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A JOHANSEN MODEL FOR REGIONAL ANALYSIS*

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A 'bottoms-up' regional model of the Johansen class allowing for price substitution, flexibility in classifying and reclassifying variables into exogenous or endogenous categories, and ease of computation, is constructed to demonstrate the attractiveness of Johansen type models for regional analysis. These models have been used extensively by international trade theorists with success, but surprisingly they have not caught the attention of regional economists.

1. Introduction

Johansen type models[1] have been used extensively in the analysis of international trade since the seminal work by Taylor and Black (1974). For example, using a Johansen type model, de Melo (1978) studied the impact of protection on resource allocation while Dervis (1980) analysed the resource pull effect of a devaluation under exchange control. Given the popularity of Johansen type models with economists studying international trade, it is perhaps surprising that these type of models have not become popular with regional economists.

In this paper, we present a skeletal version of a Johansen type model designed especially for regional analysis. Our model is different from the original model constructed by Johansen (1960) and the Johansen type models that followed it. Johansen (1960) had no theory of international trade and it was not until the Taylor and Black (1974) study that Johansen models were used to study questions on international trade. Our model differs from the earlier Johansen type models in its emphasis on regional disaggregation. Unlike other Johansen type models, we have (i) treated commodities of the same kind coming from different regions as imperfect substitutes and have modeled inter-regional commodity flows, (ii) explicit regional specific factor supply constraints, thus allowing factor prices to vary across regions, and (iii)

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[1]Named after its pioneer, Johansen (1960).

allowed government policies and other exogenous factors originating at the regional level to affect national aggregates such as aggregate employment.

Using our model, we wish to demonstrate the attractiveness of Johansen type models (especially those adopting the Johansen approximation for computation) for regional analysis. The attractiveness of Johansen type models lies in their

(i) allowance for price responsiveness and substitution in both supply and demand,
(ii) complete flexibility in reclassifying variables between the endogenous and exogenous categories,
(iii) relatively undemanding amounts of time series data compared to most economy-wide regional econometric models, and
(iv) ease of computation with the Johansen linearisation method.

2. The model

2.1. Input demand

Industry production functions are assumed to be of the form

$$X_j^s = \underset{i=1}{\overset{n+1}{\mathrm{LEONTIEF}}}(X_{ij}^{0s}), \qquad j=1,\ldots,n, \quad s=1,\ldots,m, \tag{1}$$

where X_j^s is the production of industry j in region s, and X_{ij}^{0s} is the input i used by industry j in region s. The Leontief type production technology above exhibits constant returns to scale. It embodies n material inputs and an index of primary factor.

Although the inputs are used in fixed proportions, we allow substitution between inputs of the same kind supplied by different sources and between different types of primary factors. Formally, using a constant elasticity of substitution function, we have

$$X_{ij}^{0s} = \underset{r=1}{\overset{k}{\mathrm{CES}}}(X_{ij}^{rs}), \tag{2}$$

$$i=1,\ldots,n+1, \quad j=1,\ldots,n, \quad k=m+1 \quad \text{for } i=1,\ldots,n,$$

$$k=2 \quad \text{for } i=n+1, \quad s=1,\ldots,m.$$

where X_{ij}^{rs} is the input i from source r ($i=1,\ldots,n$) or the primary factor ($i=n+1$) of type r used by industry j in region s. For the $n+1$ input, the superscript r only ranges from 1 to 2. The $n+1$ input is the index of primary factor with $r=1$ as the input of labour and $r=2$ as the input of capital. Supplies of commodities come from m domestic regions and overseas, giving a total of $m+1$ sources of supply.

Producers are assumed to be price takers and minimise their costs. Input demand equations for industry j in region s are obtained by choosing quantities of

$$X_{ij}^{rs}, \qquad i=1,\dots,n+1, \quad r=1,2 \quad \text{for } i=n+1,$$

$$r=1,\dots,m \quad \text{for } i=1,\dots,n,$$

and

$$X_{ij}^{0s}, \qquad i=1,\dots,n+1,$$

to minimise

$$\sum_{r=1}^{k}\sum_{i=1}^{n+1} P_{ij}^{rs}\chi_{ij}^{rs}, \qquad k=2 \quad \text{for } i=n+1, \quad k=m+1 \quad \text{for } i=1,\dots,n,$$

subject to eqs. (1) and (2). The resulting input demand equations in percentage change[2] for the model are

$$X_{ij}^{rs}=x_j^s-\sigma_{ij}^{0s}\left(p_{ij}^{rs}-\sum_{r=1}^{k} S_{ij}^{rs}P_{ij}^{rs}\right), \tag{3}$$

$$i=1,\dots,n+1, \quad j=1,\dots,n, \; k=2 \quad \text{for } i=n+1,$$

$$k=1,\dots,m+1 \quad \text{for } i=1,\dots,n, \; r=1,\dots,k, \quad s=1,\dots,m,$$

where σ_{ij}^{0s} is the elasticity of substitution between input i (materials and primary factor) from the different sources[3] in forming a unit of effective input i, S_{ij}^{rs} is the share of input i from source r in the total usage of input i by industry j in region s, and P_{ij}^{rs} is the price of input i from source r paid by industry j in region s.

If the prices of factors remain constant, the demand for inputs from the various sources moves at the same percentage as output, since the production technology exhibits constant returns to scale. Any change in the prices of a given input from the different sources would lead to a substitution towards the cheaper sources of supply.

[2]In this model capital and lower cases of a given letter explain the same variable. However, capital letters are used for variables measured in levels. Lower case letters show the variables in percentage form. Johansen models are non-linear. By presenting the equations in percentage form we are making the model linear.

[3]From now onwards, for $i=n+1$, source $r=1$ is the labour input, and source $r=2$ is the capital input.

2.2. Consumer demand

We assume that consumers in region s maximise a utility function of the form

$$U^s(C_1^{0s}, C_2^{0s}, \dots C_n^{0s}),$$

subject to

$$C^s = \sum_{i=1}^{n} P_i^{0s} C_i^{0s}, \tag{4}$$

and

$$C_i^{0s} - \underset{r=1}{\overset{m+1}{\mathrm{CES}}}(C_i^{rs}), \qquad i=1,\dots,n, \quad r=1,\dots,m+1, \quad s=1,\dots,m, \tag{5}$$

where C_i^{0s} $(i=1,\dots,n)$ is the effective good i in the utility function of households in region s, C^s is the total consumption expenditure in region s, P_i^{0s} is the price paid by households in region s for good i, and C_i^{rs} is the consumption of good i from source r in region s. Eq. (4) is the budget constraint. Eq. (5) introduces the possibilities of substitution in consumer demand between commodities of the same name coming from different sources. Solving the above problem will give us consumer demand functions (in percentage change) of the form

$$c_i^{0s} = \sum_{j=1}^{n} \eta_{ij}^s P_j^{0s} + \varepsilon_i^s c^s, \qquad i=1,\dots,n, \quad s=1,\dots,m, \tag{6}$$

and

$$c_i^{rs} = c_i^{0s} - \sigma_i^s \left(p_i^{rs} - \sum_{r=1}^{m+1} S_i^{rs} P_i^{rs} \right),$$

$$i=1,\dots,n, \quad r=1,\dots,m+1, \quad s=1,\dots,m, \tag{7}$$

where

$$p_i^{0s} = \sum_{r=1}^{m+1} S_i^{rs} P_i^{rs}, \qquad i=1,\dots,n. \tag{8}$$

η_{ij}^s is the price elasticity of good i with respect to a change in the price of good j in region s, ε_i^s is the household expenditure elasticity of good i in region s, S_i^{rs} is the share of source r in the total purchases of good i by region s, and σ_i^s is the elasticity of substitution between the various goods i from different sources used by households in region s.

Regional consumption expenditure C^s is explained by regional labour income.[4] Formally,

$$C^s = f\left(\sum_{j=1}^{n} p_{(n+1)j}^{1s} X_{(n+1)j}^{1s}\right). \tag{9}$$

In percentage change form, eq. (9) becomes

$$c^s = \phi^s \sum_{j=1}^{n} (p_{(n+1)j}^{1s} + x_{(n+1)j}^{1s}) S_{1j}^s, \qquad s = 1, \dots, m, \tag{10}$$

where ϕ^s is the elasticity of regional consumption expenditure in response to changes in regional income in region s and S_{1j}^s is the share of industry j in the total labour incomes of region s.

2.3. Exports

Foreign demands (in percentage change) are explained by

$$P_i^e = \Omega_i + \eta_i^e e_i, \qquad i = 1, \dots, n, \tag{11}$$

where e_i is the volume of exports from industry i, η_i^e is the reciprocal of the foreign elasticity of demand for good i, p_i^e is the foreign currency price of good i, and Ω_i is the shift variable accounting for any exogenous shifts in foreign demand.

Sales to foreigners come from m domestic regions. All goods of the same name from different domestic regions are imperfect substitutes for one another. They are linked through a CES function, We assume that foreign buyers minimise their import bill by selecting, for any good i, different quantities from the various domestic regions, i.e., they choose quantities of E_i^s $(s = 1, \dots, m)$ to minimise $\sum_{s=1}^{m} P_i^{se} E_i^s$, subject to $E_i = \mathrm{CES}_{s=1}^{m}(E_i^s)$. E_i^s is the good i from region s exported, and p_i^{se} is the price of good i from region s in foreign currency.

The resulting export demand equations (in percentage change) are

$$e_i^s = e_i - \sigma_i\left(P_i^{se} - \sum_{s=1}^{m} S_i^s P_i^{se}\right), \qquad i = 1, \dots, n, \quad s = 1, \dots, m, \tag{12}$$

where

$$P_i^e = \sum_{s=1}^{m} S_i^s P_i^{se}, \qquad i = 1, \dots, n. \tag{13}$$

[4]In making our model as simple as possible, we have ignored returns to capital and govenment transfer payments.

The variable σ_i is the elasticity of substitution of good i from different domestic sources exported overseas, and S_i^s is the share of good i from domestic region s in the total exports of good i.

At this point, it is appropriate to discuss the specifications of the model that have been formulated so far. A problem commonly encountered in international trade models is extreme patterns of specialisation. Domestic purchases of commodities are satisfied totally either by domestic production or imports. For a given commodity we have either zero domestic production or zero imports. This is unrealistic given the level of aggregation that economic models are forced to operate. A usual method of overcoming this problem is by specifying arbitrary restrictions on production and import levels. A sophisticated way of handling this problem is by introducing the idea of imperfect substitution between commodities of the same name coming from the domestic source and from imports.[5] We incorporate this idea by introducing more than one domestic source and allowing imperfect substitution between commodities of the same name coming from m domestic sources and overseas.

This will not only avoid either having domestic production or imports satisfying all of domestic purchases but also prevent a domestic region from supplying the whole domestic economy of a given commodity. The latter could be unrealistic for some economies.

In the ideal world, we would specify more general production functions, allowing perhaps substitution between every input. However, in the real world that modellers operate, this would be an unrealistic approach. The time series data required and the task of estimating would be too demanding.

The path that our model takes is one of compromise. Allowing substitution possibilities but limiting them so that the tasks of data collection and estimation would not be unduly onerous.

2.4. Price system

Firms are assumed to be price takers and earn only competitive profits. Thus prices are equated with costs. We can write

$$P_j^s X_j^s = \sum_{i=1}^{n+1} \sum_{r=1}^{k} P_{ij}^{rs} X_{ij}^{rs},$$

$$j=1,\ldots,n, \quad k=2 \quad \text{for } i=n+1,$$

$$k=m+1 \quad \text{for } i=1,\ldots,n, \quad s=1,\ldots,m, \tag{14}$$

[5]This approach was first suggested by Armington (1969). Subsequently it was adopted in other works, like Artus and Rhomberg (1973), and Dixon et al. (1982). For a discussion of the problem of extreme specialization in trade models, see Taylor (1975).

where P_j^s is the basic seller price of good j produced in domestic region s. In percentage change, eq. (14) becomes

$$P_j^s = \sum_{i=1}^{n+1} \sum_{r=1}^{k} P_{ij}^{rs} T_{ij}^{rs}, \tag{15}$$

$$j=1,\ldots,n, \quad k=2 \quad \text{for } r=i,\ k=m+1 \quad \text{for } i=1,\ldots,n, \quad s=1,\ldots,m,$$

where T_{ij}^{rs} is the share of input i, source r in the total costs of industry j in region s. With constant returns to scale production functions and competitively structured markets, commodity prices are functions only of materials and factor prices. They do not depend on production levels.

To keep our model simple, we assume away margins and taxes. This ensures basic seller prices equal purchases prices and prices paid by users in different industries and regions for a given good are the same. In other words,

$$P_{ij}^{rs} = P_i^{rs} = P_i^r, \qquad i,j=1,\ldots,n, \quad r=1,\ldots,m+1, \quad s=1,\ldots,m. \tag{16}$$

As part of an overall strategy to prevent patterns of extreme specialisation, we restrict the mobility of capital and labour, allowing them to shift only between industries.[6] Thus, we have

$$P_{(n+1)j}^{rs} = P_{(n+1)}^{rs}, \qquad j=1,\ldots,n, \quad r=1,2, \quad s=1,\ldots,m. \tag{17}$$

Domestic prices are linked with world prices through the equation

$$P_j^{m+1} = P_j^I \theta T_j, \qquad j=1,\ldots,n, \tag{18}$$

where P_j^I is the world price of good j, θ the exchange rate, and T_j the 'power of the tariff' (one plus the ad valorem trade tariff). The above equation in percentage change form is

$$P_j^{m+1} = P_j^I + \theta + t_j, \qquad j=1,\ldots,n. \tag{19}$$

Next we link prices of domestic goods to f.o.b. export prices. We assume

$$P_j^s = P_j^{se} \theta V_j^s, \qquad j=1,\ldots,n, \quad s=1,\ldots,m, \tag{20}$$

[6]Restrictions imposed on factor mobility can take many forms. For example, in Liew (1982) where there are three primary factors, machines are industry specific but allowed to move between regions, buildings are region specific but allowed to be used by different industries, and agricultural land is both region and industry specific.

where V_j^s is one plus the rate of export subsidy given to industry j in region s. We write the above equation in percentage change form to obtain

$$P_j^s = P_j^{se} + \theta + v_j^s, \qquad j=1,\dots,n, \quad s=1,\dots,m. \tag{21}$$

2.5. Market clearing equations

Total supply of commodities must be equal to the demand for it, i.e.,

$$X_i^z = \sum_{j=1}^{n} \sum_{s=1}^{m} X_{ij}^{zs} + C_i^{zs} + E_i^z, \qquad i=1,\dots,n, \quad z=1,\dots,m. \tag{22}$$

For simplicity, we have assumed zero investment and no government sector in this model.

In percentage change form, the preceding equation becomes

$$x_i^z = \sum_{j=1}^{n} \sum_{s=1}^{m} x_{ij}^{zs} J_{ij}^{zs} + \sum_{s=1}^{m} c_i^{zs} J_i^{zs} + e_i^z J_i^z,$$

$$i=1,\dots,n, \quad z=1,\dots,m, \tag{23}$$

where the Js are sales shares. For example, J_{ij}^{zs} is the share of industry j, region s, in the total sales of industry i in region z.

Next, the demands for labour and capital are set to equal their supplies. Formally,

$$\sum_{j=1}^{n} X_{(n+1)j}^{ks} = F_k^s, \qquad k=1,2, \quad s=1,\dots,m, \tag{24}$$

where F_k^s is the supply of primary factor ($k=1$ labour, $k=2$ capital) in region s. The formulation of eq. (24) does away with inter-regional mobility of primary factors, allowing only mobility between industries. Restricting the mobility of factors is often used as a means of preventing extreme specialisation resulting in unrealistic patterns of industrial structure. Eq. (24) in percentage change form is

$$\sum_{j=1}^{n} x_{(n+1)j}^{ks} L_j^{ks} = f_k^s, \qquad k=1,2, \quad s=1,\dots,m, \tag{25}$$

where L_j^{ks} is the share of industry j in the employment of primary factor k in region s.

2.6. Balance of trade

The aggregate foreign currency receipts of exports (ER) is defined as

$$ER = \sum_{i=1}^{n} \sum_{z=1}^{m} P_i^{ze} E_i^z. \tag{26}$$

In percentage change, this becomes

$$er = \sum_{i=1}^{n} \sum_{z=1}^{m} (p_i^{ze} + e_i^z) SE_i^z, \tag{27}$$

where SE_i^z is the share of good i from domestic region z in aggregate export receipts.

The aggregate demand for imported i (XM_i) is represented by the equation

$$XM_i = \sum_{j=1}^{n} \sum_{s=1}^{m} X_{ij}^{(m+1)s} + \sum_{s=1}^{m} C_i^{(m+1)s}, \qquad i = 1, \ldots, n. \tag{28}$$

Rewriting this in percentage change and we obtain

$$xm_i = \sum_{j=1}^{n} \sum_{s=1}^{m} x_{ij}^{(m+1)s} H_{ij}^s + \sum_{s=1}^{m} c_i^{(m+1)s} H_i^s, \qquad i = 1, \ldots, n, \tag{29}$$

where H_{ij}^s is the share of industry j in region s, and H_i^s is the share of households in region s of the total imports of good i.

The foreign currency cost of imports is

$$M = \sum_{i=1}^{n} P_i^I XM_i. \tag{30}$$

Expressed in percentage change, eq. (30) becomes

$$m = \sum_{i=1}^{n} (P_i^I + xm_i) M_i^f, \tag{31}$$

where M_i^f is the share of good i in the total foreign currency value of imports.

The balance of trade (BT) is defined as

$$BT = E - M. \tag{32}$$

Therefore, the change in the balance of trade is

$$\Delta BT = erER - mM. \tag{33}$$

Because the balance of trade in levels could be either negative or positive we avoid having percentage change in the balance of trade.

2.7. Indexation equations

The last set of equations for the model consists of indexation equations. They determine the real price of primary factors to users. They are of the form

$$P^{rs}_{(n+1)} = cpi + g^{rs}, \qquad r = 1, 2, \quad s = 1, \ldots, m, \tag{34}$$

where cpi is consumer price index, and g^{rs} is the real rental price of primary factor r paid for by users in region s. The consumer price index is defined by

$$cpi = \sum_{i=1}^{n} \sum_{r=1}^{m+1} D^r_i P^r_i, \tag{35}$$

where D^r_i is the share of good i from source r in total consumption expenditure.

The model that we have just presented will be considered in the field of regional macroeconomic modelling, a 'bottoms-up' model.[7] A 'bottoms-up' model is characterised by the recognition of economic agents (producers, consumers, etc.) at the regional level and the explicit modelling of their behaviour (maximising profits, utility, etc.). Variables within such a model are fully interactive with one another. The model can be used to analyse the impacts of policies specifically aimed at the regional level on all other variables of the model, in particular national performance. National aggregates are obtained by summing up the various relationships explaining the behaviour of regional variables.

With the complexity of 'bottoms-up' models, perhaps it is not surprising that such models are less popular than their 'tops-down' cousins. However, we hope to demonstrate in the next section that a 'bottoms-up' model of the Johansen class of models is flexible and not difficult to compute, and that data requirements are not insurmountable.

[7]This together with the term 'tops-down' was introduced in Klein and Glickman (1977) who gave a survey of regional econometric modelling.

3. Computing the model

The model outlined in section 2 can be represented by a system of linear equations,

$$Az = 0, \tag{36}$$

where A is an $f \times g$ matrix of coefficients (consists of shares and substitution parameters) and z is the $g \times 1$ vector of variables of the model.

A count of the number of variables in the model (q) will show that they exceed the number of equations (f). To solve the model, we will require to set $(q-f)$ variables exogenously. We can rewrite eq. (36) as

$$A_1 x_1 + A_2 x_2 = 0, \tag{37}$$

where A_1 is the $(f \times f)$ matrix of coefficients related to the $(f \times 1)$ vector of endogenous variables, and A_2 is the $(f \times (q-f))$ matrix of coefficients related to the $((q-f) \times 1)$ vector of exogenous variables.

The solution of the model is, therefore,

$$x_1 = -A_1^{-1} A_2 x_2, \tag{38}$$

where $(-A_1^{-1}A_2)$ can be considered as the solution matrix. The element $(-A_1^{-1}A_2)_{ij}$ is the elasticity of the ith endogenous variable with respect to the jth exogenous variable. By setting an exogenous variable to a non-zero percentage change, eq. (38) will solve out the percentage values of the endogenous variables resulting from the specified exogenous change. For example, we might be interested to analyse the economy-wide and regional impacts of a reduction in international trade tariffs. By setting the variables $t_j = 10$ $(j = 1, \ldots, n)$ the model through eq. (28) solves values for the endogenous variables in response to a 10 percent uniform change in 'the power of the tariff'.

A count of the number of equations and variables tells us that $2m + mn + 2n + 1$ variables have to be made exogenous. A possible choice of exogenous variables for the model is given by table 1.

An important point to bear in mind in our choice of variables to be made exogenous is that the choice must be consistent with economic theory. It is not possible, for example, to make both the power of the tariffs and outputs exogenous. Neither is it possible to fix exogenously both the export subsidies and export quantities. Subject to the constraint that any choice of variables to be made exogenous is consistent with economic theory, there is complete flexibility in reclassifying variables between the endogenous and exogenous categories.

Table 1

List of possible exogenous variables.

Symbol	Variable name	Number
g^{rs}	real rental prices of primary factors	$2m$
P_j^I	world price of imports	n
t_j	'one plus ad valorem tariffs'	n
s_j^s	export subsidies	mn
θ	exchange rate	1
Total exogenous variables		$2m+2n+mn+1$

For example, instead of setting the real rental prices of primary factors exogenously and letting the model solve for the employment of these factors, we could set employment exogenously and allow the model to solve for the required real rental prices. Similarly, the export quantities could be set exogenously and the model solving for the necessary export subsidies.

The exchange rate is used as a numeraire. In other models, for example that of Taylor and Black (1974), the money wage is used as a numeraire. We need to specify a numeraire in our model, i.e., at least one monetary variable should be treated as exogenous. This is necessary to generate a unique solution for the model as there is nothing in the model to determine the absolute price level. We can see this by setting $\theta=1$ and all other exogenous variables in table 1 to zero. By working through the equations of the model (in percentage change) we will realise that all the real endogenous variables are homogeneous of degree zero and all domestic monetary variables are homogeneous of degree 1 with respect to the exchange rate. This is characteristic of Walrasian type models with constant returns to scale technology and competitive structured markets.

The computing involved in generating a solution for a Johansen type model is relatively undemanding once the A matrix is set up. To set up the A matrix we will require a multi-region input–output data base and estimates of the various elasticities of substitution. Time series data are required only to estimate these elasticities.

Although the demands for data to implement the model seem onerous, they are not insurmountable. The difficulty of getting hold of multi-region input–output data bases is the easier problem to deal with compared to the problem of getting estimates of the substitution elasticities. Multi-region input–output data bases are a rarity. However, national input–output data bases are not. From a knowledge of regional production, regional absorption, and transportation costs one could convert a set of national input–output tables into a consistent set of multi-regional input–output tables. With varying degrees of success Polenske (1978) and Liew (1982) have constructed such data bases for the United States and Australia.

As to the substitution elasticities, the ones that we consider the most difficult to get are the elasticities measuring the substitutability of given commodities of the same kind from different sources. If data are insufficient for them to be estimated one could use different elasticities for different simulations to see how the results are affected.

Our discussion on data really boils down to the familiar question as to whether theory precedes data or vice versa. Quite often, statisticians do not collect regional data because there is no demand for them. Once sufficient interest is generated, the data would be forthcoming, if slowly. This has certainly been the experience in Australia.

Because the model is linear, size is not a serious constraint in computation. The model can be condensed by substituting away equations and variables. We can solve only for those endogenous variables that we are interested in. Those variables that have been substituted away are not 'lost'. If the need arises, we can always use the endogenous variables solved by the condensed model and substitute them back to the original full model to solve for the variables that have been substituted out.

The condensing of the model can be carried out in two steps. The first step involves a series of algebraic manipulations. When the system reaches the stage where algebraic manipulations can no longer be performed with ease because of the complexity of the equations, we rewrite the reduced system in matrix form. From this point onwards, the reduction of the system is carried out with the use of the computer.

For example, eq. (11) and (13) of our model can be condensed into

$$\sum_{s=1}^{m} S_i^s P_i^{se} = \Omega_i + \eta_i^e e_i, \qquad i=1,\ldots,n. \tag{39}$$

In matrix form the above becomes

$$H(1)P^{ze} = H(2)\Omega + H(3)E, \tag{40}$$

where

$$H(1)=\left[\begin{array}{cccc|cccc|c|cccc}
S_1^1 & & & & S_1^2 & & & & & S_1^m & & & \\
 & S_2^1 & & \mathbf{0} & & S_2^2 & & \mathbf{0} & & & S_n^m & & \\
 & & \ddots & & & \mathbf{0} & \ddots & & \cdots & & & \ddots & \\
\mathbf{0} & & & S_n^1 & & & & S_n^2 & & & & & S_n^m
\end{array}\right]_{n+mn},$$

$$H(3)=\begin{bmatrix} \eta_1^e & & & \\ & \eta_2^e & & \\ & & \ddots & \\ & & & \eta_n^e \end{bmatrix}.$$

$H(2)$ is an identity matrix and the rest of the uppercase letters denote vectors containing those variables designated by the lowercase letters. Furthermore, eq. (12) in matrix form is

$$E^e = H(4)E - H(5)P^{ze} + H(6)H(1)P^{ze}, \tag{41}$$

where

$$H(4)=\underset{mn\times n}{\begin{bmatrix} I_{n\times n} \\ I_{n\times n} \\ \vdots \\ I_{n\times n} \end{bmatrix}}, \qquad H(5)=\underset{mn\times mn}{\begin{bmatrix} \sigma_1 & & & & & & & \\ & \sigma_2 & & & & \mathbf{0} & & \\ & & \ddots & & & & & \\ & & & \sigma_n & & & & \\ & \mathbf{0} & & & \sigma_1 & & & \\ & & & & & \ddots & & \\ & & & & & & \sigma_n & \\ & & & & & & & \sigma_n \end{bmatrix}},$$

$$H(6)=\underset{mn\times n}{\begin{bmatrix} \sigma_1 & & \\ & \ddots & \mathbf{0} \\ \mathbf{0} & & \sigma_n \\ \hdashline \sigma_1 & & \\ & \ddots & \mathbf{0} \\ \mathbf{0} & & \sigma_n \\ \hdashline \sigma_1 & & \\ & \ddots & \mathbf{0} \\ \mathbf{0} & & \sigma_n \end{bmatrix}},$$

and E^e is the column vector containing the mn variables, e_i^z. We can simplify eq. (41) to

$$E^e = H(4)E + H(7)P^{ze}, \tag{42}$$

where

$$H(7)=[H(6)H(1)-H(5)]. \tag{43}$$

The matrices are formed straight from the data base of the model and stored on the disk in the computer. Condensation of the model is carried out by the multiplication, addition and subtraction of matrices such as those above.

These matrices are the submatrices of the large matrix A. The elements of A_1 and A_2 are then formed by writing the relevant submatrices into the appropriate positions of A_1 and A_2.

A problem common in computing large general equilibrium models is the checking of the solution computed. We have to be sure that the solution produced is what the model should give us and not caused by a computing error. A Johansen type model is attractive in this regard. In our discussion of the exchange rate as a numeraire, we pointed out that the real endogenous variables are homogeneous of degree zero and all domestic monetary variables are homogeneous of degree 1 with respect to the exchange rate. Thus, for every equation in our condensed system presented in matrix form, we set all the exogenous variables (except the exchange rate, which we set to 1) to zero and the domestic monetary endogenous variables to 1. If the equations are consistent, we can be reasonably sure that the computing has been done correctly.

As a final check, we could carry out a simulation with the complete model, setting the exchange rate to 1 and all other exogenous variables to zero. If the computing is correct, we will expect the model to calculate out values of zero for the endogenous real variables and 1 for the domestic endogenous monetary variables.

4. Conclusion

Despite the extensive use of Johansen type models in the study of international trade, these models are rarely constructed by regional economists. However, as this paper demonstrates, Johansen type models allowing for price responsiveness and substitution possibilities, flexibility in classifying variables into either exogenous or endogenous, and ease of computation, make them attractive as a form of regional macroeconomic modelling adopting the 'bottoms-up approach.

Appendix: Derivation of eq. (3)

The problem is to choose for industry j in region s

$$X_{ij}^{rs}, \qquad i=1,\ldots,n+1, \quad r=1,2 \quad \text{for } i=n+1,$$

$$r=1,\dots,m \quad \text{for } i=1,\dots,n,$$

and

$$X_{ij}^{0s}, \qquad i=1,\dots,n+1, \quad \text{to minimise}$$

$$\sum_{r=1}^{k}\sum_{i=1}^{n+1} P_{ij}^{rs}X_{ij}^{rs}, \qquad k=2 \quad \text{for } i=n+1, \quad k=m+1 \quad \text{for } i=1,\dots,n,$$

subject to

$$X_j^s=\underset{i=1}{\overset{n+1}{\mathrm{LEONTIEF}}}(X_{ij}^{0s}), \qquad j=1,\dots,n, \quad s=1,\dots,m, \tag{1}$$

and

$$X_{ij}^{0s}=\underset{r=1}{\overset{k}{\mathrm{CES}}}(X_{ij}^{rs}),$$

$$i=1,\dots,n+1, \quad j=1,\dots,n, \quad k=m+1 \quad \text{for } i=1,\dots,n,$$

$$k=2 \quad \text{for } i=n+1, \quad s=1,\dots,m. \tag{2}$$

To simplify our derivation, we rewrite eq. (2) as

$$Z=\left[\sum_{r=1}^{k}(Y^r)^{-h}\alpha\right]^{-1/h}, \tag{2a}$$

where α and h are the parameters of a given CES function, Z refers to X_{ij}^{0s} and Y^r refers to X_{ij}^{rs}. The elasticity of substitution (σ) can be defined as $1/1+h$. Furthermore, we rewrite P_{ij}^{rs} as p^r.

The first-order conditions of the problem are, therefore,

$$P^r-\Lambda\left\{\left[\sum_{r=1}^{k}(Y^r)^{-h}\alpha\right]^{-1/h-1}(Y^r)^{-h-1}\alpha\right\}=0, \tag{2b}$$

$$\left[\sum_{r=1}^{k}(Y^r)^{-h}\alpha\right]^{-1/h}-Z=0. \tag{2c}$$

Using (2a) we rewrite (2b) as

$$P^r=\Lambda(Z)^{1+h}(Y^r)^{-1-h}\alpha. \tag{2d}$$

In percentage change form the above becomes

$$P^r = \lambda + (1+h)z - (1+h)y^r, \tag{2e}$$

where the lower case letters show the variables in percentage change form.
Totally differentiating eq. (2c) and using (2b) gives

$$\mathrm{d}Z = \sum_{r=1}^{k} P^r \,\mathrm{d}Y^r/\Lambda.$$

Converting the above to percentage change we have

$$z = \sum_{r=1}^{k} p^r \,\mathrm{d}Y^r/\Lambda Z = \sum_{r=1}^{k} P^r Y^r y^r/\Lambda Z. \tag{2f}$$

From (2d) and using (2a) we obtain

$$\sum_{r=1}^{k} P^r Y^r = \Lambda Z^{1+h}\left[\sum_{r=1}^{k} (Y^r)^{-h}\alpha\right] = \Lambda Z.$$

Substitute this into (2f) and we have

$$z = \sum_{r=1}^{k} S^r y^r, \tag{2g}$$

where

$$S^r = P^r Y^r \Big/ \sum_{r=1}^{k} P^r Y^r.$$

Multiplying (2e) by S^r and aggregating over r, we obtain

$$\sum_{r=1}^{k} S^r p^r = \sum_{r=1}^{k} [S^r\lambda + S^r(1+h)z - S^r(1+h)y^r].$$

Rearranging the above, using the fact that $\sum_{r=1}^{k} S^r = 1$ and (2g), we obtain an expression for λ,

$$\lambda = \sum_{r=1}^{k} S^r p^r. \tag{2h}$$

Recalling that $\sigma = 1/1+h$ and substituting the above expression in (2e), we have

$$y^r = z - \sigma\left(p^r - \sum_{r=1}^{k} S^r pr\right). \tag{2i}$$

Reintroduce the original subscripts and superscripts and (2i) becomes

$$x_{ij}^{rs} = x_{ij}^{0s} - \sigma_{ij}^{0s}\left(p_{ij}^{rs} - \sum_{r=1}^{k} S_{ij}^{rs} p_{ij}^{rs}\right).$$

Using eq. (1), we can rewrite this as

$$x_{ij}^{rs} = x_j^s - \sigma_{ij}^{0s}\left(p_{ij}^{rs} - \sum_{r=1}^{k} S_{ij}^{rs} p_{ij}^{rs}\right).$$

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