IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

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A THEORETICAL MODEL FOR OCCUPATIONAL MOBILITY IN AUSTRALIA

by

Lynne S. Williams
Monash University

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IMPACT Project, Industries Assistance Commission, 608 St. Kilda Road, Melbourne, Victoria 3004 Australia
Area Code 03 Telephone 51 8611

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The model comprises the following equations :

(2) $$QM_{ij} = \mu DM_{ij} + (1-\mu)SM_{ij} \,,\quad 0 \leqslant \mu \leqslant 1 \,,\quad (i \neq j) \,.$$

(11) $$DM_{ij} = \left[\frac{SM_{ij}}{\sum\limits_{i \neq j} SM_{ij} + SE_j + SI_j} \right] D(LX)_j \,.$$

(7) $$D(LX)_j = \overline{D(LT)}_j - \overline{D(LN)}_j \,.$$

(43) $$SE_j = \overline{SE}_j \,;\quad SI_j = \overline{SI}_j \,.$$

(52) $$SM_{ij} = \left[\frac{SM_{ij}}{SM_{.j}} \right]_{s.m.} \overline{SM}_{.j} + \overline{SM}_{ij} \sum_{k \neq i} \frac{T^{(j)}}{\bar{\omega}_{ik(j)}} \left(\frac{R_{kj}}{\bar{R}_{kj}} - \frac{R_{ij}}{\bar{R}_{ij}} \right), \quad i \neq j \,; \quad i,j = 1,\dots,n.$$

(44) $$SM_{.j} = S(LT)_j - \overline{S(LN)}_j - \overline{SE}_j - \overline{SI}_j \,.$$

(39) $$S(LT)_j = \left[\frac{S(LT)_j}{S(LT)} \right]_{s.m.} \overline{S(LT)} + \overline{S(LT)}_j \sum_{i \neq j} \frac{T_{ij}}{\bar{\omega}_{ij}} \left(\frac{R_i}{\bar{R}_i} - \frac{R_j}{\bar{R}_j} \right) .$$

The endogenous variables in this system are : QM_{ij} , DM_{ij} , SM_{ij} , $D(LX)_j$, SE_j , SI_j , $SM_{.j}$ and $S(LT)_j$.

CONTENTS

5. ESTIMATION FRAMEWORK

The proposed theoretical model is summarised by equation (2), which says that observed mobility from occupation i to occupation j is a (predetermined) weighted function of both the demand for and supply of job changers functions. These functions are in turn derived from the behaviour and technology assumed appropriate to the relevant agents.

Thus, combining equations (7), (11) and (43) yields

$$(53) \qquad DM_{ij} = \left[\frac{SM_{ij}}{\sum_{i \neq j} SM_{ij} + SE_j + SI_j} \right] (\overline{D(LT)}_j - \overline{D(LN)}_j) \quad .$$

Similarly, combining and substituting equations (35), (39) and (12) into equation (52) will yield a reduced form expression for SM_{ij} .

The model can therefore be reduced to a very simple simultaneous system comprising three equations : a demand equation, a supply equation, and a "clearing" equation to determine the quantities actually moving between occupations. It is intended to use the simultaneous equation package RESIMUL[1] at the estimation stage.

1. See Wymer (1973).

The method of analysis is identical to that advanced in equations (27) to (39), and yields an equation for the supply of appropriately qualified manhours from occupation i to occupation j of the form

$$(52) \qquad SM_{ij} = \left[\frac{SM_{ij}}{\overline{SM}_{.j}}\right]_{s.m.} \overline{SM}_{.j} + \overline{SM}_{ij} \sum_{k \neq i} \frac{\tau^{(j)}}{\bar{\omega}_{ik(j)}} \left(\frac{R_{kj}}{\bar{R}_{kj}} - \frac{R_{ij}}{\bar{R}_{ij}}\right) ,$$

$$i \neq j ; \quad i, j = 1, \ldots, n ,$$

where

$$\bar{\omega}_{ik(j)} = 1 + \frac{\bar{R}^*_{ij} \, \overline{SM}_{ij}}{\bar{R}^*_{kj} \, \overline{SM}_{kj}} ,$$

in which

$$\bar{R}^*_{ij} = \text{sample mean of } R^*_{ij} ,$$

with

$$R^*_{ij} = R_{ij} (1 + \phi^*_{ij}) ,$$

in which ϕ^*_{ij} is a weighted average of the flexibilities of R_{ij} with respect to movements into j from other occupations k $(k \neq i, j)$. To a good approximation, the ϕ^*_{ij}'s in most cases could be treated as zero due to the relatively small size of occupational transferees as elements of the total labour force.

Equation (52) is the CET transform of the linear supply system (50). It involves n wage parameters (i.e., the $\tau^{(j)}$) as apposed to the original $\frac{1}{2}n(n - 1)^2$, and therefore is probably sufficiently parsimonious in the use of parameters as to allow estimation from the limited supply of data available.

Equations (39), (44) and (52) together form a supply system which yields the supply of suitably qualified transferees from occupation i to occupation j in a given time period.

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1. INTRODUCTION AND OUTLINE

This paper is concerned with formulating the structural form of a model to explain occupational mobility of labour in Australia. The equations for the demand for and supply of labour of different occupational types are specified from the employer's point of view, which is usual in this form of analysis. It is obviously impossible to treat the mobility between occupations in isolation from the other sources of labour supply, such as entrants from the immigration and education sectors. It is therefore necessary to consider demand and supply equations and their inter-relationships before a reduced form model of actual occupational changes can be interpreted. However, it must be emphasized that the primary aim is to explain the process of occupational mobility, as opposed to the far more complex problem of modelling the entire labour market. Analysis is restricted accordingly.

Observed mobility is defined as the flow of workers between any two occupations in a given time period. As such, it is neither purely demand nor purely supply determined.

The main hypothesis advanced in this paper is that occupational mobility varies with excess demand (or supply) in the aggregate labour market. An individual market for a certain occupational type is defined as a supplier's market if the demand for workers is greater than the available supply; it

* I am indebted to Dean Parham and Alan Powell for extensive comments on earlier drafts of this paper. All remaining errors are my own.

is assumed that workers' labour supply decisions will determine the amount of mobility. Conversely, employers' decisions are assumed to determine the amount of occupational mobility in a buyer's market.

The innovative element of the analysis that follows is that labour is treated in the context of a *multi-good* market, where the sub-markets for different occupations can be in divergent states of excess demand, excess supply or equilibrium. Mobility between specific occupations is comprised of movements between those occupations aggregated over all regions, some of which are in excess demand and others in excess supply. Thus total mobility between two sub-markets is modelled as a response to a weighted function of both demand and supply variables, whose relative importance is determined by the overall state of the labour market, which is treated as exogenous.

The starting point for the econometric specification is the seminal article by Fair and Jaffee (1972) on the treatment of a disequilibrium market for a single good. They specify that the quantity actually traded, denoted by Q_t, is determined as the minimum of supply and demand. Using their notation,

$$Q_t = \min\{D_t, S_t\}$$

$$= k_t\{\alpha_o X_t^D + \alpha_1 P_t + \mu_t^D\} + (1-k_t)\{\beta_o X_t^S + \beta_1 P_t + \mu_t^S\},$$

where

D_t = quantity demanded during period t;

S_t = quantity supplied during period t;

P_t = price in period t;

X_t^D, X_t^S = sets of additional exogenous variables influencing D_t and S_t respectively;

(49) $$C_{ij} = \left[\frac{SM_{ij}}{\overline{SM}_{ij}}\right]_{s.m.}$$ evaluated at sample means.

Thus equation (48) becomes

(50) $$SM_{ij} = \left[\frac{SM_{ij}}{\overline{SM}_{ij}}\right]_{s.m.} \overline{SM}_{.j} + \sum_k D_{ijk} R_{kj}, \quad (i \neq j),$$

which reduces the number of unknown parameters to $\frac{1}{2}n(n-1)(n-2)$.

Now consider the occupation-j specific skill transformation frontier $z_j(.)$. If it is assumed that it exhibits an approximately constant direct elasticity of transformation (CET) for any pairing of alternative source occupations, the basic shape of this frontier can be represented, for all pairs of source occupations and a given destination occupation j, by

(51) $$\tau^{(j)} = \frac{d\left(\frac{SM_{ij}}{SM_{kj}}\right)}{\left(\frac{\partial SM_{ij}}{\partial SM_{kj}}\right)} \cdot \frac{\frac{SM_{ij}}{\partial SM_{kj}}}{\frac{SM_{ij}}{SM_{kj}}} \quad \text{for any } i \neq k \neq j.$$

This assumption, admittedly a strong one, reduces the number of parameters to n. It is unlikely that the currently available Australian data base would support the estimation of any more highly structured transformation surface.

By tracing through the effect of an increase dR_k in R_k, (the expected certainty equivalent adjusted wage in source occupation k), the CET transform of the linear supply system (50) at some local value (e.g., sample means) can be derived.

Suppose again that the supply system (47) may be approximated by a linear one. Thus it takes the functional form

$$\text{(48)} \qquad SM_{ij} = C_{ij}\,\overline{SM}_{.j} + \sum_k D_{ijk}\,R_{kj} \;,$$

where $\{C_{ij}\}$ and $\{D_{ijk}\}$ encompass $\frac{1}{2}n(n-1)^2$ non-zero parameters.

It is highly unlikely that there exists any set of data adequate to estimate the parameters of equation (48). Further a priori economic structure would seem to be an absolute precondition to estimation.

The term $\overline{SM}_{.j}$ can be interpreted as a shift function incorporating the effects of changes in many factors such as the demographic characteristics of the population, changes in the education and immigration policy, and the like. For this fixed value of $SM_{.j}$, variations in the $\overline{SM}_{ij}$ are generated by changes in the *linear* combination of relative certainty equivalent wages, $\sum_k D_{ijk} R_{kj}$.

Consider first the C_{ij} parameters. Assume that the function $z_j(.)$ is homothetic. That is, as $\overline{SM}_{.j}$ increases, the optimal contribution from each of the source occupations required to achieve $\overline{SM}_{.j}$ increases proportionately, so that the ratio of each source specific contribution to the total remains constant.

Further, assume there exists a local correspondence between the functions $K^S_{ij}(.)$ and $z_j(.)$ at the sample means of the observed values. This correspondence coupled with the assumption that $z_j(.)$ is homothetic determines the values of the C_{ij} parameters. Using analysis identical to that presented in equations (17) to (21), it follows that

μ^D_t, μ^S_t = error terms;

$$\text{and } k_t = \begin{cases} 0 & \text{if } D_t > S_t\,; \\ 1 & D_t < S_t\,. \end{cases}$$ [1]

The problem is therefore reduced to one of defining some decision rule to determine whether the market is in excess demand or supply at different points in time, and then to estimate the parameters α_o, α_1, β_o and β_1.

If the error terms are well behaved, an appropriate method for estimating the required parameters is the maximum likelihood technique. The likelihood function for the entire sample is

$$L = (2\pi\sigma^2_D)^{\frac{-m}{2}}(2\pi\sigma^2_S)^{\frac{-n}{2}}\exp\left[-\frac{1}{2\sigma^2_D}\sum_1^m(D_t-\alpha_oX^D_t-\alpha_1P_t)^2 - \frac{1}{2\sigma^2_S}\sum_1^n(S_t-\beta_oX^S_t-\beta_1P_t)^2\right],$$

where

m = the number of times $Q_t = D_t$;

n = the number of times $Q_t = S_t$;

and σ_D, σ_S are the standard deviations of μ^D_t, μ^S_t respectively.

If the partition into supply-determined and demand-determined data points is known *a priori* (i.e., if k_t is determined *outside* the model), then L can be divided into two parts, say L^D and L^S. Independent maximization of log L^D and log L^S for given m and n yields consistent estimates of α_o, α_1, β_o and β_1 as required.[2]

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1. If $D_t = S_t (= Q_t)$ in any t, then k_t is arbitrarily set at 0 or 1.

2. Fair and Jaffee go on to discuss alternative methods of estimation which use price change, formulated as a function of excess demand, to determine whether the economy is on the demand or supply curve. These methods are not appropriate to labour market analysis, as the corresponding price mechanism (the wage rate) does not operate smoothly, continuously, or instantaneously in both directions, nor can its rate of change be expressed primarily as a function of excess demand for labour.

This method provides the basis for the model presented here, which incorporates a predetermined (i.e., known) weighting factor reflecting the state of the aggregate labour market. The analysis extends that of Fair and Jaffee's in that it deals with many occupations as opposed to the single 'commodity' case. Further, the weighting factor for each occupation aggregated over all regions relevant to any one time period is unrestricted in the range of values 0 to 1 . This flexibility is important in analysis of the labour market because it allows for the simultaneous influence of both demand and supply variables.

To use this approach for the analysis of occupational mobility, it is necessary to specify the relevant demand and supply relationships for each occupation, and the appropriate weighting factor. It is assumed that employers determine the demand for labour in each occupation, and workers the supply.

Before developing the theoretical structure of the model, it is necessary to discuss some of the more serious limitations on the scope of the analysis imposed as a result of data inadequacies.

The only data currently available on Australian occupational mobility are two Australian Bureau of Statistics (A.B.S.) surveys conducted in November 1972 and February 1976. These measure mobility in the twelve months prior to each survey date. Analysis is therefore of necessity restricted to cross-section methods. This limitation means that it is necessary to develop an aggregate as opposed to an occupation specific weighting factor.

The majority of data pertaining to the labour market are cast in terms of numbers of persons, with no distinction made between full-time and part-time employees. However, the effective work force size is better

such that

(46) $$\overline{SM}_{.j} = z_j\,(\underset{\sim}{SM}_j) ,$$

where

$\Pi_j(.)$ = a synthetic welfare index measuring the partial contribution made to total welfare by movements from other occupations into occupation j ;

$\underset{\sim}{R}_j$ = a vector of certainty equivalent wages (adjusted for private costs of training) per manhour in occupation j, with typical element R_{ij} ;

R_{ij} = the difference in certainty equivalent adjusted wages per manhour in occupation j as compared to occupation i ,

= $R_j - R_i$;

$\underset{\sim}{SM}_j$ = a vector of manhours offered for transfer into occupation j by appropriately qualified persons classified by source occupation i ;

SM_{ij} = a typical element of $\underset{\sim}{SM}_j$;

$\overline{SM}_{ij}$ = the total supply of manhours to occupation j from all other occupations, as determined at a previous level of the problem ;

and

$z_j(.)$ = a transformation function measuring the effective number of manhours of type j which can be produced by combinations of manhours supplied by transferees from other occupations.

Solving equations (45) and (46) for the SM_{ij}'s yields functions of the form

(47) $$SM_{ij} = K^S_{ij}\,(\overline{SM}_{.j}, \{R_{ij}\}) , \qquad (i \neq j) ,$$

where

$\{R_{ij}\}$ is the set of values R_{ij} in which, for fixed i , $j \neq i$, and $\overline{SM}_{.j}$ is predetermined (by equations (39) and (44)).

exists a significant time delay between the decision to immigrate and entry into the Australian work force. This analysis is tantamount to saying that occupation changing makes up the residue of optimal total labour supply $S(LT)_j$ to each occupation j, once supplies have been offered by all other sources.

Exogeneity of two of the sources of "outside" labour supply reduces equation (42) to

$$(43) \qquad S(LX)_j = SM_{.j} + \overline{SE}_j + \overline{SI}_j \quad .$$

Rearranging (43) and substituting from (41) gives

$$(44) \qquad SM_{.j} = S(LX)_j - \overline{SE}_j - \overline{SI}_j$$

$$= S(LT)_j - \overline{S(LN)}_j - \overline{SE}_j - \overline{SI}_j \quad ,$$

where $S(LT)_j$ is determined by equation (39) .

The concern now is to break the $SM_{.j}$ variable into its source occupation specific components, i.e., to determine SM_{ij} for all j, $i \neq j$.

The analysis is again presented in terms of the representative labour supplier who now has to relocate his working hours to get optimal $S(LT)_j$ in each occupation j.

The SM_{ij} functions are obtained as solutions to the first order conditions of a constrained optimization problem of the form

$$(45) \qquad \text{Max } \Pi_j(\underset{\sim}{R}_j) = \sum_{i \neq j} R_{ij} SM_{ij} \quad ,$$

represented by the number of quality adjusted manhours. In so far as it has been practical, the theory presented below has been developed in terms of this unit.[1]

This paper comprises five sections. The basic mobility equation is formally derived in section 2, where the appropriate weighting factor is presented as an aggregate not only over all regions, but also over all occupations. Section 3 contains the derivation of the occupation specific demand functions. In section 4 the corresponding supply functions are developed as the second of two stages : at the first stage occupation specific aggregate supply is derived, then this is allocated amongst the various sources of supply. The final form of the model is drawn together in section 5.

1. In order to enable empirical testing of the theory using the available data, it will be necessary to transform this basic unit to worker equivalents.

2. DERIVATION OF THE BASIC MOBILITY EQUATION

Let mobility from occupation i to occupation j be defined by a nested function of the form

$$(1) \qquad QM_{ij} = F[DM_{ij}, SM_{ij}] , \ (i \neq j) ,$$

where

QM_{ij} = total observed movement from occupation i to occupation j ;

DM_{ij} = a function representing demand by employers for labour of occupation j type from source occupation i ;

SM_{ij} = a function representing the supply of labour to occupation j from occupation i .

Suppose equation (1) takes the specific functional form

$$(2) \qquad QM_{ij} = \mu DM_{ij} + (1-\mu) SM_{ij} , \quad 0 \leq \mu \leq 1 ,$$

where

μ = a common weighting factor representing the relative importance of supply and demand functions aggregated over all regions and all occupations.

Equation (2) is defined as the basic mobility equation and is derived in the following manner: consider the individual regions, each of which is assumed to contain every occupation. For each occupation it is possible to calculate an aggregate over all regions, thus determining whether the occupation is in a situation of excess aggregate demand or excess aggregate supply. Let the intra-region[1] specific reduced form mobility equation for movement from occupation i to occupation j be

$$(3) \qquad qm^k_{ij} = \gamma dm^k_{ij} + (1-\gamma) sm^k_{ij} ,$$

1. The following treatment implicitly assumes that no inter-regional mobility occurs. If inter-regional mobility was allowed for explicitly, derivation of equation (2) would involve aggregation of the flows between regions as well as within regions, but its final estimating form would remain unchanged.

time period can be represented as

$$(41) \qquad S(LX)_j = S(LT)_j - \overline{S(LN)}_j ,$$

where $S(LT)_j$ is determined by equation (39) .

Further, "outside" supply can be expressed in terms of its component parts, thus

$$(42) \qquad S(LX)_j = \sum_{i \neq j} SM_{ij} + SE_j + SI_j = SM_{.j} + SE_j + SI_j ,$$

where $\sum_{i \neq j} SM_{ij} \equiv SM_{.j}$, SE_j and SI_j are as previously defined.

Looking at the labour market overall, then, it is hypothesized that at any particular point in time the representative agent operates so as to optimize his aggregate synthetic "welfare" function π . This may necessitate some restructuring of the occupational distribution of the work force, which can be achieved not only through the placement of new work force entrants, but also through occupational changes of the current work force. The solution process is thus a nested one whereby the same representative agent--having determined the optimal allocation mix--now has to decide the least total cost means of meeting supplies to each occupation.

It is hypothesized that any observed action in $S(LX)_j$ in a given year must result from inter-occupational movements. The planned occupation mix of entrants from the education system in any one year is largely determined by decisions taken over many previous years, and can only effect a minor variation in $S(LX)_j$ in the short run (i.e., one year). In this model it is assumed that SE_j is known and predetermined at $\overline{SE}_j$. Similarly, the occupational composition of the migrant intake is treated as predetermined (at $\overline{SI}_j$), justifiable on the grounds that there generally and substitution of (34) and (37) into (38) results in

$$(39) \qquad S(LT)_j = \left[\frac{S(LT)_j}{\overline{S(LT)}}\right]_{s.m.} \overline{S(LT)} + \overline{S(LT)}_j \sum_{i \neq j} \frac{T_{ij}}{\bar{\omega}_{ij}} \left(\frac{R_i}{\bar{R}_i} - \frac{R_j}{\bar{R}_j}\right) .$$

Equation (39) is Powell and Gruen's transform of the linear supply system (22).[1] It involves, in the case of an n-occupational model, ½n(n - 1) wage parameters as opposed to the original n(n + 1). Thus considerable parameter parsimony has been achieved. (Further savings would be possible by reparameterizing the system to reflect (say) the CRETH postulates on the functional form of h(.)).[2]

4:2 The allocation of aggregate supply amongst its alternative sources

Paralleling the analysis used to derive the occupation specific demand curves, it is posited that total labour supply to any particular occupation consists of occupation stayers, occupation changers, and entrants from the education and immigration sectors. That is,

$$(40) \qquad S(LT)_j = S(LN)_j + S(LX)_j ,$$

where

$S(LN)_j$ = the number of effective manhours worked by employees who are currently employed in occupation j and who desire to remain there (i.e., "inside" supply) ;

$S(LX)_j$ = the additional supply of effective manhours of occupation j type supplied by changers and entrants from the education and immigration sectors (i.e., "outside" supply) .

Assuming that $S(LN)_j$ is predetermined (at $\overline{S(LN)}_j$), then the aggregate additional supply function of effective manhours to occupation j for a particular

1. Powell and Gruen (1968).

2. See Dixon (1976); Vincent et al. (1978); Hanoch (1971).

where

qm^k_{ij} = the observed movement from occupation i to occupation j that occurs in region k ;

dm^k_{ij} = the demand for occupation j type labour from occupation i that occurs in region k ;

sm^k_{ij} = the supply of labour to occupation j from occupation i that occurs in region k ;

and $\gamma = \begin{cases} 0 & \text{if } dm^k_{ij} > sm^k_{ij} , \\ 1 & dm^k_{ij} < sm^k_{ij} . \end{cases}$

Aggregating equation (3) over all regions yields

$$(4) \qquad QM_{ij} = \mu_{ij} DM_{ij} + (1-\mu_{ij}) SM_{ij} , \quad 0 \leq \mu_{ij} \leq 1 ,$$

where

$$QM_{ij} = \sum_k qm^k_{ij} , \quad DM_{ij} = \sum_k dm^k_{ij} , \quad SM_{ij} = \sum_k sm^k_{ij} ,$$

and

$$(5) \qquad \mu_{ij} = \sum_k \gamma \omega^k_{ij} \text{ such that } \omega^k_{ij} = \frac{dm^k_{ij} - sm^k_{ij}}{\sum_k (dm^k_{ij} - sm^k_{ij})} \text{ and } \sum_k \omega^k_{ij} = 1 .$$

Equation (4) utilizes a different weighting factor for movement between different pairs of occupations. However, because of data limitations it is highly unlikely that sufficient information will exist to provide individual estimates of all the μ_{ij} values. Instead, a common value for μ is imposed over the whole labour market, where this parameter is predetermined and constructed so as to reflect the relative state of excess demand or excess supply aggregated over all occupations. Imposition of this single parameter μ yields the functional form of the basic labour mobility equation, equation (2).

Note that μ will only take one of its two extremes in the rare circumstance of every occupation in every region being simultaneously in excess demand or supply.

Enforcing the equality of (31) and (33) at (say) mean certainty equivalent wages $\{\overline{R_i}\}$ and mean effective manhours $\{\overline{S(LT)_i}\}$ can be achieved by setting

$$(34) \qquad B_{ij} = \frac{\overline{S(LT)}_j}{\bar{R}_i} \cdot \frac{T_{ij}}{\bar{\omega}_{ij}} ,$$

where

$$(35) \qquad \bar{\omega}_{ij} = 1 + \frac{\bar{R}_j^* \, \overline{S(LT)}_j}{\bar{R}_i^* \, S(LT)_i} .$$

By requiring that, at sample mean wages, the supply system (22) be homogeneous of degree zero in wages (i.e., an equi-proportional increase in all mean wages would leave allocation of a fixed total effective supply of labour to the various occupations unchanged) it follows that

$$(36) \qquad B_{jj} = \frac{\left(\sum_{i \neq j} B_{ij} \bar{R}_i\right)}{\bar{R}_j} ,$$

where once again the restriction has been enforced at mean wage expectations and mean effective manhours.

Substituting from (34) into (36) yields

$$(37) \qquad B_{jj} = \frac{-\overline{S(LT)}_j}{\bar{R}_j} \cdot \frac{\sum_{i \neq j} T_{ij}}{\bar{\omega}_{ij}} .$$

Equation (22) can alternatively be written as

$$(38) \qquad S(LT)_j = \left[\frac{S(LT)_j}{\overline{S(LT)}}\right]_{s.m.} \overline{S(LT)} + \sum_{i \neq j} B_{ij} R_i + B_{jj} R_j ,$$

whilst by (24)

(29) $$dS(LT)_i = \frac{R_j(1+\phi_j^*)}{R_i(1+\phi_i^*)}\, dS(LT)_j \ ,$$

where dR_j and dR_i are compensating small changes in R_j and R_i at a given level of $\overline{S(LT)}$.

Thus, combining (27), (28) and (29) yields

(30) $$\frac{dS(LT)_j}{S(LT)_j}\left(1 + \frac{R_j(1+\phi_j^*)\,S(LT)_j}{R_i(1+\phi_i^*)\,S(LT)_i}\right) = T_{ij}\,\frac{dR_i}{R_i} \ .$$

If $R_i(1+\phi_i^*)$ is written R_i^*, then, solving for $\frac{\partial S(LT)_j}{\partial R_i}$, one obtains

(31) $$\left.\frac{\partial S(LT)_j}{\partial R_i}\right|_{\text{constant } \overline{S(LT)}} = \frac{S(LT)_j}{R_i} \cdot \frac{T_{ij}}{\omega_{ij}} \ ,$$

where

(32) $$\omega_{ij} = 1 + \frac{R_j^*\,S(LT)_j}{R_i^*\,S(LT)_i} \ ,$$

and where ω_{ij} is closely related to the reciprocal of the expected share of occupation i in certainty equivalent labour income derived from i and j combined.

Partial differentiation of equation (22) gives

(33) $$\frac{\partial S(LT)_j}{\partial R_i} = B_{ij} \ .$$

3. DEVELOPMENT OF THE SOURCE - AND DESTINATION - SPECIFIC DEMAND FOR TRANSFEREES

The theory relating to the determination of demand for specific i to j transferees is developed in terms of the behaviour of the representative firm. This firm is a convenient fiction which avoids aggregation problems involved in summing over all individual firms. The difference between an individual firm and the representative one is that, whereas each individual agent is sufficiently small to be unaffected by supply constraints, the representative firm can be affected by aggregate supply constraints. In effect, then, the representative firm is just a proportionately scaled down version of the economy as a whole, and aggregation over all such (identical) firms yields the appropriately constrained behaviour of this economy.

The demand for labour of a given type is defined as a derived demand, which for the purposes of this analysis is assumed to be determined at a previous level of the firm's decision making in its production process. This enables attention to be confined to the 'allocation' of this demand to the various sources supplying labour of this particular type.

The major sources of labour supply are people who want to change their occupations, and entrants to the labour force from the education and immigration sectors.[1] It is possible that the firm may have some preference ordering as regards these alternative sources. For example, employers may feel it is easier for new graduates to assimilate into their firm than for occupation changers to adjust. Alternatively, they

1. An implicit assumption in this approach is that the participation rate is exogenous and fixed over the estimation period.

may feel that older workers may have more general experience which could increase their worth to the firm relative to younger graduates. No consistent information exists on these preferences. Further, in the Australian labour market the nature of the institutional framework is such that the Commonwealth Conciliation and Arbitration Commission determines a series of award wages for specific tasks. Firms are obliged to pay these wages, irrespective of the source of the labour, i.e., there are no source-specific wage differentials for a particular job. Thus, although it might be attractive to set up a model in which firms operate so as to minimize the cost of obtaining their additional labour requirements, the absence of wage differentials between alternative sources of labour supply (in Australia) means that any such model would yield no information on the quantities demanded from alternative sources.

It is therefore assumed that the allocation of a firm's total demand for labour of a particular type among alternative sources is determined according to the relative numbers offering from each of these sources. This implies that the elasticities of substitution between workers of the same occupation from different sources in the firm's production function are infinite. In other words, there exist no specific advantages or disadvantages to the different groups competing in the labour market - - for example, native born are not favoured over immigrants, nor new workforce entrants over occupation changers.

Denote the representative firm's demand for labour of occupation j as $D(LT)_j$. Suppose this demand is determined outside this model. This quantity can be disaggregated thus:[1]

(6) $$D(LT)_j = D(LN)_j + G^D\left(\sum_{i \neq j} DM_{ij},\ DE_j,\ DI_j\right) ,$$

1. Imagine there is a t (current time period) subscript on all variables.

$$\nu_{\alpha\beta} = \frac{S(LT)_\beta\, R_\beta}{S(LT)_\alpha\, R_\alpha} ,$$

the marginal rate of substitution on the left in (24) may be written

(25) $$\frac{\partial S(LT)_j}{\partial S(LT)_i} = \frac{-R_i\{1 + \phi^*_i\}}{R_j\{1 + \phi^*_j\}} ,$$

where

$$\phi^*_\alpha = \sum_\beta \nu_{\alpha\beta}\, \phi_{\beta\alpha} , \qquad (\alpha = i, j) .$$

In what follows the flexibilities $\phi_{\beta\alpha}$ and their weighted sums $\phi^*_\alpha = \sum_\beta \nu_{\alpha\beta}\, \phi_{\beta\alpha}$ are treated as constants. On this assumption, the proportional change in the marginal rate of transformation on the right of (24) following a proportional change of dR_i/R_i in the i^{th} wage rate is

(26) $$\frac{d(\partial S(LT)_j/\partial S(LT)_i)}{\partial S(LT)_j/\partial S(LT)_i} = \frac{dR_i}{R_i} .$$

Using (23), (26) may be rewritten

(27) $$\frac{d(S(LT)_j/S(LT)_i)}{S(LT)_j/S(LT)_i} = T_{ij}\,\frac{dR_i}{R_i} .$$

However, the left hand side of (27) can be written

(28) $$\frac{d(S(LT)_j/S(LT)_i)}{S(LT)_j/S(LT)_i} = \frac{dS(LT)_j}{S(LT)_j} - \frac{dS(LT)_i}{S(LT)_i} ,$$

formation between i and j when the other occupations are similarly fixed. The direct partial transformation elasticity T_{ij} measures the curvature of $h(.)$ in the i-j plane when other occupational levels are fixed, and hence is directly related to the right-hand derivative of (24).

The left-hand term of (24) can be obtained by first setting

$$\pi = R_i\, S(LT)_i + R_j\, S(LT)_j + \sum_{\ell \neq i,j}^{n} R_\ell\, S(LT)_\ell ,$$

and with $S(LT)_\ell$ fixed for $\ell \neq i, j$, taking derivatives with respect to $S(LT)_i$ and $S(LT)_j$. These derivatives are the marginal pay-offs in terms of the objective function π of an additional unit of occupations i and j; their values are

$$\frac{\partial\pi}{\partial S(LT)_\alpha} = R_\alpha\left\{1 + \sum_{\beta=1}^{n} \frac{S(LT)_\beta R_\beta}{S(LT)_\alpha R_\alpha}\ \frac{\partial R_\beta}{\partial S(LT)_\alpha}\ \frac{S(LT)_\alpha}{R_\beta}\right\} \quad (\alpha = i,j) .$$

The terms $\dfrac{S(LT)_\beta R_\beta}{S(LT)_\alpha R_\alpha}$ compare the relative contribution of occupation α with other occupations'; the terms $\dfrac{\partial R_\beta}{\partial S(LT)_\alpha}\ \dfrac{S(LT)_\alpha}{R_\beta}$ are the flexibilities of different wage rates with respect to the supply of labour of occupational type α. The left-hand term in (24) may be obtained by taking the ratio $-[\partial\pi/\partial S(LT)_i]/[\partial\pi/\partial S(LT)_j]$. Writing $\phi_{\beta\alpha}$ as the flexibility of the β^{th} wage rate with respect to the supply of the α^{th} occupation, and writing

where

$D(LN)_j$ = the demand for labour currently employed in occupation j (i.e., the previous period stock corrected for resignations, dismissals, retirements and deaths); for convenience, call this quantity "inside" demand;

$\sum_{i\neq j} DM_{ij}$ = the demand for occupation j type labour from all other occupations, i.e., where DM_{ij} = the demand for occupation j type labour from source occupation i;

DE_j = the demand for occupation j type labour from the education sector ;

DI_j = the demand for occupation j type labour from the immigration sector ;

$G^D(.)$ = a function combining the demands for the alternative sources of labour supply, i.e., "outside" demand.

Re-arranging equation (6) yields

(7) $$D(LX)_j = D(LT)_j - D(LN)_j = G^D\Big(\sum_{i\neq j} DM_{ij},\ DE_j,\ DI_j\Big)$$

where

$D(LX)_j$ = the demand for an additional unit of labour of occupation j type, i.e., "outside" demand.

Imposing the assumption that the quantities demanded from each source of labour are determined according to the relative supplies available from each of these sources, $G^D(.)$ can be expressed as

(8) $$G^D\Big(\sum_{i\neq j} DM_{ij},\ DE_j,\ DI_j\Big) = \sum_{i\neq j} DM_{ij} + DE_j + DI_j ,$$

and

(9) $$DM_{ij} = \alpha_{ij} D(LX)_j,\quad DE_j = \beta_j D(LX)_j,\quad DI_j = \gamma_j D(LX)_j ,$$

where

(10:1) $$\alpha_{ij} = \frac{SM_{ij}}{\sum_{i \neq j} SM_{ij} + SE_j + SI_j} = \frac{SM_{ij}}{SM_{\cdot j} + SE_j + SI_j} ,$$

(10:2) $$\beta_j = \frac{SE_j}{\sum_{i \neq j} SM_{ij} + SE_j + SI_j} = \frac{SE_j}{SM_{\cdot j} + SE_j + SI_j} ,$$

(10:3) $$\gamma_j = \frac{SI_j}{\sum_{i \neq j} SM_{ij} + SE_j + SI_j} = \frac{SI_j}{SM_{\cdot j} + SE_j + SI_j} ,$$

(10:4) $$\sum_{i \neq j} \alpha_{ij} + \beta_j + \gamma_j = 1 \quad \text{and} \quad \sum_{i \neq j} \alpha_{ij} = \alpha_{\cdot j} ,$$

where

α_{ij} = the proportion of "outside" demand for labour of occupation j type from source occupation i ;

$\alpha_{\cdot j}$ = the proportion of "outside" demand for labour of occupation j type from all *other* occupations;

β_j = the proportion of "outside" demand for entrants from the education system;

γ_j = the proportion of "outside" demand for immigrants;

SM_{ij} = the supply of labour to occupation j from occupation i ;

$\sum_{i \neq j} SM_{ij}$ = $SM_{\cdot j}$ = the supply of labour to occupation j from all *other* occupations;

SE_j = the supply of labour from the education system to occupation j ;

SI_j = the supply of labour from the immigration system to occupation j.

In (23), all levels of outputs of effective manhours are held constant, other than those of the i^{th} and j^{th} occupations.[1]

Consider the case of a fixed total supply of labour, $\overline{S(LT)}$. In terms of the linear supply system (22), movements around the transformation frontier h(.) are generated by changes in the linear combination $\sum_i B_{ij} R_i$ of certainty equivalent wages, rather than by changes in $\overline{S(LT)}$.

The analysis now traces through the effect of an exogenous small increase dR_i in R_i , the expected certainty equivalent wage (adjusted for privately met training costs) in occupation i .[2] If all other wages, including that in occupation j , remain unchanged, then the proportional increase in the relative wage ratio R_i/R_j is dR_i/R_i for any choice of j . Parameterization in terms of direct partial transformation elasticities allows the first order conditions to be solved by consideration in turn of each of the $\binom{n}{2}$ problems involving only two choice variables $S(LT)_i$ and $S(LT)_j$. For, at the initial equilibrium before the exogenous change dR_i in the wage rate of occupation i , it is true that

(24) $$\left.\frac{\partial S(LT)_j}{\partial S(LT)_i}\right|_{\pi,\ \text{other } S(LT)} = \left.\frac{\partial S(LT)_j}{\partial S(LT)_i}\right|_{h,\ \text{other } S(LT)} ,$$

where the left hand derivative is the marginal rate of substitution between occupations i and j in the synthetic welfare function π at fixed levels -- i.e., at the initially optimal levels -- of all other occupations, and the right hand derivative is the marginal rate of skill trans-

1. An alternative approach would involve parameterization in terms of Allen-Uzawa partial transformation elasticities.

2. For simplicity, 'expected certainty equivalent wage (adjusted for privately met training costs)' is referred to in the remainder of this discussion simply as 'the wage rate.'

Combining equations (19) and (20) it follows that

$$(21) \qquad A_j = \left[\frac{S(LT)_j}{\overline{S(LT)}}\right]_{s.m.}$$

(where s.m. indicates evaluation at sample means) .

Thus equation (16) becomes

$$(22) \qquad S(LT)_j = \left[\frac{S(LT)_j}{\overline{S(LT)}}\right]_{s.m.} \overline{S(LT)} + \sum_i B_{ij} R_i \quad ,$$

and by imposing homotheticity locally the number of parameters to be estimated has been reduced from $n(n + 1)$ to n^2 .

Recall assumption (b).[1] Using this, the model is parameterized initially in terms of <u>direct</u> partial transformation elasticities, T_{ij} , defined by

$$(23) \qquad T_{ij} = \frac{d\left(\frac{S(LT)_i}{S(LT)_j}\right)}{d\left(\frac{\partial S(LT)_i}{\partial S(LT)_j}\right)} \cdot \frac{\frac{\partial S(LT)_i}{\partial S(LT)_j}}{\frac{S(LT)_i}{S(LT)_j}} \quad , \qquad \text{for all } j \; ,$$

$$= T_{ji} \quad \text{(the symmetry condition) .}$$

1. The following analysis is heavily based on the approach developed in Powell and Gruen (1968).

Combining (9) and (10:1) yields

$$(11) \qquad DM_{ij} = \left[\frac{SM_{ij}}{\sum_{i \neq j} SM_{ij} + SE_j + SI_j}\right] D(LX)_j \quad , \quad (i \neq j) \; .$$

Equation (11) represents the demand by all firms for labour of occupation j type from source occupation i. Equations (7) to (11) allocate a certain additional demand[1] for labour of occupation j type to be satisfied from alternative sources. In turn, the proportions demanded from each source are determined by the relative number of workers in each source willing and able to supply jobs of occupation j type. Thus the proportions are supply or worker determined, while the total number of opportunities are demand determined.

1. It is assumed that total "outside" demand $D(LX)_j > 0$, and the case for $D(LX)_j < 0$ is not considered, though the same analysis is applicable but in reverse. If $D(LX)_j = 0$, then no net movement occurs into this occupation.

4. DEVELOPMENT OF THE SOURCE - AND DESTINATION - SPECIFIC SUPPLY OF TRANSFEREES

The supply schedule for workers who wish to shift from occupation i to occupation j is determined at the second stage of a two level process. At the first stage, aggregate supply to a particular occupation ($S(LT)_j$) is determined. This quantity is then broken down into its component sources: the stock of labour already in that occupation wishing to remain - "inside" supply ($S(LN)_j$); supply from all other occupations ($\sum_{i \neq j} SM_{ij}$), and supply from the education (SE_j) and immigration (SI_j) sectors - collectively denoted as "outside" supply.

These quantities are all assumed to respond to certain labour market signals such as occupation specific wage and unemployment rates. An element in the explanation for the existence of long run unemployment is the possibility of distortions in these signals, i.e., relative wage and unemployment rates are perceived inaccurately. However, these market imperfections are ignored in the paradigm presented here.

4:1 The derivation of aggregate supply to each occupation, $S(LT)_j$

To obtain the supply by occupation schedule for the labour force, use is again made of a fictional entity, this time called the representative labour supplier.[1] Unlike any individual agent, this worker is assumed to be capable of allocating his *fixed* hours available for work (where this quantity is derived from a standard model of labour-leisure choice) amongst *alternative* occupations so as to maximize his synthetic welfare

1. This concept and that of the skill transformation frontier used below are developed in Parham and Ryland (1978).

Suppose forther that $g_j^S(.)$ may be approximated by a linear function so that equation (15) represents a linear supply system of the form

$$(16) \qquad S(LT)_j = A_j \overline{S(LT)} + \sum_i B_{ij} R_i \quad ,$$

where A_j and B_{ij} encompass n and n^2 coefficients respectively.

If $h(.)$ is homothetic ,

$$(17) \qquad \frac{\partial \ln S(LT)_j}{\partial \ln \overline{S(LT)}} = 1 \quad ,$$

or

$$(18) \qquad \frac{\partial S(LT)_j}{\partial \overline{S(LT)}} \cdot \frac{\overline{S(LT)}}{S(LT)_j} = 1 \quad .$$

Although $h(.)$ is obviously not globally compatible with a linear $g_j^S(.)$ function, it is possible to impose a local mapping between these two functions at some central point of the observations, e.g., sample means.

Imposing homotheticity at sample means yields

$$(19) \qquad \frac{\partial S(LT)_j}{\partial \overline{S(LT)}} \left[\frac{\overline{S(LT)}}{S(LT)_j} \right]_{\text{sample means}} = 1 \quad .$$

But from equation (15),

$$(20) \qquad \frac{\partial S(LT)_j}{\partial \overline{S(LT)}} = A_j \quad .$$

agent is derived from the first-order conditions to the following constrained optimization problem :

$$(13) \qquad \operatorname{Max} \pi(\underset{\sim}{R}) = \sum_{j=1}^{n} \pi(R_j) = \sum_{j=1}^{n} R_j \, S(LT)_j$$

subject to

$$(14) \qquad \overline{S(LT)} = h(\underset{\sim}{S(LT)}) \ ,$$

where $\underset{\sim}{S(LT)}$ is the vector of supplies of manhours of different occupational types with typical element $S(LT)_j$, with the other symbols as previously defined.

Without at this stage specifying the functional form appropriate to the transformation frontier h(.) , but on the assumptions

(a) that it is homothetic[1] ,

and

(b) that, in the neighbourhood of the solution point, different pair-wise direct transformation elasticities are approximately constant (but nevertheless allowed to differ among different pairs of occupations) ,

the maximization problem (13) - (14) can be solved for effective manhours allocated to the various occupations by the representative labour supplier. Solution of the first order conditions yields a supply function of the type

$$(15) \qquad S(LT)_j = g_j^s \, (\overline{S(LT)}, \underset{\sim}{R}) \ .$$

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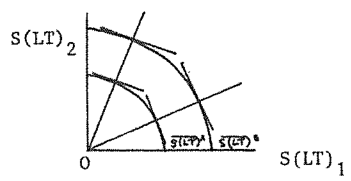
1. Homotheticity implies that as $\overline{S(LT)}$ increases, the distribution of total effective manhours amongst each alternative occupation increases by the same proportion, so that the ratio of manhours between any two occupations remains constant. Put alternatively, homotheticity implies the existence of straight line expansion paths from the origin. This can be represented for two occupations $S(LT)_1$ and $S(LT)_2$ thus



index. Thus it is possible for a representative labour supplier to work two hours a day as a brain surgeon, three hours as a garbage collector, and two hours as a school teacher, whereas any individual worker generally confines his skills to one occupation. Summation over these identical representative labour suppliers yields the aggregate n-occupation supply schedule at a particular point in time.

Underlying this approach is the concept that the potential distribution of aggregate labour supply at any point in time can be represented by a surface which has as its determinants many factors, including the demographic characteristics of the workforce, outputs of the public and private education systems, the government's funding decisions for post-secondary education, immigration, post-school investment in human capital and 'on-the-job' training. These factors combine to produce an n-dimensional occupation specific skill transformation frontier,[1] and the position actually adopted on this frontier represents the occupational distribution of the work-force at that given point in time. This position is obtained as the point of tangency between this frontier and a function determined by the relative rates of remuneration of the different occupations[2] (i.e., the synthetic welfare index referred to above). These rates of remuneration are not pure wage variables, but rather represent wages corrected for other occupation specific characteristics such as social status, the probability of getting a job in a specific occupation, the private cost of training required to enter that

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1. This frontier is regarded as predetermined at any particular point in time.

2. The analogue of this concept in the goods space is that of the economy's output mix at any point in time being determined by the point of tangency between the production possibilities frontier and a locus of relative prices, so that aggregate revenue is maximized.

occupation, and the like. The rate of remuneration appropriate to a particular occupation is therefore interpreted as the certainty equivalent wage that can be obtained there, adjusted for the private training costs.

To clarify the proposed formulation, consider the case of an economy in which the labour market includes only three occupations, A, B and C. At any point in time, the capacity of the economy to produce labour services in aggregate is fixed and in quality adjusted units is denoted by $\overline{S(LT)}$. It is possible to define a transformation frontier h(.) representing the various combinations of occupations A, B and C that this economy can 'produce' consistent with this level of short run labour capacity. It is also possible to define an aggregate objective function, $\pi(\underset{\sim}{R}) = \sum_{j=1}^{3} \pi_j(R_j)$ (where R_j, the j^{th} element of $\underset{\sim}{R}$, represents the adjusted certainty equivalent rate of remuneration per manhour attainable in occupation j) representing the trade-offs between occupations A, B, and C from the viewpoint of the representative labour supplier. The remuneration variable for any <u>particular</u> occupation j, R_j, is considered to depend on the following occupation-specific sets of variables :

(i) wages $\{W_j\}$,

(ii) unemployment $\{U_j\}$,

(iii) the cost of training required $\{Y_j\}$.

In particular,

$$R_j = W_j^*(1 - U_j) , \tag{12}$$

where

$$W_j^* = W_j - Y_j .$$

In this formulation, W_j^* is the expected after tax wage net of an allowance for privately-borne training costs of entry into j, while $(1 - U_j)$ is the probability of obtaining employment in j. R_j is thus a certainty equivalent remuneration rate.[1]

It is argued that the function h(.) is concave to the origin on the grounds that, if an attempt were made to move elements of a fixed workforce from one occupation to another, real social costs would increase, and at an increasing rate. In other words, suppose it is easier (i.e., less costly) to convert type B as compared to type C workers into type A workers. Now, as previous type B workers transfer to type A work, the value of each A worker in terms of his contribution at the margin to total (quality-adjusted) labour services available falls. Further, since any additional workers must now originate in occupation C, the marginal social cost of retraining increases. Of course, even within occupational categories, the suitability of individuals for retraining varies. This spectrum of suitability of individuals for transfer enables the aggregate skill transformation frontier h(.) to be modelled as a smooth and continuous function.

Consider now the labour supply decision of the representative agent in an n-occupational situation. It is assumed that his objective function π takes the following form

$$\pi(\underset{\sim}{R}) = \sum_{j=1}^{n} R_j \, S(LT)_j ,$$

where $S(LT)_j$ is the total number of effective manhours allocated to occupation j. The occupational allocation of labour supplied by this

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1. Application of the probability axioms proved in von Neumann and Morgenstern (1947) implies that the certainty equivalent wage in any occupation can be defined as the wage in that occupation multiplied by the probability of obtaining employment in that occupation, where this probability is equal to (1 - unemployment rate).