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ESTIMATION OF ELASTICITIES OF SUPPLY OF LABOUR HOURS FOR AUSTRALIAN WORKERS CLASSIFIED BY SEX AND MARITAL STATUS

by

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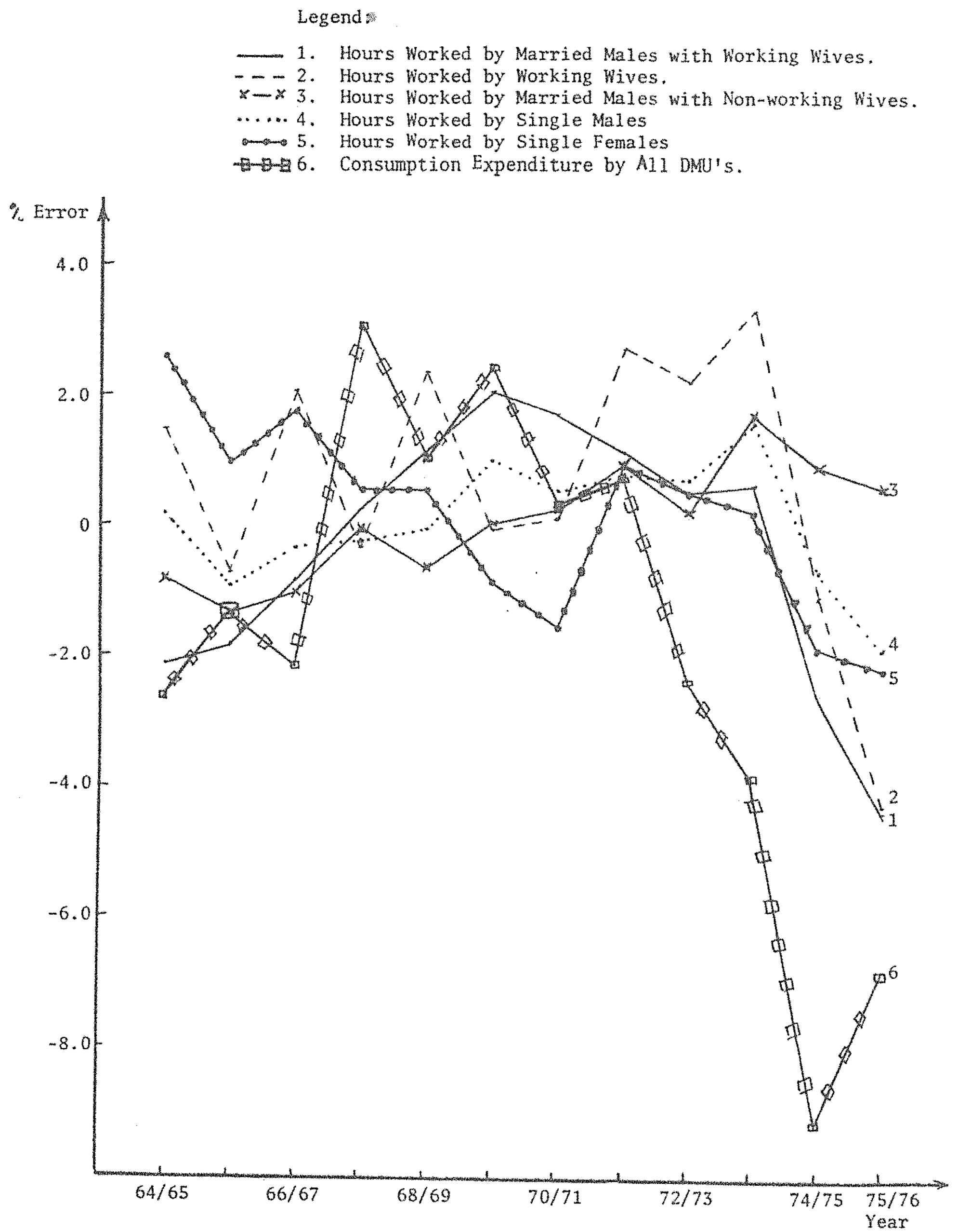

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ABSTRACT

In this paper the supply of labour hours offered by workers classified by sex and marital status has been modelled as responding to the price level, hourly wage rates and non-labour income. The working population has been divided into four types of decision making units. Separate demand systems have been developed for each type of decision making unit and these systems have been combined into a single system of equations for estimation purposes. A modified Twice Extended Linear Expenditure System for several Household types (TELESH) has been developed to take into account the above mentioned demographic factors and is fitted to national accounts and labour force data for 1964-65 to 1975-76 using full information methods. Elasticities of supply of labour hours are estimated for workers classified by sex and marital status. The estimated elasticities differ substantially depending upon the demographic characteristics of the workers.

1. INTRODUCTION

In an earlier paper[1], aggregate time series data on consumption, savings and hours worked were analysed using a Twice Extended Linear Expenditure System (TELES). In that work, only a single representative agent was distinguished. Estimates were made of his marginal propensity to consume 'leisure'[2], β_L, and the maximum number of hours, γ_H, that he would be willing to work regardless of the wage rate. Both of these parameters were

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* I am grateful to Alan Powell, Tony Lawson and Ross Williams for valuable comments on earlier drafts of this paper.

1 Ashok Tulpulé, "Revised Estimates of Labour Supply Elasticities", IMPACT Working Paper No. B-12, IMPACT Research Centre, University of Melbourne, April 1980.

2 Leisure hours include time spent on genuine leisure and also on other non-labour market activities such as housework.

taken to be constant over the sample period (1964-65 to 1975-76). The increasing rates of workforce participation of married women over this period, however, raise the question of the adequacy of a paradigm built around a single representative worker with a stationary utility function.

The minimum leisure requirements and the marginal propensity to consume leisure of a typical married female worker are likely to be different to those of male workers, and therefore, it is reasonable to expect that the leisure parameters of the utility function of the representative worker would change over time with increased participation of married women in the workforce. The maximum number of hours that a married male would be willing to work could also change when his wife joins the labour force. Thus the leisure parameters for a typical working man are also likely to change as the married female participation rates change. It would be reasonable to assume stability of parameters for families grouped into different categories such as those in which both husband and wife work, families in which only the husband works, single person families and so on. Time series data required for estimating suitably modified TELES models for different types of families are not available in Australia. Models of joint determination of the work behaviour of the husband and wife have been estimated overseas using survey data[1]. Suitable survey data also are not available in Australia. In view of the non-availability of suitable data in Australia, in this paper a model is proposed that can be estimated empirically using available National Accounts and Labour Force data.

The plan of the paper is as follows. Section 2 discusses how

1 See For example, M.D. Hurd, "Estimating the Family Labour Supply Functions Derived from the Stone-Geary Utility Function", National Bureau of Economic Research, Working Paper No. 228, Stanford, January 1978; M. Kester, "Income and Substitution Effects in a Family Labour Supply Model", Rand Corporation, Santa Monica, 1966.

in these equations were not estimated by the model but were fixed exogenously. On the basis of the above test and visual inspection of the residuals (Figure 1), it is concluded:

(a) The serial correlation, though higher than desired, is not statistically significant at a 'high' significance level (*viz.*, $\alpha < .05$) except in two cases.

(b) In these two cases (and perhaps elsewhere) the serial correlation seems to be more likely to be due to specification and/or data handling problems than to stochastic misspecification.

Let n be the number of observations and let r denote the number of changes of sign of the residuals. If it is assumed that positive and negative values are equally likely to occur, the probability of a sign change is 0.5. Thus r follows a Binomial Distribution with parameters (n-1) and p =0.5.

For large n,

$$z = \frac{r - p\,(n-1)}{\sqrt{p(1-p)(n-1)}} \sim N\,(0,1).$$

The sample size in the present case is 12 which is not very large. However when p is close to 0.5, Hoel[1] has shown that the normality assumption holds reasonably well for n=12.

Estimates of z for the six equations are shown below:

Endogenous Variable	z
1 Hours worked by married male with working wife	-2.1106
2 Hours worked by working wife	-0.3015
3 Hours worked by married male with non-working wife	-1.5976
4 Hours worked by single male	-1.5976
5 Hours worked by single female	-1.5076
6 Consumption expenditure	-2.1106

The above figures show that the serial correlation is not very strong in most equations. It is rather high for equations 1 and 6. However it may be recalled that the marginal budget shares parameters

1 Hoel, P.G., Introduction to Mathematical Statistics, Asia Publishing Company, Bombay, 1964.

the parameters of the synthetic utility function of a representative worker would change over time due to compositional changes in a working population that is divided into different types of decision making units (DMU). Modelling the behaviour of the four representative DMUs involves the specification of their utility functions and budget constraints. Problems associated with developing a single expenditure/labour supply system for several household types (TELESH) based on four separate systems for the four types of DMUs are discussed in Section 3. The required data series are derived from data available from published sources in Section 4. Section 5 gives the estimates of the coefficients of the TELESH system and in Section 6 the various labour supply elasticities are estimated. Some possible applications of the results are discussed in the concluding Section 7.

2. EFFECTS OF COMPOSITIONAL CHANGES

The 'typical' Australian household's consumption expenditure and hours worked in any one year represent the sum total of decisions made by households of several types. With increasing workforce participation of married women the structure of the 'typical' household has been changing over time. This section illustrates how the aggregation bias introduced by modelling aggregate behaviour with only one representative agent can be reduced by working at a disaggregated level. Within the latter framework we are able to capture the effects of compositional changes in the working population on the 'parameters' of the synthetic utility function of such a representative agent.

It is possible to identify several types of household units; however, for reasons of practicality only four types of households[1] are distinguished in this study and with each is associated a representative agent called a Decision Making Unit[2] (DMU). The four household types distinguished in this paper cover all the workers in Australia. The corresponding DMUs are defined in Table 2.1, where the distribution of the working population over household types in 1964-65 and 1975-76 is shown.

The table shows that during the sample period 1964-65 to 1975-76 substantial changes in the distribution of the Australian working population have occurred. The proportion of workers associated with DMU Type 1 has increased and there has been a corresponding decrease in those associated with DMU Type 2.

1 Where the word 'household' is used in the narrow technical sense as defined by the Australian Bureau of Statistics (ABS), it will appear in italics. Where not italicized, the looser (ordinary English) meaning of the term is to be understood.

2 It is assumed that the decisions regarding the work and consumption behaviour of the household represent a joint decision by its members. Thus, the quesion of how the DMU arrives at a compromise between its members is avoided.

SERIAL PROPERTIES OF THE DATA

As mentioned in Section 5 the model fits reasonably well to the sample period data. The means of residuals expressed as a percentage of the average values of the first five endogenous variables, i.e. the hours worked variables, are less than 1 per cent and for the consumption expenditure variable it is 2 per cent. For individual data points the errors in estimating the endogenous variable are shown in Figure 1, where, the residuals, i.e., the difference between the observed values and the values estimated by the system, are shown as a percentage of the observed values. In general, the errors in estimating the hours worked are within ± 4 per cent of the observed values. The hours worked equations give estimates of -wH, i.e., labour income with a negative sign. Therefore a negative residual means that the hours worked are over-estimated by the model and *vice versa*. For working wives and single persons (equations 2, 4 and 5) there are small fluctuations in the percentage residuals. For married men the assumption of $\beta_5 = 0 = \beta_7$ means that the hours worked estimated by the model remain constant. The residuals therefore, simply reflect the fluctuations in the original data series. The percentage errors in estimating consumption expenditure (equation 6) are somewhat larger than those in the hours worked estimates.

In order to test the serial properties of the data of the six endogenous variables the following non-parametric test is conducted following the method used by McAleer *et al.*[1] and by Caddy *et al.*[2]

1. McAleer, M., Powell, A.A., Dixon, P.B. and A. Lawson, "Estimation of the Consumption Function: A Systems Approach to Employment Effects on the Purchase of Durables" IMPACT Working Paper No. M-02, Industries Assistance Commission, Melbourne, February 1979.
2. Caddy, V., Jackson B. and A.A. Powell, "Determinants of Australian Migration" IMPACT Working Paper No. B-07, Industries Assistance Commission, Melbourne, June 1978.

TABLE A.6 : CONSUMPTION EXPENDITURE AND PRICE INDEX AUSTRALIA 1964-65 TO 1975-76

Year	Consumption Expenditure per DMU ($)	Price-Index (1966-67 = 100)
1964-65	3,094.34	94.03
1965-66	3,224.25	97.11
1966-67	3,422.83	100.00
1967-68	3,686.82	103.28
1968-69	3,917.48	106.11
1969-70	4,221.78	109.99
1970-71	4,574.85	116.57
1971-72	5,073.30	123.66
1972-73	5,618.79	130.61
1973-74	6,522.07	145.28
1974-75	7,867.55	169.26
1975-76	9,225.38	194.05

SOURCE: Based on Australian National Accounts, *op.cit.*.

TABLE 2.1 : DISTRIBUTION OF THE WORKING POPULATION BY HOUSEHOLD TYPE

DMU Type	Description Of Households Associated With DMU	Estimated Number Of Households In Thousands (And Percentage Distribution)				Percentage Distribution Of Total Workers By Household Type	
		1964-65		1975-76		1964-65	1975-76
1	Both husband and wife work[a]	634.65	(16.17)	1281.48	(28.46)	27.84	44.31
2	Only the husband works[b]	1648.15	(42.00)	1468.80	(32.62)	36.15	25.39
3	Household contains only a not-married male worker[c]	978.05	(24.92)	1037.17	(23.03)	21.45	17.93
4	Household contains only a not-married female worker[c]	663.73	(16.91)	715.67	(18.89)	14.56	12.37
	All types	3924.58	(100)	4503.12	(100)	100	100

a Numbers are same as the number of working married women. It is assumed that whenever a married woman is working her husband is also working.
b Numbers are estimated by subtracting the number of working married women from the number of working married men.
c 'Not-married' includes single, divorced, widowed, etc.

SOURCE: Based on Australian Bureau of Statistics, The Labour Force (Ref. No. 6.22).

Before discussing the effects of these compositional changes, it is necessary to introduce some notation and the general form of the utility function that will be associated with different types of DMUs.

2.1 Utility Functions

For each of the different types of DMUs the Klein-Rubin form of utility function is adopted. Since households associated with some types of DMUs may contain two workers, two leisure hours variables, L_1 and L_2, appear as arguments:

$$U = \beta_c \ln \left(C-\gamma_c\right) + \beta_s \ln (s) + \beta_{L1} \ln \left(L_1-\gamma_{L1}\right) + \beta_{L2} \ln \left(L_2-\gamma_{L2}\right), \quad (2.1)$$

where

C = a scalar index of the quantity of goods and services consumed by the household[1],

s = savings in $ by the household,

L_1 = leisure hours consumed by the first worker associated with the DMU,

L_2 = leisure hours consumed by the second worker,

β_c = marginal propensity to consume,

β_s = marginal propensity to save,

γ_c = 'subsistence' level of consumption,

γ_{L1} = minimum leisure hours pdrameter for the first worker,

and

1 In theory it is possible to include several commodity groups; however, in view of the lack of disaggregated data on consumption expenditure by different types of DMUs only one commodity group is considered.

TABLE A.5 : ESTIMATES OF HOURLY WAGE RATES FOR WORKERS CLASSIFIED BY SEX AND MARTIAL STATUS, AUSTRALIA 1964-65 TO 1975-76

Year	Worker Type			
	Married Male	Married Female	Single Male	Single Female
1964-65	1.34	0.85	1.08	0.71
1965-66	1.40	0.89	1.12	0.75
1966-67	1.47	0.93	1.19	0.79
1967-68	1.53	0.97	1.25	0.83
1968-69	1.64	1.02	1.35	0.87
1969-70	1.75	1.12	1.44	0.97
1970-71	1.95	1.26	1.61	1.09
1971-72	2.12	1.41	1.75	1.21
1972-73	2.37	1.63	1.96	1.40
1973-74	2.75	1.97	2.29	1.69
1974-75	3.53	2.68	2.95	2.30
1975-76	3.92	3.13	3.28	2.72

SOURCE: Derived data; see section 4.

TABLE A.4 : ESTIMATES OF AFTER TAX NON-LABOUR INCOME ($) PER DMU PER YEAR, AUSTRALIA 1964-65 TO 1975-76

Year	Decision Making Unit Type			
	Type 1	Type 2	Type 3	Type 4
1964-65	1,210.50	834.83	417.41	567.58
1965-66	1,136.16	783.56	391.78	532.82
1966-67	1,303.88	899.23	449.61	611.47
1967-68	1,192.78	822.61	411.30	559.37
1968-69	1,402.45	967.21	483.60	657.70
1969-70	1,389.24	958.10	565.28	613.18
1970-71	1,400.66	965.97	569.93	618.22
1971-72	1,649.40	1,137.52	671.14	728.01
1972-73	2,055.59	1,417.65	836.41	907.29
1973-74	2,473.16	1,705.63	1,006.32	1,091.60
1974-75	2,713.38	1,871.30	1,104.06	1,197.63
1975-76	3,411.38	2,352.67	1,388.08	1,505.71

SOURCE: Derived data; see section 4.

γ_{L2} = minimum leisure hours parameter for the second worker.

The utility function for a typical DMU with one worker will have a similar form but will contain only one leisure hour term instead of the two in (2.1).

2.2 Compositional Effects

The effects of compositional changes can be discussed in terms of movements in the minimum consumption and leisure needs of the entire population or of a single synthetic agent who represents a weighted average of the four agents. It is argued that the proposed disaggregation of the population into households associated with the four types of DMUs would avoid a specification error which would be made by working at the aggregate (single representative agent) level with stationary minimum consumption and leisure needs parameters.

Basic needs

The utility specification in (2.1) assumes that the basic consumption needs parameter for a given type of DMU is stable over time. The utility function in the previous paper[1] assumed the basic needs *per worker* of each commodity to remain constant over time. Such an assumption implies that with increasing workforce participation, the basic consumption needs

1 Ashok Tulpulē, *op. cit.*.

per head of population[1] increase over time. By subdividing the population into different types of households we are assuming that the basic consumption needs per household associated with each type of DMU are stable over time. If the number of persons per household type were stable over time the assumption would be realistic. In practice the average size is likely to have changed slowly. The assumption of stable basic needs per DMU, however, seems more plausible than stable basic needs per worker for the following reason.

Suppose that the population were divided into two types of households, those where only the husband worked and those where both husband and wife worked, with basic needs parameters of γ_c^1 and γ_c^2 respectively. If there were no difference in the average sizes of the two types of households, γ_c^1 would be likely to be less than γ_c^2. That is, we expect the basic household consumption of commodities to increase because the working wife's time constraint does not permit her to buy commodities intensive in the use of her time. She substitutes frozen vegetables and pre-cooked meals for fresh vegetables and home cooked meals, etc.. Hence the real value of the 'basic needs' bundle of the population as a whole, which is a weighted average of the two γ's, is likely to increase with increased workforce participation by married women. The assumption in the previous

1 In some applications of the linear expenditure system basic needs have been formulated to follow a trend. For example, R.A. Pollak and T.J. Wales, "Estimation of the Linear Expenditure System", Econometrica, Vol. 37, No. 4, October 1969, pp. 611-628, and L. Phlips, "A Dynamic Version of the Linear Expenditure Model", Review of Economics and Statistics, Vol. LIV, No. 4, November 1972, pp. 450-458, introduce the concept of habit formation and time subscripted basic needs parameters to capture the upward drift in socially accepted "basic needs". It is not clear whether the introduction of a trend in the basic needs parameter in fact picks up the upward drift in standards or compositional shifts such as those on which this paper is focused. For a detailed discussion on habit formation see K.W. Clements, M. Evans, D. Ironmonger and A.A. Powell, 'A Linear Expenditure System with Adjustment Costs', Economic Record, Vol. 55, No. 147, December 1978, pp. 321-333.

TABLE A.3 : ESTIMATES OF AFTER TAX LABOUR INCOME ($) PER WORKER PER YEAR FOR DIFFERENT TYPES OF WORKERS, AUSTRALIA 1964-65 TO 1975-76

	Worker Type				
Year	Married Male with Working Wife	Married Female	Married Male with Non-Working Wife	Single Male	Single Female
1964-65	2,768.00	1,434.99	2,973.00	2,273.45	1,349.18
1965-66	2,899.80	1,478.80	3,092.20	2,357.49	1,404.34
1966-67	3,085.70	1,539.89	3,264.40	2,490.17	1,475.08
1967-68	3,238.80	1,621.42	3,424.10	2,630.65	1,556.55
1968-69	3,512.80	1,678.65	3,655.10	2,830.00	1,613.43
1969-70	3,782.40	1,810.78	3,932.40	3,029.63	1,773.76
1970-71	4,208.50	2,050.71	4,394.50	3,365.66	1,989.14
1971-72	4,534.80	2,300.09	4,794.70	3,649.80	2,236.28
1972-73	5,041.80	2,559.46	5,328.80	4,034.59	2,524.58
1973-74	5,859.40	3,112.26	6,283.10	4,718.20	3,020.54
1974-75	7,276.40	4,144.40	7,986.30	5,943.13	4,032.35
1975-76	7,947.30	4,716.90	8,849.40	6,509.97	4,722.99

SOURCE: Derived data; see section 4.

TABLE A.2 : ESTIMATES OF HOURS WORKED PER WORKER PER YEAR (PER WEEK) BY TYPE OF WORKER, AUSTRALIA, 1964-65 TO 1975-76

	Worker Type				
Year	Married Male with Working Wife	Married Female	Married Male with Non-Working Wife	Single Male	Single Female
1964-65	2071.10 (39.83)	1680.13 (32.31)	2224.50 (42.78)	2113.31 (40.64)	1894.81 (36.44)
1965-66	2076.20 (39.92)	1669.71 (32.11)	2213.90 (42.57)	2096.82 (40.32)	1884.72 (36.24)
1966-67	2098.40 (40.35)	1663.33 (31.99)	2219.90 (42.69)	2095.86 (40.31)	1872.94 (36.02
1967-68	2120.90 (40.79)	1670.35 (31.12)	2242.30 (43.12)	2106.59 (40.51)	1875.63 (36.07)
1968-69	2140.90 (41.17)	1638.11 (31.50)	2227.60 (42.83)	2092.46 (40.24)	1844.54 (36.47)
1969-70	2158.60 (41.52)	1616.34 (31.08)	2244.20 (43.16)	2098.10 (40.35)	1833.88 (35.27)
1970-71	2153.90 (41.42)	1621.59 (31.18)	2249.10 (43.25)	2088.49 (40.16)	1824.81 (35.09)
1971-72	2140.70 (41.17)	1636.81 (31.48)	2263.40 (43.53)	2082.01 (40.04)	1848.29 (35.54)
1972-73	2127.40 (40.91)	1574.60 (30.28)	2248.50 (43.26)	2055.45 (39.53)	1806.70 (34.74)
1973-74	2128.30 (41.98)	1580.83 (30.40)	2282.20 (43.89)	2062.92 (39.67)	1783.22 (34.29)
1974-75	2063.30 (39.67)	1547.96 (29.77)	2264.60 (43.54)	2015.15 (38.75)	1753.11 (33.72)
1975-76	2026.80 (38.98)	1507.27 (28.99)	2256.90 (43.40)	1984.11 (38.16)	1733.12 (33.33)

SOURCE: Derived data, see section 4.

paper, *viz.*, constant basic needs *per worker* throughout the period, will tend to overstate the basic needs of the whole population in the earlier period and understate it in the later part of the sample period.

The above argument based on two types of households can be generalised for several types. It is expected that the disaggregation of the Australian population into four types of households will enable us to capture the effects of the main demographic trends in the composition of the workforce.

Minimum leisure requirements

The minimum leisure requirements of a 'typical' worker are also likely to change over time due to the compositional changes shown in Table 2.1. It is not clear whether the average of the minimum leisure requirements of the five types of workers would increase or decrease over time. If married women perform a greater share of domestic work than their husbands, their minimum 'leisure' requirements could be greater than those of married men. Similarly if married men perform a larger proportion of home duties when their wives join the workforce, they also may require more minimum leisure hours than men with non-working wives. The minimum leisure requirements of single persons are likely to be influenced by their life-styles. In view of the above factors it is not possible to postulate a definite upward or downward trend in the minimum leisure requirements of the 'typical' Australian worker.

If compositional changes cause the minimum leisure coefficient,

γ_L, of a 'typical' worker to increase over time, then γ_H (i.e., the weighted average of the maximum feasible work hours parameters) would fall over time[1]. Philips[2] allowed γ_H to vary over time by adopting the state variable approach. Using Abbott and Ashenfelter's[3] data on the U.S.A., he demonstrated that γ_H decreased slowly over the estimation period 1939 to 1967. He interpreted this fall as habit formation in the sense that minimum leisure requirements increase steadily over time.

Without ever introducing the concept of habit formation, however, it may be possible to demonstrate that for the working population as a whole γ_H could vary simply due to compositional changes. By disaggregating the working population into four household types, the constancy of parameters for workers associated with different types of DMUs can be preserved. At the same time it is possible for the average 'parameter' of the working population as a whole to change over time.

1 It is possible for γ_H to increase over time in spite of the increase in the weighted average of the minimum leisure needs parameters, if the average of total time available for work and leisure per worker increases over time.
2 Louis Philips, "The Demand for Leisure and Money", Econometrica, Vol. 46 (1978), No. 5, pp. 1025-1042.
3 M. Abbott and O. Ashenfelter, "Labour Supply, Commodity Demand and the Allocation of Time", Review of Economic Studies, 43 (1976), pp. 389-411.

APPENDIX I

DATA SERIES USED IN THE PAPER

TABLE A.1 : NUMBERS OF DECISION MAKING UNITS (IN THOUSANDS) BY TYPE*, AUSTRALIA, 1964-65 TO 1975-76

Year	DMU Type				
	Type 1	Type 2	Type 3	Type 4	TOTAL
1964-65	634.65	1,648.15	978.05	663.73	3,924.58
1965-66	683.00	1,651.70	986.45	686.60	4,007.75
1966-67	745.25	1,661.80	969.98	700.00	4,077.03
1967-68	803.25	1,650.15	982.58	702.82	4,138.80
1968-69	857.88	1,647.10	1,006.30	702.65	4,213.93
1969-70	937.60	1,642.45	1,014,20	710.10	4,304.35
1970-71	1,020.92	1,627.30	1,010.18	715.08	4,373.48
1971-72	1,058.90	1,631.20	1,002.53	684.60	4,377.23
1972-73	1,133.87	1,574.23	1,016.83	689.90	4,414.83
1973-74	1,213.95	1,537.18	1,035.40	701.92	4,506.27
1974-75	1,247.72	1,497.71	1,026.05	695.35	4,466.83
1975-76	1,281.48	1,468.80	1,037.17	715.67	4,503.12

* The four types are defined in Table 2.1.

SOURCE: Based on data on number of workers classified by sex and marital status from The Labour Force (ABS Ref. No. 6.22), *op.cit.*.

Similar experiments can be conducted by varying the male/female relativities or a mixture of changes in relativities of adults, juniors, males and females. By comparing a number of alternative schemes it would be possible to determine a mix of wage policy which is likely to be more efficient in reducing unemployment.

3. THE 'TELESH' MODEL

The population was divided into the four types of Decision Making Units (DMUs) shown in Table 2.1. The time and income allocation problem faced by each type of DMU is now discussed in turn.

3.1 Decision Making Units with Husband and Wife Working

The representative DMU's consumption and work behaviour is generated by the maximisation of the utility function

$$(3.1.1) \quad U_1 = \beta_1 \ln(x_1 - \gamma_1) + \beta_{s1} \ln s_1 + \beta_5 \ln(L^{m1} - \gamma_L^{m1}) + \beta_6 \ln(L^{f1} - \gamma_L^{f1}),$$

subject to

$$(3.1.2) \quad px_1 + s_1 = w_m^m H^{m1} + w_m^f H^{f1} + \alpha_1,$$

where

x_1 = a scalar quantity index of the bundle of goods consumed by a representative household of type 1,

s = savings in \$ by household of type 1,

γ_1 = 'subsistence' level of consumption,

β_1 = marginal propensity to consume,

β_{s1} = marginal propensity to save,

β_5 = husband-specific marginal propensity to consume leisure,

β_6 = wife-specific marginal propensity to consume leisure,

L^{ml} = number of leisure hours consumed by husband,

L^{fl} = number of leisure hours consumed by wife,

γ_L^{ml} = minimum leisure requirements for the husband,

γ_L^{fl} = minimum leisure requirements for the wife,

H^{ml} = hours worked by the husband,

H^{fl} = hours worked by the wife,

w_m^m = after tax hourly wage rate for married males,

w_m^f = after tax hourly wage rate for married females,

and

p = price index.

The Lagrangean (F_1) is

$$F_1 = U_1 + \lambda_1 (\alpha_1 + H^{ml} w_m^m + H^{fl} w_m^f - px_1 - s_1). \quad (3.1.3)$$

The first order conditions comprise (3.1.2) and those equations obtained by setting

$$\frac{\partial F_1}{\partial x_1}, \frac{\partial F_1}{\partial s_1}, \frac{\partial F_1}{\partial H^{ml}} \text{ and } \frac{\partial F_1}{\partial H^{fl}} \text{ to zero;}$$

that is, the DMU optimally chooses consumption, savings, and hours worked by both parties. These first order conditions are

$$\frac{1}{x_1 - \gamma_1} - \lambda_1 p = 0, \quad (3.1.4a)$$

Suppose that as assumed by Dixon, Powell and Parmenter[1] the labour market for all occupations is slack; assume, however, that the degree of slackness varies among the demographically disaggregated sectors of the labour market[2]. According to the ORANI model the proposed policy package which includes a certain percentage reduction in the real hourly wage rate produces the desired increase in the demand for labour. This basic scheme would lead to certain supply responses that can be estimated from the model presented in this paper. These estimates of demand for and supply of labour can be used to calculate the resultant reduction in unemployment under the basic scheme. It is possible that the resultant employment situation remains in imbalance, depending on the elasticities of substitution on the demand side between different age-sex groups.

Alternative ORANI experiments can be conducted by holding the policy package in the basic scheme unchanged but by applying differential wage rate reductions to different types of workers - say adults and juniors - such that the reduction in the overall real hourly wage cost is the same as in the basic scheme. These experiments would provide alternative sets of estimates of consequent changes in the demand for labour and give details of the composition of such changes. The results in this paper suggest that the labour supply response of married men to wage rate changes is likely to be minimal. Single persons, who are mostly juniors, however, would have a somewhat different response under each alternative scheme from the one under the basic scheme. The changes in the supply of labour hours can be estimated using elasticity estimates from this paper. These sets of estimates of changes in the demand for and supply of labour can now be used to calculate the overall effect on unemployment under the alternative schemes.

1 *Op.cit.*.
2 In recent years the unemployment rates have been higher for younger members of the workforce.

Interactive Analysis

While a proper analysis of the interaction between the demand and supply of labour in the medium and long term framework must await the development of all the modules of the IMPACT model and the interface structure, it would be possible to use the results in this paper for the analysis of short term problems on the assumption that most of the demographic factors are relatively stable in the short run (say six to eighteen months).

With this assumption, it would be possible to compare alternative wage policies within the overall economic package suggested in a recent study[1] based on the ORANI model of the IMPACT Project. This study indicated that a reduction in real hourly wage costs would help in reducing unemployment by increasing the demand for labour. If the increased demand is partly met by an increase in the number of hours worked by those already employed, the reduction in unemployment may be less than expected. The model presented in this paper would be useful to study the supply responses of the workers who are currently employed. It would be possible to investigate the effects of differential reductions in wage rates that have the same overall effect on wage costs but which would minimize the likely increase in the supply of hours by the workers who are currently employed. The analysis may be conducted along the following lines:

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1 P. Dixon, A. Powell and B. Parmenter, Structural Adaptation in an Ailing Macroeconomy, Melbourne University Press, Melbourne, 1979.

(3.1.4b) $$\frac{\beta_{s1}}{s_1} - \lambda_1 = 0,$$

(3.1.4c) $$\frac{\beta_5}{L^{ml} - \lambda_L^{ml}} - \lambda_1 w_m^m = 0,$$

and

(3.1.4d) $$\frac{\beta_6}{L^{fl} - \gamma_L^{fl}} - \lambda_1 w_m^f = 0.$$

An expression for λ_1 can be obtained by making use of the condition[1] $\beta_1 + \beta_{s1} = 1$:

(3.1.5) $$\lambda_1 = 1/\left(w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1 - p\,\gamma_1\right).$$

After substituting for λ_1 from (3.1.5) into equations (3.1.4a) to (3.1.4d) and rearranging, the expenditure system becomes:

(3.1.6) $$px_1 = p\,\gamma_1 + \frac{\beta_1}{1 + \beta_5 + \beta_6}\left(w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1 - p\,\gamma_1\right),$$

(3.1.7) $$s_1 = \frac{\beta_{s1}}{1 + \beta_5 + \beta_6}\left(w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1 - p\,\gamma_1\right),$$

(3.1.8) $$-\,w_m^m H^{ml} = -w_m^m \gamma_5 + \frac{\beta_5}{1 + \beta_5 + \beta_6}\left(w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1 - p\,\gamma_1\right),$$

and

(3.1.9) $$-\,w_m^f H^{fl} = -w_m^f \gamma_6 + \frac{\beta_6}{1 + \beta_5 + \beta_6}\left(w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1 - p\,\gamma_1\right),$$

where

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1 Note that here we are not normalizing over consumption, savings and leisure as in Tulpulē, *op. cit.*, but over consumption and savings alone. The coefficients can still be renormalised to obtain results comparable to those obtained using other normalisations.

$$\gamma_5 = T_1 - \gamma_L^{ml}, \; H^{ml} = T_1 - L^{ml};$$

$$\gamma_6 = T_2 - \gamma_L^{fl}, \; H^{fl} = T_2 - L^{fl};$$

T_1 = total hours available to the husband for work and leisure;

and

T_2 = total hours available to the working wife for work and leisure.

The total hours available for work and leisure could vary according to the demographic characteristics of the worker. As far as the derivation of the demand system is concerned, however, the number is immaterial as it does not appear in the system of equations.

3.2 Decision Making Units of Types 2, 3 and 4

Utility functions and budget constraints similar to (3.1.1) and (3.1.2) can be written for representative DMUs of types 2, 3 and 4. The utility functions will include only one leisure hour term and the budget constraint will include only one labour income term on the right hand side. Three separate expenditure systems can now be obtained by maximization of utility functions subject to the relevant budget constraints. Thus the expenditure system for DMU Type 2 becomes:

$$(3.2.1) \quad px_2 = p\,\gamma_2 + \frac{\beta_2}{1+\beta_7}\left(w_m^m\,\gamma_7 + \alpha_2 - p\,\gamma_2\right),$$

$$(3.2.2) \quad s_2 = \frac{\beta_{s2}}{1+\beta_7}\left(w_m^m\,\gamma_7 + \alpha_2 - p\,\gamma_2\right),$$

and

$$(3.2.3) \quad -\,w_m^m\,H^{m2} = -\,w_m^m\,\gamma_7 + \frac{\beta_7}{1+\beta_7}\left(w_m^m\,\gamma_7 + \alpha_2 - p\,\gamma_2\right).$$

In conjunction with other elements of the IMPACT framework, the model developed above allows a broad range of questions to be studied. Consider, for example, the large increase in the workforce participation of married women in Australia. Married women tend to work shorter hours. If the participation rate continues to rise, how many extra hours of labour will be supplied? What will be the effect on consumption expenditure? What would be the effect on hours supplied by married men? Such questions can be addressed by the model in conjunction with the rest of the BACHUROO model.[1]

Policy Issues

Due to the equal pay legislation and plateau wage indexation, male/female and adult/junior wage relativities have changed significantly since 1974. The TELESH model could be used to estimate the likely effects on hours supplied due to a further narrowing of wage relativities. It thus has potential for studying the effects of national wage and indexation decisions. The model can also be used to study the effects of different taxation policies that may affect work incentives differently. In particular, it would be interesting to compare the effects of increasing the family allowance (i.e., of changing the non-labour income) as against reducing the tax rate (i.e., changing after tax wage rates).

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1 A. Tulpulé and M.K. McIntosh, "BACHUROO - An Economic Demographic Module for Australia" IMPACT Working Paper No. B-02, Industries Assistance Commission, Melbourne, May 1976.

In a previous paper[1] the supply of labour hours was modelled in a situation where, because overtime is voluntary and payment for such work is at a higher rate than the basic hourly rate, the marginal wage rate is endogenous. Elasticity formulae for estimating the effects of changes in the conditions of employment, *viz.*, basic hourly rate, standard work hours and overtime rate, were derived[2].

Numerical estimates of the elasticities can be made if the marginal leisure preference parameter for a homogeneous group of workers facing the same conditions of employment is known. This paper provides such estimates for different types of workers. While the specification of the model in the current paper differs from that of the previous paper[3], it seems likely that the parameter values estimated in this paper would be sufficiently robust to allow their use in the more general model.

Simulation and Projection

The various elasticities in Section 6 give the estimated impact on the supply of hours of changes in one exogenous variable at a time. In practice many of the exogenous factors change simultaneously. The TELESH model of Section 3 enables one to simulate the effects on the supply of labour hours of simultaneous changes in the demographic composition of the workforce, in wage levels, in wage relativities, and in the rate of inflation.

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1 Alan A. Powell, Ashok Tulpulé and Richard J. Filmer, "Commodity Specific Subsidies, Demand Patterns and the Incentive to work", IMPACT Preliminary Working Paper No. BP-10, Industries Assistance Commission, Melbourne, November 1977.

2 Empirical estimates of these elasticities are given in Ashok Tulpulé, "Effects on the Supply of Labour Hours by Employees of Changes in Their Conditions of Employment" Paper Presented to Economic Society of Australia and New Zealand, Ninth Conference of Economists, University of Queensland, Brisbane, August 1980.

3 *Ibid.*.

The expenditure system for DMU Type 3 becomes:

$$(3.3.1) \quad px_3 = p\,\gamma_3 + \frac{\beta_3}{1+\beta_8}\left(w_s^m \gamma_8 + \alpha_3 - p\,\gamma_3\right),$$

$$(3.2.2) \quad s_3 = \frac{\beta_{s3}}{1+\beta_8}\left(w_s^m \gamma_8 + \alpha_3 - p\,\gamma_3\right),$$

and

$$(3.3.3) \quad -\,w_s^m H^{m3} = -\,w_s^m \gamma_8 + \frac{\beta_8}{1+\beta_8}\left(w_s^m \gamma_8 + \alpha_3 - p\,\gamma_3\right).$$

The expenditure system for DMU Type 4 becomes:

$$(3.4.1) \quad px_4 = p\,\gamma_4 + \frac{\beta_4}{1+\beta_8}\left(w_s^f \gamma_9 + \alpha_4 - p\,\gamma_4\right),$$

$$(3.4.2) \quad s_4 = \frac{\beta_{s4}}{1+\beta_9}\left(w_s^f \gamma_9 + \alpha_4 - p\,\gamma_4\right),$$

and

$$(3.4.3) \quad -\,w_s^f H^{f4} = -\,w_s^f \gamma_9 + \frac{\beta_9}{1+\beta_9}\left(w_s^f \gamma_9 + \alpha_4 - p\,\gamma_4\right).$$

The notation in equations (3.2.1) to (3.4.3) is as follows:

x_i = a scalar quantity index of the bundle of goods consumed by a representative household of type i, $i = 2, 3, 4$;

γ_i = 'subsistence' level of consumption for DMU Type i, $i = 2, 3, 4$;

β_i = marginal propensity to consume for DMU Type i, $i = 2, 3, 4$;

β_{si} = marginal propensity to save for DMU Type i, $i = 2, 3, 4$;

β_7 = marginal propensity to consume leisure for the worker associated with DMU Type 2;

β_8 = marginal propensity to consume leisure for the worker associated with DMU Type 3;

β_9 = marginal propensity to consume leisure for the worker associated with DMU Type 4;

γ_7 = maximum feasible work hours for the worker associated with DMU Type 2;

γ_8 = maximum feasible work hours for the worker associated with DMU Type 3;

γ_9 = maximum feasible work hours for the worker associated with DMU Type 4;

H^{m2} = hours worked by the worker in household type 2 (married male with a non-working wife);

H^{m3} = hours worked by the worker in household type 3 (single male);

H^{f4} = hours worked by the worker in household type 4 (single female);

w_m^m = as before, after tax hourly wage rate for married males;

w_s^m = after tax hourly wage rate for single males;

and

w_s^f = after tax hourly wage rate for single females.

3.3 Combined Expenditure System, TELESH, For All Decision Making Units

Suppose that there are n_1, n_2, n_3 and n_4 DMUs of types 1, 2, 3 and 4 respectively. The total number of DMUs is $N = n_1 + n_2 + n_3 + n_4$. The consumption expenditures and savings of the four types of households are not known but time series data on the average consumption and saving for all households combined can be obtained from the national accounts. A single consumption equation can be written by taking a weighted average of equations (3.1.6), (3.2.1), (3.3.1) and (3.4.1),

7. PROPOSED APPLICATIONS OF THE RESULTS

The above analysis shows that the marginal leisure preference parameters differ significantly according to workers' demographic characteristics. It would be possible to use these estimates in the calculation of elasticities of labour supply for workers classified into several groups and in the analysis of some policy related issues. Some possible uses are discussed below.

Occupation and Sex Specific Elasticities

If the marginal leisure preference parameters are assumed to be the first order parameters that are invariant with respect to other characteristics of workers such as their occupation, industry and age[1], then it would be possible to calculate occupation and sex specific elasticities of supply of labour hours. For each occupation and sex group the elasticity would be a weighted average of elasticities for married and single persons, the weights being the proportions of hours worked by married and single persons in each group.

Analysis of Changes in Conditions of Employment

In the current paper the marginal hourly wage rate for each type of worker was treated as exogenous and independent of the actual hours worked.

1 Classification of workers by marital status, to some extent, takes into consideration the effect of age.

TABLE 6.4 : OWN WAGE ELASTICITIES FOR DIFFERENT TYPES OF WORKERS CALCULATED AT SEVERAL LEVELS OF HOURS WORKED

Hours Worked		DMU and Worker Type				
		1		2	3	4
Weekly (No.)	Annual ('000)	Married Male* with Working Wife	Married Working Female	Married Male* with Non-Working Wife	Single Male	Single Female
32	1.664	0	0.2842	0	0.1835	0.0552
36	1.872	0	0.1415	0	0.0520	-0.0620
40	2.080	0	0.0274	0	-0.0532	-0.1558
44	2.288	0	-0.0660	0	-0.1393	-0.2326
48	2.496	0	-0.1439	0	-0.2110	-0.2965
52	2.704	0	-0.2097	0	-0.2717	-0.3506

* As we have assumed that $\beta_5 = 0 = \beta_7$, it is clear from equation (3.1.8) and (3.2.3) that the only permissible values of hours worked by married males, H^{m1} and H^{m2}, are γ_5 and γ_7 respectively. Thus at each level of hours worked $H^{m1} \equiv \gamma_5$ and $H^{m2} \equiv \gamma_7$. Therefore own wage elasticities for married males are zero.

The above analysis suggests that the famous backward sloping labour supply curve does not operate everywhere in the hours/wage rate space; and the extent to which it does operate depends on the demographic characteristics of the worker.

the weights being $\frac{n_1}{N}$, $\frac{n_2}{N}$, $\frac{n_3}{N}$ and $\frac{n_4}{N}$ respectively. Similarly, a single savings equation can be written. After rearranging the terms on the right hand side, the combined system, TELESH, becomes:

(3.5.1) $$-w_m^m H^{m1} = -w_m^m \gamma_5 + \frac{\beta_5}{1+\beta_5+\beta_6}\left(w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1 - p\,\gamma_1\right);$$

(3.5.2) $$-w_m^f H^{f1} = -w_m^f \gamma_6 + \frac{\beta_6}{1+\beta_5+\beta_6}\left(w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1 - p\,\gamma_1\right);$$

(3.5.3) $$-w_m^m H^{m2} = -w_m^m \gamma_7 + \frac{\beta_7}{1+\beta_7}\left(w_m^m \gamma_7 + \alpha_2 - p\,\gamma_2\right);$$

(3.5.4) $$-w_s^m H^{m3} = -w_s^m \gamma_8 + \frac{\beta_8}{1+\beta_8}\left(w_s^m \gamma_8 + \alpha_3 - p\,\gamma_3\right);$$

(3.5.5) $$-w_s^m H^{f4} = -w_s^f \gamma_9 + \frac{\beta_9}{1+\beta_9}\left(w_s^f \gamma_9 + \alpha_4 - p\,\gamma_4\right);$$

(3.5.6) $$\begin{aligned} p_x\bar{x} = {} & \gamma_1\left(1 - \frac{\beta_1}{1+\beta_5+\beta_6}\right) p\,\frac{n_1}{N} + \gamma_2\left(1 - \frac{\beta_2}{1+\beta_7}\right) p\,\frac{n_2}{N} \\ & + \gamma_3\left(1 - \frac{\beta_3}{1+\beta_8}\right) p\,\frac{n_3}{N} + \gamma_4\left(1 - \frac{\beta_4}{1+\beta_9}\right) p\,\frac{n_4}{N} \\ & + \gamma_5\left(\frac{\beta_1}{1+\beta_5+\beta_6}\right) w_m^m \frac{n_1}{N} + \gamma_6\left(\frac{\beta_1}{1+\beta_5+\beta_6}\right) w_m^f \frac{n_1}{N} \\ & + \frac{\beta_1}{1+\beta_5+\beta_6}\,\alpha_1\,\frac{n_1}{N} \\ & + \gamma_7\left(\frac{\beta_2}{1+\beta_7}\right)\frac{n_2}{N} w_m^m + \frac{\beta_2}{1+\beta_7}\frac{n_2}{N}\alpha_2 \\ & + \gamma_8\left(\frac{\beta_3}{1+\beta_8}\right)\frac{n_3}{N} w_s^m + \frac{\beta_3}{1+\beta_8}\frac{n_3}{N}\alpha_3 \\ & + \gamma_9\left(\frac{\beta_4}{1+\beta_9}\right)\frac{n_4}{N} w_s^f + \frac{\beta_4}{1+\beta_9}\frac{n_4}{N}\alpha_4; \end{aligned}$$

and

$$(3.5.7) \quad \bar{s} = \frac{\beta_{s1}}{1 + \beta_5 + \beta_6} \left\{ \gamma_5 w_m^m \frac{n_1}{N} + \gamma_6 w_m^f \frac{n_1}{N} + \alpha_1 \frac{n_1}{N} - \gamma_1 p \frac{n_1}{N} \right\}$$

$$+ \frac{\beta_{s2}}{1 + \beta_7} \left\{ \gamma_7 w_m^m \frac{n_2}{N} + \alpha_2 \frac{n_2}{N} - \gamma_2 p \frac{n_2}{N} \right\}$$

$$+ \frac{\beta_{s3}}{1 + \beta_8} \left\{ \gamma_8 w_s^m \frac{n_3}{N} + \alpha_3 \frac{n_3}{N} - \gamma_3 p \frac{n_3}{N} \right\}$$

$$+ \frac{\beta_{s4}}{1 + \beta_9} \left\{ \gamma_9 w_s^f \frac{n_4}{N} + \alpha_4 \frac{n_4}{N} - \gamma_4 p \frac{n_4}{N} \right\}.$$

In order to keep the variance-covariance matrix of the stochastic errors (which are later appended to the behavioural equations) non-singular, one of the equations must be deleted[1]. The last equation (savings equation) will be deleted and the system estimated via equations (3.5.1) to (3.5.6).

1 The choice of equation eliminated is arbitrary. The maximum likelihood estimates of the parameters of the system are invariant to the equation deleted. See Anton Barten, "Maximum Likelihood Estimation of a Complete Set of Demand Equations", European Economic Review, Vol. 1 (1969), pp. 7-73.

equivalent term $\left\{p\gamma_4 - \alpha_4\right\}$, which represents the excess of subsistence expenditure over non-labour income, is expected to be positive and hence a negative elasticity value is expected. The difference between these two cases reflects differences in the relative strengths of the income and substitution effects. In the case of a married female worker associated with DMU Type 1, the income effect of a change in her wage rate is attenuated (relatively speaking) because of the fact that her labour income represents only about a third of the DMU's labour income.

The estimates of cross wage elasticities suggest that a 10 per cent increase in a husband's wage rate would reduce his working wife's labour supply by 5½ per cent, whereas a similar increase in the wife's wage rate would have no effect on her husband's labour supply. The own wage elasticities for different types of workers shown in Table 6.3 differ partly because the average hours worked by different types of workers differ. At the same level of hours worked the differences in the wage elasticities are smaller as can be seen from Table 6.4.

The estimates in Table 6.4 show that the differences between different types of married workers are not as large as seems to be suggested by Table 6.3. The overall pattern for different types of workers is such that for higher work hour levels the elasticities become negative. In other words the elasticity estimates indicate that a worker who is initially working fewer hours per week, would supply more work hours in response to an increase in the hourly wage rate, whereas a worker who is initially working long hours, would reduce the supply of work hours in response to an increase in the hourly wage rate. This tendency is most pronounced in the case of single females and least in the case of married females.

The income elasticities for married males are assumed to be zero. For all other types of workers the estimated elasticity is negative. The numerical value is highest for married females followed by single females, and lowest for the single males. (The pure income elasticities also show a similar pattern.) The figures indicate that a 10 per cent increase in income caused by equi-proportional increases in wage rates and non-labour income would reduce the labour hours supplied by a married female by 4.3 per cent as compared to 1.2 per cent for single females and 0.9 per cent for single males. On the other hand the pure income elasticities are much higher in absolute value. In the case of the working wife associated with DMU Type 1 the pure income elasticity is estimated to be -1; that is, a one per cent increase in the household income caused solely by an increase in its non-labour component would lead to a one per cent reduction in hours of work offered by her.

The own wage elasticity for married females is positive and for single persons it is negative. This suggests that when hourly wage rates increase, married females would increase their labour supply, whereas single persons would reduce their labour supply. The result for married females is rather surprising. The reason becomes clear when the elasticity formulae is re-written by substituting for H^{f1} from (6.10) into (6.19):

$$\text{(6.19a)} \qquad \eta^{f1} = \frac{-\beta_6}{1 + \beta_5 + \beta_6} \frac{1}{w^f_m H^{f1}} \left(pY_1 - w^m_m Y_5 - \alpha_1 \right).$$

The presence of the term $w^m_m Y_5$, which is the husband's "full" labour income, is critical because it causes the term in the bracket to be negative and hence yield a positive elasticity estimate. For a single female the

4. ESTIMATES OF DATA SERIES 1964-65 TO 1975-76

Even for the simplified approach adopted in this paper, the requisite data are not readily available from published sources. In this section the assumptions made in assembling the data base are described in detail. Some readers may wish to proceed directly to Section 5, where estimation is discussed.

Number of DMUs

The number of workers by sex and marital status are available for four quarters each year since 1964 from the Labour Force Surveys[1]. It is assumed that the number of households of type 1 is the same as the number of working married women. The number of households of type 2 is assumed to be the number of married working males less the number of working married females. It is assumed that the numbers of households of the third and fourth type are the numbers of not-married working males and females respectively. Averages of four quarterly estimates were obtained for each financial year 1964-65 to 1975-76.

Hours Worked

Estimates of average weekly hours worked by males, married females and not-married females are published in the Labour Force Survey reports[2]. Estimates of average hours worked by married and other males are available for the August quarter from 1971 onwards[3]. These figures

1 Australian Bureau of Statistics, The Labour Force (Ref. No. 6.22). Figures for 1964 to 1968 are from Historical Supplement to the Labour Force, and for other years from the annual reports.
2 ABS, *ibid.*.
3 Unpublished data.

show that the average hours worked by single males have been decreasing at a faster rate than hours worked by married males; thus the ratio of hours worked by other males to hours worked by married males has slowly decreased from 0.9410 to 0.9146 between August 1971 and August 1976. Estimates of hours worked by married and other males for earlier years and all the four quarters in each year are obtained by fitting a linear regression to the data on the ratio K_t, of hours worked by married and by other males, in the t^{th} quarter:

(4.1) $$K_t = 0.9714 - 0.0010327t \qquad (R^2 = 0.7275).$$

Estimates of K_t obtained from equation (4.1), along with the data on average hours worked by all males, $\overline{H}_t$, in each quarter, were used to obtain estimates for married and other males as follows:

(4.2) $$H_{a,t} = \frac{n_{at} + n_{bt}}{n_{at} + K_t n_{bt}} \cdot \overline{H}_t,$$

(4.3) $$H_{b,t} = \frac{K_t \left(n_{at} + n_{bt}\right)}{n_{at} + K_t n_{bt}} \cdot \overline{H}_t,$$

where $H_{a,t}$ and $H_{b,t}$ are estimates of hours worked by married and single males in the t^{th} quarter and n_{at} and n_{bt} are the numbers of married and single male workers respectively. Thus $H_{a,t}$ is an estimate of the average hours worked by all married males, i.e., males associated with DMU types 1 and 2.

Estimates of annual hours worked are obtained from the estimates of number of workers, hours worked per worker per week, and weeks per quarter. Separate estimates of hours worked by married men whose wives were, or were not working, are not available. Estimates are obtained as

TABLE 6.2 : AVERAGE VALUES OF VARIABLES OVER THE SAMPLE PERIOD, 1964-65 TO 1975-76

Variable	DMU and Worker Type				
	1		2	3	4
	Married Male with Working Wife	Married Working Female	Married Male with Non-Working Wife	Single Male	Single Female
Hourly Wage Rate ($)	2.1472	1.4877	2.1472	1.7732	1.2777
Annual Hours Worked ('000)	2.1089	1.6173	2.2448	2.0743	1.8297
Labour Income ($000)	4.5130	2.3707	4.8315	3.6527	2.3082
Labour and Non-Labour Income ($000)	8.6619		6.0579	4.3439	3.1074
Commodity Price Level:			1.2416		

TABLE 6.3 : ESTIMATES OF ELASTICITIES OF WORK HOURS SUPPLIED (& ASYMPTOTIC STANDARD ERRORS) FOR WORKERS ASSOCIATED WITH DIFFERENT TYPES OF DMUS CALCULATED AT THE AVERAGE VALUES OF THE VARIABLES

Elasticity	DMU and Worker Type				
	1		2	3	4
	Married Male with Working Wife	Married Working Female	Married Male with Non-Working Wife	Single Male	Single Female
Income Elasticity	0	-0.4324 (0.0105)	0	-0.0880 (0.0035)	-0.1273 (0.0050)
Own Wage Elasticity	0	0.3213 (0.0088)	0	-0.0506 (0.0034)	-0.0403 (0.0035)
Cross Wage Elasticity	0	-0.5254 (0.0105)	-	-	-
Pure Income Elasticity	0	-1.0023 (0.0201)	0	-0.3282 (0.0119)	-0.3875 (0.0134)
Cross Price Elasticity	0	0.3875 (0.0078)	0	0.0958 (0.0035)	0.1304 (0.0045)

The wage elasticity for a given type of worker depends on the actual hours worked. For a single worker whose actual hours worked are much less than his maximum feasible hours, the elasticity would be positive; and its value would fall and become negative as the actual hours worked increase, thus reducing the ratio of maximum feasible hours to actual work hours. The cross wage elasticities are negative indicating a fall in the labour hours supplied by the married male with a working wife when the female wage rate increases, and *vice versa*.

Numerical estimates of elasticities given in Table 6.3 are calculated at the average values of the variables over the sample period shown in Table 6.2.

follows. Hours worked by married men with working wives (H^{m1}) and by those with non-working wives (H^{m2}) are derived on the assumption that the ratio of incomes of household types 1 and 2, $\frac{Y_1}{Y_2}$ is a linear function of time. The incomes of household types 1 and 2, Y_1 and Y_2 respectively, can be defined as a sum of their labour and non-labour components:

$$Y_1 = w_m^m H^{m1} + w_m^f H^{f1} + \alpha_1,$$

and

$$Y_2 = w_m^m H^{m2} + \alpha_2 ,$$

where H^{f1} is the average number of hours worked by working married females. Published data on units with a working, or with a non-working wife, are not available; however, income data for familes with one and two workers in 1968-69 and 1973-74 are available from ABS surveys[1]. It is expected that the ratio for the two types of households would be similar to that obtained for the two types of families. The survey estimates of incomes of families with two and one worker give estimates of the ratio for 1968-69 and 1973-74 as 1.4265 and 1.4326 respectively. If it is assumed that the slow trend rate of increase between the two survey years is applicable for the whole sample period 1964-65 to 1975-76, then an estimate of the ratio for each year can be obtained by making use of a trend line passing through the two data points.

Thus estimates of hours worked by married males in DMU types 1 and 2 in year t, H_t^{m1} and H_t^{m2} can be obtained by solving

$$(4.4) \quad \frac{w_{mt}^m H_t^{m1} + w_{mt}^f H_t^{f1} + \alpha_{1t}}{w_{mt}^m H_t^{m2} + \alpha_{2t}} = A + Bt,$$

1 Australian Bureau of Statistics, Income Distribution 1973-74, Part 2 (Ref. No. 17.8), Canberra, 1979; and similar publication for 1968-69 (ABS Ref. No. 17.17).

and

(4.5) $n_{1t} H_t^{m1} + n_{2t} H_t^{m2} = \left(n_{1t} + n_{2t}\right) H_{at}$,

provided the values of A and B and the wage rate and non-labour income series are known. The derivation of these two series is discussed later. In 1968-69 and 1973-74 the ratio on the left of (4.4) is known. Thus from these two data points we are able to deduce $A = 1.42046$ and $B = 0.001216$, and then to solve for H_t^{m1} and H_t^{m2} over the period 1964-65 to 1975-76. In fact, the ratio of incomes on the left of (4.4) increases slowly over time due mainly to the faster growth of female labour incomes relative to male labour incomes. The ratio of hours worked $\frac{H^{m1}}{H^{m2}}$ increases steadily until 1969-70 and then decreases as shown in Table 4.1. Thus, the estimates show that the hours worked by married males with working wives decreased from 41½ to 39 hours per week whereas for married males with non-working wives the figure remained almost stationary at 43 hours per week.

TABLE 4.1 : ESTIMATES OF AVERAGE HOURS WORKED BY MARRIED MALES WITH WORKING AND NON-WORKING WIVES

Year	Annual hours worked by married males[b] with			Ratio
	Working wife H^{m1}	Non-working wife H^{m2}	Average[a]	$\frac{H^{m1}}{H^{m2}}$
1964-65	2071.1	2224.5	2181.8	0.9310
1965-66	2076.2	2213.9	2173.7	0.9378
1966-67	2098.4	2219.9	2182.3	0.9453
1967-68	2120.9	2242.3	2202.5	0.9459
1968-69	2140.9	2227.6	2197.9	0.9611
1969-70	2158.6	2244.2	2213.1	0.9619
1970-71	2153.9	2249.1	2212.4	0.9577
1971-72	2140.7	2263.4	2215.1	0.9458
1972-73	2127.4	2248.5	2197.8	0.9461
1973-74	2128.3	2282.2	2214.3	0.9326
1974-75	2063.3	2264.6	2173.2	0.9111
1975-76	2026.8	2256.9	2149.7	0.8980

a Estimated from equation (4.2).
b Estimated from equations (4.4) and (4.5) with $A = 1.42046$ and $B = 0.001216$.

(6.24) $\eta_{hc}^{m1} = \frac{\beta_5}{1 + \beta_5 + \beta_6} \frac{pY_1}{w_m^m H^{m1}}$;

and

(6.25) $\eta_{hc}^{f1} = \frac{\beta_6}{1 + \beta_5 + \beta_6} \frac{pY_1}{w_m^f H^{f1}}$.

Formulae for the cross price elasticities for workers associated with DMU Types 2, 3 and 4 can be obtained in a similar manner. These formulae are shown in Table 6.1.

property. For DMU Type 1 when α_1 alone changes, differentiating (6.9) and (6.10) with respect to α_1 gives the pure income elasticities e_h^{*m1} and e_h^{*f1} as follows:

$$(6.22) \qquad e_h^{*m1} = \frac{\partial H^{m1}}{\partial \alpha_1} \frac{y_1}{H^{m1}} = \frac{-\beta_5}{1 + \beta_5 + \beta_6} \frac{y_1}{w_m^m H^{m1}} ,$$

$$(6.23) \qquad e_h^{*f1} = \frac{\partial H^{f1}}{\partial \alpha_1} \frac{y_1}{H^{f1}} = \frac{-\beta_6}{1 + \beta_5 + \beta_6} \frac{y_1}{w_m^f H^{f1}} .$$

For DMU Types 2, 3 and 4 the formulae are shown in Table 6.1.

Table 6.1 shows that all the income elasticities will be negative when the marginal leisure preferences are positive. The sign of an own wage elasticity can be either positive or negative depending on whether the second term in the formulae in row 2 of Table 6.1 is less than or greater than one.

Cross Price Elasticities

The effect of a change in the price index, p, on the supply of work hours by workers associated with DMU Type 1 can be estimated by differentiating (6.9) and (6.10) with respect to p. The elasticities η_{hc}^{m1} and η_{hc}^{f1} give the percentage changes in hours supplied by the male and female worker associated with DMU Type 1 respectively when the commodity price level p, alone changes by one per cent:

Hourly Wage Rates

The after-tax hourly wage rates for married and single males and females, w_m^m, w_s^m, w_m^f and w_s^f, are estimated by making use of three sets of data for 1964-65 to 1975-76:

a. after-tax labour income of all workers[1];

b. hourly wage rates for junior and adult males and females[2]; and

c. number of workers classified by age (adult and junior), sex and marital status[3].

The data on labour income provide a control total which ensures that in each year the four separate wage rates multiplied by the corresponding estimates of hours worked add up to the estimated total after tax labour income earned by all workers. The procedure adopted is as follows.

Indices of the four wage rates for married and single males and females, I_m^m, I_s^m, I_m^f and I_s^f, are obtained as weighted averages of junior and adult wage rates, the weights being the proportions of juniors and adults in each sex and marital status category. Thus the index for married males I_m^m is given by:

$$(4.6) \qquad I_m^m = \left(N_j^m w_j^m + N_a^m w_a^m\right) \Big/ \left(N_j^m + N_a^m\right),$$

where N_j^m and N_a^m are the numbers of junior and adult married males

1 Ashok Tulpulē, *op. cit.*.
2 Australian Bureau of Statistics, Survey of Weekly Earnings and Hours 1972 (Ref. No. 6.1) for 1964 to 1972; and ABS Ref. No. 6.40, for 1973 onwards.
3 ABS, The Labour Force, *op. cit.*.

respectively; and w_j^m and w_a^m are the hourly wage rates for junior and adult males respectively.

Indices for single males I_s^m, married females I_m^f and single females I_s^f are obtained for each year in a similar manner. For year t, adding a subscript t, the after tax hourly wage rates $\left(w_{mt}^m, w_{st}^m, w_{mt}^f \text{ and } w_{st}^f\right)$ are calculated by multiplying the corresponding indices $\left(I_{mt}^m, I_{st}^m, I_{mt}^f \text{ and } I_{st}^f\right)$ by an appropriate scaling factor, R_t, which is calculated from:

$$(4.7) \qquad R_t = WB_t \Big/ \left(I_{mt}^m H_{mt}^m + I_{st}^m H_{st}^m + I_{mt}^f H_{mt}^f + I_{st}^f H_{st}^f\right),$$

where WB_t is the total after tax wage bill, and H_{mt}^m is the total hours worked by all married males in year t. The other H's are similarly defined.

The above procedure ensures that the four separate after tax wage rates are consistent with the overall rate. For married males, the procedure is likely to produce an underestimate and for others an over-estimate of after tax wage rates because no account is taken of the different tax rates that may apply to different types of workers.

Non-labour Income

For four types of families, that are similar to the four types of DMUs, estimates of non-labour income in 1966 and 1971 were obtained in an earlier working paper[1]. These were in the following proportions for 1966

1 Ashok Tulpulé, "Estimation and Mapping of the Distribution of Income for the IMPACT Model", IMPACT Preliminary Working Paper No. BP-05, Industries Assistance Commission, Melbourne, November 1976.

Cross Wage Elasticities

The work hours supplied by the male worker associated with DMU Type 1 will be affected by changes in the wage rate of married females because such a change affects the household income. The cross wage elasticity for the male worker associated with DMU Type 1, η^{*m1} (i.e., the effect of a one per cent change in the married female wage rate, w_m^f, on hours worked by the male), can be obtained by differentiating (6.9) with respect to w_m^f. Similarly the cross wage elasticity for the married female worker, with respect to a change in the married male wage rate, w_m^m, can be obtained by differentiating (6.10) with respect to w_m^m as follows:

$$(6.20) \qquad \eta^{*m1} = \frac{\partial H^{m1}}{\partial w_m^f} \frac{w_m^f}{H^{m1}} = \frac{-\beta_5}{1 + \beta_5 + \beta_6} \frac{w_m^f}{w_m^m} \frac{\gamma_6}{H^{m1}} \quad ,$$

$$(6.21) \qquad \eta^{*f1} = \frac{\partial H^{f1}}{\partial w_m^m} \frac{w_m^m}{H^{f1}} = \frac{-\beta_6}{1 + \beta_5 + \beta_6} \frac{w_m^m}{w_m^f} \frac{\gamma_5}{H^{f1}} \quad .$$

Pure Income Elasticities

The income elasticities given by (6.13) and (6.14) for DMU Type 1 and similar formulae for DMU Types 2, 3 and 4 assume an equi-proportional change in the non-labour income and wage rates. The pure income elasticity is the income derivative of hours supplied after compensation for changes in the ratio of the market price of commodities to the shadow price of leisure. In the case of DMU Type 1, the pure income elasticity is computed under conditions in which the male/female wage relativity is also frozen. In all cases the appropriately compensated derivative can be identified with the derivative of hours supplied with respect to that part of income whose changes induce no alteration in relative prices, and hence no substitution effects. The exogenous component of income, α, has this

TABLE 6.1 : ELASTICITY OF WORK HOURS SUPPLIED FORMULAE FOR DIFFERENT TYPES OF DECISION MAKING UNITS

Elasticity	DMU and Worker Types: 1		2	3	4
	Married Male with Working Wife	Married Working Female	Married Male with Non-Working Wife	Single Male	Single Female
Income Elasticity	$\frac{-\beta_5\ p\gamma_1}{w_m^m\gamma_5+w_m^f\gamma_6+\alpha_1}\ \frac{y_1}{w_m^m H^{m1}}$	$\frac{-\beta_6\ p\gamma_1}{w_m^m\gamma_5+w_m^f\gamma_6+\alpha_1}\ \frac{y_1}{w_m^f H^{f1}}$	$\frac{-\beta_7\ p\gamma_2}{w_m^m\gamma_7+\alpha_2}\ \frac{y_2}{w_m^m H^{m2}}$	$\frac{-\beta_8\ p\gamma_3}{w_s^m\gamma_8+\alpha_3}\ \frac{-y_3}{w_s^m H^{m3}}$	$\frac{-\beta_9\ p\gamma_4}{w_s^f\gamma_8+\alpha_4}\ \frac{y_4}{w_s^f H^{f4}}$
Own Wage Elasticity	$-1+\frac{\gamma_5}{H^{m1}}\left(1-\frac{\beta_5}{1+\beta_5+\beta_6}\right)$	$-1+\frac{\gamma_6}{H^{f1}}\left(1-\frac{\beta_6}{1+\beta_5+\beta_6}\right)$	$-1+\frac{\gamma_7}{H^{m2}}\left(1-\frac{\beta_7}{1+\beta_7}\right)$	$-1+\frac{\gamma_8}{H^{m3}}\left(1-\frac{\beta_8}{1+\beta_8}\right)$	$-1+\frac{\gamma_9}{H^{f4}}\left(1-\frac{\beta_9}{1+\beta_9}\right)$
Cross Wage Elasticity	$\frac{-\beta_5}{1+\beta_5+\beta_6}\ \frac{w_m^f\gamma_6}{w_m^m H^{m1}}$	$\frac{-\beta_6}{1+\beta_5+\beta_6}\ \frac{w_m^m\gamma_5}{w_m^f H^{f1}}$			
Pure Income Elasticity	$\frac{-\beta_5}{1+\beta_5+\beta_6}\ \frac{y_1}{w_m^m H^{m1}}$	$\frac{-\beta_6}{1+\beta_5+\beta_6}\ \frac{y_1}{w_m^f H^{f1}}$	$\frac{-\beta_7}{1+\beta_7}\ \frac{y_2}{w_m^m H^{m2}}$	$\frac{-\beta_8}{1+\beta_8}\ \frac{y_3}{w_s^m H^{m3}}$	$\frac{-\beta_9}{1+\beta_9}\ \frac{y_4}{w_s^f H^{f4}}$
Cross Price Elasticity	$\frac{\beta_5}{1+\beta_5+\beta_6}\ \frac{p\gamma_1}{w_m^m H^{m1}}$	$\frac{\beta_6}{1+\beta_5+\beta_6}\ \frac{p\gamma_1}{w_m^f H^{f1}}$	$\frac{\beta_7}{1+\beta_7}\ \frac{p\gamma_2}{w_m^m H^{m2}}$	$\frac{\beta_8}{1+\beta_8}\ \frac{p\gamma_3}{w_s^m H^{m3}}$	$\frac{\beta_9}{1+\beta_9}\ \frac{p\gamma_4}{w_s^f H^{f4}}$

and 1971 respectively:

1.45 : 1.00 : 0.50 : 0.68; and

1.47 : 1.00 : 0.59 : 0.64.

Estimates of non-labour incomes α_1, α_2, α_3 and α_4 for the four types of DMUs were obtained by assuming that the 1966 proportions apply for years 1964-65 to 1968-69 and the 1971 proportions apply for 1969-70 onwards, subject to the overall values of non-labour income per worker used in the previous paper[1].

The derived data series are given in Appendix I. It is clear from the above comments that some of the data series used are rather crude estimates obtained under assumptions that cannot be tested. It would be desirable to try to obtain better data series; in particular, on hours worked by married males with working and non-working wives. In principle, it would be possible to retrieve much of the missing information from Labour Force Surveys.

1 Tulpulĕ, "Revised Estimates of ... Elasticities", *op. cit.*.

5. ESTIMATION OF THE TELESH MODEL

In the absence of data on consumption and savings by households associated with the four types of DMUs it is not possible to obtain estimates of the consumption parameters, β_1, β_2, β_3, β_4 and γ_1, γ_2, γ_3, γ_4 for the four types of DMUs from the model described in Section 3. It is therefore necessary to obtain estimates of these parameters from alternative sources. Exogenous information is used to fix the relative magnitudes of the subsistence parameters and it is ensured that the values for the four DMUs are consistent with the overall value previously obtained.[1] By making use of the method described later, the relative magnitudes of the minimum needs parameters can be obtained from cross sectional estimates of average *household* sizes and of average propensities to consume for different types of *households* for 1966-67 obtained by Williams[2,3]. These papers give estimates for *households* with a working wife, *households* with non-working wife and single (working) person *households*. For the single person *households* separate estimates for males and females are not available. In the absence of estimates for DMUs the parameters for DMU Types 1 and 2 will be based on Williams' estimates for *households* with a working wife and with a non-working wife respectively. Parameters for DMU Types 3 and 4 will be based on single person *households*. As the definition of a *household* differs from that of a DMU the assumptions are not strictly valid. Some of the differences and similarities between DMUs and *households* are listed below:

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1 Tulpulé, "Revised Estimates of ... Elasticities" *op. cit.*.
2 R.A. Williams, "Household Consumption in Australia : An Examination of Patterns Across Socio-Economic Classes", IMPACT Preliminary Working Paper, No. SP-04, Industries Assistance Commission, Melbourne, May 1976.
3 R.A. Williams, "Wants and Working Wives : Household Demand and Saving in Australia", Economic Repord, Vol. 54, No. 145, April 1978, pp. 32-44.

and making use of the appropriate commodity demand equation, savings equation and labour supply equation. These results are summarised in Table 6.1.

Wage Elasticities

The uncompensated wage elasticity of hours worked by the male worker associated with DMU Type 1, η^{ml}, gives the percentage change in hours worked by the male worker due to a one per cent change in his wage rate, w_m^m, when commodity prices, the married female wage rate and non-labour income remain constant, i.e.,

$$(6.16)\quad \eta^{ml} = \frac{\partial H^{ml}}{\partial w_m^m} \frac{w_m^m}{H^{ml}}$$

From (6.9) we get

$$(6.17)\quad \frac{\partial H^{ml}}{\partial w_m^m} = \frac{-1}{w_m^m} H^{ml} + \frac{\gamma_5}{w_m^m}\left(1 - \frac{\beta_5}{1 + \beta_5 + \beta_6}\right)$$

Therefore :

$$(6.18)\quad \eta^{ml} = -1 + \frac{\gamma_5}{H^{ml}}\left(1 - \frac{\beta_5}{1 + \beta_6 + \beta_6}\right)$$

Similarly, the wage elasticity for the female worker associated with DMU type 1 is given by

$$(6.19)\quad \eta^{f1} = -1 + \frac{\gamma_6}{H^{f1}}\left(1 - \frac{\beta_6}{1 + \beta_5 + \beta_6}\right)$$

Wage elasticity formulae for workers associated with DMU Types 2, 3 and 4 are derived similarly and are shown in Table 6.1.

When y_1 changes as a result of equi-proportional changes in α_1, w_m^m and w_m^f, H^{m1} changes by δH^{m1} and H^{f1} changes by δH^{f1}. Expressions for δH^{m1} and δH^{f1} are obtained from (6.9) and (6.10) respectively:

$$(6.11) \quad \delta H^{m1} = \frac{\beta_5}{1 + \beta_5 + \beta_6} \left[\frac{-1}{w_m^m} \left(\frac{p}{w_m^m} \gamma_1 \right) \right] \delta w_m^m \quad ,$$

and

$$(6.12) \quad \delta H^{f1} = \frac{\beta_6}{1 + \beta_5 + \beta_6} \left[\frac{-1}{w_m^f} \left(\frac{p}{w_m^f} \gamma_1 \right) \right] \delta w_m^f \quad .$$

Income elasticities for the male and female workers associated with DMU Type 1 can be obtained by substituting from (6.8) into (6.11) and (6.12) and taking limits as $\delta y_1 \to 0$:

$$(6.13) \quad e_h^{m1} = \frac{\partial H^{m1}}{\partial y_1} \frac{y_1}{H^{m1}} = \frac{-\beta_5 p \gamma_1}{w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1} \frac{y_1}{w_m^m H^{m1}} ,$$

and

$$(6.14) \quad e_h^{f1} = \frac{\partial H^{m1}}{\partial y_1} \frac{y_1}{H^{f1}} = \frac{-\beta_6 p \gamma_1}{w_m^m \gamma_5 + w_m^f \gamma_6 + \alpha_1} \frac{y_1}{w_m^f H^{f1}} .$$

Income elasticity formulae for workers associated with DMU Types 2, 3 and 4 can be derived in a similar fashion by assuming

$$\frac{\delta\alpha_2}{\alpha_2} = \frac{\delta w_m^m}{w_m^m} ,$$

$$(6.15) \quad \frac{\delta\alpha_3}{\alpha_3} = \frac{\delta w_s^m}{w_s^m} ,$$

$$\frac{\delta\alpha_4}{\alpha_4} = \frac{\delta w_s^f}{w_s^f} ,$$

1. The four types of DMUs cover the entire working population whereas the three types of *households* do not.
2. *Households* with a working wife and with a non-working wife may include some families where the husband is not employed. The definition of DMUs Type 1 assumes that when a married woman is working her husband is also working.
3. DMU Types 3 and 4 include single working person *households* as well as single working persons who are living with other families (e.g., parents) and also groups of single persons living together. The subsistance parameter for these different types of persons included in DMU Types 3 and 4 would be different. It is not possible to make an allowance for such differences.

In spite of the above problems, estimates of the average propensity to consume for the three types of *households* and estimates of the average size of *households* with working and non-working wives respectively are used in estimating the values of parameters for the four types of DMUs because other suitable estimates are not available.

Subsistence Parameters

The subsistence parameter, γ_1, can be interpreted as the minimum quantity of a bundle of goods and services required by DMU Type 1. γ_2, γ_3 and γ_4 are similar quantities for DMU Types 2, 3 and 4 respectivel The average subsistence expenditure over the sample period for DMU of type i would be $\bar{p}\gamma_i$, where $\bar{p}$ is the average of the price index of all goods and services over the sample period. An estimate of the subsistence expenditure,

$\bar{p}\,\gamma_i$, can be obtained from the estimate of the Frisch[1] parameter ω_i for DMU Type i, which in turn can be estimated by making use of the relationship between ω and after tax income per head[2]:

$$(5.1) \qquad \omega_i = K\,\bar{Y}_i^{-0.36},$$

where

$\bar{Y}_i$ = average after tax income per head over the sample period in current Australian dollars for household type i;

and

K = -28.62.

This value of K ensures that the overall value of ω for all DMUs combined is consistent with the overall estimate of ω (-1.8744) obtained previously[3]. ω_i is related to sample mean expenditure, the price level and the subsistence parameter by

$$(5.2) \qquad \omega_i = -\,\bar{V}_i/(\bar{V}_i - \bar{p}\gamma_i),$$

where

$\bar{V}_i$ = average consumption expenditure of household type i over the sample period.

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1 R. Frisch, "A Complete System for Computing all Direct and Cross Elasticities in a Model with Many Sectors", Econometrica, 27 (1959), pp. 177-196.

2 Equation (5.1) is obtained from a similar relationship (5.1a) between ω and GNP per head in 1970 US dollars, X, by replacing X by Y and by scaling the constant term in

$$(5.1a) \qquad \omega = -\,36\,X^{-0.36}.$$

Equation (5.1a) was obtained using time series data for 14 countries by C. Lluch, A. Powell and R. Williams, Patterns in Household Demand and Saving, Oxford University Press, New York, 1977, pp. 76-77.

3 Tulpule, "Revised Estimates of ... Elasticities", *op. cit.*.

$$(6.5) \qquad e_h^{f1} = \frac{\partial H^{f1}}{\partial y_1}\,\frac{y_1}{H^{f1}}$$

Since $\beta_1 + \beta_{s1} = 1$, an expression for y_1 can be obtained by adding (3.1.6) and (3.1.7):

$$(6.6) \qquad y_1 = \left\{v_1 + s_1\right\} = p\,\gamma_1 + \frac{1}{1+\beta_5+\beta_6}\left\{w_m^m\,\gamma_5 + w_m^f\,\gamma_6 + \alpha_1 - p\gamma_1\right\}.$$

If there are small changes in α_1, w_m^m and w_m^f, denoted by $\delta\alpha_1$, δw_m^m and δw_m^f respectively, then assuming no change in the price level, y_1 will change by δy_1, where

$$(6.7) \qquad \delta y_1 = \frac{1}{1+\beta_5+\beta_6}\left\{\delta w_m^m\,\gamma_5 + \delta w_m^f\,\gamma_6 + \delta\alpha_1\right\}.$$

After substituting from (6.1), (6.7) becomes,

$$(6.8) \qquad \delta y_1 = \frac{1}{1+\beta_5+\beta_6}\left\{\gamma_5 + \frac{w_m^f}{w_m^m}\,\gamma_6 + \frac{\alpha_1}{w_m^m}\right\}\delta w_m^m.$$

The labour supply equations for the male and female worker associated with DMU Type 1 can be written by dividing equations (3.1.8) and (3.1.9) by $-w_m^m$ and $-w_m^f$ respectively:

$$(6.9) \qquad H^{m1} = \gamma_5 + \frac{\beta_5}{1+\beta_5+\beta_6}\,\frac{1}{-w_m^m}\left\{w_m^m\,\gamma_5 + w_m^f\,\gamma_6 + \alpha_1 - p\gamma_1\right\},$$

and

$$(6.10) \qquad H^{f1} = \gamma_6 + \frac{\beta_6}{1+\beta_5+\beta_6}\,\frac{1}{-w_m^f}\left\{w_m^m\,\gamma_5 + w_m^f\,\gamma_6 + \alpha_1 - p\gamma_1\right\}.$$

6. ELASTICITIES OF SUPPLY OF LABOUR HOURS

Formulae for income and own wage elasticities of work hours supplied are given below for each type of DMU. In the case of DMU Type 1, cross wage elasticities are also given for the hours supplied by the male and female workers.

Income Elasticities

The income elasticity measures the percentage change in hours supplied by a given DMU type when its income changes by one per cent. In practice it is unlikely that the income will change without a change in the wage rates. The income elasticity formulae are derived on the assumption that the non-labour income and the hourly wage rates change by the same proportion. Thus for DMU Type 1 it is assumed that

$$(6.1)\qquad \frac{\delta\alpha_1}{\alpha_1} = \frac{\delta w_m^m}{w_m^m} = \frac{\delta w_m^f}{w_m^f}.$$

Total income for DMU Type 1 is given by

$$(6.2)\qquad y_1 = \alpha_1 + w_m^m H^{m1} + w_m^f H^{f1} \quad \text{(=non-labour + labour income)},$$

and

$$(6.3)\qquad y_1 = v_1 + s_1 \quad \text{(=total expenditure + savings)}.$$

The income elasticities of hours supplied by the male and female worker associated with DMU Type 1 are given by (6.4) and (6.5) respectively:

$$(6.4)\qquad e_h^{m1} = \frac{\partial H^{m1}}{\partial y_1}\frac{y_1}{H^{m1}},$$

and

After substituting the value of ω_i from (5.1) into (5.2), the subsistence expenditure, $\bar{p}\gamma_i$, is given by:

$$(5.3)\qquad \bar{p}\gamma_i = \bar{V}_i\left(1 + \frac{1}{\omega_i}\right).$$

The average price $\bar{p}$ of all goods and services over the sample period, is estimated from National Accounts[1] data on total consumption expenditure at current and constant prices. Estimates of $\bar{V}_i$ for household type i can be obtained from data on the average income, $\bar{Y}_i$, and the average propensity to consume, c_i.

For the four types of households the average income figures over the sample period, $\bar{Y}_1$, $\bar{Y}_2$, $\bar{Y}_3$ and $\bar{Y}_4$, are obtained from the data on labour income and non-labour income. Thus

$$(5.4)\qquad \begin{aligned} \bar{Y}_1 &= \overline{w_m^m}\,\overline{H^{m1}} + \overline{w_m^f}\,\overline{H^{f1}} + \bar{\alpha}_1, \\ \bar{Y}_2 &= \overline{w_m^m}\,\overline{H^{m2}} + \bar{\alpha}_2, \\ \bar{Y}_3 &= \overline{w_s^m}\,\overline{H^{m3}} + \bar{\alpha}_3 \quad \text{and} \\ \bar{Y}_4 &= \overline{w_s^f}\,\overline{H^{f4}} + \bar{\alpha}_4. \end{aligned}$$

The average propensities to consume for *households* with a working wife, a non-working wife and for single person *households* obtained from data in papers by Williams are shown in Table 5.1.

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1 Australian Bureau of Statistics, "Australian National Accounts, National Income and Expenditure 1975-76" (Ref. No. 7.1), Canberra, 1977.

TABLE 5.1: ESTIMATES OF INCOME, TOTAL EXPENDITURE AND AVERAGE PROPENSITY TO CONSUME BY *HOUSEHOLD* TYPE, 1966-68

Household Type	Mean Income ($)	Mean Total Expenditure ($)	Average Propensity to Consume
Male head, working wife	5011	4621	0.922
Male head, non-working wife	4380	4164	0.950
Single working person	2662	2526	0.949
Weighted Average*			0.94326

* Average weights for 1964-65 to 1975-76, based on numbers of DMUs, are used.

Source: R.A. Williams, "Wants and Working Wives", *op.cit.*, and "Household Consumption in Australia." *op.cit.*.

The overall average propensity to consume figure in Table 5.1 is grossly overestimated as compared to the National Accounts estimate because in the survey the income figures are understated. However the relative values of average propensities to consume for the three types of *households* (as estimated by Williams) can be used to estimate the average consumption expenditure for the four types of DMUs. In the absence of separate data for single males and females the same average propensity to consume is assumed for DMU Types 3 and 4. Given the estimates of incomes and the average propensities, consumption expenditure per household associated with each type of DMU can be obtained. These preliminary estimates are then scaled so that the overall estimate of expenditure for all households combined is the same as the national average ($4992.21) over the sample period as determined from National Accounts data.

squares reasonably well with the long term experience: recall that the rate of decline in average hours worked by married men was almost zero (see Table 4.1). Maximum feasible hours for married females are estimated (perhaps surprisingly) to exceed those of single workers. In most empirical studies of consumer demand using LES type models, there are difficulties in estimating the subsistence parameters. These parameters are often unstable depending on factors such as commodity aggregation.' In this study the estimates of the minimum leisure requirements parameters for married females seem to be rather high; however the overall average of all workers is controlled in line with the previous results.[1] In general, when the estimated or assumed marginal leisure preference is high, so is the estimated maximum hours parameter[2]. In other words the estimates of marginal leisure preference parameters are positively correlated with the estimated maximum work hours parameter.[3] It is interesting (but not surprising) to note that the estimated marginal leisure preference parameters are high when the difference between the average hours worked and maximum feasible hours is large. The parameter estimates given in Table 5.5 can be used to calculate the various elasticities of supply of labour hours.

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1 Tulpulé, "Revised Estimates of ... Elasticities", *op.cit.*.

2 The same tendency was observed in the model for a typical worker using different commodity groupings; see Figure 1 in Ashok Tulpulé, "Empirical Estimation of Labour Supply Elasticities", IMPACT Project <u>Preliminary Working Paper</u> No. BP-12, Industries Assistance Commission, Melbourne, 1979, p. 17.

3 Reasons for this tendency are discussed briefly in Tulpulé, "Revised Estimates of ... Elasticities", *op.cit.*, p. 14.

Table 5.5 shows that the unconstrained marginal leisure preference parameter for the married female is slightly greater than the corresponding constrained estimate, for the single females the unconstrained estimate is much greater, and for single males it is somewhat lower. Estimates of the maximum feasible hours in the two experiments are similar, except for DMU type 4 (single females). In the constrained version most of the asymptotic t-values have improved (although clearly their nominal values greatly overstate the precision of estimation in all cases). The results obtained by constraining β_5 and β_7 to zero are now discussed.

The model fits the sample period data reasonably well. The means of residuals expressed as a percentage of the average values of the first five endogenous variables, i.e., the hours worked variables, are less than 1 per cent, while for the consumption expenditure variable the mean residual is 2 per cent. For individual data points the errors in estimating the endogenous variable are shown in Figure 1 in Appendix II where the size of error and the serial properties of the data are discussed.

The re-normalized marginal propensity to consume leisure for the working wife is similar to that for the single male. The estimate for the single female is somewhat higher than for the single male and for the married female. All the non-zero estimates are higher than the aggregate estimate obtained in the earlier study using only one representative worker. Thus out of an exogenous endowment of an extra dollar of full income[1], a single female is likely to spend 29 cents on leisure time as against 28 cents for the single male and 27 cents for the working wife. By constraining β_5 and β_7 to zero, we have assumed that married men will not spend any part of their additional full income on extra leisure time. This assumption

1 'Full Income' is the sum of actual labour and non-labour income and an imputed value of income that can be earned if no time is spent on leisure.

In order to calculate the estimates of after tax income per person, values of average sizes of household types 1 and 2 are required. For household types 3 and 4 the size is assumed to be 1. The average size of *households* with working and non-working wives, according to survey data presented in Williams[1], are 3.45 and 4.01 respectively. The average number of persons over the sample period in household types 1 and 2 combined is 4.1989 on the assumption that the average size of household types 3 and 4 is 1. If the sizes of household types 1 and 2 are assumed to be proportional to the sizes of the two types of *households*, estimates of average sizes of households associated with DMU Types 1 and 2 would be 3.8131 and 4.4320 respectively.

Estimates of after tax incomes obtained from (5.4), income per head, and the total expenditure estimates for households associated with each type of DMU are shown in Table 5.2.

1 R.A. Williams, "Wants and Working Wives", *op.cit.*, p. 9.

TABLE 5.2: ESTIMATES OF AVERAGE AFTER-TAX INCOME AND EXPENDITURE AND AVERAGE SIZE FOR HOUSEHOLDS ASSOCIATED WITH THE 4 TYPES OF DMUs

DMU Type	After-tax Income per Household, Y ($)	Expenditure per Household, V ($)	Average size	After-tax Income per Head ($)
1	8661.90	7440.57	3.8131	2271.62
2	6057.86	5361.21	4.4320	1366.85
3	4343.97	3840.50	1	4343.97
4	3107.40	2747.25	1	3107.40
All Types	5680.90	4992.21	2.9245	1942.52

Estimates of Frisch parameters and subsistence expenditures for the four types of DMUs can now be obtained from equations (5.1) and (5.3) respectively by making use of the data in Table 5.2. The weighted average of the four subsistence expenditure estimates obtained from equation (5.3) is about 3 per cent lower than the value implied by the overall estimate of ω, *viz.*, -1.8744. In order to achieve consistency, the initial estimates of subsistence expenditures were scaled. The resulting estimates, along with the estimated Frisch Parameters, ω_i, subsistence parameters, γ_i, and estimates of subsistence expenditure calculated at the average price level, $\bar{p}$, for all goods and services over the sample period, $\bar{p}\gamma_i$, are shown in Table 5.3.

TABLE 5.5: FIML PARAMETER ESTIMATES OF MARGINAL PROPENSITIES TO CONSUME LEISURE AND MAXIMUM FEASIBLE WORK HOURS PER YEAR FOR WORKERS ASSOCIATED WITH DIFFERENT TYPES OF DMUs

DMU Type & Worker Type	Parameter Estimates (and asymptotic t-values): Constrained with $\beta_5 = \beta_7 = 0$: Marginal Propensities for Leisure: First Normalization[a]	Second Normalization[b]	Maximum Feasible Hours per year (Thousands)	Unconstrained Estimates: Marginal Propensities for Leisure, First Normalization	Maximum Feasible Hours per year (Thousands)
1 Married male with working wife	0	0	2.1144 [c] (230.05)	-0.0443 (7.02)	1.9955 (157.45)
Working wife	0.3780 (36.27)	0.2743 (49.98)	2.9446 (83.47)	0.4092 (40.34)	3.0490 (88.57)
2 Married male with non-working wife	0	0	2.2416 [c] (582.50)	-0.1041 (10.35)	2.0993 (137.55)
3 Single male	0.3812 (19.95)	0.2760 (27.56)	2.7201 (82.40)	0.2824 (12.48)	2.5487 (67.66)
4 Single female	0.4042 (20.63)	0.2879 (28.96)	2.4658 (83.00)	0.7797 (16.83)	3.0601 (40.73)
Aggregate result* =		0.2436	2.4735		

* Tulpulé, "Revised Estimates of ... Elasticities", *op.cit.*, p. 16.

a First normalization assumes that for each DMU type, the sum of the marginal propensities to consume and to save is one.

b For males and females in DMU Type 1 the re-normalized marginal propensities to consume leisure are $\frac{\beta_5}{1+\beta_5+\beta_6}$ and $\frac{\beta_6}{1+\beta_5+\beta_6}$ respectively and for DMU Types 2, 3 and 4 they are $\frac{\beta_7}{1+\beta_7}$, $\frac{\beta_8}{1+\beta_8}$ and $\frac{\beta_9}{1+\beta_9}$ respectively. The marginal propensities to consume leisure for the married males are assumed to be zero.

c With their corresponding β values set to zero, these values lose a utility interpretation, becoming instead estimates of average hours worked. See equations (3.5.1) and (3.5.3).

of demand for leisure is negative. That is, that married men demand less leisure or supply more labour hours when their non-labour income increases. Because negative marginal budget shares destroy the concavity properties of the Klein-Rubin utility function, this result is unacceptable. Therefore the model was re-estimated, constraining the values of the relevant marginal leisure preference parameters to zero. This left only seven free parameters to be estimated in the model. These estimates and the unconstrained FIML estimates are shown in Table 5.5. It may be recalled from Section 3 that the normalization used assumes that, like in the ELES models, the sum of the marginal propensities to consume and to save is equal to one. It is necessary to re-normalize the marginal budget shares so that the sum of marginal propensities to consume goods and services, and leisure, and the marginal propensity to save, add up to one. After re-normalization the results can be compared with those obtained from aggregated data for all workers presented in Tulpulé[1].

1 Tulpulé, "Revised Estimates of ... Elasticities", *op. cit.*.

TABLE 5.3: ESTIMATES OF FRISCH PARAMETER, SUBSISTENCE EXPENDITURE AND SUBSISTENCE PARAMETERS FOR 4 TYPES OF DMUs

DMU Type	Frisch Parameter	Subsistence Expenditure ($)	Subsistence* parameter
1	-1.7717	3349.56	2.6977
2	-2.1272	2936.14	2.3647
3	-1.4028	1268.05	1.0213
4	-1.5826	1045.24	0.8418
All Types	-1.8744	2328.85	1.8756

* Calculated as Subsistence Expenditure in thousand dollars divided by $\bar{p}$ (1.24163), the average price index over the sample period.

Marginal Propensity to Consume

An estimate of the marginal propensity to consume goods and services for each type of DMU can be obtained from a relationship between the marginal propensity to consume, μ, the Frisch parameter, ω, and the average propensity to consume, V/Y, given by Lluch, Powell and Williams[1]; *viz.*,

$$(5.5) \qquad (1-V/Y) = \frac{1}{1-\psi}; \quad \psi = \frac{-\omega\mu}{1-\mu}.$$

After substituting the values of ω from Table 5.3 and of V/Y derived from Table 5.2 into equation (5.5), estimates of the marginal propensities to consume can be obtained for the four types of DMUs. The weighted average of

1 Lluch, Powell and Williams, *op.cit.*, p. 25.

these four initial estimates is 0.4 per cent lower than the overall estimate obtained previously[1], *viz.*, 0.7795. The initial estimates were scaled in order to achieve consistency. The scaled estimates of the marginal propensities to consume goods and services along with the average propensities derived from Table 5.2 are shown in Table 5.4.

TABLE 5.4: ESTIMATES OF AVERAGE AND MARGINAL PROPENSITIES TO CONSUME GOODS AND SERVICES FOR THE 4 TYPES OF DMUs

DMU Type	Average Propensity[a]	Marginal Propensity[b]
1	0.8590	0.7475
2	0.8850	0.7863
3	0.8841	0.8478
4	0.8841	0.8313
Average*	0.8787	0.7795

* Tulpulé, "Revised Estimates of ... Elasticities", *op.cit.*.
a Derived from Table 5.2.
b Derived from equation (5.5).

Maximum hours per worker

The maximum hours per worker parameters ($\gamma_5, \ldots, \gamma_9$) are estimated by the model subject to

$$(5.6) \qquad \bar{n}_1\gamma_5 + \bar{n}_1\gamma_6 + \bar{n}_2\gamma_7 + \bar{n}_3\gamma_8 + \bar{n}_4\gamma_9 = \bar{N}\gamma_H,$$

1 Tulpulé, "Revised Estimates of ... Elasticities" *op.cit.*, p. 18.

where

$\bar{n}_i$ = the average number of DMUs of type i over the sample period,

$\bar{N} = 2\bar{n}_1 + \bar{n}_2 + \bar{n}_3 + \bar{n}_4$,

and

γ_H = externally supplied estimate of maximum hours per worker[1] = 2.4735.

Thus the overall average of the maximum hours parameters for all DMUs is made consistent with the estimate obtained by using aggregate data for the same sample period. One of the parameters can now be written as a function of the other parameters as follows:

$$(5.7) \qquad \gamma_9 = \frac{1}{\bar{n}_4}\left[\bar{N}\gamma_h - \bar{n}_1\gamma_5 - \bar{n}_1\gamma_6 - \bar{n}_2\gamma_7 - \bar{n}_3\gamma_8\right].$$

FIML Estimates

With this much background on the assumed values of consumption parameters, the rest of the parameters of the system, *viz.*, the labour supply parameters, can now be estimated by the Full Information Maximum Likelihood method using Wymer's RESIMUL computer program.[2] For married males the initially obtained estimates of the marginal propensity to consume leisure were negative and the maximum feasible hours parameters were less than the actual hours worked at the midpoint of the sample period. A negative parameter for marginal leisure preference implies that the income elasticity

1 The value of this parameter was obtained from Tulpulé, "Revised Estimates of ... Elasticities", *op.cit.*, p. 16.
2 C.R. Wymer, "Computer Programs: RESIMUL Manual" (Washington, D.C.: International Monetary Fund, 1977) (mimeo).