (1 - \tau_{ij}^{t})/\Pi_{ij}^{t-\ell} \qquad i=1,\ldots,m \quad j=1,\ldots,n; \quad (A2.5)$$

$$\beta_{ij}^{t} = (1 - \ell d_{ij}^{t})\ \Pi_{ij}^{t}/(\ell\Pi_{ij}^{t-\ell}) \qquad i=1,\ldots,m \quad j=1,\ldots,n; \quad (A2.6)$$

$$\gamma_{ij}^{t} = -\alpha_{ij}^{t} - \beta_{ij}^{t} \qquad i=1,\ldots,m \quad j=1,\ldots,n; \quad (A2.7)$$

$$\lambda_{ij}^{t} = -\bar{P}_{ij}^{t}/\Pi_{ij}^{t-\ell} \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n; \end{array} \quad (A2.8)$$

and

$$\psi_{ij}^{t} = -\Pi_{ij}^{t}/\ \Pi_{ij}^{t-\ell} \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n. \end{array} \quad (A2.9)$$

The above notation can be explained as follows. The lower-case variables are the percentage changes over the period t to t+$\ell$ in the respective upper-case variables; and $100\Delta R_{ij}{}^{t}$, $100\Delta\tau_{ij}{}^{t}$, and $100\Delta d_{ij}{}^{t}$ are the percentage-point changes over the period t to t+$\ell$ in the rate of return on capital, the tax rate, and the depreciation rate, respectively.

It is assumed that the percentage change in rentals earned by corporation i in industry j, $p_{ij}{}^{t}$ adjusts at a constant rate over the period t to t+$\ell$ (cf. Figure 5.1):

$$\bar{p}_{ij}^{t} = p_{ij}^{t}/2 \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n. \end{array} \quad (A2.10)$$

Note that in future work, it may be possible to empirically determine a value for the division in (A2.10) which treats the adjustment path in a less arbitrary manner; see Cooper, McLaren, and Powell (1985). To allow for this possibility we will write this divison as L in what follows.

The next step is to substitute equation (A2.10) into equation (A2.4) and express the coefficients of the resulting equation in terms of the observed prices times quantities:

$$100\Delta R_{ij}^{t} = p_{ij}^{t}\ \alpha_{ij}^{t}/L + \pi_{ij}^{t}\beta_{ij}^{t} + \pi_{ij}^{t-\ell}\ \gamma_{ij}^{t}$$

$$+\ 100\Delta\tau_{ij}^{t}\lambda_{*j}^{t} + 100\Delta d_{ij}^{t}\ \psi_{ij}^{t} \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n; \end{array} \quad (A2.11)$$

where

$$\alpha_{ij}^{t} = \bar{P}_{ij}^{t}\ K_{ij}^{t-\ell}(1 - \tau_{ij}^{t})\ /\ (\Pi_{ij}^{t-\ell} K_{ij}^{t-\ell}) \qquad i=1,\dots,m \quad j=1,\dots,n; \quad (A2.12)$$

$$\beta_{ij}^{t} = (1 - \ell d_{ij}^{t})\Pi_{ij}^{t}\ K_{ij}^{t-\ell}\ /\ (\ell\Pi_{ij}^{t-\ell}\ K_{ij}^{t-\ell}) \qquad i=1,\dots,m \quad j=1,\dots,n; \quad (A2.13)$$

$$\gamma_{ij}^{t} = -\alpha_{ij}^{t} - \beta_{ij}^{t} \qquad i=1,\dots,m \quad j=1,\dots,n; \quad (A2.14)$$

$$\lambda_{ij}^{t} = -\ \bar{P}_{ij}^{t}\ K_{ij}^{t-\ell}\ /\ (\Pi_{ij}^{t-\ell}\ K_{ij}^{t-\ell}) \qquad i=1,\dots,m \quad j=1,\dots,n; \quad (A2.15)$$

and

$$\psi_{ij}^{t} = -\ \Pi_{ij}^{t}\ K_{ij}^{t-\ell}\ /\ (\Pi_{ij}^{t-\ell}\ K_{ij}^{t-\ell}) \qquad i=1,\dots,m \quad j=1,\dots,n. \quad (A2.16)$$

Equations (A2.12) to (A2.16) are based on the following two assumptions. The first is that at time t we can observe the replacement value of the initial capital stock, $\Pi_{ij}{}^{t}K_{ij}{}^{t-\ell}$. The second assumption concerns when the depreciation of the capital stock actually occurs (cf. Figure 5.2). Here we assume that the depreciation $\ell d_{*j}{}^{t}$ occurs just before the point in time t. As a result of this last assumption, the average annual rental that is earned over the period t-ℓ to t is approximately equal to the total rental earned over the period t-ℓ to t, $P_{ij}{}^{t}K_{ij}{}^{t-\ell}$, divided by the number of years, ℓ.

It is assumed that changes in the component of corporation i located in industry j are related to the changes projected by the CGE model for industry j as follows:

$$p_{ij}^{t} = p_{*j}^{t} + h_{ij}^{(p)t} \qquad i=1,\dots,m \quad j=1,\dots,n; \quad (A2.17)$$

$$\pi_{ij}^{t} = \pi_{*j}^{t-\ell} + h_{ij}^{(\pi)t} \qquad i=1,\ldots,m \quad j=1,\ldots,n; \quad (A2.19)$$

$$\pi_{ij}^{t-\ell} = \pi_{*j}^{t-\ell} + h_{ij}^{(\pi)t-\ell} \qquad i=1,\ldots,m \quad j=1,\ldots,n; \quad (A2.19)$$

$$100\Delta\tau_{ij}^{t} = 100\Delta\tau_{*j}^{t} + h_{ij}^{(100\Delta\tau)t} \qquad i=1,\ldots,m \quad j=1,\ldots,n; \quad (A2.20)$$

$$100\Delta d_{ij}^{t} = 100\Delta d_{*j}^{t} + h_{ij}^{(100\Delta d)t} \qquad i=1,\ldots,m \quad j=1,\ldots,n; \quad (A2.21)$$

where the h's are shift variables and the other variables are defined above. The h's have been included to allow for differential responses between corporations located within one industry. For example, it may be that due to, say, better management techniques, the component of corporation i located in industry j earns 5 per cent more in rentals than the average for industry j. This situation could be handled by setting $h_{ij}{}^{(p)t}$ equal to 5.0. Note that the shift variables are not functions of the exogenous variables of the CGE model.

For consistency between an industry projection and the projections of the corporations located within that industry the following conditions on the h's must hold:

$$\sum_{i=1}^{m} h_{ij}^{(p)t} \, (P_{ij}^{t}K_{ij}^{t}/(P_{*j}^{t}K_{*j}^{t})) = 0 \qquad j=1,\ldots,n; \ (A2.22)$$

$$\sum_{i=1}^{m} h_{ij}^{(\pi)t} \, (\Pi_{ij}^{t}K_{ij}^{t}/(\Pi_{*j}^{t}K_{*j}^{t})) = 0 \qquad j=1,\ldots,n; \ (A2.23)$$

$$\sum_{i=1}^{m} h_{ij}^{(\pi)t-\ell} \, (\Pi_{ij}^{t-\ell}K_{ij}^{t-\ell}/(\Pi_{*j}^{t-\ell}K_{*j}^{t-\ell})) = 0 \qquad j=1,\ldots,n; \ (A2.24)$$

$$\sum_{i=1}^{m} h_{ij}^{(100\Delta\tau)t} \, (P_{ij}^{t} \, K_{ij}^{t}/(P_{*j}^{t}K_{*j}^{t})) = 0 \qquad j=1,\ldots,n; \ (A2.25)$$

and

$$\sum_{i=1}^{m} h_{ij}^{(100\Delta d)t} \; (\Pi_{ij}^{t} K_{ij}^{t} / \; (\Pi_{*j}^{t} K_{*j}^{t})) = 0 \qquad j=1,\ldots,n. \quad (A2.26)$$

The next step is to substitute equations (A2.17) to (A2.21) into equation (A2.11):

$$100\Delta R_{ij}^{t} = (p_{*j}^{t} + h_{ij}^{(p)t}) \; \alpha_{ij}^{t}/L + (\pi_{*j}^{t} + h_{ij}^{(\pi)t}) \; \beta_{ij}^{t}$$

$$+ \; (\pi_{*j}^{t-\ell} + h_{ij}^{(\pi)t-\ell}) \; \gamma_{ij}^{t} + (100\Delta\tau_{*j}^{t} + h_{ij}^{(100\Delta\tau)t}) \; \lambda_{ij}^{t}$$

$$+ \; (100\Delta d_{*j}^{t} + h_{ij}^{(100\Delta d)t}) \; \psi_{ij}^{t} \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n. \end{array} \quad (A2.27)$$

The percentage changes in the rental and creation costs of capital in each industry are estimated by the CGE model; see equations (5.16) and (5.17), respectively. These projected changes can be substituted into equation (A2.27):

$$100\Delta R_{ij}^{t} = (\alpha_{ij}^{t}/L)(\sum_{v=1}^{k} \eta_{P_{*j}F_v} \; f_v^{t} + h_{ij}^{(p)t})$$

$$+ \; \beta_{ij}^{t} \; (\sum_{v=1}^{k} \eta_{\Pi_{*j}F_v} \; f_v^{t} + h_{ij}^{(\pi)t}) + C_{ij}^{t} \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n; \end{array} \quad (A2.28)$$

where

$$C_{ij}^{t} = (\pi_{*j}^{t-\ell} + h_{ij}^{(\pi)t-\ell})\gamma_{ij}^{t} + (100\Delta\tau_{*j}^{t} + h_{ij}^{(100\Delta\tau)t})\lambda_{ij}^{t}$$

$$+ \; (100\Delta d_{*j}^{t} + h_{ij}^{(100\Delta d)t})\psi_{ij}^{t} \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n. \end{array} \quad (A2.29)$$

Note that $\eta_{P_{*j}F_v}$ and $\eta_{\Pi_{*j}F_v}$ and the elasticities of the rental and creation cost, respectively, in industry j with respect to the $v^{th}$

exogenous variable (i.e., factor); and $f_v{}^t$ is the percentage change in the $v^{th}$ exogenous variable originating at t and sustained at least until $t+\ell$. Recall from section 5 that it is assumed that $\pi *_j{}^{t-\ell}$, $\Delta\tau *_j{}^t$, and $\Delta d *_j{}^t$ are known from the data base.

Next we substitute equation (A2.28) into equation (A2.3):

$$100\Delta R^t_{i*} = \sum_{j=1}^{n} [(\alpha^t_{ij}/L)(\sum_{v=1}^{k} \eta_{P_{*j}F_v} f^t_v + h^{(p)t}_{ij})$$

$$+ \beta^t_{ij} (\sum_{v=1}^{k} \eta_{\Pi_{*j}F_v} f^t_v + h^{(\pi)t}_{ij}) + C^t_{ij}] S^t_{ij}$$

$$+ 100\Delta R^t_{in+1} S^t_{in+1} \qquad i=1,\ldots,m. \qquad (A2.30)$$

Furthermore, equation (A2.30) can be written:

$$100\Delta R^t_{i*} = \sum_{j=1}^{n} [\beta^t_{ij1} f^t_1 + \ldots + \beta^t_{ijk} f^t_k + (\alpha^t_{ij}/L) h^{(p)t}_{ij}$$

$$+ \beta_{ij} h^{(\pi)t}_{ij} + C^t_{ij}] S^t_{ij}$$

$$+ 100\Delta R^t_{in+1} S^t_{in+1} \qquad i=1,\ldots,m; \qquad (A2.31)$$

where

$$\beta^t_{ijv} = (\alpha^t_{ij}/L) \eta_{P_{*j}F_v} + \beta^t_{ij} \eta_{\Pi_{*j}F_v} \qquad \begin{array}{l} i=1,..,m \\ j=1,..,n \\ v=1,..,k. \end{array} \qquad (A2.32)$$

Finally, we can substitute equation (A2.31) into equation (A2.2):

$$100R_{i*}^{t+\ell} = 100R_{i*}^{t} + \sum_{j=1}^{n} [\beta_{ij1}^{t} f_1^t + \ldots + \beta_{ijk}^{t} f_k^t$$

$$+ (\alpha_{ij}^{t}/L)\, h_{ij}^{(p)t} + \beta_{ij}\, h_{ij}^{(\pi)t} + C_{ij}^{t}]\, S_{ij}^{t}$$

$$+ 100\Delta R_{in+1}^{t} S_{in+1}^{t} \qquad i=1,\ldots,m. \quad (A2.33)$$

The similarity between equation (A2.33) and the return generating process as expressed in section 3, equation (3.8), can be more clearly seen if we assume that all the shift variables are set to zero change and we rewrite equation (A2.33):

$$100R_{i*}^{t+\ell} = 100R_{i*}^{t} + \sum_{j=1}^{n} [C_{ij}^{t} + b_{ij1}^{t}\, (F_1^{t+\ell} - F_1^{t}) + \ldots$$

$$+ b_{ijk}^{t}(F_k^{t+\ell} - F_k^{t})]\, S_{ij}^{t} + 100\Delta R_{in+1}^{t}\, S_{in+1}^{t} + u_{i*}^{t+\ell}$$

$$i=1,\ldots,m; \quad (A2.34)$$

where

$$b_{ijv}^{t} = B_{ijv}^{t}\, f_v^t\, F_v^t \,/\, 100 \qquad \begin{array}{l} i=1,\ldots,m \\ j=1,\ldots,n \\ v=1,\ldots,k. \end{array} \quad (A2.35)$$

Two final points can be made concerning equation (A2.34). The first is that it contains the term ${u_{i*}}^{t+\ell}$ which is the realized error at time $t+\ell$. The second concerns the role of the term ${C_{ij}}^{t}$. This term is implicitly contained in equation (3.8), where the factors are changes in: the creation costs of capital over the period $t-\ell$ to $t$, the tax rate over the period t to $t+\ell$, and the depreciation rate over the period t to $t+\ell$.

## A.3 ANALYTICAL DERIVATION OF THE EFFICIENT PORTFOLIO FRONTIER

In this appendix we follow Merton (1972) and analytically derive the efficient portfolio frontier when agents are allowed to borrow and short-sell all assets. Suppose that at instant t there are m+1 risky assets with the expected average annual rate of return on the $i^{th}$ asset over the period t to t+2 denoted by $E^t(R_{i*}{}^{t+2,s})$, the covariance of returns between the $i^{th}$ and the $j^{th}$ assets denoted by $Cov^t(R_{i*}{}^{t+2,s}, R_{j*}{}^{t+2,s})$, and the variance of the return on the $i^{th}$ asset denoted by $Var^t(R_{i*}{}^{t+2,s}, R_{i*}{}^{t+2,s})$. Let $S_{(p)i}{}^t$ equal the share of the value of portfolio p invested in the $i^{th}$ asset at t. Then, the frontier of efficient portfolios can be described as the set of portfolios that satisfy the following constrained minimization problems (each conditioned on a specific value of the expected rate of return):

Choose $S^t_{(p)1}, \ldots, S^t_{(p)m+1}$

to minimize $\frac{1}{2} Var^t(R^{t+2,s}_{(p)})$ ; (A3.1)

subject to

$$Var^t(R^{t+2,s}_{(p)}) = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} S^t_{(p)i}\, S^t_{(p)j}\, Cov^t(R^{t+2,s}_{i*}, R^{t+2,s}_{j*}) \quad ; \qquad (A3.2)$$

$$E^t(R^{t+2,s}_{(p)}) = \sum_{i=1}^{m+1} S^t_{(p)i}\, E^t(R^{t+2,s}_{i*}) \quad ; \qquad (A3.3)$$

and

$$1 = \sum_{i=1}^{m+1} S^t_{(p)i} \quad . \qquad (A3.4)$$

Using Lagrangian multipliers we can write:

$$\Gamma = \tfrac{1}{2} \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} S_{(p)i}^{t} S_{(p)j}^{t} \mathrm{Cov}^{t}(R_{i*}^{t+2,s}, R_{j*}^{t+2,s})$$

$$+ \lambda_1 (E^{t}(R_{(p)}^{t+2,s}) - \sum_{i=1}^{m+1} S_{(p)i}^{t} E^{t}(R_{i*}^{t+2,s}))$$

$$+ \lambda_2 (1 - \sum_{i+1}^{m+1} S_{(p)i}^{t}) \quad . \qquad \text{(A3.5)}$$

The resulting first-order conditions are:

$$\partial\Gamma/\partial S_{(p)i}^{t} = \sum_{j=1}^{m+1} S_{(p)j}^{t} \mathrm{Cov}^{t}(R_{i*}^{t+2,s}, R_{j*}^{t+2,s})$$

$$- \lambda_1 E^{t}(R_{i*}^{t+2,s}) - \lambda_2 = 0 \qquad i=1,\ldots,m+1; \text{ (A3.6)}$$

$$\partial\Gamma/\partial\lambda_1 = E^{t}(R_{(p)}^{t+2,s}) - \sum_{i=1}^{m+1} S_{(p)i}^{t} E^{t}(R_{i*}^{t+2,s}) = 0 \quad ; \qquad \text{(A3.7)}$$

and

$$\partial\Gamma/\partial\lambda_2 = 1 - \sum_{i=1}^{m+1} S_{(p)i}^{t} = 0 \quad . \qquad \text{(A3.8)}$$

The $S_{(p)i}{}^{t}$'s that satisfy (A3.6), (A3.7) and (A3.8) minimize (A3.1) and are unique if no asset can be represented as a linear combination of the other assets (i.e., if the variance-covariance matrix of returns is nonsingular).

Next we note that equation (A3.6) can be written in matrix notation as:

$$\mathrm{Cov}^{t}(R_{.*}^{t+2,s})\, S_{(p).}^{t} = \lambda_1 E^{t}(R_{.*}^{t+2,s}) + \lambda_2 \underline{1} \quad ; \qquad \text{(A3.9)}$$

where $Cov^t(R_{.*}{}^{t+2,s})$ is an $((m+1)\times(m+1))$ variance-covariance matrix of the rates of return of the $(m+1)$ securities; $S_{(p).}{}^t$ is a $((m+1)\times 1)$ vector of the shares of the value of portfolio p invested in the $(m+1)$ securities; $E^t(R_{.*}{}^{t+2,s})$ is an $((m+1)\times 1)$ vector of the expected returns of the $(m+1)$ securities; and $\underline{1}$ is an $((m+1) \times 1)$ vector every element of which is unity.

Following equation (A3.9), the vector $S_{(p).}{}^t$ can be expressed:

$$S^t_{(p).} = \lambda_1 \, [Cov^t(R^{t+2,s}_{.*})]^{-1} \, E^t(R^{t+2,s}_{.*})$$

$$+ \lambda_2 [Cov^t(R^{t+2,s}_{.*})]^{-1} \underline{1} \qquad . \qquad (A3.10)$$

Let $v_{ij}$ be the $ij^{th}$ element of $[Cov^t(R_{.*}{}^{t+2,s})]^{-1}$. Equation (A3.10) can then be written:

$$S^t_{(p)i} = \lambda_1 \sum_{j=1}^{m+1} v_{ij} \; E^t(R^{t+2,s}_{j*})$$

$$+ \lambda_2 \sum_{j=1}^{m+1} v_{ij} \qquad i=1,\ldots, m+1. \quad (A3.11)$$

Now multiply equation (A3.11) by $E^t(R_{i*}{}^{t+2,s})$ and then sum over i:

$$\sum_{i=1}^{m+1} E^t(R^{t+2,s}_{i*}) \, S^t_{(p)i} = \lambda_1 \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} \, E^t(R^{t+2,s}_{j*}) \, E^t(R^{t+2,s}_{i*})$$

$$+ \lambda_2 \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} \, E(R^{t+2,s}_{i*}) \qquad . \qquad (A3.12)$$

Next we sum equation (A3.11) over i:

$$\sum_{i=1}^{m+1} S_{(p)i}^{t} = \lambda_1 \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} E^t(R_{j*}^{t+2,s})$$

$$+ \lambda_2 \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} \quad . \qquad (A3.13)$$

It is possible to ease the notational burden by making the following definitions:

$$X = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} E^t(R_{j*}^{t+2,s}) \quad ; \qquad (A3.14)$$

$$Y = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} E^t(R_{j*}^{t+2,s}) E^t(R_{i*}^{t+2,s}) \quad ; \qquad (A3.15)$$

and

$$Z = \sum_{i=1}^{m+1} \sum_{j=1}^{m+1} v_{ij} \quad . \qquad (A3.16)$$

Using equations (A3.14), (A3.15), and (A3.16), equation (A3.12) can be written:

$$\sum_{i=1}^{m+1} E^t(R_{i*}^{t+2,s}) S_{(p)i}^{t} = \lambda_1 Y + \lambda_2 X \quad . \qquad (A3.17)$$

Similarly we can rewrite equation (A3.13):

$$\sum_{i=1}^{m+1} S_{(p)i}^{t} = \lambda_1 X + \lambda_2 Z \quad . \qquad (A3.18)$$

Using equation (A3.3) we can rewrite equation (A3.17):

$$E^t(R_{(p)}^{t+2,s}) = \lambda_1 Y + \lambda_2 X \qquad . \qquad (A3.19)$$

Using equation (A3.4) we can rewrite equation (A3.18):

$$1 = \lambda_1 X + \lambda_2 Z \qquad . \qquad (A3.20)$$

From (A3.20) we can solve for $\lambda_1$:

$$\lambda_1 = (1 - \lambda_2 Z)/X \qquad . \qquad (A3.21)$$

Substitute (A3.21) into (A3.19):

$$E^t(R_{(p)}^{t+2,s}) = (1 - \lambda_2 Z)Y/X + \lambda_2 X \qquad . \qquad (A3.22)$$

Rearrange equation (A3.22):

$$\lambda_2 = (E^t(R_{(p)}^{t+2,s}) X - Y)/(X^2 - YZ) \qquad . \qquad (A3.23)$$

Substitute (A3.23) into (A3.21):

$$\lambda_1 = (X - E^t(R_{(p)}^{t+2,s})Z)/(X^2 - YZ) \qquad . \qquad (A3.24)$$

Now substitute equations (A3.23) and (A3.24) into equation (A3.11):

$$S_{(p)i}^t = \{(X - E^t(R_{(p)}^{t+2,s})Z) \sum_{j=1}^{m+1} v_{ij} \, E^t(R_{i*}^{t+2,s})$$

$$+ (E^t(R_{(p)}^{t+2,s})X - Y) \sum_{j=1}^{m+1} v_{ij}\}/(X^2 - YZ) \qquad i=1, \ldots, m+1. \quad (A3.25)$$

Equation (A3.25) can also be written:

$$S^t_{(p).} = \{(X - E^t(R^{t+2,s}_{(p)})Z)/(X^2 - YZ)\}[Cov^t(R^{t+2,s}_{.*})^{-1}]\ E^t(R^{t+2,s}_{.*})$$

$$+ \{(E^t(R^{t+2,s}_{(p)})X - Y)/(X^2 - YZ)\}[Cov^t(R^{t+2,s}_{.*})^{-1}]\ \underline{1} \quad ; \qquad (A3.26)$$

where $S_{(p).}{}^t$ is an ((m+1)×1) vector of the shares of each of the (m+1) risky assets held in the frontier portfolio p; the values enclosed within curly brackets "{}" are scalars; $[Cov^t(R_{.*}{}^{t+2,s})^{-1}]$ is the((m+1)×(m+1)) inverse of the variance-covariance matrix of the rates of return of the (m+1) assets; $E^t(R_{.*}{}^{t+2,s})$ is an ((m+1)×1) vector of the expected rates of return of the (m+1) assets; and $\underline{1}$ is an ((m+1)×1) unit vector. Equation (A3.26) is important as it solves for shares of each asset held in the frontier portfolio with an expected return $E^t(R_{(p)}{}^{t+2,s})$.

Finally, we will solve explicitly for the efficient frontier. First we multiply equation (A3.6) by $S_{(p)i}{}^t$ and then sum over i:

$$\sum_{i=1}^{m+1}\sum_{j=1}^{m+1} S^t_{(p)i}\, S^t_{(p)j}\, Cov^t(R^{t+2,s}_{i*},\ R^{t+2,s}_{j*}) =$$

$$\lambda_1 \sum_{i=1}^{m+1} S^t_{(p)i}\, E^t(R^{t+2,s}_{i*}) + \lambda_2 \sum_{i=1}^{m+1} S^t_{(p)i} \quad . \qquad (A3.27)$$

From equations (A3.2), (A3.3), and (A3.4) we can rewrite equation (A3.27):

$$Var^t(R^{t+2,s}_{(p)}) = \lambda_1\, E^t(R^{t+2,s}_{(p)}) + \lambda_2 \quad . \qquad (A3.28)$$

Next substitute equations (A3.23) and (A3.24) into equation (A3.28):

$$Var^t(R_{(p)}^{t+2,s}) = [ZE^t(R_{(p)}^{t+2,s})^2 - 2XE^t(R_{(p)}^{t+2,s}) + Y]/(YZ - X^2) \quad . \qquad (A3.29)$$

This gives the variance associated with the optimal portfolio p yielding an expected return $E^t(R_{(p)}{}^{t+2,s})$. To solve for the maximum expected return obtainable for a given variance, we proceed as follows.

From equation (A3.29) we can write:

$$aE^t(R_{(p)}^{t+2,s})^2 + bE^t(R_{(p)}^{t+2,s}) + c = 0 \quad ; \qquad (A3.30)$$

where $a = Z$ ; (A3.31)

$b = -2X$ ; (A3.32)

and

$$c = Y - Var^t(R_{(p)}^{t+2,s})(YZ - X^2) \quad . \qquad (A3.33)$$

Noting that equation (A3.30) is quadratic, we can write the equation for $E^t(R_{(p)}{}^{t+2,s})$:

$$E^t(R_{(p)}^{t+2,s}) = -b/(2a) \pm (b^2 - 4ac)^{1/2}/(2a) \quad . \qquad (A3.34)$$

Finally, substitute (A3.31), (A3.32), and (A3.33) into (A3.34):

$$E^t(R_{(p)}^{t+2,s}) = X/Z + Max\{\pm\ [(Z\ Var^t\ (R_{(p)}^{t+2,s}) - 1)\ (YZ - X^2)]^{1/2}/Z\} \quad . \qquad (A3.35)$$

NOTES

1. For surveys of CGE models see, for example, Shoven and Whalley (1984), Borges (1986), and Decaluwe and Martens (1986).

2. For more details see, for example, Harrington (1987).

3. Readers please note that it is not possible with our current word-processing system to represent in the text in the same vertical space a superscript and subscript. To overcome this limitation, the convention of first typing the subscripts and then the superscripts was adopted.

4. Systematic risk is an estimate of how the expected returns from an asset or portfolio will move relative to the returns from the market portfolio. As an aside, note that the CAPM designated systematic risk as beta; see equation (3.21).

5. The unsystematic risk of an asset is caused by changes that are specific to the asset (i.e., not related to the market). For example, changes in a firm's management may affect its returns independent of changes in the market's rate of return.

6. Note that it is only with the introduction of a concave utility function that the greater sensitivity translates into "risk". In the pure APT with no utility function, a downward sloping capital market line is possible.

7. See Higgs and Powell (forthcoming).

8. A complete list of the exogenous variables is given in Higgs (1987a).

9. The real exchange rate is defined to be equal to the nominal exchange rate times the world price index divided by the domestic price index. Note that, in all the simulations, any changes in the world price index have been assumed to be negligible.

10. Note that this is consistent with the ORANI theory of investment; see Dixon, Parmenter, Sutton, and Vincent (1982, pp. 94-5).

11. Note that we have implicitly assumed that the period during which the unit of capital is held (one year) is long enough to avoid any capital gains tax.

12. The derivation of equation (5.3) is given in Appendix A.1.

13. With $\ell$ set equal to a value no smaller than the length of the ORANI short run, no further adjustment occurs beyond this point under the conditions of a standard short-run closure.

14. Suppose there were no changes between t and t+$\ell$ in any of the factors (i.e., $F_v^{t+\ell} = F_v^t$ for all v = 1,..., k) and that there were no change in the tax rate or the depreciation rate. Thus the only component capable of generating a change in $R_{*j}$ is the first term on the right of equation (5.19); namely, the inherited initial growth rate of capital goods prices. If there is no change from one period to the next in this growth rate (and

therefore no change in the initial conditions), then the rates of return are also stationary.

15. Note that estimates of the value of capital stocks are contained in the ORANI model. However, these values were principally estimated to provide a commodity breakdown of the creation of a unit of investment in each industry rather than to estimate the total value of the capital stock in each industry; see Hourigan (1980). Rather than use these estimates the values of the capital stocks at time t-2 and t were calibrated to achieve the rates of return shown in Table 5.2. For more details see Higgs (1987a).

16. Recall from section 4 that a one per cent increase in the nominal exchange rate (i.e., the numeraire) has the effect of increasing all prices by one per cent but leaves all real variables unaffected.

17. For an example where the future developments in the exogenous variables are correlated see Higgs (1987b).

18. If $A = (a_{ij})$ and $B = (b_{ij})$ be each $(m \times n)$ matrices, their Hadamard product is the $(m \times n)$ matrix of elementwise products; $A \circ B = (a_{ij} b_{ij})$. For more details see, for example, Rao and Mitra (1971).

19. Notice that since $R_{\cdot *}^{t+2} = R_{\cdot *}^{t} + \Delta R_{\cdot *}^{t}$, the conditional covariance matrix among the elements of $R_{\cdot *}^{t+2}$, given the realization $R_{\cdot *}^{t}$, is identically the conditional covariance matrix for $\Delta R_{\cdot *}^{t}$.

20. To solve this problem use was made of a quadratic programming package based on Land and Powell (1973).

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