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A Commonwealth Government inter-agency project in co-operation with the University of Melbourne, to facilitate the analysis of the impact of economic demographic and social changes on the structure of the Australian economy

THE *ORANI-MACRO* INTERFACE: AN ILLUSTRATIVE EXPOSITION

by

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6. For a technical description of continuous time econometric models, see Bergstrom (1976).

7. For details of RBA79, see Jonson and Trevor (1981) and Jonson, McKibbin and Trevor (1980).

The ORANI-MACRO Interface: An Illustrative Exposition

by

Russel J. Cooper and Keith R. McLaren*

I. Introduction

A convenient device for thinking about the IMPACT Project's medium term model is in terms of three distinct modules:

(i) ORANI, a general equilibrium model specifying the sectoral composition of output, the aggregate volumes and composition of imports and exports, occupation specific demands for labour, and relative commodity prices;

(ii) MACRO, a macroeconomic model determining aggregate levels of real private consumption and investment, and modelling the financial and monetary markets;

and

(iii) BACHUROO, a demographic model endogenizing the supply of labour disaggregated by nine occupational groups.[1]

In the early stages, the development of each of these three components proceeded independently. This paper is concerned with the interfacing of the ORANI and MACRO modules.

The idea of constructing ORANI and MACRO as separate modules was appealing from the point of view of division of labour, but may be open to criticism on at least two grounds. The first concerns estimation. Given that ORANI and MACRO do not constitute a fully block recursive model, their joint estimation would be required to ensure cross-equation consistency and efficiency. However, in the absence (among other things) of

(a) an integrated data base - - ORANI is based largely on input-output data from a given historical year, while MACRO uses quarterly aggregate time series national accounts data,

(b) a theory of estimation which handles the two classes of data,

and

(c) computer hardware capable of handling such a massive task,

reversion to the second-best alternative, namely, separate estimation of each module, is the appropriate course.

The second criticism, made from an economic-theoretic standpoint, is that certain endogenous variables should be determined compatibly with the mechanisms posited in both modules, an outcome which cannot be guaranteed when each is developed as a separate model in its own right. This difficulty is the challenge to be faced in linking the two modules, at which stage the possible interdependencies between them must be incorporated.

From this latter perspective, the theory of the interface between the modules becomes a critical methodological issue. This theory must,

Footnotes

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1. For more detail on the original design of IMPACT's medium term model, see Powell (1977).

2. The class of applied general equilibrium models pioneered by Leif Johansen in Johansen (1960).

3. For a comprehensive technical description of the forerunner of ORANI 78, see Dixon, Parmenter, Ryland and Sutton (1977). The technical detail of ORANI 78 is given in Dixon et. al. (1981).

4. For a statement of these arguments, see Powell (1981).

5. For a detailed critical assay of the design and performance of the prototype, see Norton (ed) (1977).

VI Conclusion

This paper has presented the methodology by which the MACRO and ORANI modules of the IMPACT Project have been interfaced to form a composite macroeconomic general equilibrium model of the Australian economy. It has been found that a combined system in which, at the interface, MACRO provides the domestic macroeconomic environment and ORANI provides the components of the export sector, constitutes a reasonably well integrated model of the real sector of the economy. The details of this interfaced system are illustrated by experiment A2 of Section V. The methodology outlined there, in which we have analysed the responses of certain aggregate variables to a government spending shock, is immediately applicable to a variety of shocks and responses. Thus, for example, sectoral specific responses to either aggregate or sectoral specific shocks may be analysed within the context of an endogenous macroeconomic environment.

of course, reflect the theoretical structures of the two modules. Of the various sub-classes of models in the class of general equilibrium models, ORANI is of the Johansen type.[2] A second generation working version of ORANI, ORANI 78, is now available.[3] In the terminology of Challen and Hagger (1979) MACRO might be chosen either from the Keynes-Klein (KK) class, or the Phillips-Bergstrom (PB) class (although there are some strong arguments for choosing the latter).[4] It is proposed that MACRO should be based on the RBII structure, a disequilibrium model of the PB class developed by Jonson and co-workers at the Reserve Bank of Australia.[5] The particular features of this model - - its formulation as a disequilibrium system in continuous time, its linearity in logarithms - - suggests that this model (and others in the same sub-class) be referred to as a Bergstrom (or perhaps Bergstrom-Wymer) model.[6]

The structure of the remainder of this paper is as follows. In Section II, salient features of the Bergstrom and Johansen sub-classes of models are described. Both problems and the potential for reconciliation suggested by these features are discussed in Section III. Section IV is devoted to a simplified diagrammatic exposition of the methodology of the interface. Then in Section V we illustrate the interface by a concrete application to the MACRO and ORANI models.

II The Two Models

MACRO

This model is to be identified with RBA79, a model which has been documented in a number of papers.[7] As it appears in these papers, the basic structural form of MACRO can be represented as:

$$DY_M(t) = E_M \, DY_M(t) + F_M \, Y_M(t) + G_M \, Z_M(t) \qquad (1)$$

where the subscript M is used throughout to denote MACRO variables or parameters; Y_M refers to the vector of MACRO endogenous variables; Z_M is the vector of MACRO exogenous variables; D denotes differentiation with respect to time. As a general rule, variables are expressed as logarithms in the general specification (1).

MACRO identifies 26 endogenous variables and 47 exogenous variables, and models such macro-aggregates as consumption, investment, labour demand, output, prices, taxes, the exchange rate, various interest rates and various monetary aggregates. The typical model specification is one of partial adjustment toward equilibrium values. The reader is referred to the papers by Jonson et al. for more details on the form of the system (1).

For our purposes it is more useful to transform system (1) to "reduced form" by premultiplying through by $(I - E_M)^{-1}$. This gives the system:

Table 2

A Typical Outcome for Experiment B3(b)

Double Endogeneity		
Variable	Model Endogenizing Variable	$t^* = 8$ quarters
Output	MACRO	1.74
	ORANI	1.81
Labour	MACRO	1.31
	ORANI	2.68
Real Wages	MACRO	0.29
	ORANI	-1.40

(b) Interpretation is as for Table 1, except that the parameters $t^* = 9.0$, $\beta_1 = \beta_2 = \beta_3 = 0.5$ are arbitrarily chosen.

Table 1

Results of Experiment A2 [a]

Double Endogeneity		ORANI Short Run (Quarters) t^*		
Variable	Model Endogenizing Variable	8.0	8.25	8.5
Output	MACRO	0.79	0.77	0.75
	ORANI	0.88	0.87	0.85
Labour	MACRO	0.79	0.79	0.79
	ORANI	1.27	1.24	1.22
Imports	MACRO	2.28	2.17	2.06
	ORANI	2.28	2.30	2.31
Prices	MACRO	0.87	0.90	0.93
	ORANI	3.18	3.24	3.30
Weighted Sum of Squared Differences		0.24	0.23	0.25

(a) The number in the body of the table is the percentage increase (relative to what it would otherwise have been) in the indicated endogenous variable t^* quarters after the instant at which the path of government spending is lifted 10% above the path it would otherwise have followed. These results are conditional upon the jointly determined optimal adjustment rates $\beta_1 = 0.99$, $\beta_2 = 0.99$.

$$DY_M(t) = A_M Y_M(t) + B_M Z_M(t) \tag{2}$$

which we treat as the representative form of a Bergstrom type, linear in logarithms, continuous time dynamic model. Now (2) is a system of linear differential equations which can be solved explicitly for a time path of endogenous variables $Y_M(t)$, $t \geq 0$, given an initial condition $Y_M(0)$ and a time path of exogenous variables $Z_M(t)$, $t \geq 0$. Let the "control" solution path $Y_M^C(t)$ correspond to the "control" time path of exogenous variables $Z_M^C(t)$, and define a "shocked" path of exogenous variables by $Z_M^S(t) = Z_M^C(t) + z_M$. Then the deviation of the shocked path of endogenous variables from the control path can be written as

$$y_M(t) = Y_M^S(t) - Y_M^C(t)$$

and can be expressed analytically as

$$y_M(t) = C_M(t)\, z_M , \tag{3}$$

where

$$C_M(t) = A_M^{-1} [\exp(A_M t) - I] B_M . \tag{4}$$

For more details see Cooper and McLaren (1980). Equation (3) may be interpreted as follows. Since Y_M and Z_M are logarithms of variables, y_M and z_M correspond to proportional changes. By z_M is meant proportional changes which occur in the levels of the exogenous variables at time 0 and which are maintained throughout the entire interval $[0, t]$-- e.g. government spending is increased by 10% at time

0 and is then held at a 10% higher level than it would otherwise have been. On the other hand, $y_M(t)$ represents the accumulated proportional changes in the MACRO endogenous variables up to the point t in response to the shock z_M. Thus it should be noted that MACRO can answer such questions as: "if government spending is increased by 10%, what will be the percentage impact t quarters later on prices, employment, output etc." without requiring explicit simulation of the levels of the variables. All relevant information is contained in the matrix of elasticities $C_M(t)$, which we note is a function of the time lag t.

ORANI

This model is to be identified with ORANI 78, which is fully documented in Dixon et al. (1981). The structure of ORANI can be represented by the system:

$$y_0 = C_0 z_0 \qquad (5)$$

where the subscript 0 is used throughout to denote ORANI variables and parameters; y_0 represents a vector of proportional changes in ORANI endogenous variables; z_0 represents a vector of proportional changes in ORANI exogenous variables; and C_0 is the matrix of ORANI elasticities.

ORANI provides quite a deal of flexibility in the allocation of variables to the endogenous/exogenous categories, but there are typically many thousands in each category. Thus C_0 is a "large"

Turning now to the difference in results for the price variable, the incompatibility between the two models seems to be quite extreme. The first point to make is that, since in this configuration ORANI is homogeneous of degree one in nominals, the exchange rate from MACRO acts as numeraire. Hence the ORANI price response depends critically on the MACRO exchange rate response, a sector of the macroeconomy that is notoriously difficult to model. It is for this reason that we feel justified in concentrating on real variables and relative prices at the interface, and hence giving prices a low weighting. The actual size of the price response difference may be attributed to a number of factors. For example, in ORANI all indirect taxes are passed directly into prices, whereas this is not the case in MACRO. Also, ORANI is based on a production function elasticity of substitution of 0.5, whereas the MACRO value is 1.0. Thus a larger increase in price relative to wage rates is required by ORANI in order to induce the extra supply to meet the increase in demand generated by the increased government spending.

in which the real wage rate was exogenous to MACRO and endogenized by ORANI. However the results were discouraging.

Our preferred configuration for the interface is therefore one in which the ORANI exogenous variables real wages, consumption demand, investment demand and the exchange rate are endogenized by MACRO, and the MACRO exogenous variables price of wool, price of exports and quantity of exports are endogenized by ORANI. Our preferred estimate of the value of the ORANI short run is 8.25 quarters, and at this value the double endogeneities output and imports virtually correspond for the two models. Thus all elements of the national income identity correspond for the ORANI and the MACRO parts of the overall ORANI-MACRO model (recall that government spending, consumption, investment and exports are common to the two parts). We turn now to the two double endogeneities which give incompatible results.

For the double endogeneity labour, the differing results between the two models can be attributed to the fact that the production function constraint is applied differently in the two models. In ORANI the production function always holds, whereas in MACRO the production function only holds at a long-run equilibrium. Thus in the experiment considered, the shock to government spending led to a 0.77% increase in output in MACRO. Since the effect on capital stock is negative, the MACRO production function would suggest a need to increase employment by at least (1/0.7) = 1.4 times the increase in output, or 1.1%, which is fairly close to the ORANI value of 1.24%.

matrix. ORANI can provide results of the following form: "given a policy change A in the macroeconomic environment B, then in the short-run, variable C will differ by x% from the value it would have had in the absence of the policy change ..." (Dixon et al. (1981), ch.3). Thus we note that system (5) has an interpretation quite analogous to that of (3), provided that we note the following two essential points. Firstly, ORANI in stand-alone form takes the macroeconomic climate as given, so that many variables which appear in the vector z_0 also appear in the vector y_M. It is the purpose of the interface to produce a combined ORANI-MACRO model in which those macroeconomic variables which appear as ORANI exogenous variables are endogenized at the interface by MACRO. Secondly, whereas the elasticity matrix for MACRO is explicitly time dependent, the ORANI elasticity matrix is constant. But implicit in the calculation of the ORANI elasticity matrix is the notion of the ORANI short-run. The short-run is defined as a period long enough for prices to adjust, for output to be expanded using given plant, for new investment plans to be made but not completed etc., but not long enough for changes in the size of capital stock in use. The length of the ORANI short run will be designated by the symbol t^*, and to indicate the dependence of the matrix of ORANI elasticities on the value of t^*, we will henceforth write (5) as

$$y_0 = C_0(t^*)z_0 . \qquad (6)$$

III Problems and Potentialities

A comparison of (3) and (6) reveals some potential for compatibility of ORANI and MACRO as modules of a combined system. The final form of each model is linear in percentage changes. Additionally, MACRO treats as endogenous those macroeconomic variables which are exogenous to ORANI. However, in order to consider the problems of combining systems (3) and (6) it is necessary to introduce some new notation. Consider firstly the set of MACRO endogenous variables y_M. Variables in this set may also be:

(i) endogenous to ORANI, i.e. in y_0. The set of such "double endogeneities" will be denoted $y_M y_0$

(ii) exogenous to ORANI, i.e. in z_0. The set of such variables exogenous to ORANI but endogenized by MACRO will be denoted by $y_M z_0$.

(iii) not appearing in ORANI. If we let x_0 represent the set of all variables in ORANI, i.e.,

$$x_0 = \begin{bmatrix} y_0 \\ z_0 \end{bmatrix},$$

and let $\tilde{x}_0$ be the set of all variables in the combined system not appearing in ORANI, then the set of variables endogenous to MACRO but not appearing in ORANI may be represented as $y_M \tilde{x}_0$.

Using a combination of standard vector and set notation, the vector y_M may be decomposed into three disjoint vectors,

gives the best overall results, except that prices seem to be inconsistent. To avoid the price effect dominating, it was given a zero weighting in the iterative process, with output, employment and imports weighted equally. The result was an optimal value of t^* of 8.25 quarters, and speeds of adjustment of $\beta_1 = 0.99$, $\beta_2 = 0.99$ (corner solution). Behaviour in the neighbourhood of t^* is illustrated in Table 1. From Table 1, it is seen that the response for imports is quite compatible between the two models, and so experiment A3 gives quite similar results. Unlike the export sector, which forms a relatively separate part of MACRO, the import sector enters into a number of other equations and forms an integral part of the MACRO model. Thus it was felt that there were no advantages, and some disadvantages, in experiment A3 compared with A2. The preferred interface for this configuration of ORANI is therefore experiment A2.

In the B type experiments, ORANI takes the price level as given, and endogenizes the real wage rate. Two general features of the results of this experimental configuration are that in all the experiments there is typically less compatibility between the double endogeneities than in the corresponding A type experiments, and that the double endogeneity real wages tends to be typically negative in ORANI and positive in MACRO. This latter problem of course corresponds to the incompatibility of the price variable in the A experiments. As an illustration, Table 2 presents the results for a t^* value of 8 quarters, and with $\beta_1 = \beta_2 = \beta_3 = 0.5$. No locally optimal results could be obtained within a reasonable range of t^* values. To try to improve on these results, a fourth experiment with the B configuration was carried out, response to equal proportionate changes in prices and the exchange rate. Since MACRO does not directly provide an estimate of the rate of adjustment of real wages, the corresponding ORANI+ parameter β_3 is freely estimated together with β_1, β_2 and t^*. The adjustment parameters for output, employment and quantity of exports are constrained as in the A experiments.

Results

For any given choices of the speed of adjustment parameters and the value of t^*, the combined ORANI - MACRO model can be simulated in a manner quite analogous to the MACRO response (3). Thus a variable exogenous to both models (or exogenous to one model and excluded from the other) may be subjected to a shock, and the responses of the endogenous variables calculated. These endogenous variables contain the set of double endogeneities, and the compatibility of these variables provides a measure of the appropriateness of the choice of speed of adjustment parameters and t^*. Specifically, government spending g was subjected to a shock of 10%, and a weighted sum of squared differences between doubly endogenous variables was used as a criterion function. An iterative procedure was then used to choose the optimal values for the adjustment parameters and t^*.

As expected, experiment A1 leads to results in which the export sector variables are quite inconsistent between the two models. Thus for a value of $t^* = 8$, $\beta_1 = \beta_2 = 0.5$, ORANI gives an export response of -5.48%, while the MACRO figure is -0.38%. Experiment A2

$$y_M = \begin{bmatrix} y_M y_0 \\ y_M z_0 \\ y_M \tilde{x}_0 \end{bmatrix}$$

in which each element of y_M is further classified according to its role in ORANI. The ordering of the terms in the expression $y_M y_0$ is significant. The notations $y_M y_0$ and $y_0 y_M$ both identify the subset of variables in the system which are doubly endogenous. These double endogeneities are to be retained in the combined system. Thus $y_M y_0$ is the vector of <u>values</u> of the doubly endogenous variables as endogenized, within the combined system, by the MACRO model, while $y_0 y_M$ is the vector of such values within the combined system as endogenized by ORANI.

Extending this notation analogously to the vectors y_0, z_M and z_0, and suppressing reference to t and t^* allows (3) and (6) to be written as:

$$\begin{matrix}(a)\\(b)\\(c)\end{matrix} \quad \begin{bmatrix} y_M y_0 \\ y_M z_0 \\ y_M \tilde{x}_0 \end{bmatrix} = C_M \begin{bmatrix} z_M y_0 \\ z_M z_0 \\ z_M \tilde{x}_0 \end{bmatrix} \qquad (7)$$

$$\begin{matrix}(a)\\(b)\\(c)\end{matrix} \quad \begin{bmatrix} y_0 y_M \\ y_0 z_M \\ y_0 \tilde{x}_M \end{bmatrix} = C_0 \begin{bmatrix} z_0 y_M \\ z_0 z_M \\ z_0 \tilde{x}_M \end{bmatrix} . \qquad (8)$$

The combined system (7) and (8) can be interpreted for present purposes as the structural form of the linked ORANI-MACRO system (a structural form because the right-hand side variables in the sets $z_M y_O$ and $z_O y_M$ are endogenous to the system as a whole). However, three (interrelated) issues must be considered before (7) and (8) can be put into a reduced form analogous to (3) or (6).

Firstly, there is the problem of double endogeneity. Equations (7) and (8) separately represent closed linear models, each having as many linearly independent equations as endogenous variables. Certain of the latter will be endogenous in both models, leading to separate equations (7)(a) and (8)(a) respectively for $y_M y_O$ and $y_O y_M$. Whilst these refer to the same set of variables there are no 'across-models' constraints built into the estimation of either MACRO or ORANI to ensure that $y_M y_O$ and $y_O y_M$ take the same values. In this sense there is scope for incompatibility between the two models. Short of a competitive evaluation of the performance and of the plausibility of the mechanisms of the two models, there can be no justification for deleting either of (7) (a) or (8) (a) while maintaining the other. The only sensible reduced form that is implied by the joint system is that which includes both $y_M y_O$ and $y_O y_M$. It will be shown below that there is scope for minimizing the extent of this incompatibility.

Secondly, the elasticities in (8) are elements of the matrix $C_O(t^*)$, so (8) represents the accumulated response of the endogenous variables y_O after an elapsed time t^*, where t^* is the ORANI short-run. The elasticities in (7) are elements of the matrix $C_M(t)$, which

This choice of ORANI endogenous/exogenous variables corresponds to the standard ORANI experiment. The elasticities matrix is homogenous of degree one in nominal variables and homogeneous of degree zero in real variables. The exchange rate, provided by MACRO, acts as numeraire.

The classification of variables for experiment A2 is as for experiment A1, except that price of exports and quantity of exports are moved from category (a) to category (d). Experiment A3 is generated by also moving quantity of imports from category (a) to category (d).

The imposed speeds of adjustment are (i) output, employment and quantity of exports : a β value of .12 which corresponds to the estimated MACRO $\ell n\beta$ coefficient on the output equation of -2.14; (ii) the domestic price level : a β value of .59 corresponding to the MACRO $\ell n\beta$ estimate of -0.53. This leaves three parameters to be freely estimated, (i) β_1 : the speed of adjustment of all export prices; (ii) β_2 : the speed of adjustment of the quantity of imports; (iii) t^* : the value in quarters of the ORANI short run.

Category B Experiments

The categorisation of variables for Experiment B1 is as in Experiment A1, except that prices and real wages are interchanged in classes (a) and (c). This choice is based on the argument that MACRO should provide "the" price level directly to ORANI, rather than indirectly through the exchange rate. In this case the ORANI elasticity matrix is homogeneous (of degree one in nominals and zero in reals) in

<u>Category A Experiments</u>

Experiment A1 is defined by the following classification of variables:

(a) Variables endogenous to both models:
- output
- employment
- domestic price level
- price of exports
- quantity of exports
- quantity of imports

(b) Variable exogenous to both models and subject to shock:
- government spending

(c) Variables exogenous to ORANI endogenized by MACRO:
- exchange rate
- real wages
- consumption
- investment

(d) Variables exogenous to MACRO endogenized by ORANI:
- price of wool

requires an explicit value of t. If the combined system (7) - (8) is to make sense, we need to set $t = t^*$, and so an explicit estimate of t^* is required. While general knowledge of the approximate length of the ORANI short-run is implicit in the model builders' provision of the elasticities matrix $C_0(t^*)$, at this point a precise number for t^* is required. Of course, some flexibility in choice of t^* may introduce a means of reducing problem one, the possible incompatibility of the double endogeneities. This point is taken up in the following section.

Thirdly, any attempt to put (7) and (8) directly into reduced form would ignore a timing problem. Consider for example the set of variables endogenous to MACRO but exogenous to ORANI. Where these variables appear in $z_0 y_M$ as variables exogenous to ORANI, the ORANI elasticities $C_0(t^*)$ assume they are of the form of a shock at time 0 sustained throughout the period 0 to t^*. But where these variables appear in $y_M z_0$ as variables endogenous to MACRO they represent the accumulated response after t periods, and inspection of the structure of $C_M(t)$ indicates that this is not in the form of a constant sustained shock, but rather a continuously accumulating path.

This last point, which is at the heart of the problem of linking the two types of models, will be pursued further in the next section, where an approach will be illustrated which simultaneously solves the above problems and allows the development of an appropriately interfaced model.

IV The Interface: A Simplified Exposition

Assumed ORANI Dynamics

Although (3) and (6) cannot be combined directly, their similarity suggests that ORANI may be endowed with a simple dynamic structure which is compatible with MACRO. By analogy with MACRO, the system:

$$DY_0(t) = A_0Y_0(t) + B_0Z_0(t) \qquad (9)$$

may be regarded as the implicit structural form underlying (6), where A_0, B_0 satisfy:

$$C_0(t^*) = A_0^{-1}[\exp(A_0t^*) - I]B_0 \,. \qquad (10)$$

Given $C_0(t^*)$ as data, (but with t^* not known) A_0 and t^* may be freely chosen, with B_0 following from (10). To reduce the dimensionality of the choice problem, in what follows A_0 will be restricted to diagonal form. This has an added interpretational advantage. Since the general solution for y_0 corresponding to the structure (9) is:

$$y_0(t) = C_0(t)z_0(t)$$

where:

$$C_0(t) = A_0^{-1}[\exp(A_0t) - I]B_0 \,,$$

demand rather than the level. In addition, this parameter is estimated freely, so that the production function constraint does not hold along the path of adjustment. To the extent to which a production function constraint is imposed in MACRO, it is based on the linear in logarithms Cobb-Douglas production function:

$$y = 0.7\ell + 0.3k \,,$$

(expressed in proportional change form). This is reasonably compatible with the implied aggregate production function in ORANI. For example, in the standard ORANI experiment a shock to government spending produces an ORANI stand alone result in which the proportional change in output is .68 times the proportional change in employment. The linear in logarithms production function constraint is built in to the ORANI+ model by constraining the speed of adjustment coefficient on labour to be equal to that on output.

In the case of variables endogenous to ORANI and either exogenous to MACRO or not appearing explicitly as endogenous variables in MACRO the ORANI+ speeds of adjustment may be estimated in such a way as to improve the compatibility of the models in the interface. Two prior constraints which were imposed were the equality of the speeds of adjustment on price of exports and price of wool, and the identification of the speed of adjustment of the quantity of exports with that of output.

MACRO and ORANI was the export sector, and that ORANI could provide a more satisfactory explanation of this sector than could MACRO. Thus in experiments identified by a 2, exports and price of exports were treated as exogenous in MACRO and endogenized by ORANI. For completeness, a third experiment was considered in which quantity of imports as well as the price and quantity of exports were treated in this way.

ORANI

The ORANI model is ORANI 78. The choice of the endogenous/exogenous split leads to a further categorization of experiments, and the two possible specifications considered below are identified by the symbols A and B.

ORANI is converted to ORANI+ by the assumption of explicit dynamics. For both MACRO and ORANI a typical equation may be conceptualised in the dynamic form:

$$DY_i = (\ell n\beta_i)(Y_i - \hat{Y}_i), \qquad 0 < \beta_i < 1,$$

where $\hat{Y}_i$ is the equilibrium level of Y_i, and β_i will be referred to as the speed of adjustment coefficient.

In the case of double endogeneities such as output and the domestic price level, the ORANI speed of adjustment coefficients may be constrained to equality with the corresponding MACRO estimates. With regard to the employment variable, however, we note that MACRO parameterises the speed of adjustment of the rate of change of labour

in view of the fact that

$$z_0(t) = z_0(t^*) \text{ for } t \leqslant t^*,$$

we have:

$$y_0(t) = [\exp(A_0t) - I][\exp(A_0t^*)-I]^{-1}y_0(t^*), \qquad (11)$$

so that, in the case of diagonal A_0, each ORANI endogenous variable is assumed to follow a simple growth law from 0 at $t = 0$ to the ORANI stand-alone result at $t = t^*$. Thus, for a typical endogenous variable such as ℓ, the proportional response of labour demand to a given shock, the response path is depicted in Figure 1.

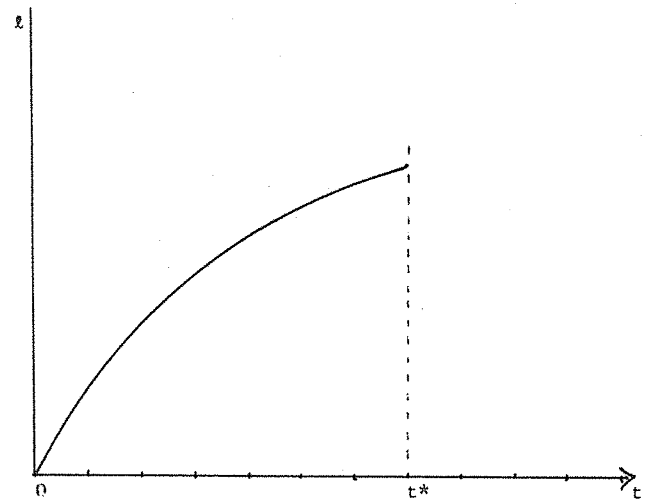

Figure 1 : The Assumed Response Path of Labour Demand in ORANI+

More generally, in the case of the i^{th} variable within $y_0(t)$, the growth law represented by (11) is

$$y_{0i}(\tau) = \frac{\beta_i^{\tau}-1}{\beta_i^{t^*}-1}\, y_{0i}(t^*), \quad 0 \leqslant \tau \leqslant t^*, \quad 0 < \beta_i < 1 ,$$

where $(\ell_n\beta_i)$ is the $(i,i)^{th}$ element of the diagonal matrix A_0. By making an appropriate choice of β_i the within-short-run dynamics assumed for ORANI can be varied from instantaneous adjustment [(a) in Figure 2] to linear adjustment [(d) in Figure 2]. The ORANI model with these simplified assumed dynamics appended will be referred to as ORANI+.

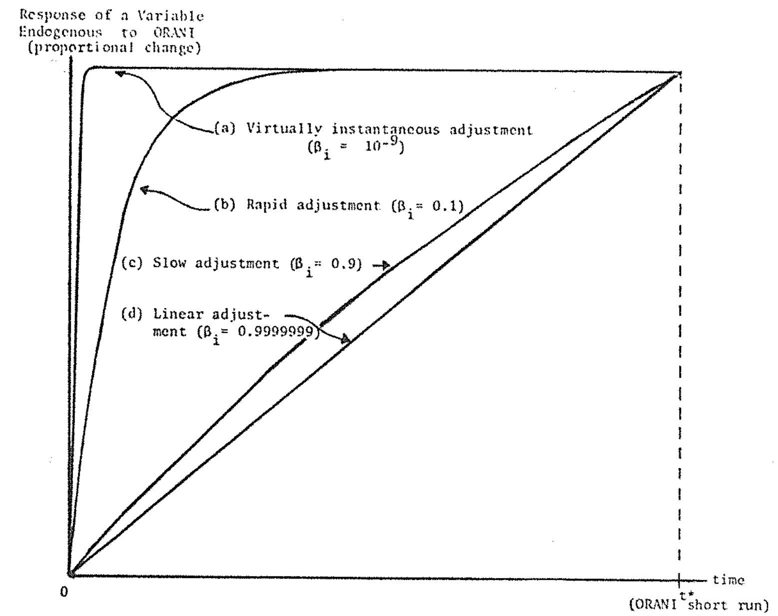

Figure 2: Range of Possible Within-Short-Run Adjustment Dynamics for ORANI's Endogenous Variables

V Experimental Specification and Results

MACRO

The MACRO model is essentially RBA79. However, to provide consistency between the definitions of MACRO endogenous variables and ORANI variables, a number of identities are appended to RBA79 to define new variables. These identities define:

(a) the proportional change in net investment in construction and equipment as the sum of the proportional change in capital stock K and the proportional change in the ratio of net investment to capital stock, k.

(b) gross investment in construction and equipment as the sum of net investment and the depreciated component of the capital stock.

(c) total investment as the sum of gross investment in construction and equipment, and investment in dwellings, where the latter is a fraction of consumption spending.

(d) real wages in terms of nominal wages and prices.

(e) real output as the sum of real output net of depreciation and the depreciated component of the capital stock. Where necessary, proportions are defined at the sample means for the MACRO estimation period. More details of these constructions may be found in Cooper and McLaren (1980).

One MACRO exogenous variable, the price of wool, is endogenized by ORANI. This basic configuration is identified by the subscript 1. However, it was felt that a source of basic incompatibility between

If the interfaced ORANI-MACRO model is internally compatible, this path should compare favourably with that of $\ell \in y_M y_0$ in ORANI - MACRO, the "MACRO version" of the labour demand response. In the present example, because of the assumed recursive nature of the interface, with MACRO driving ORANI, the latter path is equivalent to that of $\ell \in y_M y_0$ in MACRO in stand alone form (Figure 4).

In practical application, of course, allowance needs to be made for feedbacks from ORANI to MACRO. The price of wool, for example, is an element of $z_M y_0$, the set of variables exogenous to MACRO which are endogenised by ORANI. Aggregate exports and imports, as an alternative to treatment as double endogeneities, may also be dealt with in this way by first exogenising them in MACRO for purposes of simulation. The diagrammatic exposition could in principle be extended with now not only ORANI+ but also MACRO being subjected to a series of sustained overlapping shocks in addition to the initial government spending shock. The computer program INTER takes advantage of the continuous time formulation of MACRO and the assumed continuous time formulation of ORANI+ to solve out the interfaced paths simultaneously. The continuous time formulation also allows a precise limiting specification of the concept of overlapping sustained shocks. Derivation of the interfacing formulae used in INTER is contained in Cooper and McLaren (1980).

A Simple ORANI-MACRO Linkage

In order to concentrate on the basic ideas, consider the simplest case in which the linkage is one way, that is MACRO drives ORANI, and assume for simplicity that all the MACRO variables also appear as ORANI variables. Thus in the notation of Section III the sets $y_0 z_M$, $y_M \tilde{x}_0$ and $z_M \tilde{x}_0$ are empty. The combined system (7) - (8) simplifies to:

$$\begin{matrix}(a)\\ \\(b)\end{matrix} \quad \begin{bmatrix} y_M y_0 \\ \\ y_M z_0 \end{bmatrix} = C_M \quad (z_M z_0) , \qquad (11)$$

$$(y_0 y_M) = C_0 \begin{bmatrix} z_0 y_M \\ \\ z_0 z_M \end{bmatrix} , \qquad (12)$$

where for notational simplicity C_M and C_0 are retained for the appropriate submatrices of the C_M and C_0 of (7) and (8).

The variables which appear in the set $z_M z_0$ are exogenous to both models. A typical example of such a variable is government spending, g. The set $y_M z_0$ consists of those variables which are exogenous to ORANI but are endogenised by MACRO. A typical example of one of these variables is aggregate consumer demand, d. The set $y_M y_0$ consists of those variables which are endogenous to both MACRO and ORANI, and a typical example here is labour demand, ℓ.

Consider now the following experiment: a variable in the set $z_M z_0$ -- for concreteness say g -- is subjected to a sustained shock of 10%. The MACRO and ORANI stand alone responses at t^* may be read off (11) (a) and (12) respectively, with (11) evaluated at $t = t^*$. In a linked system, however, this is not the whole story, for a shock to g will cause a response also to d in (11) (b), varying for $t \in [0,t^*]$ and creating a series of further shocks to (12), where d appears in $z_0 y_M$. The implications for the linked ORANI - MACRO model are best illustrated diagrammatically.

The 10% shock to government spending is represented in Figure 3 both for $g \in z_0 z_M$ in ORANI and for $g \in z_M z_0$ in MACRO.

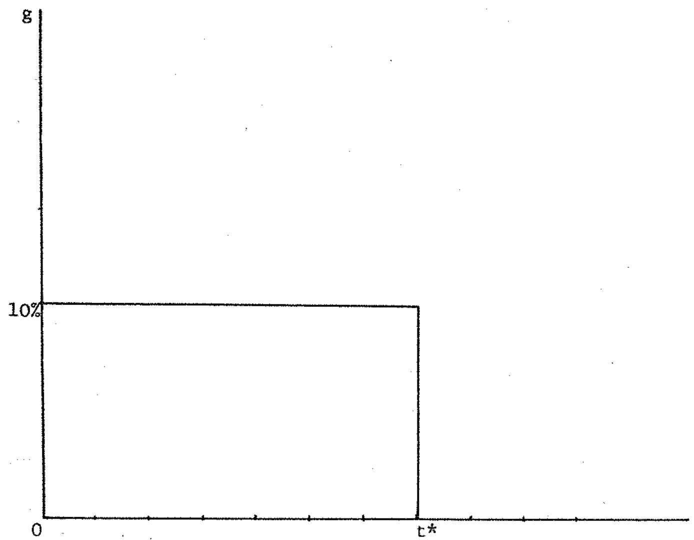

Figure 3 : The Exogenous Shock to Government Spending

Thus the overall response path of the interfaced ORANI variables may well exhibit cyclical behaviour induced by MACRO. In the example, the response path of labour demand would tend to take the shape induced by the summation of the curves in Figure 7. This is depicted in Figure 8 for $\ell \in y_0 y_M$ in ORANI - MACRO.

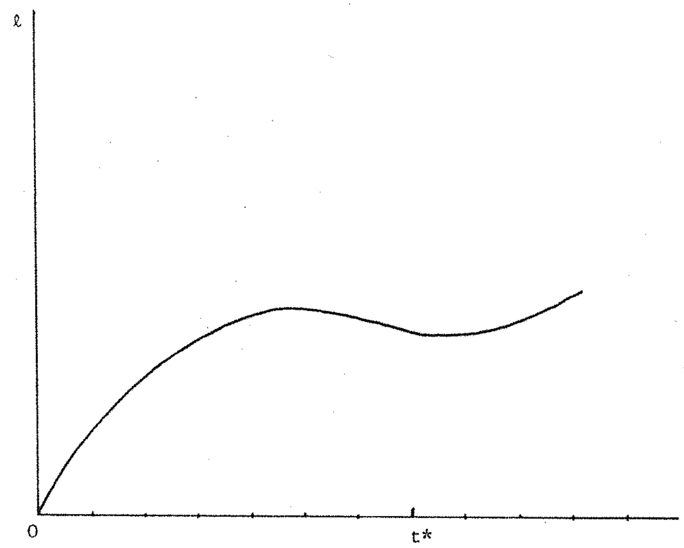

Figure 8 : The "ORANI version" of the Labour Demand Response Path in ORANI - MACRO

Now the interfaced ORANI - MACRO response at t^* may be viewed as the sum of successive ORANI+ stand alone responses to the above shocks. The ORANI+ responses of labour demand to each shock are collectively depicted in Figure 7.

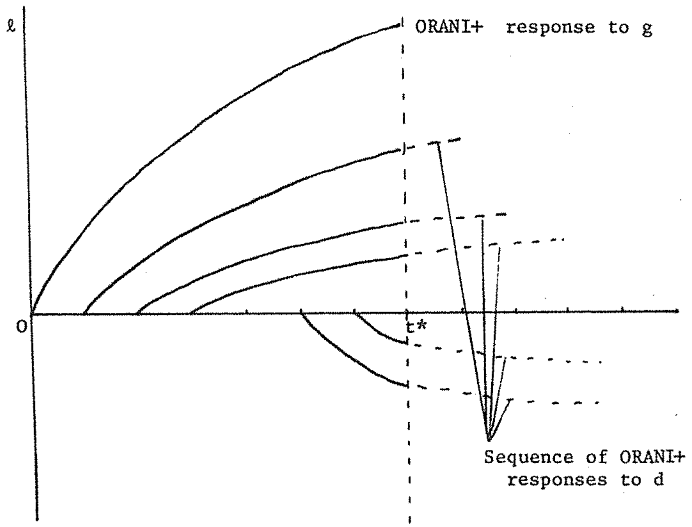

Figure 7 : ORANI+ Responses of Labour Demand to the Succession of Shocks

The ORANI+ stand alone response has been given in Figure 1 for labour demand, ℓ. For MACRO, the stand alone response might be something like the path depicted in Figure 4 for labour demand $\ell \in y_M y_0$,

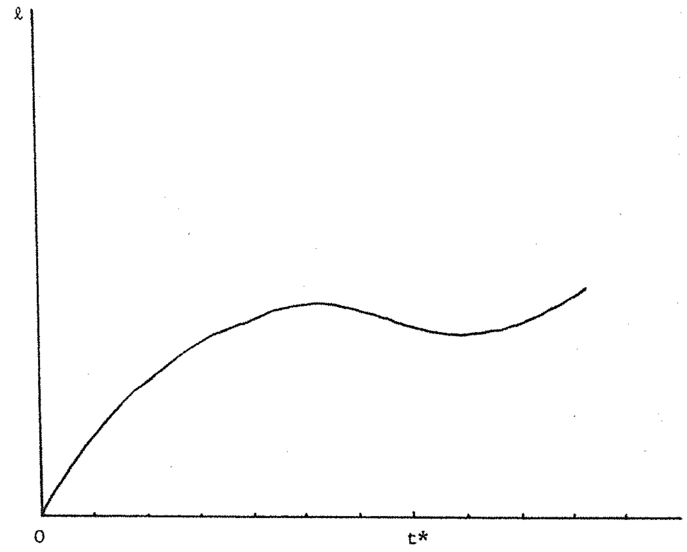

Figure 4 : The Response Path of Labour Demand in MACRO

and like the path depicted in Figure 5, for the aggregate consumption demand response, $d \in y_M z_0$ in MACRO.

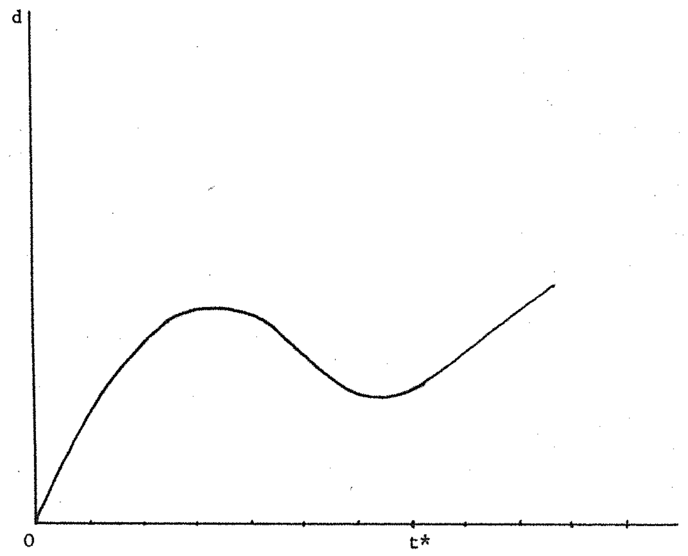

Figure 5 : The Consumption Response Path in MACRO

However, viewing d as an element of $z_0 y_M$ in ORANI, Figure 5 may be approximated by a series of positive and negative overlapping sustained shocks, such as is depicted in Figure 6.

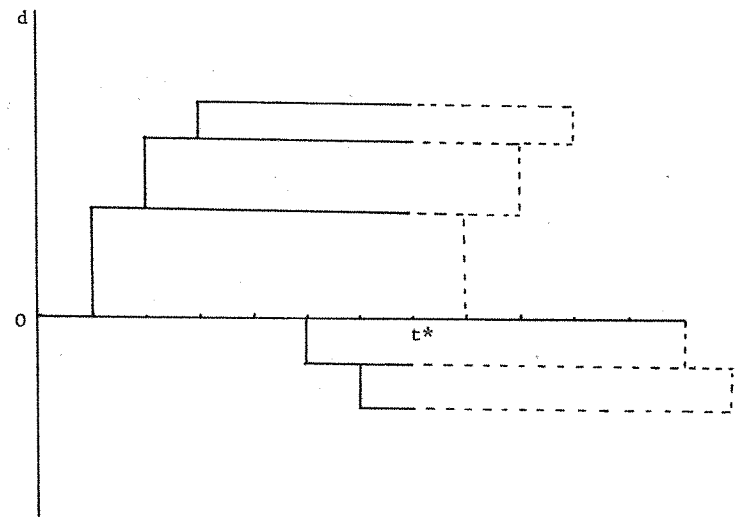

Figure 6 : The MACRO Consumption Response Represented as Overlapping Sustained Shocks to ORANI