CENTRE of POLICY STUDIES and the IMPACT PROJECT

Eleventh Floor, Menzies Building
Monash University, Wellington Road
CLAYTON Vic 3800 AUSTRALIA

Telephone: (03) 9905 2398, (03) 9905 5112 — from overseas: 61 3 9905 2398 or 61 3 9905 5112

Fax: (03) 9905 2426 — 61 3 9905 2426

e-mail: impact@buseco.monash.edu.au

Internet home page: http//www.monash.edu.au/policy/

Theoretical Structure of the FAGE Model

by

JINGLIANG XIAO
Centre of Policy Studies
Monash University

General Paper No. G-198 March 2010

ISSN 1 031 9034

ISBN 978 1 921654 05 3

Theoretical Structure of the FAGE Model*

Jingliang Xiao

Centre of Policy Studies, Monash University

Abstract

This paper explains the theoretical framework of the Financial Applied General Equilibrium (FAGE) model as developed in Xiao (2009). FAGE is a MONASH-style dynamic CGE model for China with a detailed financial extension. In section 1, we discuss a stylized version of the financial module. Section 2 discusses the important aspects of the full version of the FAGE model, such as, the database and investment theory.

JEL Classification: C68, D58, E44, E52, E62, F31

Key words: dynamic CGE, financial market, monetary policy

*I would like to thank Professor Peter Dixon for his help in developing the FAGE model and Heinrich Bohlmann for his valuable comments on this paper.

Contents

List of Tables

1. A stylized financial extension

Section 1 illustrates the linkages between the i) balance sheets of agents, ii) input-output tables, iii) social accounting matrix (SAM), and iv) equations provided in this section. In section 1.1, we discuss the database for the stylized model, which includes the stocks, e.g. assets and liabilities of each agent, and the flows, e.g. savings or deficits of each agent. The model equations and optimising behaviour of agents are discussed in sections 1.2 and 1.3, respectively.

1.1 Accounting conundrums

1.1.1 Balance sheets of agents

There are six agents in the stylized financial model. They are HouseHolds (HH), the INDustries (IND), the CoMmercial Banks (CMB), the CenTral Bank (CTB), the GOVernment (GOV), and the Rest Of the World (ROW). Table 1 lists the balance sheets of each agent. The variables on the left are assets and the variables on the right are liabilities. Items are denominated in Chinese currency, i.e. RMB or Chinese Yuan, unless otherwise stated.

Table 1 Agents' balance sheets (items are in Chinese currency, i.e. ￥)

HH		**IND**		**CMB**	
EQT		qPK	BLN	BLN	DEP
DEP			FLN	GLN	CLM
CUR			EQT	RR, XR	
NW = EQT+DEP+CUR		NW = qPK-BLN-FLN-EQT		NW = BLN+GLN+RR+XR-DEP-CLM	

CTB		**GOV**		**ROW**	
$FXR^{\$}/E$	CUR		GLN	FLN	$FXR^{\$}/E$
CLM	RR, XR		GTLN		
GTLN					
NW = $FXR^{\$}/E$+CLM+GTLN-CUR-RR-XR		NW = -GLN-GTLN		NW = FLN-$FXR^{\$}/E$	

note: assets are on the left and liabilities on the right.
EQT: The value of share held by Households;
DEP: The amount of domestic deposits saved by Households;
CUR: Currency held by Households;
qPK: The value of the physical capital in Industries;
BLN: Loan from Commercial Banks to Industries;
FLN: Foreign loan to Industries;
GLN: Government bond held by Commercial Banks;
RR: Required reserves;
XR: Excess reserves;
CLM: Central Bank's claims on the Commercial Banks;
$FXR^{\$}$: Foreign exchange reserves in Central Bank (in foreign currency, e.g. $);
GTLN: Government bond held by Central Bank.
E: Nominal exchange rate in indirect quotation, i.e., FgnCur/DomCur, or $/￥.

Households have three financial assets: equities issued by industries (*EQT*), deposits in the domestic commercial banks (*DEP*) and currency issued by the central bank (*CUR*). We split up households' behaviour into real estate investment and the behaviour in financial portfolios. Households' investments in real estate are treated as a separate industry called "dwellings". There are no mortgages as a liability in households' balance sheet. Rather, their mortgages are regarded as liabilities in the "dwellings" industry. ***Industries*** have financial liabilities which include loans from the domestic commercial bank (*BLN*), loans from Rest of World (*FLN*) and shares owned by households (*EQT*). Industries' assets are the value of its capital stock (*qPK*). The replacement value of industries' capital stock is *PK*, and *q* represents the valuation ratio or level of "*Tobin's q*"[1]. The value of *q* can be linked to investment decisions by industries. For the ***Commercial Banks***, assets consist of loans to industries (*BLN*) and government (*GLN*), together with holdings in the central bank, including required reserves (*RR*) and excess reserves (*XR*). On the liability side, the commercial banks have loans from the central bank (*CLM*) with interest charged at the discount rate. It also offers households a rate of interest to attract their savings, so that the major liability of the commercial banks is household deposits. The assets in the ***Central Bank's*** balance sheet are claims on the commercial banks, some government bonds (*GTLN*), and foreign exchange reserves converted to Chinese currency ($FXR^{\$}/E$). Foreign exchange reserves in U.S. dollar ($FXR^{\$}$) need to be converted to RMB by dividing by the exchange rate, *E*. The exchange rate is quoted indirectly, that is, how many dollars required to buy one Chinese Yuan. ***Government*** has only liabilities (*GLN* and *GTLN*). ***ROW*** has assets (*FLN*) and liabilities consisting of the reserves of the central bank and loans from Chinese households.

At the bottom of each agent's balance sheet, we calculate their net worth (*NW*) as the sum of assets minus liabilities. Lastly, we note three balancing conditions.

- The commercial banks have zero net worth reflecting the zero pure profit condition;
- The industries have zero net worth, e.g. $qPK = BLN+EQT+FLN$;
- The sum of the financial net worth (*FNW*)[2] of all of the agents is equal to zero.

1.1.2 Input-Output (I-O) table

In this section, we use an I-O table to describe how agents' income flow helps to move the balance at the start of the period to the balance at the end of the period. Before moving to discuss the I-O table, it would be appropriate to list the assumptions of the stylized model. Unlike the full version of FAGE, the stylized model has no margins, no inventory stock, no tariffs, no household income tax, and no capital gains tax, etc.

---

[1] In this application, Tobin's q should be interpreted as the ratio of the market value of industry equities & bonds to the replacement cost of industry fixed assets.

[2] Financial net worth (*FNW*) differs from net worth (*NW*) because it does not take the physical capital, e.g. *qPK*, into account. For example, the *FNW* of industries is the total liability: $–BLN–EQT–FLN$. But for other agents, their financial net worth is just as the same as their net worth because they hold no physical assets.

Table 2 shows the I-O table, which represents the flow of economic transactions between major economic agents. The headers of columns show the absorption of the products. The 5 columns represent the 5 demand categories or users: i) domestic producers, *i* industries; ii) investors who carry out the physical capital formation, one for each of the *i* industries; iii) a single representative household; iv) a foreign purchaser of exports; and v) the government.

Table 2 Input-output table of the stylized model

		Absorption Matrix				
		1	**2**	**3**	**4**	**5**
		Producers	**Investors**	**Household**	**Export***	**Government**
	size	**I**	**I**	**1**	**1**	**1**
Basic flows domestic	**C**	V1BAS("dom")	V2BAS("dom")	V3BAS("dom")	V4BAS("dom")	V5BAS("dom")
Basic flows imported	**C**	V1BAS("imp")	V2BAS("imp")	V3BAS("imp")		V5BAS("imp")
Taxes	**C*S**	V1TAX	V2TAX	V3TAX	V4TAX	V5TAX
Labour	**O**	V1LAB				
Capital	**1**	V1CAP				
Land	**1**	V1LND				
Production Tax	**1**	V1PTX				

C=Number of commodities

I=Number of Industries

S=2: Dom and Imp

O=Number of Occupation Types

*Note: Export column is for domestic goods only

Most of the cells can be interpreted as a matrix, with the dimension of each read from the size of the columns and rows. For instance, *V1BAS(“dom”)* is a *C*I* matrix, which shows the intermediate usage of *dom*estically produced commodities *c*, ($c \in \text{Set of } C \text{ commodities}$), by industry *i* ($i \in \text{Set of } I \text{ industries}$). *V1BAS(“imp”)* represents the *imp*orted commodities, *c*, used in industry *i*, as intermediate inputs. We assume that only domestically produced goods are exported, therefore, *V4BAS(“imp”)* is zero. In the row of “Taxes”, *V1TAX…V5TAX*, the source-specific commodities can potentially be taxed differently by purchaser. As well as intermediate inputs, current production requires inputs of three kinds of primary factors: i) labour, ii) physical capital, and iii) agricultural land. The payments to them are *V1LAB*, *V1CAP* and *V1LND*, respectively. Finally, the non-user-specific production tax, *V1PTX* is included in the last row in column one.

Detailed descriptions of the database for standard CGE models including margins, inventories, tariffs etc., are documented in Dixon *et al.* (1982) and Horridge (2000).

1.1.3 Social Accounting Matrix (SAM)

Table 4 sets out the country's flow accounts in the form of a social accounting matrix (SAM). The shadowed part of the table is similar to the I-O table previously discussed. The difference is that the SAM reclassifies the content in column 1 in Table 2 to show the income generation more explicitly. But the total value of output in cell (N,1)[3], *PX*, is the same as the total of column 1 in the I-O table, and is also equal to the amounts in cell (A,15) in Table 4. Row (A) shows that the domestic outputs are used for its usual purposes i.e., intermediate usage in industries [*V1BAS("dom")*], investment or capital formation [*V2BAS("dom")*], private consumption [*V3BAS("dom")*], exports [(*V4BAS("dom")*] and government spending [*V5BAS("dom")*].

<u>*Incomes: Rows (B)-(G)*</u>

One basic rule is that cells in the top half in Table 4 represent the payments from the agents of the column headers to the agents of the row headers. For instance, transfers in cell (C,5) are the government's payments to households.

As in an I-O table, column (1) shows costs of production or payments by producers to agents identified by the rows. Compared to the I-O table, however, there are some notable differences in a SAM. To demonstrate how we reclassify income generated by industries to different agents in a SAM, we start by looking at the accounting conventions in corporate finance. The left hand side (LHS) column in Table 3 shows the conventional financial income statement, whilst the right hand side (RHS) column shows the adjusted income statement which is used in our model.

Table 3 Income statement of industries

Conventional Statement	**Adjusted Statement from FAGE**
Revenue	*PX*
<u>*- Operating Expense*</u>	<u>*- (V1BAS +V1TAX+V1LAB+V1PTX)*</u>
EBITDA	*V1CAP+V1LND*
<u>*- Depreciation*</u>	<u>- $(i^{FLN}*FLN + i^{BLN}*BLN)$</u>
EBIT	*EBTDA*
<u>*- Interest payment*</u>	<u>**(1-TAX_K_R)*</u>
EBT	*EBDA*
<u>**(1-TAX_K_R)*</u>	<u>*- DEP+RALPH*DEP*TAX_K_R*</u>
Net Income	*Net Income*
<u>*- Dividend*</u>	<u>**(1-DivRate)*</u>
Retained Earnings	*Retained Earnings*
<u>*+Depreciation*</u>	<u>*+DEP*</u>
Discretionary Cash Flow	*Discretionary Cash Flow*

[3] N,1 stands for the cell located in row N and column 1.

Table 4 SAM with production, income, expenditure, and flow of funds accounts (items are in Chinese currency, i.e. ￥)

		Current expenditures							
		Industries as Producers	Industries as Investors	Households	ROW	Government	Commercial Banks	Central Bank	Total
		1	2	3	4	5	6	7	15
A	Producers	V1BAS("dom")	V2BAS("dom")	V3BAS("dom")	V4BAS("dom")	V5BAS("dom")			PX
Incomes									
B	Industries as investors	NI*(1-DivRate)+DEP							Y_{ind}
C	Households	V1LAB+NI*DivRate	Cap_Gain			Transfer	i^{DEP}*DEP	0*CURR	Y_{hh}
D	ROW	V1BAS("imp") + i^{FLN}*FLN	V2BAS("imp")	V3BAS("imp")		V5BAS("imp")			Y_{row}
E	Government	∑V1TAX+∑V1PTX+(EBTDA-RALPH*DEP)*TAX_K_R	∑V2TAX	∑V3TAX	∑V4TAX	∑V5TAX			Y_{gov}
F	Commercial Banks	i^{BLN}*BLN				i^{G}*GLN		0*RR+i^{XR}*XR	Y_{cmb}
G	Central Bank				REV+i^{FX}*FXR$^{\$}$/E	i^{G}*GTLN	i^{CLM}*CLM		Y_{ctb}
Flows of funds									
H	Industries as investors		S_{ind}						
I	Households			S_{hh}					
J	ROW				S_{row}				
K	Government					S_{gov}			
L	Commercial Banks						S_{cmb}		
M	Central Bank							S_{ctb}	
N	Total	PX	Y_{ind}	Y_{hh}	Y_{row}	Y_{gov}	Y_{cmb}	Y_{ctb}	

(..Continue)

		Changes in National Claims					Changes in Foreign Claims		
			Commercial Banks		Central Bank Liabilities				
		Industrial Equity	Assets	Liabilities	Assets	Liabilities	National Assets	National Liability	Total
		8	9	10	11	12	13	14	15
A	Producers								
Incomes									
B	Industries as investors								
C	Households								
D	ROW								
E	Government								
F	Commercial Banks								
G	Central Bank								
Flows of funds									
H	Industries as investors	dEQT	dBLN					dFLN	0
I	Households	-dEQT		-dDEP		-dCUR			0
J	ROW						dFXR$^{\$}$/E	-dFLN	0
K	Government		dGLN		dGTLN				0
L	Commercial Banks		-dBLN -dGLN -dRR -dXR	dDEP + dCLM					0
M	Central Bank		dRR + dXR	-dCLM	-dGTLN	dCUR	-dFXR$^{\$}$/E		0
N	Total	0	0	0	0	0	0	0	0

Assumptions: no tariffs, no inventories, no margins, no household income tax, no capital gains tax; *Sind* usually is negative, means deficit; V5TAX = 0; *Scmb* = 0 (zero profit conditions); E = FgnCur/DomCur (e.g.=$/￥)

The first line of both income statements is the revenue, i.e. sales or *PX* in CGE modelling. Revenue less operating expense is earnings before interest, taxes, depreciation and amortisation (*EBITDA*). From the RHS, we notice that operating expenses is the sum of *V1BAS*, *V1TAX*, *V1LAB*, and *V1PTX*, and that *EBITDA* is the sum of *V1CAP* and *V1LND*. In finance terms as shown in the LHS, depreciation, interest payment, and taxes are taken away from *EBITDA* in sequence, giving us net income (*NI*). Conversely, in Xiao (2009), we first subtract from *EBITDA* the interest payment (i^{FLN}*FLN + i^{BLN}*BLN[4]), then taxes, and lastly, the depreciation. Since the depreciation is only excluded after taxes, an adjustment term, such as, *RALPH*DEP*TAX_K_R*, needs to be added back. *RALPH* is a coefficient to capture the tax benefit of depreciation, and it is usually set at either zero or one. A detailed description of the *RALPH* term can be found in Dixon and Rimmer (2002, p.193). Net income is the profit for shareholders, but it may not be fully transferred to shareholders immediately. Based on the dividend rate, firms might only distribute some part of the net income as dividends. The remaining income after dividends is called retained earnings (*RE*). But retained earnings are not the cash flows which a firm can use to finance projects yet. To calculate the discretionary cash flows (*DCF*), we should add back depreciation to the retained earnings. The discretionary cash flows, in addition to the new issuance of equity and new loans from banks, are the total amount of funds available to finance investments. In Table 4, cell (B,1) shows the discretionary cash flows, as *NI*(1–DivRate) + DEP*. Apart from discretionary cash flows, investors receive no any other payments.

Row (C) shows the components of households' income. The largest amount of income is in cell (C,1) which is made up of wages (*V1LAB*) and dividends (*NI*Div_Rate*) from industries. *Cap_Gain* in cell (C,2) represents the capital gain on the industries' equities held by households, which can be thought of as a type of transfer from the industries. The value of *Cap_Gain* is equal to the retained earnings of the industries plus the revaluation of the physical capital. That is,

$$Cap\_Gain = RE + (P_I^1 - P_I^0)\cdot K^0(1 - DEP) + (P_I^1 - P_I\ )\cdot I$$

where *RE* is retain earnings, *DEP* represents the depreciation rate of the physical capital, K^0 is the physical capital at the beginning of the year, *I* is investments, P_I^1, P_I^0, and P_I are the replacement cost of unit of capital at the end of the year, start of the year, and middle of the year. (C,6) shows the households' interest received from commercial banks (i^{DEP}*DEP*). The government's transfers are in cell (C,5) as mentioned earlier. Combined these cells add up to the total income Y_{hh} [cell (C,15)]. The figures in cells ((B…G),15) represent the total income of each agent, and they should be equal to the entries in cells (N,(2…7)), respectively.

Row (D) shows the payments to the ROW in the current account. It includes the value of imported goods, *V1BAS("imp")…V5BAS("imp")*[5] and income payable, i^{FLN}*FLN*,

[4] i^{BLN} is the interest rate on domestic loan, *BLN*; i^{FLN} is the interest rate on the foreign loan, *FLN*. We use the same notation in column (1) of Table 4,

[5] As mentioned above, we assume that no imported goods are redirected to export, which implies V4BAS("imp") = 0 and not shown in cell (D,4).

in cell (D,1). Row (E) shows the revenue of government, which includes the Value Added Taxes (*∑V1TAX…∑V5TAX*[6]), Tax on production *∑V1PTX* in cell (E,1), and the corporation tax, i.e. *(EBTDA – RALPH*DEP) * TAX_K_R*.

Commercial banks receive interest paid by the central bank on excess reserves (cell (F,7), i.e. $i^{XR}*XR$)[7], interest from loans to industries (cell (F,1), i.e. $i^{BLN}*BLN$), and interest from loans to government (cell (F,5), i.e. $i^{G}*GLN$). The central bank receives interest from three sources: from ROW on foreign reserves (cell (G,4), i.e. $i^{FX}*FXR^{\$}/E$); from bonds issued by government (cell (G,5), i.e. $i^{G}*GTLN$); and from claims on commercial banks (cell (G,6), i.e. $i^{CLM}*CLM$). The central bank's total income also depends on whether we write down the revaluation effect on its foreign assets. To simplify, we assume that all of the central bank's foreign assets are U.S. government bonds. For example, if the Chinese currency appreciates against the dollar, i.e. *E* increases, there will be a loss to the central bank in its foreign-denominated assets. The total effect of the revaluation term (*REV*) in cell (G, 4) should be calculated as follows

$$REV = FXR^{\$,0} * (\frac{1}{E^1} - \frac{1}{E^0}) + dFXR^{\$} * (\frac{1}{E^1} - \frac{1}{E})$$

where $FXR^{\$,0}$ is the foreign reserves in dollar at the beginning of year, $dFXR^{\$}$ is the change of foreign reserves in dollar during the year, E^0, E^1, E are the exchange rate at the start of the year, end of the year and middle of the year. So there is no capital gain in the revaluation term (*REV*) apart from the exchange rate effect.

Savings:

In order to calculate the savings of agents, we need to also consider their expenditures in addition to their income discussed previously. Industries in their role as producers have zero saving. However, industries in their role as investors usually spend more than their income, i.e. *V2BAS*[8]+ *V2TAX* is greater than discretionary cash flows, Y_{ind} in cell (B,15). The income minus total spending and the transfers to household (i.e. *Cap_Gain*) gives us the saving of industries, S_{ind} [cell (H,2)], which is normally negative (indicating a deficit).

$$S_{ind} = NI * (1 - DivRate) + DEP - (V2BAS + V2TAX + Cap\_Gain) \qquad (1)$$

Savings of households, S_{hh} in cell (I,3), are equal to their income, Y_{hh}, minus their total spending, *V3BAS + V3TAX*.

$$\begin{aligned} S_{hh} &= V1LAB + NI * DivRate + Cap\_Gain + Transfer + i^{DEP} * DEP \\ &\quad - (V3BAS + V3TAX) \end{aligned} \qquad (2)$$

[6] ∑ViTAX, i = 1…5, represent the column sum of the matrix ViTAX.
[7] We assume there is no interest on required reserves (*RR*).
[8] V2BAS = V2BAS("dom") + V2BAS("imp").

S_{row} in cell (J,4) shows the current account deficit (*CAD*) and revaluation of foreign reserves (*REV*) of China, sometimes referred to as foreign saving in terms of Chinese currency. This is calculated as imports, $\sum_{i}^{5} \mathrm{V}iBAS(\text{"}imp\text{"})$, plus income payables (*IP*, e.g. $i^{FLN}*FLN$) minus the exports, *V4BAS* + *V4TAX*, income receivables (*IR*, e.g. $i^{FX}*FXR^{\$}/E$) and the revaluation term (*REV*). Mathematically, $S_{row} = M + IP - (X + IR + REV) = CAD - REV$.

$$S_{row} = \sum_{i=1...5} ViBAS(\text{"}imp\text{"}) + i^{FLN} * FLN \\ - (V4BAS + V4TAX + REV + i^{FX} * FXR^{\$} / E) \tag{3}$$

Savings for government, S_{gov} in cell (K,5), is income made up of taxes and interest received, i.e. the entries in row (E), minus expenditures, interest paid, and transfers, i.e. the entries in column (5).

$$S_{gov} = \sum_{i=1..5} ViTAX + V1PTX + (EBTDA - RALPH * DEP) * TAX\_K\_R \\ - (V5BAS + V5TAX + Transfer + i^{G} * GLN + i^{G} * GTLN) \tag{4}$$

Savings for commercial banks, S_{cmb} in cell (L,6) equal interest received, i.e. $i^{BLN}*BLN + i^{G}*GLN + i^{XR}*XR$, minus interest paid, i.e. $i^{DEP}*DEP + i^{CLM}*CLM$. It is worth noting again that the savings for commercial banks should be equal to zero because of the zero pure profit condition in the private banking sector.

$$S_{cmb} = 0 \tag{5}$$

Savings for the central bank, S_{ctb} in cell (M,7) equal the difference between the total income, Y_{ctb}, and interest paid on the reserves, $0*RR + i^{XR}*XR$. Savings for the central bank, S_{ctb}, are usually not zero in the short run. At least two reasons can explain this. First, countries with persistent accumulation of foreign reserves, like China, would usually undertake a massive sterilisation to mop up the liquidity in the domestic market. In this case, the central bank usually runs a deficit because the interest paid on sterilisation is more than the interest received on the foreign reserves. Second, there is no guarantee that the revaluation term (*REV*) will be zero. For a country with huge foreign currency denominated foreign assets, any small change of the exchange rate will cause serious losses or gains in *REV* and their savings.

$$S_{ctb} = REV + i^{FX} * FXR^{\$} / E + i^{G} * GTLN + i^{CLM} * CLM - i^{XR} * XR \tag{6}$$

Flow of funds: Rows (H) - (M)

After agents determine their savings, the question of how they are going to allocate these savings to specific assets arises. A potential solution is given by the optimal portfolio theory of diversification discussed in detail in Section 1.3. But first, we set some constraint for the stock (e.g. assets or liabilities) and flow (e.g. savings or deficits) relationships.

Row (H) shows that the saving of investors (S_{ind} in cell (H,2), deficits actually) is financed by three sources: the new issues of equity, *dEQT* in cell (H,8), new borrowing from commercial banks, *dBLN* in cell (H,9), and new borrowing from abroad, *dFLN* in cell (H,14). So the sum of these items in row (H) is equal to zero, namely, $S_{ind} + dEQT + dBLN + dFLN = 0$.

Households, in row (I), use their savings to build up currency holdings, *dCUR* in cell (I,12), to acquire new equity, *dEQT* in cell (I,8), and to save more in domestic banks, *dDEP* in cell (I,10). That is, $S_{hh} – dEQT – dDEP – dCUR = 0$.

Row (J) shows the “balance of payments” constraint, which is $S_{row} + dFXR\$/E – dFLN = 0$. As we discussed, savings S_{row} represent the current account deficit of the domestic country. *dFLN* shows the capital account surplus, namely the private capital inflow. The identity above also can be written as $dFXR\$/E = - S_{row} + dFLN$, which means that the increase in official foreign reserves is equal to current account surplus plus capital account surplus.

Row (K) shows that, if the government is running a budget deficit (a negative S_{gov}), it needs to obtain financing by borrowing from commercial banks, *dGLN*, or from the central bank, *dGTLN*. That is, $S_{gov} + dGLN + dGTLN = 0$.

Under the assumption that commercial banks earn zero pure profit, S_{cmb} in cell (L,6) equals zero. Row (L) implies that the outflow of funds of commercial banks, i.e. the increase in assets ($dBLN + dGLN + dRR + dXR$) is offset by the inflow of funds, i.e. the increase in liabilities ($dDEP + dCLM$). Implicitly, the restriction that applies to row (L) can be written as $dBLN + dGLN + dRR + dXR = dDEP + dCLM$.

Row (M) shows the constraint, $S_{ctb} + dRR + dXR – dCLM – dGTLN + dCUR – dFXR^{\$}/E = 0$, as it applies to the central bank. The above equation can be rearranged as $dCUR = dFXR^{\$}/E + dCLM + dGTLN – (dRR + dXR + S_{ctb})$, which is essentially a money supply equation. The central bank’s usual instruments for altering money supply are i) open market operations (changes in *dGTLN*); ii) adjustment of the funds rate, which affect the central bank’s claims on the commercial banks, *dCLM*; and iii) a change in reserve requirements (*dRR*). The constraint also tells us the supply of money is additionally determined by the change in foreign reserves. It means that accumulation of reserves implies an increase in money supply if the reserves are absorbed without any sterilisation policy. Saving for the central bank, S_{ctb}, is usually a small number relative to other terms in the equation.

The zero-sum of all the entries in rows (H) to row (M), namely, $S_{ind} + S_{hh} + S_{row} + S_{gov} + S_{cmb} + S_{ctb} = 0$, leads to the standard saving-investment identity. At first glance, this result may not be obvious. However, when we assume that the discretionary cash flow of industries is only the depreciation, i.e. *Dep* in cell (B,1), (implying a 100% dividend payout), the savings of industries are

$$S_{ind} = Dep - GFCF^{9} - Cap\_gain = - \text{net investment } (PI^{n}) - Cap\_gain$$

Given that the capital gain in cell (C,2) is zero, the savings of industries are

$$S_{ind} = - \text{net investment } (PI^{n})$$

Moving the savings of industries to the right hand side, the zero-sum equation mentioned before can be written as:

$$S_{hh} + S_{row} + S_{gov} + S_{cmb} + S_{ctb} = - S_{ind}$$

or alternatively stated,

$$S_{hh} + S_{row} + S_{gov} + S_{cmb} + S_{ctb} = PI^{n}$$

From this equation, we can interpret that economic-wide net saving should equal net investment.

[9] Gross fixed capital formation is *V2BAS+V2TAX*.

1.2 Model equations and variables

1.2.1 Equations and variables

The equations listed in Table 5 describe the stylized financial extension and can be classified into three categories. One is used to illustrate the stock and flow relationships, e.g. Equation (7), (11), etc. The flows (e.g. savings, *S*) appear in Equations (1) - (6) in Section 1.1.3. The second category, such as, Equations (8) - (10), explain how to determine the end-of-year stocks that appeared in Table 1, such as, *CUR*, *EQT*, etc. The third category described by Equations (27) - (36) contains the market clearing and zero pure profit conditions.

The FAGE model is dynamic and therefore all of the stock variables have subscript *t* and superscript 0 or 1, which either represent the start or end of year *t*. A detailed description of each variable that appears in the list of equations can be found in Table 6. The derivations of Equations (8) - (10), (12) - (16), and (23), which describe the Agents' optimising behaviour, will be discussed in detail in Section 1.3. The superscript "+/-" on the variables in the equations denotes the relationship between the function and the variable. For example, the superscript "+" on Z_t^{EQT} in Equation (9) means $\frac{\partial EQT_{hh,t}^{1}}{\partial Z_t^{EQT}} > 0$, i.e., households increase their equity holding when the yield on the equity rises relative to the yield on other assets. Stock variables are denominated in RMB, except where denoted with a "$" superscript, e.g. $FLN_t^{\$,1}$ is the end of year foreign reserve in $ terms, while FLN_t^1 is denominated in RMB.

Table 5 Equations for stylized financial extension

Households:

$$FNW_{hh,t}^{1} = FNW_{hh,t}^{0} + S_{hh,t} \quad (7)$$

$$CUR_t^1 = P_t \cdot \Psi_{CUR}[\overset{+}{C_t}, \overset{-}{i_t^{DEP}}] \quad (8)$$

$$EQT_{hh,t}^{1} = (FNW_{hh,t}^{1} - CUR_t^1) \cdot \Psi_{EQT,hh}[\overset{+}{Z_t^{EQT}}, \overset{-}{Z_t^{DEP}}] \quad (9)$$

$$DEP_t^1 = (FNW_{hh,t}^{1} - CUR_t^1) \cdot \Psi_{DEP}[\overset{-}{Z_t^{EQT}}, \overset{+}{Z_t^{DEP}}] \quad (10)$$

Industries:

$$FNW_{ind,t}^{1} = FNW_{ind,t}^{0} + S_{ind,t} \quad (11)$$

$$BLN_{ind,t}^{1} = -FNW_{ind,t}^{1} \cdot \Psi_{BLN,ind}[\overset{-}{i_t^{BLN}}, \overset{+}{i_t^{FLN}}, \overset{+}{i_t^{EQT}}] \quad (12)$$

$$FLN_{ind,t}^{1} = -FNW_{ind,t}^{1} \cdot \Psi_{FLN,ind}[\overset{+}{i_t^{BLN}}, \overset{-}{i_t^{FLN}}, \overset{+}{i_t^{EQT}}] \quad (13)$$

$$EQT_{ind,t}^{1} = -FNW_{ind,t}^{1} \cdot \Psi_{EQT,ind}[\overset{+}{i_t^{BLN}}, \overset{+}{i_t^{FLN}}, \overset{-}{i_t^{EQT}}] \quad (14)$$

Commercial Banks:

$$BLN^1_{cmb,t} = (DEP^1_t + CLM^1_t - RR^1_t - GLN^1_t)\cdot\Psi_{BLN,cmb}[\overset{+}{Z^{BLN}_t}, \overset{-}{Z^{XR}_t}] \tag{15}$$

$$XR^1_t = (DEP^1_t + CLM^1_t - RR^1_t - GLN^1_t)\cdot\Psi_{XR}[\overset{-}{Z^{BLN}_t}, \overset{+}{Z^{XR}_t}] \tag{16}$$

$$CLM^1_t = P_t \cdot \Psi_{CLM}[\overset{-}{i^{CLM}_t}] \tag{17}$$

$$RR^1_t = \delta_t * DEP^1_t \tag{18}$$

Government:

$$FNW^1_{gov,t} = FNW^0_{gov,t} + S_{gov,t} \tag{19}$$

$$GTLN^1_t = -FNW^1_{gov,t} - GLN^1_t \tag{20}$$

Central Bank:

$$FNW^1_{ctb,t} = FNW^0_{ctb,t} + S_{ctb,t} \tag{21}$$

$$FXR^1_t = FNW^1_{ctb,t} - CLM^1_t - GTLN^1_t + CUR^1_t + RR^1_t + XR^1_t \tag{22}$$

Rest of World:

$$FLN^{\$,1}_{row,t} = TF^{\$,1}_{row,t}\cdot\Psi_{row}[\overset{+}{Z^{FLN}_t}, \overset{-}{Z_{row,t}}] \tag{23}$$

$$dFLN^{\$}_{row,t} = FLN^{\$,1}_{row} - FLN^{\$,0}_{row} \tag{24}$$

$$FLN^1_{row,t} = FLN^0_{row,t} + dFLN^{\$}_{row,t} / E_t \tag{25}$$

$$FNW^1_{row,t} = FLN^1_{row,t} - FXR^1_t \tag{26}$$

Market Clearing:

$$BLN^1_{ind,t} = BLN^1_{cmb,t} \tag{27}$$

$$EQT^1_{ind,t} = EQT^1_{hh,t} \tag{28}$$

$$FLN^1_{ind,t} = FLN^1_{row,t} \tag{29}$$

Zero Pure Profit Condition:

$$BLN^1_{cmb,t} * i^{BLN}_t + XR^1_t * i^{XR}_t + GLN^1_t * i^G_t = DEP^1_t * i^{DEP}_t + CLM^1_t * i^{CLM}_t \tag{30}$$

Others:

$$Z^{FLN}_t = \left.\frac{(1+i^{FLN}_t)*APP^e_t}{1+\dot{P}^{row}_t}\right/ 1+\bar{r}^{FLN}_t \tag{31}$$

$$Z^s_t = \left.\frac{1+i^s_t}{1+\dot{P}_t}\right/ 1+\bar{r}^s_t \qquad s \in \{EQT, DEP, BLN, XR, CLM\} \tag{32)-(36}$$

Table 6 Descriptions for variables in the stylized financial extension

Name	Descriptions
$FNW_{j,t}^{1}$	Financial net worth of agent j at end of yr t, $j \in \{hh, ind, gov, ctb, row\}$
$FNW_{j,t}^{0}$	Financial net worth of agent j at start of yr t, $j \in \{hh, ind, gov, ctb\}$
$S_{j,t}$	Saving of agent j in yr t, $j \in \{hh, ind, gov, ctb\}$
$\Psi 's$	Functions
$EQT_{hh,t}^{1}$	Demand of equities held by Households at end of yr t
DEP_{t}^{1}	Households deposits in Commercial Banks at end of yr t
CUR_{t}^{1}	Currency demand by Households at end of yr t
C_{t}	Real private consumption in yr t
P_{t}	Consumer price index in yr t
i_{t}^{s}	Nominal rate of return of security s in yr t, $s \in \{G, FLN, EQT, DEP, BLN, XR, CLM\}$
Z_{t}^{s}	Power of yield of security s in yr t, $s \in \{FLN, EQT, DEP, BLN, XR, CLM\}$
$Z_{row,t}$	Power of yield of foreign investor's ROW portfolio in yr t
$BLN_{ind,t}^{1}$	Demand of domestic loans of Industries at end of yr t
$FLN_{ind,t}^{1}$	Demand of foreign loans of Industries at end of yr t
$EQT_{ind,t}^{1}$	Demand of equities of Industries at end of yr t
$BLN_{cmb,t}^{1}$	Supply of loans from Commercial Banks to industry at end of yr t
CLM_{t}^{1}	Central Bank's claims on Commercial Banks at end of yr t
RR_{t}^{1}	Required reserves in Central Bank at end of yr t
GLN_{t}^{1}	Government bonds held by Commercial Banks at end of yr t
XR_{t}^{1}	Excess reserves in Central Bank at end of yr t
δ_{t}	Rate of required reserves in yr t
$GTLN_{t}^{1}$	Government bonds held by Central Bank at end of yr t
FXR_{t}^{1}	Foreign exchange reserves in RMB at end of yr t
$FLN_{row,t}^{\$,1}$	Foreign loans to Industries in \$ at end of yr t
$TF_{row,t}^{\$,1}$	Total fund of ROW in \$ at end of yr t
$dFLN_{row,t}^{\$}$	Change in foreign loans in \$ to Industries in yr t
$FLN_{row,t}^{\$,0}$	Foreign loans to Industries in \$ at start of yr t
$FLN_{row,t}^{1}$	Foreign loans to Industries in RMB at end of yr t
$FLN_{row,t}^{0}$	Foreign loans to Industries in RMB at start of yr t
E_{t}	Average nominal exchange rate in yr t, (i.e. \$/￥)
APP_{t}^{e}	Expected appreciation of exchange rate
$\dot{P}_{t}$	Expected inflation rate of domestic country
$\dot{P}_{t}^{row}$	Expected inflation rate of ROW
$\overline{r}_{t}^{s}$	Required real yield of security s in yr t, $s \in \{FLN, EQT, DEP, BLN, XR, CLM\}$

1.2.2 Explanation of equations

For households, Equation (7) shows that the end-of-year financial net worth ($FNW^1_{hh,t}$) is the initial financial net worth, $FNW^0_{hh,t}$, plus savings during that year (i.e. $S_{hh,t}$). Equation (8) indicates that real money demand (nominal demand, *CUR*, divided by price level, *P*) depends on the real private consumption and the nominal interest rate. At the end of each year, households will reallocate their total end-of-year financial net worth less currency holdings according to their expected power of yields[10] (e.g. *Z's*) as described in Equations (9) and (10). For example, if the increase in the expected yield of equities is higher than deposits, households would allocate more of their funds to the stock market. An elaborated analysis of this optimisation is described in Section 1.3.

Equation (11) is similar to Equation (7), which calculates the end-of-year financial net worth of industries ($FNW^1_{ind,t}$). The end-of-year demands for loans (*BLN*, *FLN*) and equities (*EQT*) are determined by Equations (12) - (14). The reason for having a negative sign in front of *FNW* in these three equations is described in Footnote 2.

Because normally the sum of *BLN*, *EQT* and *FLN* is positive, the industries' *FNW* is usually less than zero. Therefore, –*FNW* can be explained as the industries' liabilities, i.e. *BLN* + *EQT* + *FLN*. The saving of industries, $S_{ind,t}$, as shown in Equation (1) usually is less than zero thereby indicating a deficit rather than saving. For example, an increase in industries' investment will lower the saving (i.e. a more negative $S_{ind,t}$), and lead to a larger negative $FNW^1_{ind,t}$, thereby raising the external funding requirement (either by increase of *EQT*, *BLN*, or *FLN*).

However, this raises the question of whether industries should demand more loans or equities. The answer will depend on the borrowing cost of each source of finance. As borrowing cost minimisers, industries would prefer the cheaper way to borrow. This is specified through demand functions of the form (12) - (14). For example, if there is a sharp decrease in the required rate of return on foreign loans, i.e. lower i^{FLN}_t, industries tend to borrow more from foreigners, i.e. $FLN^1_{ind,t}$ increases.

Equations (15) and (16) state that the supply of funds from the commercial banks to industries and the excess reserves to the central bank, depends on the total available funds ($DEP^1_t + CLM^1_t - RR^1_t - GLN^1_t$) in the account of the commercial banks and the expected power of yields (e.g., *Z's*). To simplify, we assume that the claims (*CLM*) from the central bank are only determined by the domestic price level (*P*) and the nominal discount rate (i^{CLM}_t), which is explained by Equation (17). However, in reality, we might expect that the demand for the claims from the central bank is also affected by the market interest rate (i^{BLN}_t). If that is the case, we need to adopt new optimisation behaviour for commercial banks and change Equations (15) - (17).

[10] We adopted the static approach to handle the expected value in this model, which means the most reasonable estimate for the year t+1 value is the value in year t. The forward looking approach can also be considered. More details can be found in Dixon and Rimmer (2002, pp.193-198).

Equation (18) states that the commercial banks are required to deposit a certain proportion (δ_t) of total household deposits (*DEP*) as required reserves (*RR*) in the central bank.

Similar to Equations (7) and (11), Equation (19) expresses the relationship of government's end-of-year financial net wealth and its savings (i.e. budget surplus). The government's demand for domestic credit at the end of year t, $-FNW^1_{gov,t}$, consists of government loans from commercial banks (*GLN*) and borrowings from the central bank (*GTLN*) . Either *GLN* or *GTLN*, shown in Equation (20), must be set exogenous in the model closure.

Similar to other agents, Equation (21) reveals the stock and flow relationship of the financial net worth of the central bank. The balance sheet of the central bank listed in Table 1 give us the following identity:

$$FNW^1_{ctb,t} = FXR^1_t + CLM^1_t + GTLN^1_t - CUR^1_t - RR^1_t - XR^1_t$$

All the terms in the above identity, except the end-of-year foreign reserves in RMB (FXR^1_t), have already been determined by other equations in Table 5. So we can calculate FXR^1_t with an alternative form of the above identity shown in Equation (22). The alternative approach to determine FXR^1_t is dividing the end-of-year reserves in dollar $FXR^{\$,1}_t$ by the exchange rate, i.e. $FXR^1_t = FXR^{\$,1}_t / E^1$. The end-of-year reserves in dollar ($FXR^{\$,1}_t$) can be figured out via adding the current account surplus and net capital inflows to the start-of-year reserves in dollar ($FXR^{\$,0}_t$).

The ROW section shows the optimising behaviour of foreign investors and the determination of the Chinese-owned net foreign liabilities. Equation (23) shows that foreign loans in dollars at the end of year depend on the foreign investors' total funds ($TF^{\$,1}_{row,t}$) and the differences in yields between Chinese assets and ROW assets. The detailed derivation of Equation (23) is in Section 1.3.4. The stock and flow relationship of foreign loans in dollar is explained by Equation (24).

In the stylized model, we assume that the foreign loans to Chinese industries are denominated in Chinese currency, RMB. Therefore industries pay the interest and repay the principle of loans in RMB terms. This assumption is justified for the Chinese economy. As described by Equation (25), the value of the end-of-year foreign loans in RMB is equal to the start-of-year foreign loans in RMB, $FLN^1_{row,t}$, plus the new foreign loans industries borrowed during year t, $dFLN^{\$}_{row,t} / E_t$.

Foreigner's financial net worth (FNW_{row}) in Chinese currency, namely, Chinese-owned net foreign liabilities in RMB, is defined by Equation (26). We expect China would have huge negative net foreign liabilities (i.e. $FNW^1_{row} < 0$), because its foreign reserves (*FXR*) are much bigger than its loans from overseas (*FLN*).

Equations (27) - (29) are the market clearing equations and ensure that demand equals supply. Equation (30) states the zero pure profit condition in the banking sector. Equations (31) - (36) show the definition of the expected power of yields for different securities. It is worth noting that the required real yields, $\bar{r}_t^s$, $s \in \{FLN, EQT, DEP, BLN, XR, CLM\}$, are implemented as preference variables (i.e. real interest rates plus risk premiums). They are naturally exogenous and can therefore be shocked. For instance, a reduction in the required real yield on *EQT* (usually occurring during economic booms) means that households might have a taste shift towards equities from other assets. *Ceteris paribus*, households might still like to purchase more equities because of the reduction in $\bar{r}_t^{EQT}$. These variables are potentially endogenous in the historical closure via exogenising the actual value of different assets.

1.3 Portfolio optimising behaviour of agents

This section mainly concentrates on explaining Equations (8) - (10), (12) - (16), and (23). Four of the six Agents (households, industries, commercial banks, ROW) have optimising behaviour. The behaviour of the central bank and government are exogenous as policy instruments. Households, the commercial banks and ROW (i.e. foreign investors) maximise their total expected yield on assets by changing the shares of each type of financial asset in their portfolio. Industries minimise their borrowing cost by having suitable amounts of financing sources (e.g. *BLN*, *FLN* and *EQT*), which are influenced by the nominal interest rate of borrowing funds (*i's*).

1.3.1 Households

The optimisation problem for households is to maximise their utility, given a CES function (Arrow *et al.*, 1961) where consumers must choose between two types of assets, *DEP* and *EQT*, based on annual rate of returns.[11] This functional form has been readily adopted in financial CGE models related to portfolio optimisation since Rosensweig and Taylor (1990). In our model, utility is maximised subject to a constraint to households' total financial net worth (*FNW*) minus their currency holding.

$$\text{Max: } E(\text{expected return}_{t+1}) = CES[Z_t^{DEP} \times DEP_t^1, Z_t^{EQT} \times EQT_t^1]$$
$$\text{St: } FNW_t^1 - CUR_{h,t}^1 = DEP_t^1 + EQT_t^1$$

Note: $E(X_{t+1})$ means the expectation for X in year t+1.

By solving this mathematical problem, we get the demand functions of equities (*EQT*) and deposits (*DEP*) at the end of year t. Both demand equations for equities and

[11] The CES utility function form can not only reflect the expected return, but also capture the risk-averse characteristics of investors. It assumes that investor use the risk diversified strategies, which is similar to the Mean-Variance Portfolio Choice's theory. For example, although some assets, e.g. equities, may give investors higher expected profit; they still keep some other less risky assets, e.g. bonds, for diversifying risks.

deposits are a function of financial net worth less currency (*FNW – CUR*) and *Z* ratios, described as Equations (9) and (10).

Transactions demand for currency (*CUR*) is determined by Equation (8). There are at least two schools of thought about the demand function of local currency. The first is the empirical model, which can be written as

$$\log \frac{CUR_t}{P_t} = \gamma_0 + \gamma_1 \log C_t - \gamma_2 \log i_t^{DEP},$$

where *CUR* is the demand for currency, *P* the consumer price index, *C* the real expenditure, and i^{DEP} the nominal interest rate. The parameter, γ_0 determines the velocity of money, whilst the parameters, γ_1 and γ_2 are generally between 0 and 1. For example, empirical studies (McCallum, 1989) suggest that the U.S. economy from 1895-1958 had the currency demand function:

$$\log \frac{CUR_t}{P_t} = 8.4 + 0.27 \log C_t - 0.32 \log i_t^{DEP}$$

The second approach to modelling currency demand is that of Baumol (1952) and Tobin (1956). Adapted from the optimisation theory of inventory demand in Operations Research, the currency demand formula implied by the Baumol-Tobin model is quite similar to the empirical model, and can be written as

$$\log \frac{CUR_t}{P_t} = 0.5 \log \frac{\delta}{2} + 0.5 \log C_t - 0.5 \log i_t^{DEP}$$

where δ reflects the real cost of transaction involved in obtaining currency (e.g. visiting a bank), and the other terms are the same as in the empirical model.

Therefore, under either approach to determine the money demand, we would still be able to write down the functional form of currency demand as shown in Equation (8).

1.3.2 Industries

Industries face the problem of minimising their total annual cost of capital, described by a CET function (Powell & Gruen, 1968), by changing the proportion of capital held from three sources, namely equities (*EQT*) and debts (local debts, *BLN*, and foreign debts, *FLN*). The following optimisation problem produces the demand functions for *EQT*, *BLN* and *FLN* as shown by Equations (12) - (14) in Table 5.

$$\text{Min: } E(\text{cost of capital}_{t+1}) = CET[i_t^{BLN} \times BLN_{ind,t}^1, i_t^{FLN} \times FLN_{ind,t}^1, i_t^{EQT} \times EQT_{ind,t}^1]$$

$$\text{St: } -FNW_{ind,t}^1 = BLN_{ind,t}^1 + FLN_{ind,t}^1 + EQT_{ind,t}^1$$

1.3.3 Commercial Banks

Like households, the commercial banks have the following optimisation problem.

$$\text{Max: } E(\text{expected return}_{t+1}) = CES[Z_t^{BLN} \times BLN_{cmb,t}^1, Z_t^{XR} \times XR_t^1]$$
$$\text{St: } DEP_t^1 + CLM_t^1 - RR_t^1 - GLN_t^1 = BLN_{cmb,t}^1 + XR_t^1$$

The commercial banks choose the optimal amount of *BLN* and *XR* to maximise their utility given their available credits (*DEP* + *CLM* – *RR* – *GLN*), with the two *Z* variables representing the power of yields. The solutions to this optimisation are the demand functions for *BLN* and *XR*, as shown by Equations (15) and (16) in Table 5.

1.3.4 Rest of the World

To maximise its returns, the rest of the world (ROW) decides on the optimal amount of loans to Chinese industries and ROW industries based on their total funds and corresponding expected rate of returns.

$$\text{Max: } E(\text{expected return}_{t+1}) = CES[Z_t^{FLN} \times FLN_{row,t}^{\$,1}, Z_{row,t} \times ASST_{row,t}^{\$,1}]$$
$$\text{St: } TF_{row,t}^{\$,1} = FLN_{row,t}^{\$,1} + ASST_{row,t}^{\$,1}$$

Foreign investors regard the power of their real yield as

$$Z_t^{FLN} = \frac{\frac{(1+i_t^{FLN})*APP_t^e}{1+\dot{P}_t^{row}}}{1+\bar{r}_t^{FLN}},$$

which depends on the rates of return of *FLN* (i^{FLN}), expected appreciation of the exchange rate (APP^e), and the expected inflation rate in ROW ($\dot{P}^{row}$). For example, if foreign investors expect the exchange rate to appreciate (i.e. the Chinese RMB appreciate or *E* increase), they would expect higher yields on *FLN* (Z^{FLN}) and increase lending to China.

In this system, we only pay attention to the foreign loans given to China (*FLN*), but not the remaining part of ROW assets (*ASST*). Therefore, we only keep the supply function of *FLN* in dollar terms in our model, which yields Equation (23).

2. Essentials of the full version of FAGE

2.1 Database

The elaborated theory of constructing the database for MONASH-type models is documented by Dixon & Rimmer (2002). The core database of the model is based on the 2002 Input-Output (I-O) table of China and was prepared by Horridge and Wittwer (2008).[12] In this section we will discuss the relevant database components of the financial extension to the model.

In Section 1, we discussed the balance sheets of agents, the I-O table, and SAM for the stylized model. Table 1 contains only 6 agents and a small number of securities, e.g. CUR, EQT, and BLN and so forth. For the full FAGE model, we have 38 industries and 5 other agents, i.e. 43 agents in total. If we still adopt double-entry balance sheets, in which everything appears twice, this would overcomplicate matters. In this section, we introduce the Financial Accounting Matrix (FAM) to record the financial stocks we need to model in FAGE.

2.1.1 Some characteristics of FAM

The Financial Accounting Matrix (FAM) is inspired by the Social Accounting Matrix (SAM). In the same way that a SAM is a systematic listing of monetary flows in an economy, the FAM is a systematic listing of the financial stocks in a square table. For instance, if a model has 43 agents, FAM would have 43×43 possible entries (Holder/Issuer combinations). A flow of money always goes from an agent to another agent; a financial instrument is a liability to one agent and an asset to another. In principle, any non-diagonal cell of the FAM could be occupied (diagonal cell's values are irrelevant since they represent "agents' liabilities towards themselves").

In the FAM, assets of agents are arranged in rows, and liabilities in columns. For example, the intersection of a row titled "Households" and a column titled "Commercial Banks" would be deposits held by households (asset of households) at the commercial banks (liability of commercial banks).

While the SAM and the FAM thus share some common features, there are also important differences between them. Possibly the most important one is that the FAM need not fulfil any "balancing" requirement. In fact, the difference between the total assets (row sum) and total liabilities (column sum) of an agent equals the agent's financial net worth, and can be positive or negative.

In addition, FAM accounts only for financial assets/liabilities and excludes physical capital. For example, the physical capital term *qPK* listed in industries' balance sheet in Table 1 is not included in the FAM. As a result, we would expect that the financial net worth of industries, i.e. its row sum minus column sum, is negative.

[12] The initial MONASH-type model was adapted for China by Dr Glyn Wittwer.

2.1.2 *The FAM in FAGE model*

Using TABLO notation, a three dimensional coefficient called *ASST0(h, i, s)*[13] plays a central role in the financial extension. Agent *h* at the beginning of the year holds asset *s* issued by agent *i*:

$$ASST0(h,i,s), h \in AGENT, i \in AGENT, s \in SECURITY$$

Set *AGENT* contains 43 agents (*IND1 to IND38*, *HH*, *GOV*, *CMB*, *CTB* and *ROW*) and set *SECURITY* contains 3 securities (currency, bond and equity). We have a 43× 43 square table for each type of security. Table 7 shows the database for currency, bond and equity. The three tables contain the information of balance sheets of each agent, which readers could compare the corresponding entries appearing in Table 1 for the stylized model in Section 1. We define the row names, e.g. the first dimension of *ASST0*, as the Holders of the assets, and the column names, e.g. the second dimension of *ASST0*, denote the Issuers. For example, the hypothetical figure contained in the shadowed cell in the right-hand corner of the "***Currency***" table, cell (39,AP), means that households hold currency issued by the central bank, which corresponds to "*CUR*" in the agent's balance sheet in Table 1. Similarly, cell (41,AP) is equivalent to "*RR*" in Table 1.

Table 7 FAM in the FAGE model

ASST0(Holder, Issuer, ***Currency****)*

Currency			Issuer							
			IND1	…	IND38	HH	GOV	CMB	CTB	ROW
			A	…	AL	AM	AN	AO	AP	AQ
Holder	IND1	1								
	…	…								
	IND38	38								
	HH	39							CUR	
	GOV	40								
	CMB	41							RR	
	CTB	42								
	ROW	43								

ASST0(Holder, Issuer, ***Bond****)*

Bond			Issuer							
			IND1	…	IND38	HH	GOV	CMB	CTB	ROW
			A	…	AL	AM	AN	AO	AP	AQ
Holder	IND1	1								
	…	…								
	IND38	38								
	HH	39						DEP		
	GOV	40								
	CMB	41	BLN	…	BLN		GLN		XR	
	CTB	42					GTLN	CLM		FXR
	ROW	43	FLN	…	FLN					

[13] The "0" in ASST0 stands for the start-of-year. We use "1" for end-of-year- value. For example, ASST1 is the assets at the end of any year.

*ASST0(Holder, Issuer, **Equity**)*

Equity		Issuer							
		IND1	…	IND38	HH	GOV	CMB	CTB	ROW
		A	…	AL	AM	AN	AO	AP	AQ
Holder: IND1	1								
…	…								
IND38	38								
HH	39	EQT	…	EQT					
GOV	40								
CMB	41								
CTB	42								
ROW	43								

In the second table for "***Bond***", cells (41,A) to (41,AN) denotes the commercial banks' loans to industries, households and government. On the other hand, cells from (1,AO) to (40,AO) represent the deposits of industries, households and government that they hold in the commercial banks. The shadowed cells in Tables 7 contain a non-zero figure in FAGE. Most of them are endogenised through the optimising behaviour of agents as discussed in the stylized model. Other non-zero figures are either exogenous or linked to the GDP growth rate (e.g. we assume the deposits of the industries in the commercial banks behave as working capital and grow with GDP).

In the table for "***Equity***", cells (39, A) to (39, AL) show the industries' equities held by households. Additionally, shadowed cells (43, A) to (43, AL) implies that ROW also holds some domestic industries' equities.

After creating a database for the coefficient matrix, *ASST0*, then we can simply give values to the coefficient for the liabilities, *LIAB0*, by

$$(all, h, AGENT)(all, i, AGENT)(all, s, SCRT)LIAB0(h,i,s) = ASST0(i,h,s)$$

Data source for the FAM used in FAGE model include the Chinese Statistic Bureau and the People's Bank of China.

2.2 Adjusted MONASH-type investment theory in FAGE

The determination of investment of each industry in MONASH-type models depends on expected rates of return ($EROR_t$). Under the assumption of static expectations, expected rates of return can be expressed as following (Dixon & Rimmer, 2002, pp. 193-195). The derivation of Equation (37) is in Appendix.

$$EROR_t = -1 + \frac{\frac{P_t^K}{P_t^I} * (1 - T_t) + \frac{P_t^I}{P_t^I} * (1 - D) + RALPH * T_t * \frac{P_t^I}{P_t^I} * D}{\frac{1 + INT_t * (1 - T_t)}{1 + INF_t}} \qquad (37)$$

where

P_t^I is the cost of buying a unit of capital in year *t*;

D is the rate of depreciation;

P_t^K is the rental rate on capital in year *t*, i.e. the user cost of a unit of capital;

RALPH is a coefficient showing the proportion of depreciation that is tax deductible;

T_t is the tax rate applying to capital income in year *t*;

INT_t is the nominal rate of interest in year *t*;

INF_t is the inflation rate in year t.

In MONASH, the nominal interest rate and the inflation rate in Equation (37) are exogenous, because there is no market for credit and currency. After we introduce the credit market and currency market in a MONASH-type model, the interest rates and price level become endogenous. So some adjustments for the investment theory are needed in FAGE.

In FAGE, industries have at least two external financing sources: one is through the equity market (with rate r_t^{eqt}), the other through bond market (with rate r_t^{bnd}). FAGE follows the standard financial textbooks to define the net present value of investment, which use the weighted average cost of capital as discounted factor rather than post-tax interest rate, given as

$$NPV_t = -P_t^I + \frac{P_{t+1}^K(1-T_{t+1}) + P_{t+1}^I(1-D) + RALPH * T_{t+1} * P_{t+1}^I * D}{1+WACC_t} \tag{38}$$

where $WACC_t$ is the weighted average cost of capital (i.e. $WACC_t = S^{eqt} r_t^{bnd} + S^{eqt} r_t^{eqt}$).

Diving both side of Equation (38) by P_t^I, and assuming the next year rental rates and asset prices rise with the rate of inflation, it yields the following adjusted expected rate of return.

$$EROR_t = -1 + \frac{P_t^K / P_t^I * (1-T_t) + (1-D) + RALPH * T_t * D}{\frac{1+WACC_t}{1+INF_t}} \tag{39}$$

Furthermore, we substitute the inflation term (INF_t) in Equation (39) by the expected inflation (INF_t^e). We now have the final formula for expected rate of return in FAGE model: [14]

$$EROR_t = -1 + \frac{P_t^K / P_t^I * (1-T_t) + (1-D) + RALPH * T_t * P_t^I / P_t^I * D}{\frac{1+WACC_t}{1+INF_t^e}} \tag{40}$$

[14] It is worth noting that we need to recalibrate a parameter – normal rate of return, *RORN* – in Equation (A.2) in Appendix accordingly after we changed the definition of EROR from Equation (37) to (40).

There are two reasons for this substitution. The first is to create a more realistic assumption. Some studies, for example, Cagan (1956), Goldfeld (1973) and Robert (1997), show that people are adjusting their inflation expectations gradually and slowly, i.e. adaptive expectations instead of rational expectations. The second reason we make the adjustment is to avoid the model breaking down because of its high sensitivity to inflation. In MONASH, the consumer price index (CPI) is exogenous, and, in turn, inflation does not change. However, after we endogenised the CPI, the model became too sensitive to the inflation rate. The results of the simulations suggest that we need the adaptive expectation mechanism to smooth the inflation fluctuations.

3. Conclusions

This paper covers the theoretical framework of the FAGE model used in Xiao (2009). We introduced a stylized model with only six agents and a small number of assets and liabilities. For the financial extension, a discussion of the balance sheets of each agent was included. Balance sheets can be considered a snap shot of stock variables, i.e. the assets and liabilities that each agent holds at a particular point in time, whilst a SAM shows the accounting that records the flow of funds for each agent. The combination of start-of-year balance sheets and a SAM would then yield end-of-year balance sheets for each agent.

In the second part of this paper, we focussed on the essential issues of the full financial extension. Firstly, we explained how the data concerning assets and liabilities for all 43 agents in the FAGE model were recorded. We then discussed what adjustments to the capital-supply function were needed from the standard MONASH-type model.

One of the main contributions of the FAGE model is to allow us to investigate the economic impacts, on both a macro- and industry-level, of monetary and exchange rate policies. For example, the FAGE model can be used to evaluate the combined effect of fiscal and monetary stimulus packages during a financial crisis similar to that of 2008-09. Xiao (2009) also gives some examples of the application of the FAGE model regarding issues such as the Chinese external imbalance and an undervalued currency.

4. References

Arrow, K.J., H.B. Chenery, B.S. Minhas and R.M. Solow (1961) Capital-Labor Substitution and Economic Efficiency. *The Review of Economics and Statistics*, 43: 225-250.

Baumol, W.J. (1952) The Transactions Demand for Cash: An Inventory Theoretic Approach. *Quarterly Journal of Economics,* 66: 545-556.

Cagan, P. (1956) The Monetary Dynamics of Hyperinflation, in M. Friedman (ed.), *Studies in the Quantity Theory of Money*, Chicago: University of Chicago Press

Dixon, P.B., B.R. Parmenter, J. Sutton and D.P. Vincent (1982) *ORANI: A Multisectoral Model of the Australian Economy*, Amsterdam: North-Holland Publishing Company.

Dixon, P.B. and M.T. Rimmer (2002) *Dynamic General Equilibrium Modelling for Forecasting and Policy: A Practical Guide and Documentation of MONASH*, Amsterdam: North-Holland Publishing Company.

Goldfeld, S.M. (1973) The Demand for Money Revisited. *Brookings Papers on Economic Activity*, 3: 577-638

Horridge, M. (2000) ORANI-G: A General Equilibrium Model of the Australian Economy. *CoPS/IMPACT Working Paper OP-93*, Centre of Policy Studies, Monash University.

Horridge, M. and G. Wittwer (2008) SinoTERM, a Multi-Regional CGE Model of China. *China Economic Review*, 19:628-634.

McCallum, B.T. (1989) *Monetary Economics Theory and Policy*. New York: Macmillan Publishing Company.

Powell, A.A. and F.H.G. Gruen (1968) The Constant Elasticity of Transformation Production Frontier and Linear Supply System. *International Economic Review*, 9: 315-328.

Robert, J.M. (1997) Is Inflation Sticky? *Journal of Monetary Economics,* 39:173-196.

Rosensweig, J.A. and L. Taylor (1990) Devaluation, Capital Flows and Crowding-Out: A CGE model with Portfolio Choice for Thailand, in L. Taylor (ed.), *Socially Relevant Policy Analysis: Structuralist Computable General Equilibrium Models for the Developing World*, pp. 302-332. Cambridge: MIT Press.

Xiao, J. (2009) *A Dynamic Financial Applied General Equilibrium Model of the Chinese economy*, Unpublished Ph.D. dissertation, Monash University.

Tobin, J. (1956) The Interest-Elasticity of Transactions Demand for Cash. *Review of Economics and Statistics*, 38: 241-247.

Appendix

A.1 The investment theory in MONASH

Capital-supply function

In MONASH, the capital-supply function describes the relationship between expected rate of return (*EROR*) and proportionate growth in capital stock between the beginning and the end of the year (Dixon & Rimmer, 2002).

Growth in capital stock is defined as:

$$K\_GR = \frac{K^1}{K^0} - 1 \tag{A.1}$$

where K^1 is the end-of-year capital stock, K^0 the beginning-of-year capital stock.

The capital-supply function defines the *EROR* as an inverse logistic function of the proportionate growth in capital stock:

$$\begin{aligned} EROR = RORN + (1/C)*[&\ln(K\_GR - K\_GR\_MIN) - \ln(K\_GR\_MAX \\ &- K\_GR) - \ln(TREND\_K - K\_GR\_MIN) + \ln(K\_GR\_MAX \\ &- TREND\_K)] \end{aligned} \tag{A.2}$$

where

$RORN$ is the historically normal rate of return;

K_GR_MIN is the minimum possible rate of growth of capital equal to the negative of the depreciation rate in each industry;

K_GR_MAX is the maximum feasible rate of capital growth;

$TREND\_K$ is the industry's historically normal capital growth rate;

C is a positive parameter.

The definition of the end-of-year capital stock can be written as

$$K^1 = K^0(1 - D) + I \tag{A.3}$$

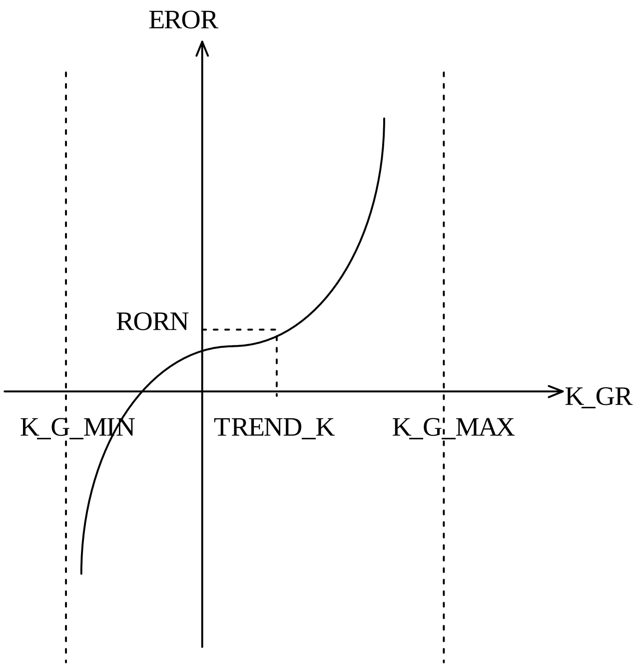
where D is the depreciation rate and I the investment. From the three Equations (A.1) - (A.3), we could express investment as a function of the expected rate of return according to:

$$I = \Psi[\overset{+}{EROR}] \tag{A.4}$$

where Ψ has a positive first derivative.

Figure A.1 shows the diagrammatical relationship between the growth in capital stock (*K_GR*) and the expected rate of return (*EROR*).

Figure A.1 Capital-supply function in MONASH



Actual and expected rates of return

The MONASH definition of actual rates of return starts with the calculation of the net present value (*NPV*) in year *t* of purchasing a unit of physical capital for use.

$$NPV_t = -P_t^I + \frac{P_{t+1}^K(1-T_{t+1}) + P_{t+1}^I(1-D) + RALPH * T_{t+1} * P_{t+1}^I * D}{1 + INT_t * (1-T_{t+1})} \qquad \text{(A.5)}$$

where

P_t^I and P_{t+1}^I are the cost of buying a unit of capital in year *t* and *t+1*;
D is the rate of depreciation;
P_{t+1}^K is the rental rate on capital in year *t+1*, i.e. the user cost of a unit of capital;
RALPH is a coefficient showing the proportion of depreciation that is tax deductible;
T_{t+1} is the tax rate applying to capital income in year *t+1*;
INT_t is the nominal rate of interest in year *t*.

To derive a rate of return we divide both sides of the above Equation (A.5) by P_t^I, i.e., we define the actual rate of return, ROR_ACT_t, in year t on physical capital as the net present value of an investment of one dollar. Then it gives us:

$$ROR\_ACT_t = -1 + \frac{P_{t+1}^K / P_t^I * (1 - T_{t+1}) + P_{t+1}^I / P_t^I * (1 - D) + RALPH * T_{t+1} * P_{t+1}^I / P_t^I * D}{1 + INT_t * (1 - T_{t+1})} \qquad \text{(A.6)}$$

The determination of investment in MONASH depends on expected (rather than actual) rates of return ($EROR_t$) as described by Equation (A.4). To estimate the expected rates of return, we adopt static expectations to handle the value in year *t+1* of the variables with *t+1* subscript in Equation (A.6). We assume that investors expect no change in the tax rate, and that rental rates and asset prices will increase by the current inflation rate (INF_t), i.e. $T_{t+1} = T_t$, $P_{t+1}^K / P_t^I = P_{t+1}^I / P_t^I = 1 + INF_t$. The formula for the expected rate of return is shown as:

$$EROR_t = -1 + \frac{P_t^K / P_t^I * (1 - T_t) + P_t^I / P_t^I * (1 - D) + RALPH * T_t * P_t^I / P_t^I * D}{\dfrac{1 + INT_t * (1 - T_t)}{1 + INF_t}}$$