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THE MEASUREMENT OF EFFICIENCY WHERE THERE ARE MULTIPLE OUTPUTS

by

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The Measurement of Efficiency where there are Multiple Outputs

Abstract

This paper is motivated by the empirical observation that in many studies the elasticity of output with respect to labour is often negative and/or insignificant. The present study applies multiple output models to estimate the technical efficiency of enterprises in the international electricity, gas and telecommunications industries. The results support the contention that single output production models may yield misleading results in respect of the elasticities of inputs such as labour. The results also suggest that relatively simple DEA and ordinary least squares models may be preferred to more complex stochastic frontier models in estimating the technical efficiency of enterprises.

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Keywords: multiple output, data envelopment analysis, stochastic production frontier, distance function, ray frontier, technical efficiency.

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1. Introduction

Single output data envelopment analysis (DEA) and stochastic production frontier (SPF) models are now being used more and more by academic and applied economists, operations researchers and management science practitioners to measure the technical efficiency of enterprises.[1] A major problem encountered, particularly with the SPF models, is the insignificance or negativity of the coefficient of labour inputs in the production process. This outcome suggests that production is inelastic with respect to labour inputs or worse that labour has a negative elasticity with respect to production.[2] A typical explanation for this phenomenon is that labour is a congested input particularly in the Australian electricity industry where labour unions are powerful and enterprises are often government owned. Coelli (1998, p12) suggested that a lot of this surplus labour was shed in the reforms of the 1990s (approximately 40%) without any reduction in output. However Whiteman (1999, p.20) notes that this problem has continued to persist. Quiggin (1997, p.264) suggests that measuring output simply as electricity generated fails to account for differences in the numbers of customers served. In other words the labour input is more closely associated with the distribution of electricity than with the generation of electricity while capital which is traditionally measured as generating capacity is almost exclusively associated with electricity generation. Accordingly it is not surprising that the elasticity of electricity generated with respect to the labour input is statistically insignificant and/or negative.

Many industries like the electricity industry are characterised by multiple outputs. As a result the measurement of technical efficiency with respect to a single output may yield misleading results. While non-parametric techniques such as data envelopment analysis are able to quite easily cope with multiple outputs, parametric efficiency measurement is only just beginning to grapple with this problem. In this paper the multiple output efficiencies of enterprises in the electricity, gas and telecommunications industries are examined utilising a number of parametric and non-parametric models. The various methodologies are outlined in sections II to IV. The data and sources are discussed in section V. The results are outlined in section VI. Section VII provides a summary with concluding remarks.

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[1] Coelli (1998), Electricity Supply Association of Australia Limited (1994) and Whiteman (1999).

[2] Cowing and Smith (1980)

2. Data Envelopment Analysis (DEA)

DEA is usually attributed to Charnes, Cooper and Rhodes (1978). More recently Fare, Grosskopf and Lovell (1985) have extended DEA to measure the impact of scale on technical efficiency. Following Fare, Grosskopf and Lovell (1985), the input oriented measure of technical efficiency of a supplier k (k=1,…,N) is calculated as the solution (TE_k) to the following mathematical programming problem:

$$\text{Choose } z \text{ to minimise } \lambda \qquad (1)$$

$$\text{s.t.} \quad y^{k} \leq \mathbf{Y}z$$

$$\mathbf{X}z \leq \lambda x^{k}$$

$$z \in R_{+} \quad \text{and}$$

$$TE_k = \text{minimum value of } \lambda.$$

y^{k} represents a (Px1) vector of the outputs of supplier k with elements y_p^k (p=1,…,P). x^k is a (Mx1) vector of supplier k's inputs with elements x_j^k (j=1,…,M). **Y** is a (PxN) matrix of the outputs of all suppliers with elements y^i (i=1,N). **X** is a (MxN) matrix of the inputs of all suppliers with element x_j^i. z is a (Nx1) vector of weights z_i to be determined. λ is a scalar value denoting the proportional reduction in all inputs, holding the relative factor proportions and output constant.

The minimum value of λ that satisfies the mathematical programming problem (i.e. TE_k) is called the Farrell radial measure of technical efficiency.[3] This represents the proportional reduction in inputs that can be achieved through the adoption of the best practices of the suppliers in the sample. It can be shown that assuming variable returns to scale requires the sum of the weights (z_i) to equal unity.[4] Accordingly, in order to estimate technical efficiency (i.e. TE_k) exclusive of any inefficiency due to scale, the following constraint is added to the mathematical programming problem (1) above: $l'z = 1$. The DEA estimate of technical efficiency from the model (1) above is called the constant returns to scale (CRS) measure of technical efficiency. The DEA measure of technical efficiency from the model (1) above plus the latter constraint is called the variable returns to scale (VRS) measure of technical efficiency.

3. Multi-output distance functions

Following Shephard (1970), the output distance function is defined on the output set P(x) as follows:

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[3] Farrell (1957).

[4] Fare, Grosskopf and Lovell (1985).

$$D_0(x,y) = \min\{\theta : (y/\theta) \in P(x)\} \qquad (2)$$

where $D_0(x,y) \leq 1$ if $y \in P(x)$ and $D_0(x,y) = 1$ if y is on the production frontier.[5]

The Cobb-Douglas output oriented distance function for the case of P outputs and M inputs is specified as follows:

$$\ln D_0(x^i,y^i) = \alpha_0 + \sum_{p=1}^{P} \alpha_p \ln y^i_p + \sum_{j=1}^{M} \beta_j \ln x^i_j\,, \qquad i = 1,\ldots,N \qquad (3)$$

Following Lovell, Richardson, Travers and Wood (1994) homogeneity implies that:

$$D_0(x^i, y^i/y^i_k) = D_0(x^i,y^i)/y^i_k \qquad (4)$$

so that

$$-\ln(y^i_k) = \alpha_0 + \sum_{p=1}^{P-1} \alpha_p \ln y^{*i}_p + \sum_{j=1}^{M} \beta_j \ln x^i_j - \ln D_0(x^i,y^i)\,, \qquad i = 1,\ldots,N \qquad (5)$$

where $y^{*i}_p = y^i_p / y^i_k$

The corrected ordinary least squares (COLS) method[6] is used to estimate the parametric deterministic output frontier (5). The distance measure for the ith firm is calculated as the exponent of the COLS residual.[7]

Following Coelli and Perelman (1996), the parametric stochastic output frontier is obtained by adding a symmetric error term v_i to equation (5) to account for the stochastic error , so that

$$-\ln(y^i_k) = \alpha_0 + \sum_{p=1}^{P-1} \alpha_p \ln y^{*i}_p + \sum_{j=1}^{M} \beta_j \ln x^i_j + v_i + u_i\,, \qquad i = 1,\ldots,N \qquad (6)$$

where $u_i = -\ln D_0(x^i,y^i)$

Assuming that v_i are iid $N(0,\sigma_v^2)$ and u_i are iid $|N(0,\sigma_u^2)|$, the parameters of (6) are estimated using maximum likelihood. Following Battese and Coelli (1988), the output distance function value for observation i is obtained from the conditional expectation

$$D_{oi} = E[\exp(-u_i)/e_i]$$

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[5] Lovell, Richardson, Travers and Wood (1994).

[6] Following Green (1980), OLS is used to estimate the parameters of the distance function (5) with the final term - $D_0(x^i,y^i)$ interpreted as the error term.

[7] The largest negative OLS residual is added to the OLS estimate of the intercept term α_0 to obtain the COLS residual.

$$= \{[1-\Phi(\sigma_A-\gamma e_i/\sigma_A)]/[\ 1-\Phi(\gamma e_i/\sigma_A)]\}\exp(\gamma e_i+\sigma^2_A/2) \qquad (7)$$

where $\Phi(.)$ is the distribution function of a standard normal random variable,

and $\gamma = \sigma_u^2 / \sigma^2$

$\sigma^2 = \sigma_u^2 + \sigma_v^2$

$\sigma_A = \{\gamma(1-\gamma)\sigma^2\}^{1/2}$

The maximum likelihood estimates of the parameters of (6) and the distance function estimates (7) are obtained using the computer program FRONTIER, Version 4.1.[8]

The COLS method is also used to estimate the parametric deterministic input frontier[9]

$$-\ln(x^i_k) = \alpha_0 + \sum_{p=1}^{P} \alpha_p \ln y^i_p + \sum_{j=1}^{M-1} \beta_j \ln x^{*i}_j - \ln D_0(x^i,y^i), \qquad i = 1,\ldots,N \qquad (8)$$

where $x^{*i}_j = x^i_j / x^i_k$.

Likewise the parametric stochastic input frontier is estimated in a similar way to (6) with the non-positive error term subtracted from the equation.

$$-\ln(x^i_k) = \alpha_0 + \sum_{p=1}^{P} \alpha_p \ln y^i_p + \sum_{j=1}^{M-1} \beta_j \ln x^{*i}_j + v_i - u_i, \qquad i = 1,\ldots,N \qquad (8)$$

where $D_{Ii} = E[\exp(u_i)/e_i]$

and $e_i = v_i - u_i$.

4. Multiple output ray frontier production model

Löthgren (1997) has generalised the single-output stochastic frontier model of Aigner, Lovell and Schmidt (1977) to a multiple output stochastic ray frontier production model based on polar coordinates. This generalised model enables simultaneous identification, estimation and testing of

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[8] See Coelli (1996).

[9] In the case of the COLS estimate of the input distance function, the OLS estimate of the intercept term is adjusted by adding the largest positive residual.

production frontiers and firm specific technical efficiency for multiple-input, multiple-output technologies.

Following Löthgren the multiple-output vector in polar-coordinate form is represented as

$$y = \iota \bullet m(\theta) \tag{9}$$

where $\iota = ||y|| = (\sum_{p=1}^{P} (y_p^2))^{1/2}$ is the Euclidean norm of the output vector y.

The function m : $[0, \pi/2]^{P-1} \rightarrow [0,1]^P$, defined by

$m_i(\theta) = \cos\theta_i \prod_{j=0}^{i-1} \sin\theta_j$, $i = 1,\ldots,P$, $\theta \in [0, \pi/2]^{P-1}$, $\sin\theta_0 = \cos\theta_P = 1$ represents a transformation of the polar coordinate angle vector $\theta \in [0, \pi/2]^{P-1}$ to the output mix vector $m(\theta) = y/\iota$.[10]

The polar coordinate angles θ are obtained as the following solution

$$\theta_I = \cos^{-1}(y/\iota \prod_{j=0}^{i-1} \sin\theta_j) \quad , i = 1,\ldots,P \text{ where } \sin\theta_0 = \cos\theta_P = 1. \tag{10}$$

In polar coordinate representation the ray production function is defined

$$\mathbf{f}(x, \theta) = \max\{\iota \in R+ : \iota \bullet m(\theta) \in P(x)\} \tag{11}$$

where $P(x) = \{y \in R+ : x \text{ can produce } y\}$is the output set.

Given the output mix represented by the polar coordinate angles θ and inputs x, this function gives the maximum norm of attainable outputs.[11]

Introducing the composed error term as in (6) above, the stochastic ray frontier model is specified as

$$\iota = \mathbf{f}(x, \theta)\exp(v-u) \tag{12}$$

The distance function is given by the ratio of the frontier norm to the observed norm,

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[10] Mardia, Kent and Bibby (1979).

[11] Note that for a technology with three outputs, the first angle θ_1 represents the angle from the y_1 axis towards the plane spanned by the y_2 and y_3 axis. The angle θ_2 represents the angle between y_2 and y_3 in the y_2-y_3 plane.

i.e. $$D_0(x, y) = \iota^f/\iota = \exp(-u) \qquad (13)$$

This measure represents the radial distance from the output vector to the frontier of the output set and therefore corresponds to the technical efficiency measure of Farrell (1957) and the output distance function defined in Shephard (1970).

Continuing to follow Löthgren (1997), a log linear functional form is imposed on the ray function (12).

$$\ln \iota_i = \alpha_o + \sum_{j=1}^{M} \beta j \ln x^{i} j + \sum_{j=M+1}^{M+P-1} \beta_j \ln \theta^{i}_{j-P} + v_i - u_i \ , \quad i = 1,\ldots,N \qquad (14)$$

The ray function (14) and firm specific technical efficiencies are estimated using the program FRONTIER 4.1 referred to above.

5. Data

The data covers three industries: electricity, gas and telecommunications. The electricity industry data comprises 41 suppliers, each characterised by two outputs - electricity generated and customers served – and three inputs – hydro capacity, other capacity and full time employee equivalents. The gas industry data comprises 51 suppliers, each characterised by two outputs – gas sales and customers served – and three inputs - distribution mains, transmission pipeline and employees. The telecommunications industry comprises 31 suppliers, each characterised by five outputs – public pay phones, residential main lines, other main lines, international traffic, and cellular mobile subscribers – and two inputs – digital lines and employees. The data for each industry is summarised in tables 1 to 3.

The international electricity data was obtained mainly from the Electricity Association (1996) and the annual report of China Light & Power Limited (1997). The Australian electricity supplier data was obtained from the Electricity Supply Association of Australia (1998). Most of the international data is centred around the year ending 1996. The Australian data is for the financial year ended 30 June 1997.

The international data on gas suppliers was collected from a variety of sources including ANZ McCaughan (1992), American Gas Association (1993), Canadian Gas Association (1992), the Monopolies and Mergers Commission (1993) and the Japan Gas Association (1992). The Australian data was obtained from the Australian Gas Association (1994).

The data on telecommunications was obtained in electronic form from the International Bank for Reconstruction and Development/World Bank (1992).

6. Results

Three estimation techniques have been used in this study. These are non-parametric mathematical programming, parametric deterministic corrected ordinary least squares, and parametric stochastic maximum likelihood methods. Accordingly, eight different multiple-output technical efficiency models have been estimated for each of the electricity, gas and telecommunications industries. These models are:

1. Constant returns to scale (CRS) DEA,
2. Variable returns to scale (VRS) DEA,
3. Deterministic input distance function,
4. Deterministic output distance function,
5. Deterministic ray frontier function,
6. Stochastic input distance function,
7. Stochastic output distance function,
8. Stochastic ray frontier function.

DEA results

The DEA results for the CRS and VRS models are provided in tables 7 to 9. On average the DEA estimates of technical efficiency are considerably higher than the estimates yielded by the stochastic frontier methodologies for all three industries. This runs counter to intuition which would suggest that the DEA and COLS estimates of technical efficiency fail to exclude the impact of stochastic factors and consequently should be somewhat lower, on average, than the corresponding estimates yielded by the stochastic distance and ray frontier models. Also, scale appears to play a bigger role in the gas industry than in the electricity and telecommunications industries according to the DEA results.

COLS results

The COLS estimates of the parameters of the deterministic Cobb-Douglas production frontier for the three industries are given in tables 4 to 6. The individual results are given for the input distance function, output distance function and the ray frontier function. The R^2 and many of the t-statistics are relatively high suggesting a reasonable fit of the data.

The coefficient of employment (β_3) in the COLS output distance function for electricity is both significant and negative. The dependent variable in the equation is the negative of the natural logarithm of customer numbers. This implies that the elasticity of customer numbers with respect to employment is significant and positive. Likewise the coefficient of labour (β_1) in the COLS output distance function for gas is negative and statistically significant. The dependent variable in this case is the negative of the natural logarithm of gas sales. This therefore implies that the elasticity of gas sales with respect to employment is significant and positive. The coefficient of employment (β_1) in the COLS output distance function for telecommunications is negative and significant implying a positive elasticity of residential lines with respect to labour.

The coefficient of employment (β_3) in the COLS ray frontier function for electricity is significant and negative. In this case the dependent variable in the ray frontier model is the natural logarithm of the norm defined over all outputs. This implies that the elasticity of the multiple output with respect to labour is negative. In the case of gas, the coefficient of labour (β_3) is significant and positive implying the expected positive elasticity of output with respect to labour. Likewise the coefficient of labour (β_1) in the COLS ray function for telecommunications is significant and positive.

In the COLS distance equations for the gas industry, the coefficients of the transmission mains (β_3) are statistically insignificant as is the coefficient of distribution mains (β_2) in the input distance equation. Likewise the coefficient of transmission mains (β_2) in the COLS ray frontier equation for the gas industry is also statistically insignificant. Otherwise the input coefficients in the COLS equations are of expected sign and are statistically significant.

The coefficients of pay phones (α_1), international minutes (α_4) and cellular mobile phones (α_5) in the COLS distance equations for telecommunications are statistically insignificant. This suggests that the significant outputs for the telecommunications industry are residential mainlines (α_2) and other main lines (α_3).

The average technical efficiency measured by the COLS output distance equation exceeds the technical efficiency measured by the COLS input distance equation for the electricity and gas industries. In theory output or input orientation should have no effect on resulting estimates of technical efficiency. In the case of the telecommunications industries the average technical efficiencies of the output and input distance functions are equal. The distance function estimates of technical efficiency on average exceed the ray frontier estimates. Again in theory estimates of the ray frontier and distance function models should be similar. However in the present case the ray frontier model has a single aggregated output whereas the distance function and the DEA estimates are based specifically on multiple outputs.

ML results

Stochastic input and output distance functions and stochastic ray frontier functions were estimated for each industry. However the input and output distance equations for the electricity industry, the output distance equation and the ray frontier equation for the gas industry, and the output distance equation for the telecommunications industry were found to have LLF values that were no different from the values obtained for the COLS equations. These results suggest that decomposition of the error into a systematic stochastic component (v_i) and a one-sided technical efficiency component (u_i) did not improve the results over those given by the deterministic COLS models. The corresponding estimates of the proportion of the error variance due to technical inefficiency (γ) were not statistically different from zero in the estimated equations. Accordingly the technical efficiency estimates yielded by these models have been ignored.

The estimate of the proportion of the variance due to technical inefficiency (γ) for the stochastic ray frontier model of the electricity industry was estimated at 0.927. The corresponding estimate for the gas industry stochastic output distance function was 0.710. These results imply that while technical inefficiency accounts for most of the error, some stochastic error is present in the estimated models. The estimates of the proportion of the variance due to technical inefficiency (γ) for the stochastic input distance and ray frontier models of the telecommunications industry were estimated at unity implying that there was no stochastic error associated with the estimated equations. This is a rather extreme result.

Coelli and Perelman (1996) also noted the unusual behavior of the ML estimation in yielding values of the parameter γ that were either zero or unity. They suggested that one possible cause for the ML estimator, selecting extreme values of γ, was the wide range in scales of operation of the railroad companies along with the second-order flexibility of the translog functional form. They contended that these factors could have resulted in the ML method adjusting the second order coefficients so that the translog function bent at extremities of the data range. As a result extreme observations would be quite close to the frontier thereby resulting in a distribution of residuals closely resembling a half-normal distribution. They supported this explanation with the observation that the same problems did not occur when the analysis was repeated with the simpler Cobb-Douglas form. However in the present study the Cobb-Douglas form has been used and we experience similar problems. Also, as outlined above, less extreme values of γ have been obtained for the stochastic ray frontier model of the electricity industry and the stochastic output distance function of the gas industry.

Comparison of technical efficiency results

Correlation matrices of the technical efficiency results for each industry are provided in tables 10–12. These matrices suggest that the technical efficiency estimates of the COLS input and output distance functions are highly correlated. Also there appears to be strong positive correlation between the technical efficiency estimates of the ML stochastic distance equations and the estimates yielded by the COLS distance equations. For instance, the COLS output distance technical efficiency estimates are perfectly correlated with the ML output distance technical efficiency estimates in the gas industry table 11. This suggests that given the problems with the ML estimator, either of the COLS distance functions could be preferred over the stochastic distance functions for estimating the technical efficiency of enterprises.

The technical efficiency estimates yielded by the ray frontier models for the electricity industry are relatively uncorrelated with estimates yielded by DEA and the distance functions for the electricity industry in table 10. However the technical efficiency estimates yielded by the ray frontier models for gas and telecommunications are correlated with counterpart estimates yielded by the DEA and distance models in tables 11 and 12. This seems to suggest that the persistent finding of a negative and/or insignificant elasticity of output with respect to labour may be more a characteristic result for the electricity supply industry than other industries. A more focussed examination of technical efficiency in the electricity industry may be required.

7. Summary and conclusions

This paper was motivated by the empirical observation that in many single output production function studies the elasticity of output with respect to labour was found to be insignificant and/or negative. It was suggested that many industries were characterised by multiple outputs and consequently that the application of single output models may yield misleading results particularly in relation to the sign and significance of the labour exponent of conventional production functions. Accordingly a number of multiple output efficiency measurement models have been applied to international data on the electricity, gas and telecommunications industries. The results suggest that the multiple output models are more likely to yield coefficients of labour with the expected positive sign. In an industry such as electricity, outputs include electricity generated as well as the number of customers served. Accordingly single output models focussing simply on electricity generated are likely to seriously underestimate the contribution of labour.

In examining the above question a number of multiple output models were used. These included DEA models, deterministic distance function models, stochastic distance function models and deterministic and stochastic ray frontier models. The results suggest that the technical efficiency estimates yielded by the deterministic and stochastic models are highly correlated. Therefore given problems involved in estimating the stochastic models, the deterministic or corrected ordinary least squares models for estimating efficiency are to be preferred.

Altogether the technical efficiency results, apart from those for the electricity industry, seem to be correlated. The DEA estimates of technical efficiency are, surprisingly, higher than comparable estimates yielded by the parametric methods. The main criticism of the DEA procedure has been that it does not exclude stochastic error and consequently that it would be likely to underestimate technical efficiency. Given the relatively high correlation between the technical efficiency estimates yielded by the deterministic and the frontier production models, the existence of stochastic error appears to have little impact on resulting estimates of technical efficiency. Also there appear to be major problems in estimating multiple output stochastic frontier models which make the parametric deterministic models or DEA preferred methodologies.

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Table 1: Summary of data on Electricity Suppliers

No.	*Supplier*	*Sales per customer (MWh)*	*Productivity (MWh)*	*Capacity factor (%)*	*Hydro capacity (%)*
1	Austria	10	1617	32	68
2	Denmark	11	3385	41	0
3	Finland	25	6009	48	18
4	France	13	3938	53	23
5	Germany	10	2606	51	8
6	Greece	6	1455	48	29
7	Ireland	11	2184	51	13
8	Italy	7	1844	37	30
9	Netherlands	11	2911	46	0
10	Oslo Energi (Norway)	25	6537	34	100
11	Poland	7	1004	46	7
12	Portugal	6	1757	38	49
13	UNESA (Spain)	7	3443	39	37
14	Vattenfall (Sweden)	107	12501	50	53
15	United Kingdom	11	3568	47	6
16	BC Hydro	36	9250	57	90
17	Hydro-Quebec	43	8218	51	93
18	Ontario Hydro	33	6201	53	23
19	TransAlta	64	15574	56	13
20	Carolina P&L	46	6698	56	2
21	Duke Power	42	5211	50	16
22	Los Angeles Dept. W&P	13	5720	36	26
23	Chubu Electric	12	5141	42	18
24	Kansai Electric	11	4860	39	20
25	Tokyo Electric	10	5484	50	14
26	NSW	20	4715	52	3
27	VIC	16	6487	58	6
28	QLD	19	4600	51	8
29	SA	13	2718	32	0
30	WA	15	3432	30	1
31	TAS	37	5708	43	89
32	NT	22	2726	28	0
33	SMHEA	1652667	11133	15	100
34	New Zealand	19	8096	51	67
35	Argentina	5	20303	40	48
36	Israel	16	4099	47	0
37	Singapore	22	8233	45	0
38	ESKOM (South Africa)	77	4786	58	5
39	South Korea	12	7404	60	6
40	Taiwan	12	6403	55	17
41	China Light & Power	13	4031	37	8
	Average	*40331*	*5658*	*45*	*27*

Table 2: Summary of data on Gas Suppliers

No.	*Supplier*	*Throughput per employee (TJ)*	*Customers per employee*	*Throughput per km of pipe (TJ)*	*Customers per km of pipe*	*Proportion of transmission mains (%)*
1	AGL	55	391	4.3	31	1.4
2	Allgas/GCQ	18	222	3.3	40	0.0
3	GFCV	68	450	8.0	53	8.9
4	SAGASCO	52	438	5.7	48	0.0
5	SECWA	230	497	12.9	28	16.8
6	NICOR (IL)	126	654	7.7	40	0.0
7	New Jersey Resources (NJ)	68	390	6.4	37	0.0
8	Southwest Gas (NV)	43	391	3.5	32	0.0
9	Atmos Energy (TX)	66	374	3.4	20	0.0
10	Indiana Energy (IN)	75	358	5.8	28	0.0
11	NUI (NJ)	70	328	8.9	42	0.0
12	Atlanta Gas Light (GA)	48	318	4.3	28	0.0
13	Oneok (OK)	80	303	4.9	18	44.6
14	Peoples Energy (IL)	55	281	29.4	149	0.0
15	Brooklyn Union Gas (NY)	34	301	20.0	175	0.0
16	Cascade Natural Gas (WA)	113	249	9.5	21	0.0
17	Washington Energy (WA)	70	282	7.3	30	0.0
18	Laclede Gas (MO)	44	280	8.2	52	0.0
19	Energen (AL)	32	273	3.7	31	0.0
20	Northwest Natural Gas (OR)	43	264	3.8	24	0.0
21	Bay State Gas (MA)	39	231	5.7	34	0.0
22	Providence Energy (RI)	39	226	7.7	45	0.0
23	South Jersey Industries (NJ)	39	230	5.1	30	0.0
24	WICOR (WI)	37	161	9.4	41	0.0
25	Washington Gas Light (DC)	45	227	9.6	48	6.5
26	Piedmont Natural Gas (NC)	62	226	7.1	26	0.0

Table 2 (continued)

No.	*Supplier*	*Throughput per employee (TJ)*	*Customers per employee*	*Throughput per km of pipe (TJ)*	*Customers per km of pipe*	*Proportion of transmission mains (%)*
27	Connecticut Energy (CT)	39	245	7.6	47	0.0
28	Questar (UT)	32	197	4.4	27	20.7
29	BC Gas	100	406	9.0	37	16.0
30	Centra Gas BC	71	262	2.8	10	10.1
31	Northland Utils BC	86	344	3.3	13	15.8
32	Pacific Northern Gas	225	164	13.7	10	61.7
33	Bonnyville Gas Co	118	380	1.2	4	8.0
34	Canadian Western Natural Gas Co	75	252	5.6	19	15.1
35	Centra Gas Alberta	74	295	0.7	3	7.2
36	City of Medicine Hat Gas Utility	617	275	29.6	13	66.4
37	Federation of Alberta Gas Coops	67	266	0.3	1	0.0
38	Northwestern Utils	88	275	6.2	19	23.3
39	SaskEnergy	119	547	1.1	5	0.0
40	Centra Gas Manitoba	91	353	11.6	45	19.9
41	Centra Gas Ontario	127	215	16.3	28	0.0
42	The Consumers Gas Co	97	305	15.4	49	0.0
43	Corporation of the City of Kitchener	196	742	12.0	46	1.9
44	Public Utils Comm (Kingston)	98	212	15.9	34	3.9
45	Union Gas	107	256	12.5	30	16.3
46	Gaz Metropolitain	131	103	28.5	22	9.8
47	Tokyo Gas Co	20	598	5.9	175	0.8
48	Osaka Gas Co	25	607	4.6	114	0.0
49	Toho Gas	16	385	3.5	86	0.0
50	Saibu Gas Co	23	481	5.2	107	0.0
51	British Gas	28	244	8.2	71	2.1
	Average	*85*	*329*	*8.4*	*42*	*7.4*

Table 3: Summary of data on Telecommunications suppliers

No.	*Country*	*Public Pay phones per Public pay phones per residential line (%)*	*Main lines per employee*	*Digital lines as a proportion of mainlines (%)*	*International traffic (hours) per employee*	*Cellular mobiles as proportion of mainlines (%)*
1	Argentina	1.3	101	28	58	1.3
2	Australia	0.6	117	26	148	6.0
3	Austria	1.1	190	27	650	5.0
4	Belgium	0.4	159	48	566	1.4
5	Brazil	3.4	111	22.2	29	0.6
6	Canada	1.5	176	80	130	6.3
7	Denmark	0.4	174	39.2	410	7.0
8	Finland	0.9	170	50.5	243	14.1
9	France	0.7	193	83.2	263	1.5
10	Germany	0.5	153	25	295	2.8
11	Hong Kong	0.3	177	98	1192	8.3
12	Ireland	0.8	85	68	379	4.0
13	Israel	1.3	174	61.2	247	2.0
14	Italy	2.1	262	48.4	271	3.3
15	Japan	2.1	228	60	85	3.0
16	Korea (Rep.)	2.1	249	93	82	1.7
17	Luxembourg	0.5	267	70	3912	0.6
18	Malta	0.8	79	100	171	2.3
19	Netherlands	0.2	229	83	585	2.2
20	New Zealand	0.4	124	95	230	6.5
21	Norway	0.8	153	50	393	12.3
22	Portugal	1.3	133	54	121	1.2
23	Singapore	3.7	158	77	745	10.3
24	South Africa	2.0	57	56	60	0.4
25	Spain	0.4	185	36.4	180	1.3
26	Sweden	0.7	174	53.9	339	11.1
27	Switzerland	1.8	196	42	1189	5.1
28	Taiwan, China	2.1	205	58	169	5.2
29	Thailand	2.6	72	75	88	13.9
30	United Kingdom	1.6	160	64	278	5.5
31	United States	1.5	208	59.7	243	7.7
	Average	1.3	165	59.1	444	5.0

Table 4: Estimated parameters for alternative models: Electricity*

	Corrected ordinary least squares (deterministic model)			*Maximum Likelihood (stochastic model)*		
	Input distance	*Output distance*	*Ray frontier*	*Input distance*	*Output distance*	*Ray frontier*
α_0	-2.936 (-2.7)	0.926 (0.9)	-7406.110 (-2.8)			-7405.297 (-14680.7)
α_1	-0.527 (-5.1)	0.596 (8.0)				
α_2	-0.487 (-6.7)					
β_1	0.017 (1.7)	-0.023 (-2.6)	0.835 (4.7)			0.812 (8.7)
β_2	0.218 (5.4)	-0.198 (-5.4)	0.031 (1.9)			0.034 (3.0)
β_3		-0.636 (-10.0)	-0.273 (-2.9)			-0.287 (-5.4)
β_4			16429.238 (2.8)			16429.386 (17190.1)
LLF(df)	-21.565 (36)	-18.119 (36)	-40.544 (36)	-21.565** (34)	-18.119** (34)	-38.023 (34)
R^2	0.914	0.979	0.720			
σ^2	0.191	0.161	0.482			1.020 (3.1)
γ				0.000 (0.0)	0.000 (0.0)	0.927 (10.7)

* t-statistics are presented in brackets.

**The likelihood value is less than that obtained from the OLS model.

*Table 5: Estimated parameters for alternative models: Gas**

	Corrected ordinary least squares (deterministic model)			*Maximum Likelihood (stochastic model)*		
	Input distance	*Output distance*	*Ray frontier*	*Input distance*	*Output distance*	*Ray frontier*
α_0	5.376 (13.2)	-5.084 (-15.7)	5.470 (15.9)		-4.790 (-15.7)	
α_1	-0.769 (-9.3)	0.718 (9.9)			0.714 (10.3)	
α_2	-0.222 (-2.1)					
β_1		-0.845 (-16.5)	0.106 (1.7)		-0.854 (-19.0)	
β_2	0.019 (0.3)	-0.144 (-2.45)	0.003 (1.2)		-0.136 (-2.6)	
β_3	0.003 (1.1)	-0.002 (-1.07)	0.871 (15.8)		-0.002 (-1.0)	
β_4			-0.182 (-2.1)			
LLF(df)	-18.296 (46)	-13.798 (46)	-17.130 (46)	-18.296** (44)	-13.283 (44)	-17.130** (44)
R^2	0.956	0.949	0.958			
σ^2	0.133	0.112	0.127		0.183 (2.6)	
γ				0.000 (0.0)	0.710 (3.0)	0.000 (0.0)

* t-statistics are presented in brackets.

**The likelihood value is less than that obtained from the OLS model.

Table 6: Estimated parameters for alternative models: Telecommunications*

	Corrected ordinary least squares (deterministic model)			*Maximum Likelihood (stochastic model)*		
	Input distance	*Output distance*	*Ray frontier*	*Input distance*	*Output distance*	*Ray frontier*
α_0	0.405 (0.5)	-1.277 (-1.4)	1392.712 (2.2)	1.211 (2.2)		1392.5 (1514.2)
α_1	-0.0001 (-0.001)	0.004 (0.05)		0.008 (0.2)		
α_2	-0.605 (-4.5)			-0.580 (-4.2)		
α_3	-0.352 (-2.7)	0.406 (2.9)		-0.395 (-4.0)		
α_4	0.088 (1.2)	-0.046 (-0.5)		0.076 (1.2)		
α_5	-0.212 (-0.4)	0.011 (0.203)		-0.043 (-2.5)		
β_1		-0.622 (-6.1)	0.345 (2.0)			0.432 (4.3)
β_2	0.403 (4.8)	-0.450 (-5.0)	0.649 (4.1)	0.421 (9.2)		0.617 (7.5)
β_3			-3344.730 (-2.2)			-3345.027 (-3399.0)
β_4			133.396 (3.1)			133.121 (137.1)
β_5			140.752 (1.3)			140.305 (139.1)
β_6			-0.202 (-2.3)			-0.304 (-4.4)
LLF(df)	2.080 (24)	-0.783 (24)	2900.000 (24)	10.643 (22)	-0.783** (22)	4200.000 (22)
R^2	0.973	0.972	0.882			
σ^2	0.066	0.079	0.230	0.103 (3.075)		0.570 (5.0)
γ				0.9996 (517.0)	0.000 (0.0)	0.99999 (62.0)

* t-statistics are presented in brackets.

**The likelihood value is less than that obtained from the OLS model.

Table 7: Technical efficiency for alternative models: Electricity

	Supplier	*Data envelopment analysis*		*Corrected ordinary least squares*			*Maximum likelihood*
		CRS (%)	*VRS (%)*	*Input distance (%)*	*Output distance (%)*	*Ray frontier (%)*	*Stochastic Ray frontier (%)*
1	Austria	65	66	19	28	14	31
2	Denmark	89	89	30	43	20	45
3	Finland	81	82	34	50	23	49
4	France	89	100	31	60	44	80
5	Germany	91	100	27	53	47	82
6	Greece	100	100	22	33	21	45
7	Ireland	87	95	21	28	12	27
8	Italy	93	100	26	44	41	77
9	Netherlands	100	100	45	77	48	83
10	Oslo Energi (Norway)	100	100	100	97	2	5
11	Poland	95	100	14	26	31	65
12	Portugal	100	100	26	36	18	39
13	UNESA (Spain)	100	100	39	62	32	67
14	Vattenfall (Sweden)	98	100	29	45	57	83
15	U. K.(Public Distribution)	83	100	32	59	41	78
16	BC Hydro	100	100	60	80	14	29
17	Hydro-Quebec	100	100	55	85	20	42
18	Ontario Hydro	89	89	30	49	35	69
19	TransAlta	100	100	37	49	36	68
20	Carolina P&L	100	100	28	42	30	61
21	Duke Power	85	86	23	37	33	65
22	Los Angeles Dept. W&P	73	81	40	49	13	29
23	Chubu Electric	78	78	41	62	26	56
24	Kansai Electric	79	79	41	63	28	61
25	Tokyo Electric	100	100	50	81	36	73
26	NSW	91	91	29	44	23	50
27	VIC	97	100	39	53	19	41
28	QLD	85	89	27	38	19	40
29	SA	80	100	35	46	20	43
30	WA	62	66	23	29	14	29
31	TAS	76	100	36	41	8	16
32	NT	60	100	21	24	12	25
33	SMHEA	100	100	32	49	100	82
34	New Zealand	98	98	53	67	13	28
35	Argentina	100	100	79	100	23	48
36	Israel	100	100	42	66	35	71
37	Singapore	100	100	60	85	32	67
38	ESKOM (South Africa)	97	97	16	31	78	90
39	South Korea	100	100	57	90	33	69
40	Taiwan	94	94	49	74	26	56
41	China Light & Power	62	65	29	38	16	34
	Average	*90*	*94*	*37*	*54*	*29*	*54*

Table 8: Technical efficiency for alternative models: Gas

Supplier	*Data envelopment analysis*		*Corrected ordinary least squares*			*Maximum likelihood*
	CRS (%)	*VRS (%)*	*Input distance (%)*	*Output distance (%)*	*Ray frontier (%)*	*Stochastic Output distance*
1 AGL	53	57	41	45	38	46
2 Allgas/GCQ	35	100	19	22	19	40
3 GFCV	69	81	51	59	49	51
4 SAGASCO	82	100	63	66	58	53
5 SECWA	97	100	90	95	78	61
6 NICOR (IL)	100	100	89	93	80	61
7 New Jersey Resources (NJ)	73	84	52	57	48	50
8 Southwest Gas (NV)	62	66	47	49	44	47
9 Atmos Energy (TX)	58	65	50	50	44	47
10 Indianna Energy (IN)	66	73	50	54	45	48
11 NUI (NJ)	79	88	46	53	43	48
12 Atlanta Gas Light (GA)	54	55	41	45	39	45
13 Oneok (OK)	49	62	39	46	37	46
14 Peoples Energy (IL)	100	100	40	55	44	49
15 Brooklyn Union Gas (NY)	100	100	37	51	43	47
16 Cascade Natural Gas (WA)	89	100	42	46	38	46
17 Washington Energy (WA)	66	71	41	46	38	46
18 Laclede Gas (MO)	68	72	37	44	37	45
19 Energen (AL)	48	55	33	37	32	43
20 Northwest Natural Gas (OR)	47	54	35	37	32	43
21 Bay State Gas (MA)	53	66	31	36	30	43
22 Providence Energy (RI)	58	94	31	36	30	43
23 South Jersey Industries (NJ)	50	100	31	35	29	42
24 WICOR (WI)	52	55	23	30	24	41
25 Washington Gas Light (DC)	56	58	28	35	28	42
26 Piedmont Natural Gas (NC)	59	100	34	39	32	43
27 Connecticut Energy (CT)	61	100	33	38	32	43
28 Questar (UT)	33	35	23	28	23	41
29 BC Gas	64	73	52	60	48	51
30 Centra Gas BC	36	48	35	34	27	42
31 Northland Utils BC	46	100	45	43	34	45
32 Pacific Northern Gas	48	56	32	40	40	44
33 Bonnyville Gas Co	53	100	52	41	34	45
34 Canadian Western Natural Gas Co	39	44	34	38	31	43
35 Centra Gas Alberta	40	41	37	30	26	41

Table 8 (Continued)

Supplier	Data envelopment analysis		Corrected ordinary least squares			Maximum likelihood
	CRS (%)	VRS (%)	Input Distance (%)	Output distance (%)	Ray Frontier (%)	Stochastic Output distance
36 City of Medicine Hat Gas Utility	100	100	62	76	100	57
37 Federation of Alberta Gas Coops	53	100	37	27	25	41
38 Northwestern Utils	43	49	37	42	34	45
39 Sask Energy	94	100	75	60	55	51
40 Centra Gas Manitoba	68	73	46	55	44	49
41 Centra Gas Ontario	100	100	39	46	38	46
42 The Consumers Gas Co	100	100	47	58	47	50
43 Corporation of the City of Kitchener	100	100	100	100	79	62
44 Public Utils Comm (Kingston)	81	100	33	39	29	44
45 Union Gas	66	95	37	45	36	46
46 Gaz Metropolitain	100	100	20	27	27	41
47 Tokyo Gas Co	100	100	48	60	57	50
48 Osaka Gas Co	100	100	58	66	64	53
49 Toho Gas	75	77	37	43	41	45
50 Saibu Gas Co	91	100	48	56	52	49
51 British Gas	55	100	26	35	30	42
Average	68	81	43	48	41	47

Table 9: Technical efficiency for alternative models: Telecommunications

		Data envelopment analysis		*Corrected ordinary least squares*			*Maximum likelihood (Stochastic)*	
	Country	*CRS (%)*	*VRS (%)*	*Input distance (%)*	*Output distance (%)*	*Ray frontier (%)*	*Input distance (%)*	*Ray frontier (%)*
1	Argentina	83	89	78	76	68	87	68
2	Australia	100	100	79	83	80	97	85
3	Austria	100	100	87	89	97	97	100
4	Belgium	80	80	68	70	53	77	48
5	Brazil	100	100	87	89	56	99	53
6	Canada	82	82	63	63	45	76	52
7	Denmark	94	95	85	85	75	98	84
8	Finland	100	100	84	82	64	97	81
9	France	82	98	44	45	32	51	29
10	Germany	100	100	64	69	76	78	67
11	Hong Kong	100	100	59	60	69	70	74
12	Ireland	61	62	50	48	45	55	45
13	Israel	79	80	80	77	36	87	39
14	Italy	100	100	83	88	43	99	45
15	Japan	100	100	75	79	88	94	96
16	Korea (Rep. of)	97	97	70	68	55	79	62
17	Luxembourg	100	100	100	100	94	99	83
18	Malta	37	100	49	41	23	47	26
19	Netherlands	88	97	65	66	42	75	42
20	New Zealand	56	56	54	50	36	60	42
21	Norway	98	98	76	75	67	88	80
22	Portugal	69	69	68	66	34	75	35
23	Singapore	100	100	69	68	56	78	65
24	South Africa	56	56	40	40	17	44	14
25	Spain	100	100	84	89	64	99	61
26	Sweden	94	100	72	73	62	86	72
27	Switzerland	100	100	82	88	100	96	98
28	Taiwan, China	96	96	83	83	43	97	51
29	Thailand	78	81	47	43	28	54	34
30	United Kingdom	73	87	55	58	40	68	41
31	United States	100	100	59	67	51	80	52
	Averages	87	91	70	70	56	80	59

Table 10: Correlation table of alternative technical efficiency measures: Electricity

	DEA CRS	*DEA VRS*	*COLS Input distance*	*COLS Output distance*	*COLS Ray frontier*	*ML Ray frontier*
DEA CRS	1.00					
DEA VRS	0.75	1.00				
COLS Input distance	0.40	0.26	1.00			
COLS Output distance	0.51	0.33	0.92	1.00		
COLS Ray frontier	0.37	0.28	-0.24	-0.02	1.00	
ML Ray frontier	0.41	0.30	-0.24	0.08	0.85	1.00

Table 11: Correlation table of alternative technical efficiency measures: Gas

	DEA CRS	*DEA VRS*	*COLS Input distance*	*COLS Output distance*	*COLS Ray frontier*	*ML Output distance*
DEA CRS	1.00					
DEA VRS	0.68	1.00				
COLS Input distance	0.57	0.34	1.00			
COLS Output distance	0.69	0.38	0.94	1.00		
COLS Ray frontier	0.69	0.38	0.87	0.94	1.00	
ML Output distance	0.69	0.39	0.94	1.00	0.95	1.00

Table 12: Correlation table of alternative technical efficiency measures: Telecommunications

	DEA CRS	*DEA VRS*	*COLS Input distance*	*COLS Output distance*	*COLS Ray frontier*	*ML Input distance*	*ML Ray frontier*
DEA *CRS*	1.00						
DEA *VRS*	0.73	1.00					
COLS *Input distance*	0.69	0.51	1.00				
COLS *Output distance*	0.77	0.56	0.98	1.00			
COLS *Ray frontier*	0.71	0.56	0.69	0.74	1.00		
ML *Input distance*	0.79	0.58	0.96	0.98	0.72	1.00	
ML *Ray frontier*	0.72	0.57	0.69	0.71	0.96	0.73	1.00