IMPACT OF DEMOGRAPHIC CHANGE ON INDUSTRY STRUCTURE IN AUSTRALIA

A joint study by the Australian Bureau of Statistics, the Department of Employment and Industrial Relations, the Department of Environment, Housing and Community Development, the Department of Industry and Commerce and the Industries Assistance Commission

COMMODITY-SPECIFIC SUBSIDIES, DEMAND PATTERNS, AND THE INCENTIVE TO WORK

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The views expressed in this paper do not necessarily reflect the opinions of the participating agencies, nor of the Australian government.

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1. SCOPE AND PURPOSE OF THE PAPER

Our aim is to develop a framework of demand analysis sufficiently flexible to deal with the following two problems :

(i) the effects on demand patterns and hours worked of the payment of a commodity-specific subsidy (such as a housing rent subsidy) ;

(ii) modelling the supply of hours worked in a situation where, because overtime is voluntary and payment for such work is at a higher rate than payment for standard hours, the marginal wage received is endogenous.

Our analysis of problem (i) hopefully will be of use in the design of the Commonwealth Government's HAVE (Household Allowance Voucher Experiment) scheme; (ii) is needed for IMPACT's medium-term model in order to determine how changes in schedules of wage rates affect the supply of hours worked.

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The form of the subsidies considered in this paper is discussed in Section 2. The income and substitution effects of these subsidies (including their effects on the incentive to work) are then developed in Section 3 within the context of a systems approach to the determination of the pattern of commodity demands and the supply of hours worked. The vehicle of analysis used is a suitably extended version of the familiar linear expenditure system (LES) of Richard Stone.[1]

In Section 4 we focus our attention on the demand for leisure and the supply of hours worked in the context of a household having only one breadwinner. Some functional forms for the schedule of wages paid to an individual for varying hours worked per week are introduced in this section. The parameters of this schedule are beyond the control of the consumer/worker whose behaviour is modelled; the elasticity of supply of hours worked is developed in terms of responses to exogenous changes in these parameters. An example of such an exogenous change would be the handing down of a new award by a wage fixing tribunal.

The supply of hours worked depends on the opportunity cost of the decision-maker's time. In families with small children, for instance, a wife may require relatively more time at home than otherwise. This can be modelled in our extended version of the LES by making the 'minimal required amounts of leisure time' (where 'leisure' is shorthand for time spent in any way except working in paid employment) a function of family

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1. Richard Stone, "Linear Expenditure Systems and Demand Analysis : An Application to the Pattern of British Demand," Economic Journal, Vol. 64, No. 255 (September 1954), pp. 511-27.

size. A similar treatment of minimal requirements of food, clothing, etc., has been adopted by a number of writers,[1] and is also relevant here. Age of household head may be an important demographic variable : suitable techniques for incorporation of this variable into the analysis of cross-sectional expenditure data have also been developed by Williams.[2] In Section 5 these issues are discussed in the context of a household having two potential breadwinners.

In Section 6, we briefly recapitulate the main points of theoretical analysis and draw up an agenda for empirical work.

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1. Constantino Lluch, Alan A. Powell and Ross A. Williams, Patterns in Household Demand and Saving (New York : Oxford University Press, 1977);

   Ross Williams, "Wants and Working Wives : Household Demand and Saving in Australia," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. SP-06, Industries Assistance Commission, Melbourne, December, 1976 (mimeo), pp. 34.

   David Ryan, "Effect of Ethnic Origin on Household Consumption Patterns in Australia," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. SP-10, Indsutries Assistance Commission, Melbourne, June, 1977 (mimeo), pp. 29.

2. Ross Williams, "Household Consumption in Australia : An Examination of Patterns Across Socio-Economic Classes", Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. SP-04, Industries Assistance Commission, Melbourne, May, 1976 (mimeo), pp. 38.

2. FORM OF SUBSIDY

The subsidy considered in this paper is one where the household can fall into one of three categories (Case A, B or C) dependent on the level (πS) of household expenditure on the subsidized good. The level of the subsidy (V) is determined by the values of the three parameters σ, δ and τ of the scheme, Y the household income (inclusive of transfer payments other than the commodity specific subsidy under consideration) and π , the price before subsidy of the subsidized commodity. The form of the subsidy is defined by the relationships :

(2.1) $\quad V = 0 \quad$ if $\pi S/Y \leqslant \delta/\sigma$; (A)

(2.2) $\quad V = \sigma\pi S - \delta Y \quad$ if $\pi S/Y > \delta/\sigma$ and $\tau \geqslant \pi S$; (B)

(2.3) $\quad V = \sigma\tau - \delta Y \quad$ if $\pi S/Y > \delta/\sigma$ and $\tau < \pi S$. (C)

The subsidy scheme described above yields as special cases the three types of housing allowance payments which it is proposed to consider in the HAVE Scheme under which a sample of some 3000 households will receive voucher payments and a further 1500 families will be monitored. The parameters of the scheme will be varied among households in order to obtain experimental data which can be used to estimate consumer reaction to a possible national scheme with as yet undetermined parameters.

When $\delta = 0$ the system is equivalent to a <u>per cent of rent</u> scheme where the household receives a subsidy of ($100.\sigma$) per cent of their expenditure on rent provided they spend less then τ dollars on rental accommodation. Rent expenditures at a level greater than or equal to τ dollars are eligible for a maximum subsidy of $(\sigma\ \tau)$ dollars. The per cent of rent scheme can be expressed as :

(2.4)	$V = \sigma\pi S$	if $\tau \geqslant \pi S$;	(B)
(2.5)	$V = \sigma\tau$	if $\tau < \pi S$.	(C)

(Note that case A does not apply under this scheme because $(\pi S)/Y > 0 = \delta/\sigma)$.

When $\sigma = 1$ the system is equivalent to a full housing gap scheme where, provided the household spends more than a given proportion, δ, of their income on rent, and rental payments do not exceed τ dollars, the excess of rental payments over and above $(\delta\ Y)$ dollars will be refunded. The justification used for such a scheme is that it is argued that no family should be required to spend more than $(100\ \delta)$ per cent of their income to obtain 'reasonable' accommodation. The housing gap scheme can be expressed as :

(2.6)	$V = 0$	if $\pi S/Y \leqslant \delta$;	(A)
(2.7)	$V = \pi S - \delta Y$	if $\pi S/Y > \delta$ and $\tau \geqslant \pi S$;	(B)
and			
(2.8)	$V = \tau - \delta Y$	if $\pi S/Y > \delta$ and $\tau < \pi S$;	(C)

where τ is interpreted as that level of expenditure required to obtain 'reasonable' accommodation.

When $1 > \sigma > \delta > 0$ the system is equivalent to a <u>partial housing gap</u> scheme where, if the household spends more than a given proportion, δ/σ, of their income on rent and rental payments do not exceed τ dollars, then a proportion, σ, of the extent to which rental payments exceed $[(\delta/\sigma).Y]$ dollars will be refunded. The justification used for such a scheme is that it is argued that those families which are placed in the position of having to pay a large proportion (i.e., greater than δ/σ) of their income on rent in order to obtain 'reasonable' accommodation should receive assistance with the additional rental burden involved in acquiring 'reasonable' accommodation. The partial housing gap scheme can be expressed as :

(2.9) $$V = 0 \qquad \text{if } \pi S/Y \leq \delta/\sigma \text{ ;} \qquad \text{(A)}$$

(2.10) $$V = \sigma(\pi S - (\delta/\sigma)Y) \qquad \text{if } \pi S/Y > \delta/\sigma \text{ and } \tau \geq \pi S \text{ ;} \qquad \text{(B)}$$

(2.11) $$V = \sigma(\tau - (\delta/\sigma)Y) \qquad \text{if } \pi S/Y > \delta/\sigma \text{ and } \tau < \pi S \text{ .} \qquad \text{(C)}$$

3. THE LINEAR EXPENDITURE SYSTEM WITH ENDOGENOUS WAGE RATES UNDER COMMODITY-SPECIFIC SUBSIDIES

In this section we model the behaviour of a consumer/worker who is the decision-maker (viz., economic agent) for a typical household's expenditure decisions. This decision-maker is taken to be the one who decides whether (and if so, how much) overtime is worked.

Earnings/Hours Schedule

We assume that different rates of pay apply to overtime and to standard hours of work. Whilst for an individual agent the schedule of rates offered will be a staircase function containing one or more steps for time and a half, double time, etc., in applied work we model a representative agent who is a composite of many such individuals. To a good approximation, therefore, the schedule of after-tax earnings, $G(H)$, may be treated as a continuous function of hours per week H actually worked. This is because the location and size of steps in the staircase will vary as a result of minor differences in awards, in practices of employers concerning over-award payments, etc.. The function $G(H)$ cannot be influenced either by the individual nor by the representative agent. Even granting that some ambiguity might exist in the case of the latter (because of such things as union activity), the proposition is certainly valid for the short run (three months to a year), which is our period of analysis.

Whilst the representative agent has no control over the form of G, he does, of course, determine its value $G(H)$ by choosing the number of hours per week, H, that are worked. We will need a notation for the marginal earnings schedule and the average earnings schedule; these we will denote by $\psi(H)$ and $\phi(H)$ respectively. The three schedules are related by :

$$(3.1) \qquad G(H) = \int_0^H \psi(h)dh = H\phi(H) \quad ;$$

$$(3.2) \qquad G'(H) = \psi(H) = H\phi'(H) + \phi(H) \quad .$$

The elasticity of the total earnings schedule at the point H is given by

$$(3.3) \qquad \eta(H) = \frac{\partial \ \ell n(G)}{\partial \ \ell n(H)} = \frac{G'(H).H}{G(H)} = \frac{\psi(H)}{\phi(H)} \quad .$$

Utility Function

We introduce an $(n + 2)$ partition of the goods and services consumed by the representative agent : n unsubsidized commodity and service items, one subsidized commodity, and 'leisure time.' The last-mentioned is broadly defined to mean any time not actually spent in paid employment. This 'spare' time might be spent in genuine leisure or in household and home duties, possibly associated with caring for children. To be fully concrete, we have in mind that the commodity groups should be broadly defined, so that n is a number like 6 or 8, and strongly additive utility postulates therefore applicable.[1] The particular functional form chosen for the utility function is the Klein-Rubin[2] or Stone-Geary function,

$$(3.4) \qquad U = \sum_{i=1}^{n+2} \beta_i \, \ell n(x_i - \gamma_i) \qquad \left(\sum_{i=1}^{n+2} \beta_i \equiv 1\right) ,$$

in which x_i is the quantity of the i^{th} good or service consumed, and $\underline{\underline{\beta}} = (\beta_1, \ldots, \beta_n ; \beta_{n+1} ; \beta_{n+2})'$ and $\underline{\underline{\gamma}} = (\gamma_1, \ldots, \gamma_n ; \gamma_{n+1} ; \gamma_{n+2})'$ are parameters.[3] The betas turn out to have interpretation as the marginal budget shares of the various goods and services; the gammas may be interpreted as the minimal amounts of these goods and services necessary to support some 'subsistence' or 'socially accepted minimum' standard of living.[4] To keep the notation both flexible and mnemonic, we duplicate the symbols for the subsidized good and for leisure time as follows :

$$(3.5) \qquad \text{[subsidized good]} \quad S \equiv x_{n+1} ; \beta_{n+1} \equiv \beta_S ; \gamma_{n+1} \equiv \gamma_S ;$$

$$(3.6) \qquad \text{[leisure time]} \quad L \equiv x_{n+2} ; \beta_{n+2} \equiv \beta_L ; \gamma_{n+2} \equiv \gamma_L .$$

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1. Because the utility function (3.4) which we choose below rules out (net) complementarity, the classification of commodities should avoid items like 'Sporting and Recreational Equipment' which would be complementary with leisure time. Hopefully complements with leisure time would be spread fairly thinly over a number of broad commodity groups.

2. L. R. Klein and H. Rubin, "A Constant-Utility Index of the Cost of Living," Review of Economic Studies, Vol. XV, 1948-49, pp. 84-87.

3. Because of the normalization constraining the sum of the betas to unity, there are only 2(n + 2) - 1 'free parameters.'

4. This interpretation is not obligatory. The LES model will not give price elasticities exceeding unity unless some gammas are allowed to be negative, in which case the subsistence interpretation of the gammas fails. See Alan A. Powell, Empirical Analytics of Demand Systems (Lexington, Mass.: D. C. Heath, 1974), p. 38.

Among the n unsubsidized commodities,[1] there may or may not occur durable goods. In this paper, however, we do not make any special allowance for durability. This is because even though, in principle, the way in which to proceed is clear,[2] explicit treatment of durability would greatly complicate the analysis and obscure our primary focus on subsidies and labour supply.

Budget Constraint

There are two complications which make the standard treatment of the LES budget constraint inappropriate here.

The first is the endogeneity of income in the current problem. Although in the context of a fixed hourly wage rate, income (and the demand for leisure) have been endogenized by Betancourt[3] and others,[4] the wage rate itself remained exogenous in those analyses (that is, $\psi(h) = \Psi$, a constant, for all (H)).

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1. For shorthand we use 'commodities' to mean 'goods and/or services' from this point on.

2. Peter B. Dixon and Constantino Lluch, "Durable Goods in the Complete Systems Approach to Applied Demand Theory," Paper presented to the Third World Congress of the Econometric Society, Toronto, August, 1975; published in heavily abridged form as "Durable Goods in the Extended Linear Expenditure System" in the Review of Economic Studies, Vol. XLIV, No. 2 (June, 1977), pp. 381-4.

   John Iacono, "Durables in the Consumption Function," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. SP-05, Industries Assistance Commission, Melbourne, September, 1976 (mimeo), pp. 38.

   Alan A. Powell, Peter B. Dixon and Michael McAleer, "Durables in the Consumption Function : A Systems Approach to Employment Effects," Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. MP-02, Industries Assistance Commission, Melbourne, February, 1977 (mimeo), pp. 26.

3. Roger R. Betancourt, "Report of a Study of Chilean Household Data Incorporating Leisure and Demographic Variables into Consumption and Savings Behaviour," Part of a report of Phase 1 of study entitled "Consumption and Savings in the Development Process," Development Research Center, World Bank, Washington, D.C. (January, 1973).

4. Michael Abbott and Orley Ashenfelter, "Labour Supply, Commodity Demand and the Allocation of Time," Review of Economic Studies, Vol. XLIII, No. 135 (October, 1976), pp. 389-411.

The second is the fact that under the subsidy arrangements outlined in Section 2, the effective purchasing power of the representative agent depends also on his expenditure S on the subsidized good.

As pointed out in Section 2, the value of the subsidy might be (A) zero, (B) a certain fraction of expenditure on the subsidized good less a certain fraction of income; or (C) a certain dollar amount less a certain fraction of income. Our decision-making paradigm is a two-step one in which the representative agent first locates himself in a region (A), (B) or (C) of the decision space, and having done so, optimizes his position within that region. Whereas calculus techniques will take care of the second step, a discrete comparison of alternatives is necessary for the first.[1] The decision variables which the agent adjusts to reach his preferred position are the quantities of goods and services consumed (especially of the subsidized commodity), and hours worked.

To this point, no mention has been made of savings. A proper treatment of the intertemporal trade-off requires the use of calculus of variations or control-theoretic techniques, and (in the context of the Klein-Rubin utility function) leads to Lluch's extended linear expenditure system (ELES).[2] Use of this approach would seriously complicate the present analysis. As pointed out by Howe, moreover, the savings/consumption choice can be recognized in a rudimentary way by use of a short-cut method.[3] In this approach one 'commodity' (say the n^{th}) becomes savings. In the

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1. The order of the steps is arbitrary. Readers may prefer to think in terms of stage one as the optimization within (A), (B) and (C) and to think of stage 2 as ranking these sub-optima and consequent choice of the global optimum.

2. Constantino Lluch, "The Extended Linear Expenditure System," European Economic Review, Vol. 4, No. 1 (April, 1973), pp. 21-32.

3. Howard Howe, "Development of the Extended Linear Expenditure System from Simple Saving Assumptions," European Economic Review, Vol. 6, No. 3 (July, 1975), pp. 305-310.

utility function γ_n becomes zero (no subsistence level of savings being essential from the viewpoint of the current period), and the price of the n^{th} commodity becomes unity (i.e., one dollar, the numeraire). The parameter β_n becomes the marginal propensity to save, whilst the marginal shares in the budget (rather than in income) of the other items become $\beta_i/(1 - \beta_n)$. If the agent whose behaviour is under analysis is too poor to save, then β_n is zero. Whether or not to include savings in this way can be decided by the empirical investigator in the light of the particular data set and problem to hand.

With this amount of background we can write the budget constraint as :

$$(3.7) \qquad \alpha + G(T) = \int_{T-L}^{T} \psi(h)\, dh + \sum_{i=1}^{n} p_i x_i + \pi S \;;$$

$$(3.8) \qquad \alpha + G(T) + \sigma\pi S - \delta Y = \int_{T-L}^{T} \psi(h)\, dh + \sum_{i=1}^{n} p_i x_i + \pi S \;;$$

$$(3.9) \qquad \alpha + G(T) + \sigma\tau - \delta Y = \int_{T-L}^{T} \psi(h)\, dh + \sum_{i=1}^{n} p_i x_i + \pi S \;;$$

in which Y is defined by

$$(3.10) \qquad Y = \alpha + G(T - L) \;,$$

which is cash income (inclusive of transfer payments other than the commodity specific subsidy under discussion). Above, (3.7) denotes the zero subsidy case and (3.8) and (3.9) the other two cases discussed in Section 2.

Additional notation appearing in (3.7-8-9) is as follows. α is the sum of non-labour income (if any) from assets, plus those government transfer payments (other than unemployment benefits)[1] which would be made even if the representative agent does not work (H=0). The value of α would therefore depend on household composition. T is the total weekly time budget, in the sense that it is the number of hours during which it would be physically possible to work. In the zero subsidy case the expression

$$\alpha + G(T)$$

is termed "full income." It consists of transfer payments plus the labour income that could be earned by a person who totally renounces 'leisure' (on our broad definition). Finally $\pi \equiv p_{n+1}$ is the price (before subsidy) of the subsidized good or service.

We now interpret each of (3.7), (3.8) and (3.9) individually. The first term on the right of (3.7) is the market valuation of leisure time 'purchased' by the representative agent by virtue of his deciding to work no more than (T - L) hours. The second term is the value of purchases of unsubsidized goods and services, plus savings (if any). The final term is the value of purchases of the subsidized good (although, of course, in case (A) the subsidy does not apply).

The left of (3.8) and (3.9) is again 'full income,' but this time inclusive of the subsidies paid. The right hand sides of (3.8) and (3.9) are identical with that of (3.7).

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1. All unemployment is assumed to be involuntary. Modification of the framework to accommodate unemployed decision makers is discussed later.

Lagrangean

We can cover all these cases of the budget constraint by constructing an artificial budget constraint

$$(3.11) \qquad \alpha + G(T) + a(b\pi S + \rho) - kY = \int_{T-L}^{T} \psi(h)\, dh + \sum_{i=1}^{n} p_i x_i + \pi S \ .$$

Case (3.7), the zero subsidy case,[1] is given by $a = k = 0$, case (3.8) by $\rho = 0$, $b = 1$, $a = \sigma$, $k = \delta$; and case (3.9) by $b = 0$, $\rho = \tau$, $a = \sigma$, $k = \delta$. Keeping in mind that

$$(3.12) \qquad G(T) - \int_{T-L}^{T} \psi(h)\, dh \equiv G(T - L) \equiv G(H) \equiv Y - \alpha \ ,$$

where $H \equiv T - L$ is the actual number of hours worked, the Lagrangean made up from (3.4) and (3.12) is

$$(3.13) \qquad F(\lambda, x_1, \ldots, x_n \ ; \ S \ ; \ L)$$

$$= \sum_{i=1}^{n} \beta_i \, \ell n(x_i - \gamma_i) + \beta_S \, \ell n(S - \gamma_S) + \beta_L \, \ell n(L - \gamma_L)$$

$$+ \lambda \left\{ (1 - k)[G(T - L) + \alpha] + a(b\pi S + \rho) - \sum_{i=1}^{n} p_i x_i - \pi S \right\} .$$

1. The other two subsidy cases described in Section 2 can be covered by putting values of a, b, ρ and k as follows :

Subsidy Case	Equation No.	Values of a	b	ρ	k
B	2.2	σ	1	0	δ
C	2.3	σ	0	τ	δ
B	2.4	σ	1	0	0
C	2.5	σ	0	τ	0
B	2.7	1	1	0	δ
C	2.8	1	0	τ	δ
B	2.10	σ	1	0	δ

First Order Conditions

The first order conditions for a maximum consist of (3.11) (obtained by differentiating (3.13) with respect to the Lagrangean multiplier λ), plus the following :

$$(3.14) \qquad \partial F/\partial x_i = 0 = \frac{\beta_i}{(x_i - \gamma_i)} - \lambda p_i \qquad (i = 1, \ldots, n) \; ;$$

$$(3.15) \qquad \partial F/\partial S = 0 = \frac{\beta_S}{(S - \gamma_S)} - \lambda\pi(1 - ab) \; ;$$

$$(3.16) \qquad \partial F/\partial L = 0 = \frac{\beta_L}{(L - \gamma_L)} - \lambda(1 - k)\psi(T - L) \; .$$

Marginal Utility of Money

Rewriting (3.14-15-16) as equations containing only betas on the left and summing these equations, we obtain

$$(3.17) \qquad \sum_{i=1}^{n} \beta_i + \beta_S + \beta_L \equiv 1 = \lambda \left\{ \sum_{i=1}^{n} p_i \, x_i + (1-ab)\pi S + (1 - k)\psi(T - L)L \right\}$$
$$- \lambda \left\{ \sum_{i=1}^{n} p_i \, \gamma_i + (1 - ab)\pi\gamma_S + (1 - k)\psi(T - L) \, \gamma_L \right\} .$$

But from the budget constraint,

$$(3.18) \qquad \sum_{i=1}^{n} p_i \; x_i + (1 - ab) \; \pi S \; = \; (1 - k)\{G(T - L) + \alpha\} + a\rho \; .$$

Substituting (3.18) into (3.17), we obtain the following solution for the marginal utility of money :

$$(3.19) \qquad \lambda = 1/\left\{ (1 - k)\,[G(T - L) + L\psi(T - L) + \alpha] + a\rho - \sum_{i=1}^{n} p_i \; \gamma_i - (1 - ab)\,\pi\gamma_S - (1 - k)\psi(T - L)\gamma_L \right\}$$

$$(3.19') \qquad = 1/(Z - P'\Gamma)$$

in which Z is a full income measure which aggregates the various components of income at the shadow prices induced by the distortions introduced by the subsidy; i.e.,

$$(3.20) \qquad Z = (1 - k)G(T - L) + (1 - k)\,[L\psi(T - L) + \alpha] + a\rho \; ;$$

and where

$$(3.21) \qquad P' = (p_1, \ldots, p_n \; ; \; (1 - ab)\,\pi \; ; \; (1 - k)\psi(T - L)) \; ,$$

with

$$(3.22) \qquad \Gamma = (\gamma_1, \ldots, \gamma_n \; ; \; \gamma_S \; ; \; \gamma_L)' \; .$$

Price Distortions

In the first term on the right of (3.20), income from work is evaluated at $(1 - k)$ times the market valuation; this is because under the subsidy arrangements a fraction k of additional earned income is lost. The second term on the right represents the income (and imputed income) available from non-wage sources (including leisure), after adjustment for the fraction k which is lost under the subsidy arrangements. The formulation indicates that at market prices the valuation of leisure would be higher than under the subsidy. Without the subsidy leisure is valued at $\psi(T - L)$, which is the cash income foregone by declining to work an additional hour; under the subsidy, however, this is reduced to $(1 - k)\psi(T - L)$. The reason is as previously. Transfer income α is similarly undervalued at $(1 - k)\alpha$; any decline in transfer income not related to the subsidy is partially made good by an increase in the subsidy.

The term $P'\Gamma$ on the right of (3.19') can be interpreted as the cost of subsistence, where cost is reckoned at the (distorted) shadow prices induced by the subsidy. The only new feature here is that, relative to market prices, the subsidized good is undervalued at $(1 - ab)\pi$, whereas the market valuation is π. These various distortions mean that the consumer/worker will reach a lower level of utility than if untied cash transfers were paid. Measurement of the utility foregone in terms of the index U would be straightforward.[1]

In the form (3.19') the expression for λ can be recognized as a standard LES result. If we make L exogenous, ψ a constant, and $a = k = 0$. (3.19') reduces to the LES/ELES result.

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1. The method would consist of comparative evaluations of the indirect utility function.

Demand Functions

Substitution of (3.19) into (3.14) yields the first n demand functions of the system :

$$x_i = \gamma_i + \frac{\beta_i}{p_i}(Z - P'\Gamma)$$

$$= \gamma_i + \frac{\beta_i}{p_i}\Big\{ a\rho + (1-k)[G(T-L) + L\psi(T-L) + \alpha]$$

(3.23)
$$- \sum_{j=1}^{n} p_j\gamma_j - (1-ab)\,\pi\gamma_S - (1-k)\psi(T-L)\gamma_L \Big\}$$

$$(i=1, \ldots, n) .$$

Similarly,[1]

$$S = \gamma_S + \frac{\beta_S}{(1-ab)\pi}(Z - P'\Gamma)$$

(3.24)
$$= \gamma_S + \frac{\beta_S}{(1-ab)\pi}\Big\{ a\rho + (1-k)[G(T-L) + L\psi(T-L) + \alpha]$$

$$- \sum_{j=1}^{n} p_j\gamma_j - (1-ab)\pi\gamma_S - (1-k)\psi(T-L)\gamma_L \Big\} ;$$

and

$$L = \gamma_L + \frac{\beta_L}{(1-k)\psi(T-L)}(Z - P'\Gamma)$$

(3.25)
$$= \gamma_L + \frac{\beta_L}{(1-k)\psi(T-L)}\Big\{ a\rho + (1-k)[G(T-L) + L\psi(T-L) + \alpha]$$

$$- \sum_{j=1}^{n} p_j\gamma_j - (1-ab)\pi\gamma_S - (1-k)\psi(T-L)\gamma_L \Big\} .$$

1. An evaluation of the demand function for the subsidized good under the operation of a full housing gap scheme shows that households electing to locate in region B (i.e., $\rho = 0$, $b = 1$, $a = \sigma$, $k = \delta$) will locate at the boundaries of B and C where they will make the maximum expenditure on housing services ($\pi S = \rho$) commensurate with satisfying the conditions required to be in region B (eqn. 2.7). That is, under a full housing gap scheme, no household will opt for the interior of region B.

All of these are greatly simplified in the case where the hourly wage rate is independent of the number of hours worked, in which case we would have $\psi(T - L) = \psi$, and $G(T - L) = (T - L)\psi$. Using M to denote exogenous constraints, (3.23) (for instance) becomes

$$(3.26a) \qquad x_i p_i = \gamma_i p_i + \beta_i \left\{ M_1 + (1 - k)M_2 - \sum_{i=1}^{n} p_i \gamma_i - (1 - ab)\pi\gamma_S - (1 - k)\psi\gamma_L \right\} \qquad (i = 1, \ldots, n) ,$$

in which

$$(3.26b) \qquad M_1 = a\rho \quad \text{and} \quad M_2 = T\psi + \alpha .$$

Households Having no Members Receiving Labour Income[1]

Demand behaviour of these households (having no one in employment) is obtained by redefining α as the sum of asset income (if any) plus transfers inclusive of welfare payments (e.g., unemployment benefits, old age or invalid pensions, etc.), and putting ψ equal to zero. The demand for leisure function drops out of the system; what is left is

$$(3.28) \qquad x_i p_i = \gamma_i p_i + \beta_i \left\{ (1 - k)\alpha \quad \sum_{i=1}^{n} p_i \gamma_i - (1 - ab)\pi\gamma S + a\rho \right\} \qquad (i = 1, \ldots, n) ,$$

plus the demand for the subsidized good; namely ,

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1. Households receiving no labour income and households with exogenous labour income are formally equivalent. Equations (3.28) and (3.29) are just the LES under the commodity subsidy.

$$(3.29) \qquad S\pi = \gamma_S\pi + \frac{\beta_S}{(1-ab)}\left\{(1-k)\alpha - \sum_{i=1}^{n} p_i\gamma_i - (1-ab)\pi\gamma_S + a\rho\right\} .$$

Elasticity Formulae : Variations in Exogenous Variables

To find the demand elasticities of the endogenous variables x_i $(i = 1, \ldots, n)$, S and L with respect to any of the exogenous variables,[1] it is useful to introduce an intermediate elasticity of the form $E_{(\ )\Theta} = \partial \ell n$ (endogenous variable)/$\partial \ell n\ \Theta$ where Θ is the following function of the exogenous variables :

$$(3.30) \qquad \Theta = [(1-k)\alpha + a\rho - \sum_{j=1}^{n} p_j\gamma_j - (1-ab)\pi\gamma_S]/(1-k) .$$

The elasticities of leisure demand and of hours supplied with respect to Θ are shown in Table 1, along with the elasticities of demand for subsidized and unsubsidized commodities. Various individual elasticities of interest can then be calculated from the table in conjunction with the chain rule ,

$$\frac{\partial\ \ell n(\text{endogenous variable})}{\partial\ \ell n(\text{exogenous variable})} = E_{(\ )\Theta} \cdot \frac{\partial\ \ell n\Theta}{\partial\ \ell n(\text{exogenous variable})} .$$

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1. The exogenous variables are $\{p_1, \ldots, p_n ; \pi ; \alpha\}$.

Note however that in the case of commodity demands the elasticities $E_{(x_i)\Theta}$ and $E_{(S)\Theta}$ in Table 1 have been based on special assumptions (namely that $dp_i = 0$ in the former case, and that $d\pi = 0$ in the latter). The table should only be used,

Table 1

Elasticities of Endogenous Variables with Respect to the Function Θ of Exogenous Variables

Endogenous Variable ( )	Elasticity with respect to Θ $E_{(\ )\Theta}$
Unsubsidized Commodities,* x_i $(i = 1, \ldots, n)$	$E_{(x_i)\Theta} = \frac{(1-k)\beta_i}{p_i}\left[1 - \frac{(L-\gamma_L)\psi'}{\left\{\frac{\psi}{\beta_L} - \frac{\psi'}{\psi}[\Theta + G]\right\}}\right]\frac{\Theta}{x_i}$
Subsidized Commodity,* S	$E_{(S)\Theta} = \frac{(1-k)\beta_S}{(1-a)\pi}\left[1 - \frac{(L-\gamma_L)\psi'}{\left\{\frac{\psi}{\beta_L} - \frac{\psi'}{\psi}[\Theta + G]\right\}}\right]\frac{\Theta}{S}$
Leisure Time L	$E_{(L)\Theta} = \Theta \Big/ \left\{L\left(\frac{\psi}{\beta_L} - \frac{\psi'}{\psi}[\Theta + G]\right)\right\}$
Hours Worked $(T - L) \equiv H$	$E_{(H)\Theta} = -\Theta \Big/ \left\{H\left(\frac{\psi}{\beta_L} - \frac{\psi'}{\psi}[\Theta + G]\right)\right\}$

* The chain rule should not be used with this elasticity to derive own-price elasticities.

therefore, to generate cross price elasticities. Own price elasticities for x_i and S, although cumbersome, are recorded for completeness :

$$(3.31) \qquad \frac{\partial \ell n\, x_i}{\partial \ell n\, p_i} = -\frac{\beta_i}{x_i}\left[\gamma_i\left\{1 - \frac{(L - \gamma_L)\psi'}{\frac{\psi}{\beta_L} - \frac{(\Theta + G)\psi'}{\psi}}\right\} + (1 - k)\frac{[\Theta + G + (L - \gamma_L)\psi]}{p_i}\right] \quad ;$$

$$(3.32) \qquad \frac{\partial \ell n\, S}{\partial \ell n\, \pi} = \frac{-\beta_S}{S}\left[\gamma_S\left\{1 - \frac{(L - \gamma_L)\psi'}{\frac{\psi}{\beta_L} - \frac{(\Theta + G)}{\psi}\psi'}\right\} + \frac{(1 - k)[\Theta + G + (L - \gamma_L)\psi]}{(1 - ab)\pi}\right] \quad .$$

Elasticity Formulae : Variations in Parameters of Earnings Schedule

In the present model there is no uniquely defined elasticity of supply of hours worked with respect to "the wage rate." The same lack of definition applies to all of the demand elasticities with respect to the same variable. The reason is simply that "the wage rate" is endogenous; a variety of responses in the other endogenous variables is possible depending on how (and why) the average or marginal wage rate changes. The exogenous influences relating to earnings power which may lead to a change in average or or marginal wages are changes in the parameters of the (after-tax) earnings schedule. Elasticities of the endogenous variables with respect to these entities are uniquely defined.

Let the parameters of the (after tax) earnings schedule be $\mu \equiv (\mu_1, \ldots, \mu_N)$. In order to make their dependence on μ explicit, the total marginal and average earnings schedules will now be written as $G(H; \mu)$, $\psi(H; \mu)$ and $\phi(H; \mu)$ respectively.[1] We now wish to investigate the incremental changes in the endogenous variables generated by a spontaneous infinitesimal change $d\mu$ in μ . We start with the supply of hours worked, H, as modelled via (3.25) . After substitution from (3.30), we obtain

$$H \equiv T - L$$

$$(3.33) \qquad = T - \gamma_L - \frac{\beta_L}{\psi}(\Theta + G + (L - \gamma_L)\psi) \ ;$$

$$(3.33') \qquad = T - \gamma_L - \frac{\beta_L}{\psi}\ \chi \ \text{(say)} \quad .$$

Using Δ to indicate small changes, and taking the total differential of (3.33), we obtain

$$(3.34) \qquad \Delta H \doteq -\beta_L \left[\frac{\Delta\chi}{\psi} + \chi\,\Delta\left(\frac{1}{\psi}\right)\right] \ .$$

Because χ appears in all of our behavioural equations, it is worthwhile to develop its differential separately for use in later formulae :

1. Notice that functions derived from G such as ψ, ψ', etc., may depend only on a subset of μ .

(3.35) $$\Delta\chi \doteq \Delta\Theta + \Delta G + (L - \gamma_L)\ \Delta\psi + \psi\Delta L\ .$$

The first term on the right vanishes because Θ contains only exogenous variables which are independent of the function G . The total differential in G is composed of the change in G induced by the change in H consequent upon the parametric change, plus the change in G due directly to the parametric change; i.e.,

$$\Delta G \doteq \left[\frac{\partial G}{\partial H}\Delta H + \sum_{j=1}^{N}\frac{\partial G}{\partial \mu_j}\Delta\mu_j\right] ,$$

i.e.,

(3.36) $$\Delta G \doteq \frac{\partial G}{\partial H}\sum_{j=1}^{N}\frac{\partial H}{\partial \mu_j}\Delta\mu_j + \sum_{j=1}^{N}\frac{\partial G}{\partial \mu_j}\Delta\mu_j ,$$

in which the partial derivatives $\frac{\partial H}{\partial \mu_j}$, $j = 1, \ldots, N$, are the very responses we are seeking to determine. A similar expression can easily be developed for $\Delta\psi$.

We now restrict attention to a change in one parameter of G only, say the j^{th}. With this simplification $\Delta\chi$ becomes

$$\Delta\chi \doteq \left\{\psi\frac{\partial H}{\partial \mu_j} + \frac{\partial G}{\partial \mu_j} + (L - \gamma_L)\left[\psi'\frac{\partial H}{\partial \mu_j} + \frac{\partial \psi}{\partial \mu_j}\right] - \psi\frac{\partial H}{\partial \mu_j}\right\}\Delta\mu_j ,$$

i.e.,

(3.37) $$\Delta\chi \doteq \left\{\frac{\partial G}{\partial \mu_j} + (L - \gamma_L)\left[\psi'\frac{\partial H}{\partial \mu_j} + \frac{\partial \psi}{\partial \mu_j}\right]\right\}\Delta\mu_j\ .$$

Returning now to the other component of (3.34), we note that

$$\text{(3.38)} \qquad \Delta\left(\frac{1}{\psi}\right) \doteq -\frac{1}{\psi^2}\left[\psi' \frac{\partial H}{\partial \mu_j} + \frac{\partial \psi}{\partial \mu_j}\right] \Delta\mu_j \quad .$$

Substituting from (3.38) and (3.37) into (3.34), we obtain :

$$\text{(3.39)} \qquad \frac{\Delta H}{\Delta \mu_j} \doteq -\frac{\beta_L}{\psi}\left[\frac{\partial G}{\partial \mu_j} - \frac{\Theta + G}{\psi}\left\{\psi' \frac{\partial H}{\partial \mu_j} + \frac{\partial \psi}{\partial \mu_j}\right\}\right] .$$

Taking the limit as $\Delta\mu_j \to 0$, and rearranging, we obtain :

$$\text{(3.40)} \qquad \frac{\partial H}{\partial \mu_j}\,\frac{\mu_j}{H} = \frac{\left[\frac{(\Theta + G)}{\psi}\,\frac{\partial \psi}{\partial \mu_j} - \frac{\partial G}{\partial \mu_j}\right]\frac{\mu_j}{H}}{\left[\frac{\psi}{\beta_L} - \frac{\psi'}{\psi}(\Theta + G)\right]}$$

as the elasticity of the supply of hours with respect to the j^{th} parameter of the total earnings schedule, G . The right of this expression involves the first and second derivatives $\left(\frac{\partial G}{\partial \mu_j}\,,\ \frac{\partial \psi}{\partial \mu_j} \equiv \frac{\partial^2 G}{\partial H\,\partial \mu_j}\right)$ of this schedule.

Other elasticities for L, S and $x_1, \ldots, x_n$ can be found using similar methods (and in particular, by using (3.37) where appropriate).

These further results are summarized (together with (3.40)) in Table 2. Particularizations of these formulae are taken up below in Section 4, where explicit functional forms for G are suggested. Notice that given a knowledge of G , the only pieces of information on behavioural parameters needed to compute the elasticity of the supply of hours are β_L and $\{\gamma_1, \ldots, \gamma_n; \gamma_S\}$. Strong priors on the latter might be developed from studies of the linear expenditure system;[1] priors on β_L might be developed from the existing empirical literature of leisure demand.[2]

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1. E.g., Constantino Lluch, Alan Powell and Ross A. Williams, Patterns in Household Demand and Saving (New York : Oxford University Press, 1977), pp. 288.

2. E.g., Abbot and Ashenfelter, op. cit.; and Nicholas M. Kiefer, "A Bayesian Analysis of Commodity Demand and Labor Supply," International Economic Review, Vol. 18, No. 1 (February, 1977), pp. 209-218.

Table 2

Elasticities of Endogenous Variables
with Respect to the Parameters
$\{\mu_1, \ldots, \mu_N\}$ of the Total (after-tax)
Earnings Schedule, G

Endogenous Variables	Elasticity with respect to the j^{th} parameter, μ_j, of the total earnings schedule, G
Unsubsidized Commodities, x_i $(i = 1, \ldots, n)$ (demand) *	$\frac{\partial \ln x_i}{\partial \ln \lvert\mu_j\rvert} = \frac{\beta_i (1-k)\mu_j}{p_i x_i} \left\{ \frac{\partial G}{\partial \mu_j} + (L-\gamma_L) \left[\psi' \frac{\partial H}{\partial \mu_j} + \frac{\partial \psi}{\partial \mu_j} \right] \right\}$
Subsidized Commodity S (demand)	$\frac{\partial \ln S}{\partial \ln \lvert\mu_j\rvert} = \frac{\beta_S (1-k)\mu_j}{(1-ab)\ \pi\ S} \left\{ \text{ditto} \right\}$
Leisure L (demand)	$\frac{\partial \ln L}{\partial \ln \lvert\mu_j\rvert} = -\frac{\mu_j}{L} \frac{\left[\frac{\Theta+G}{\psi} \frac{\partial \psi}{\partial \mu_j} - \frac{\partial G}{\partial \mu_j} \right]}{\left[\frac{\psi}{\beta_L} - \frac{\psi'}{\psi} (\Theta+G) \right]}$
Hours worked H (supply)	$\frac{\partial \ln H}{\partial \ln \lvert\mu_j\rvert} = \frac{\mu_j \left[\frac{(\Theta+G)}{\psi} \frac{\partial \psi}{\partial \mu_j} - \frac{\partial G}{\partial \mu_j} \right]}{H \left[\frac{\psi}{\beta_L} - \frac{\psi'}{\psi} (\Theta+G) \right]}$

* For further substitutions for $\partial H/\partial \mu_j$, see last row of this table.

4. THE SUPPLY OF HOURS WORKED

An Earnings/Hours Schedule

In the absence of personal income tax collections, the marginal earnings-hours schedule facing an individual consumer/worker might look something like the unbroken-line graph in Figure 1.

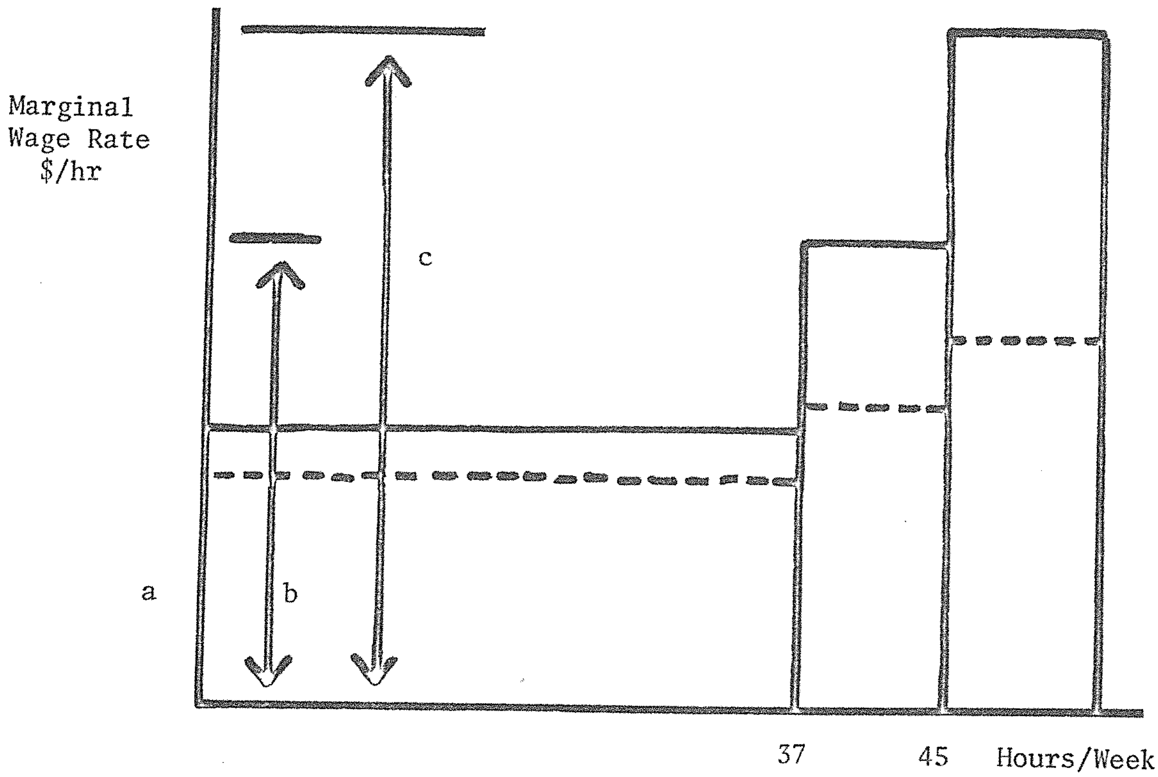

Figure 1 : Hypothetical Before-Tax Marginal Earnings Schedule

The superimposition of a progressive income tax schedule will complicate the picture considerably. If it just so happened that the switch points for different marginal tax brackets corresponded to gross weekly incomes of $(37a) and $[(37a) + (8b)], then the after-tax marginal earnings schedule might be given by the dotted lines in Figure 1. For sufficiently well-paid workers - -

e.g., skilled pipeline welders -- the after-tax staircase may descend, rather than ascend.

In practice, the switch points for different tax brackets will occur at arbitrary points, and lead to irregularities in the marginal earnings function. If the switch points in the tax schedule were at gross weekly incomes of $\$(\frac{37a}{2})$ and at $\$[(37a) + (4b)]$, for instance, then, for the before-tax marginal wage rate schedule shown in Figure 1, the resultant after-tax schedule might look something like Figure 2.

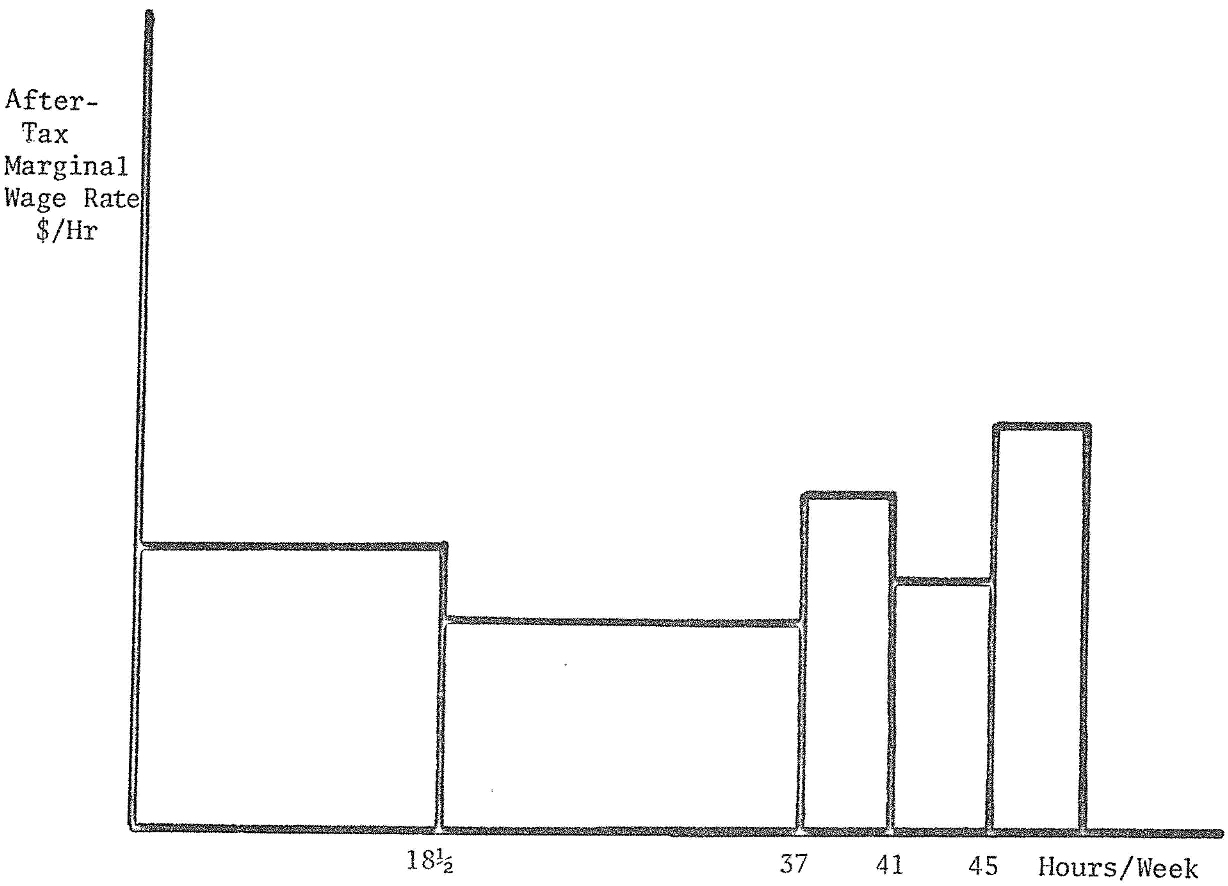

Figure 2 : Hypothetical After-Tax Marginal Earnings Schedule

Clearly, at this detailed (individual agent) level, the after-tax earnings/hours schedule will be too intractable for analysis. At least in the standard-hours-plus region of the schedule (where the bulk of the decision-makers are likely to be bunched) monotonic behaviour (either increasing or decreasing) is a reasonable starting point for analysis. For lower paid workers, it is also reasonable to suppose that overtime penalty rates, rather than marginal tax rates, will dominate in the shape of the schedule.

Deep analysis in formal aggregation theory would be needed for a rigorous justification (or refutation) of our decision to model the 'as if' earnings/hours schedule of the representative agent[1] as a continuous function. It is, nevertheless, extremely difficult to envisage how fruitful analysis could proceed without such an assumption.

To keep matters simple, in the remainder of this paper we confine our attention to a marginal earnings schedule of the shape shown in Figure 3. The schedule can be represented by

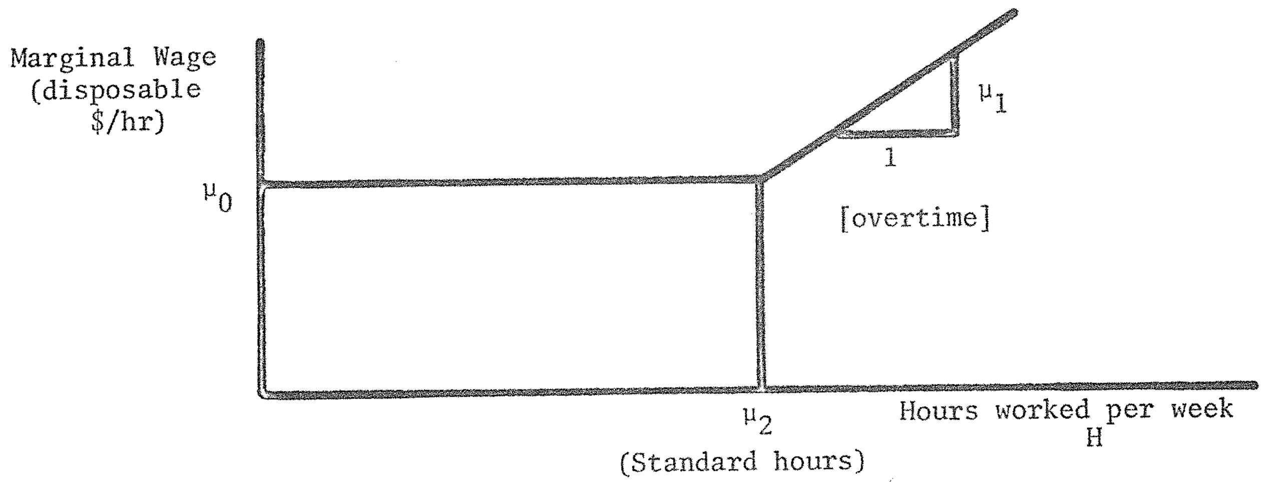

Figure 3 : Simplified After-Tax Marginal Earnings Schedule

1. The representative agent is defined as a fictitious decision maker whose behaviour is just a scaled-down version of market behaviour.

(4.1) $$\psi(H) = \mu_0 \qquad \text{when } H < \mu_2 ,$$

and

(4.2) $$\psi(H) = \mu_0 + \mu_1(H - \mu_2) \qquad \text{when } H > \mu_2 .$$

The chosen form[1] of $\psi(H)$ reflects the assumption that the marginal hourly wage remains constant as H increases from 0 to standard hours μ_2, and then rises continuously over the relevant range of variation in hours worked. The parameters can be interpreted as follows : μ_0 is the standard hourly wage rate for work done up to the standard number of hours per week, μ_2 ; and μ_1 is the parameter reflecting the steepness of the overtime progression.

From Figure 3 we can see that the total earnings schedule G(H) is given by

(4.3) $$G(H) = H\mu_0 \qquad \text{when } H < \mu_2 ;$$

whilst

(4.4) $$G(H) = \mu_0 H + \tfrac{1}{2}\mu_1(H - \mu_2)^2 \qquad \text{when } H > \mu_2 .$$

The average earnings schedule is

(4.5) $$\phi(H) = \frac{G(H)}{H} ,$$

which for $H < \mu_2$ is given by

---

1. Global differentiability in the neighbourhood of standard hours cannot be obtained from the form of $\psi(H)$; however, it is possible to overcome this difficulty by considering a form of $\psi(H)$ discussed in Appendix A.

(4.5a) $$\phi(H) = \mu \ ,$$

and for $H > \mu_2$, by

(4.5b) $$\phi(H) = \mu_0 + \frac{\mu_1}{2H}(H - \mu_2)^2 \ .$$

Wage Elasticity of Hours Worked

In Table 2 general formulae are given for the elasticity of labour hours supplied. Given the specific forms of $G(H)$ and $\psi(H)$ it is possible to work out the elasticities of hours worked (and of commodity demands) with respect to parameters μ_0, μ_1, μ_2 of $G(H)$. In order to derive these expressions, in addition to $G(H)$ and $\psi(H)$ we require $\psi'(H)$, as well as the following :

$$\frac{\partial G}{\partial \mu_j} \quad \text{and} \quad \frac{\partial \psi}{\partial \mu_j} \qquad (j = 0,\ 1 \text{ and } 2) \ .$$

Differentiating (4.1) and (4.2) with respect to H we get

(4.6) $$\psi'(H) = 0 \qquad \text{when } H < \mu_2 \ ,$$

and

(4.6a) $$\psi'(H) = \mu_1 \qquad \text{when } H > \mu_2 \ .$$

The above expressions and the others required for Table 2 are summarized in Table 3 for $H < \mu_2$ and for $H > \mu_2$.

Table 3

Expressions for Various Terms in the Elasticity Formulae of Table 2 when the Earnings Schedule is as in Figure 3

Term	Range of H (Hours worked per week)	
	$H < \mu_2$ (less than standard hours)	$H > \mu_2$ (overtime is worked)
$\psi(H)$	μ_0	$\mu_0 + \mu_1(H - \mu_2)$
$G(H)$	$\mu_0 H$	$\frac{\mu_1}{2}(H - \mu_2)^2 + \mu_0 H$
$\psi'(H)$	0	μ_1
$\frac{\partial\psi}{\partial\mu_0}$	1	1
$\frac{\partial\psi}{\partial\mu_1}$	0	$H - \mu_2$
$\frac{\partial\psi}{\partial\mu_2}$	0	$-\mu_1$
$\frac{\partial G}{\partial\mu_0}$	H	H
$\frac{\partial G}{\partial\mu_1}$	0	$\frac{(H - \mu_2)^2}{2}$
$\frac{\partial G}{\partial\mu_2}$	0	$-\mu_1(H - \mu_2)$

We now neglect the influence of commodity subsidies and make the assumption that the worker/consumer whose behaviour we are studying has negligible non-labour income. The basic formula of relevance to the supply of hours worked in Table 2, namely,

$$(4.7) \qquad \frac{\partial\, \ell n\, H}{\partial\, \ell n \left|\mu_j\right|} = \frac{\mu_j \left[\frac{\Theta+G}{\psi} \frac{\partial\psi}{\partial\mu_j} - \frac{\partial G}{\partial\mu_j}\right]}{H\left[\frac{\psi}{\beta_L} - \frac{\psi'}{\psi}(\Theta+G)\right]} ,$$

involves the variable Θ . Under the assumptions introduced above,

$$(4.8) \qquad \Theta = -\sum_{j=1}^{n+1} p_j \gamma_j \quad ;$$

thus $(\Theta+G)$ may be interpreted, in terms of the extended linear expenditure system as 'supernumerary' or discretionary income. The results obtained by substituting values from Table 3 into the above expression (4.7) are shown in Table 4.

Table 4

Elasticities of Labour Hours Supplied with respect to the Parameters of the (after-tax) Earnings Schedule, G

Parameter	Elasticities of Hours Supplied for different values of H	
	$H < \mu_2$ (less than standard hours are worked)	$H > \mu_2$ (overtime is worked)
μ_0 (basic hourly wage)	$\frac{\beta_L \Theta}{G}$	$\frac{\mu_0 \left[\frac{\Theta+G}{\psi} - H\right]}{H\left[\frac{\psi}{\beta_L} - \frac{\mu_1}{\psi}(\Theta+G)\right]}$
μ_1 (overtime progression)	0	$\frac{\mu_1 \left[\frac{\Theta+G}{\psi}(H-\mu_2) - \frac{(H-\mu_2)^2}{2}\right]}{H\left[\frac{\psi}{\beta_L} - \frac{\mu_1}{\psi}(\Theta+G)\right]}$
μ_2 (standard hours)	0	$\frac{\mu_2 \mu_1 \left[-\frac{\Theta+G}{\psi} + (H-\mu_2)\right]}{H\left[\frac{\psi}{\beta_L} - \frac{\mu_1}{\psi}(\Theta+G)\right]}$

A Hypothetical Example

In order to obtain a feeling for the orders of magnitude involved we shall work through a numberical example using some assumed values of the parameters of the earnings schedule.

Suppose that a skilled worker's weekly standard hours are 40 and that he faces the following after tax earnings schedule :

G(H) { he can earn \$4 per hour after tax for the first 40 hours of work, \$6 per hour for the next 5 hours and \$10 per hour for the next 5 hours.

Thus he can earn \$160 by working 40 hours, \$190 (160 + 5 × 6) by working 45 hours, or \$240 by working 50 hours per week. A good approximation for this type of earnings schedule can be obtained by using $\mu_0 = 4$, $\mu_1 = 0.8$ and $\mu_2 = 40$.

Now suppose that our hypothetical subject works 45 hours per week. His elasticity of labour supply with respect to the parameters of G can be calculated provided that we are able to estimate values for β_L and Θ .

Econometric evidence on β_L is scant; using time series data, Abbott and Ashenfelter suggest a value of about 0.12.[1] We adopt this value in the example below.

In approaching the estimation of Θ, some care is needed. In Table 4 we obtain different results for those working up to standard hours and those working overtime. To interpret correctly the numerical examples given below, it is important to ask why a particular individual would choose to work a particular number of hours per week. In the context of our example, the initial position of the worker/consumer is one of equilibrium with the given earnings schedule G. Differences in G, therefore, are ruled out as an explanation of the worker's initial number of hours supplied. Hence in terms of Table 4, the agent will be either in the $(H < \mu_2)$ column, or in the $(H > \mu_2)$ column, his exact position depending only upon β_L and Θ.

The utility parameter β_L is the marginal preference for leisure. Undoubtedly such a value would vary from individual to individual, and from community to community. In what follows, we abstract from all such variations, and use $\beta_L = 0.12$, which we provisionally assume to be a reasonable average value for Australia (although derived from U.S. data).

As a fraction of disposable income, the variable Θ, which is the cost of purchasing 'basic needs' (other than leisure) at current market prices, is related to the Frisch parameter, ω.

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1. Abbott and Ashenfelter, op. cit.

If G were the sole source of income, and if we took the linear expenditure system literally, we would have

$$(4.9) \qquad -\omega = \frac{(1-s)G}{\Theta + (1-s)G} ,$$

where s is the average propensity to save.[1] Because a literal interpretation of the LES would lead to wider variations in ω within national communities than is consistent either with international comparisons of mean ω values for countries at different stages of development[2] or with Frisch's famous initial conjecture,[3] we prefer to treat ω as a constant throughout our example. The individual whose behaviour is modelled, therefore, is one whose welfare level (as measured by ω) is at about the Australian average. For Australia in 1977 a reasonable value of $-\omega$ would be around 2.[4] The hypothetical agent whose behaviour is given in Table 5 has an $-\omega$ value equal to 2. His Θ, however, is allowed to vary in order to satisfy (4.9).[5]

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1. See e.g., A.S. Goldberger, "Functional Form of Utility : A Review of Consumer Demand Theory", University of Wisconsin, Systems Formulation, Methodology and Policy Workshop Paper 6703, October 1967.

2. Constantino Lluch, Alan Powell and Ross Williams, Patterns in Household Demand and Savings, (New York : Oxford University Press, 1977).

3. Ragnar Frisch, "A Complete Scheme for Computing All Direct and Cross Demand Elasticities in A Model with Many Sectors", Econometrica, Vol. 27 (April 1959), pp. 177-196.

4. Lluch, Powell and Williams, op. cit.

5. Where there are 2 or more wage earners in the same decision-making unit, the use of (4.9) with fixed ω and variable Θ can be regarded as a crude device for maintaining realism in the operation of the formulae given in Table 4.

An average value for the average propensity to save, s , can be estimated from the income and expenditure data of an average household in 1974-75.[1] If G is after tax income, and if payments for life insurance, superannuation and capital housing costs[2] are treated as savings, for an average household s would be 0.12 . Thus for $s = 0.12$ and $-\omega = 2$,

$$(4.10) \qquad \Theta = -0.44G \ .$$

We now have assembled enough information to calculate the elasticities of hours supplied for the hypothetical individual described above. Recapitulating, we use the following numerical values of the components in the elasticity formulae of Table 4 in order to compute the numerical values of the elasticities shown in Table 5 :

$\mu_0 = 4$,	$\mu_1 = 0.8$,	$\mu_2 = 40$
$s = 0.12$,	$\beta_L = 0.12$,	$\Theta = -0.44G$

The elasticities in Table 5 show that in our hypothetical example for a person working 45 hours per week, a 10 per cent increase in the basic hourly wage rate would cause a 0.50 per cent reduction in labour hours supplied, whereas a 10 per cent increase in the parameter of overtime

1. Australian Bureau of Statistics, Household Expenditure Survey 1974-75, Bulletin 5, Canberra, 1977.

2. Includescosts of outright purchase, alterations and additions, etc., but does not include mortgage repayments.

Table 5

Elasticities of Labour Hours Supplied with respect to the Parameters of (after-tax) Earnings Schedule - A HYPOTHETICAL Numerical Example

Parameter	Elasticities of Hours Supplied for different values of H							
	H < 40 (less than standard hours are worked)	Value of H (overtime is worked, i.e., H > 40)						
		41	42	43	44	45	48	50
μ_0 (basic hourly wage)	-0.053	-0.086	-0.072	-0.063	-0.056	-0.050	-0.039	-0.034
μ_1 (overtime progression)	0	0.015	0.018	0.019	0.018	0.017	0.013	0.011
μ_2 (standard hours)	0	-0.575	-0.344	-0.224	-0.152	-0.105	-0.032	-0.008

progression would cause a 0.17 per cent increase in hours supplied. A 10 per cent reduction in standard hours, on the other hand, would bring about a 1 per cent increase in hours supplied.

The results can be rationalized, in part, by reference to Table 4. For $H > \mu_2$ -- i.e., when overtime is worked -- the three elasticity formulae have numerators whose sign is determined partly by the magnitude of $H^* = (\Theta+G)/\psi$. This term is interpreted as the number of hours which would have to be worked in order to earn a supernumerary income of $(\Theta+G)$ if all such work were paid at the marginal wage. Accepting that the denominator in the $(H > \mu_2)$ column of Table 4 is likely to be positive under Australian conditions, it follows that the elasticity of supply of hours with respect to the basic hourly rate will be positive, zero, or negative, depending on whether $H^* > H$, $H^* = H$, or $H^* < H$. In developed countries like Australia, the U.S., the U.K., etc., H^* will typically be a number of the order of one half the actual hours worked. This establishes the likely sign of the elasticity of hours supplied with respect to the basic hourly wage rate as negative, a result which is the counterpart of the 'backward bending supply curve' of the literature.

Here the similarity to the traditional theory ends. The elasticity of hours supplied with respect to the overtime progression paramater (see Table 4) is :

$$(4.11) \qquad E_{H\mu 1} = \frac{\mu_1 (H-\mu_2) \left[H^* - \frac{(H-\mu_2)}{2}\right]}{H \left[\frac{\psi}{\beta_L} - \mu_1 H^*\right]} \quad .$$

Given that the denominator is likely to be positive in a country like Australia, the sign of this elasticity will be positive, zero, or negative, depending on whether H^* exceeds, is equal to, or is exceeded by, one half of the overtime hours. The elasticity of hours supplied with respect to standard hours, on the other hand, is positive, zero or negative depending on whether overtime hours exceed, are equal to, or are exceeded by H^*. Given that in our example,

$$\frac{H - \mu_2}{2} < H^* < \mu_2 ,$$

we obtained the results reported above.

All of these results can be interpreted in terms of traditional income and substitution effects. For this purpose the marginal wage is to be construed as the price of leisure. In the case of a rise in the basic hourly wage rate, the income effect (which accrues on all hours worked) is quite large, and sufficient to swamp the substitution effect : although leisure is relatively more expensive after the rise, more leisure is 'purchased' (i.e., fewer hours are worked) due to the extra income received.

When the overtime progression parameter is increased, however, the income effect is much smaller : the additional hourly amount is earned only on overtime hours, not total hours. The rise in the shadow price of leisure, in this case, is sufficient to swamp the income effect.

Consider now a lowering of standard hours. Again the income effect is generated only by hours supplied in excess of the (new) standard. Again the substitution effect swamps the income effect, leading to an increase in hours supplied. (Whether an increase is actually observed in any historical context will, of course, be dependent as well on what happens to the demand for labour.) It should be noted that in the examples in which parameters μ_1 and μ_2 are changed the effect on total earnings (G) is small and in both these cases the elasticity of hours supplied is positive; in other words, leisure is a normal good, the demand for which falls when the price rises.

To illustrate the inescapable ambiguity in commonly used elasticity concepts such as "the elasticity of labour supply with respect to earnings," we show in Table 6 some of the different results which can be obtained by varying the source of the increased earnings.

Effects of Taxation

In the above hypothetical example separate tax parameters are not included but it is possible to draw some inference about the likely effect of changes in the tax schedule on hours supplied. The figures in Table 5 suggest that a reduction in the average tax rate is likely to reduce the supply of hours worked, whereas a reduction in the marginal tax rate is likely to increase the supply of hours worked.

We considered an after-tax earnings schedule in the above discussion, and assumed that the marginal hourly earnings remained constant up to a certain number of hours per week. In practice, of course, it is possible for the after-tax marginal hourly rate to fall initially. Alternative functional forms of the earnings schedule that

Table 6

Effect of a 10 per cent Change in Parameters
of the Earnings Schedule on Hours Worked and on Earnings

Parameter	Parameter value Before	Parameter value After	Hours Worked (H) Before	Hours Worked (H) After	Earnings (G) Before	Earnings (G) After	Elasticity $\frac{\Delta H}{H} / \frac{\Delta G}{G}$
μ_0 (basic hourly wage)	4	4.4	41	40.65	164.4	179.02	-0.097
			43	42.73	175.6	190.99	-0.072
			45	44.78	190.0	206.13	-0.059
			48	47.81	217.6	234.80	-0.049
μ_1 (overtime Progression)	0.8	0.88	41	41.06	164.4	164.75	0.705
			43	43.08	175.6	176.51	0.367
			45	45.08	190.0	191.88	0.204
			48	48.06	217.6	220.85	0.087
μ_2 (standard hours)	40	36	41	43.36	164.4	195.08	0.308
			43	43.96	175.6	201.19	0.154
			45	45.73	190.0	217.78	0.072
			48	48.15	217.6	251.69	0.020

include parameters representing tax rates will be considered when data on before-tax earnings and hours and on tax rates are available. The use of modified earnings schedules will enable us to estimate the effects of income tax on hours supplied.

Estimation and Data

Data on weekly before-tax earnings and hours worked by persons classified by sex, occupation, industry and age (junior/adult) will be obtained from special tabulations of the Earnings and Hours Survey.[1] Given total weekly earnings and a table of weekly tax deductions it will be possible to fit an earnings schedules for each occupation and sex group. It will be assumed that persons in an occupation/sex group face the same earnings schedule but are located at different points on the schedule due to the influence of their different socio-economic and demographic characteristics on the value of Θ . For short term work it is reasonable to assume that the demographic parameters are stable but it will be necessary to study the trends in these characteristics for long term work.

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1. Australian Bureau of Statistics, Earnings and Hours of Employees : Distribution and Composition, May 1975, Canberra, 1976.

5. DEMOGRAPHIC INFLUENCES ON BEHAVIOUR

In the discussion above we have not dealt explicitly with the influence of family size and composition on demand and work behaviour; nor have we treated explicitly households having two or more breadwinners. Formally the problem involves respecification of the utility function and the budget constraints. We retain the Klein-Rubin form in our specification of the family utility function.

Family Utility Function

We assume that decisions on work and expenditure are taken centrally after a compromise among the competing interests of the individual members of the family has been worked out. To simplify exposition, we will consider a family consisting of two adults (mother, father), plus $\tilde{M}$ children. The adults are assumed to dominate the household's consumption and workforce decision making.

Our formulation below ensures that all goods purchased may contain a public-good aspect from the viewpoint of the members of the family. (Examples of goods which are private in the context of the market, but which are at least partially public goods, from the internal viewpoint of members of a family, are housing space, the ownership of a refrigerator, of a T.V. set, of a washing machine, etc.)

Basic Needs

Let γ_{i1} , γ_{i2} be the basic requirements of good i needed by the husband for his own use, and by the wife for her own use, as perceived respectively by themselves. Then the adults' jointly perceived total basic requirements of good i for use by the adults is some kind of compromise. If C^A is a function determining the outcome of the compromise, we write

$$(5.1) \qquad \gamma_i^A = C^A(\gamma_{i1}, \gamma_{i2}) \qquad (i = 1,\ldots,n+1) \quad ,$$

as the value of the adults' jointly perceived needs of i for use by the adults. Let γ_i^c be the public-good component of γ_i^A ; that is, that part of γ_i^A which can be consumed by any number of members of the family without diminishing another member's consumption of it. Then if the compromise function C^A is linear, we might write

$$(5.2) \qquad \gamma_i^A = \gamma_i^c + w_1(\gamma_{i1} - \gamma_i^c) + w_2(\gamma_{i2} - \gamma_i^c) \quad ,$$

where w_1 and w_2 are the weights of the husband and wife, respectively, in the compromise function $(w_1 + w_2 = 1)$. For the children $j = 1,\ldots,\tilde{M}$, let γ_{i1j} and γ_{i2j} respectively be the basic needs of the j^{th} child as perceived by the father and the mother. Then, after a compromise C^L has been worked out, the parents' joint perception of the basic needs of their children will be

$$(5.3) \qquad \gamma_i^L = C^L(\gamma_{i11},\ldots,\gamma_{i1\tilde{M}}; \gamma_{i21},\ldots,\gamma_{i2\tilde{M}}) \quad .$$

If the compromise function C^L is also linear, (5.3) might be written

$$(5.4)\qquad \gamma_i^L = \gamma_i^c + \left[w_1 \sum_{j=1}^{\tilde{M}} W_{i1j}(\gamma_{i1j} - \gamma_i^c) + w_2 \sum_{j=1}^{\tilde{M}} W_{i2j}(\gamma_{i1j} - \gamma_i^c) \right]$$

where $\sum_{j=1}^{\tilde{M}} W_{i1j} = \sum_{j=1}^{\tilde{M}} W_{i2j} = 1$ for all i . The parents' joint perception of the family's total basic needs of good i is obtained by adding (5.2) and (5.4), and then eliminating double-counting of the public good component :

$$(5.5)\qquad \gamma_i^F = \gamma_i^c + \sum_{k=1}^{2} w_k \left[(\gamma_{ik} - \gamma_i^c) + \sum_{j=1}^{\tilde{M}} W_{ikj}(\gamma_{ikj} - \gamma_i^c) \right]$$

$$= \sum_{k=1}^{2} w_k \left[\gamma_{ik} + \sum_{j=1}^{\tilde{M}} W_{ikj}\, \gamma_{ikj} \right] - \gamma_i^c \quad .$$

If all children's individual basic needs for each commodity are perceived to be identical by each parent - - viz., $W_{ikj} = W_{ik}$, and $\gamma_{ikj} = \tilde{\gamma}_{ik}$, for all j - - then (5.5) has the form

$$(5.6)\qquad \gamma_i^F = \tilde{a}_i + \tilde{b}_i \tilde{M} \qquad (\tilde{a}_i, \tilde{b}_i, \text{ constants}), \ (i=1,\ldots,n+1),$$

which is linear in the number of children. This is the formulation of family size effects adopted by Ryan.[1]

If the parents do distinguish different basic needs of dependent children of pre-school age, primary school age, secondary school age, and tertiary education age, but do not discriminate among children

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1. David Ryan, "Effect of Ethnic Origin on Household Consumption Patterns in Australia", Impact of Demographic Change on Industry Structure in in Australia, Preliminary Working Paper No. SP-10, Industries Assistance Commission, Melbourne, June, 1977 (mimeo), pp. 26.

within these four age groups, then (5.5) becomes

$$(5.7) \qquad \gamma_i^F = \tilde{a}_i + \sum_{m=1}^{4} \tilde{b}_{im} \tilde{M}_m \qquad (i=1,\ldots,n+1),$$

where $\tilde{M}_m$ is the number of children in the family belonging to the m^{th} age group.

We are restricting our attention to a household with two potential bread winners, both adult. In this situation it is reasonable treat leisure-time of children as a free good, and to exclude its determination from the analysis. We continue to use 'leisure' to cover both genuine leisure and other non-labour-market usage of adults' time. 'Leisure' in this broad sense will be needed for three purposes : (a) to provide physical and emotional recreation and family contact for the husband; (b) to provide the same for the wife; and (c) to care for house, children, and other dependants. In the context of (c), we assume that with respect to any particular home task, the husband's time at home is either a perfect substitute for his wife's, or else is no substitute at all. Let the total time required for family care be $\gamma_{L\cdot}^F$, and adopt the following notation for its components :

γ_{L1}^H : Father-specific adult time needed for the care of home, children, etc;

γ_{L2}^H : Mother-specific adult time needed for care of home, children, etc;

$\gamma^F_{L..} = (\gamma^F_{L.} - \gamma^H_{L1} - \gamma^H_{L2})$: non-sex-specific adult time needed for home duties ;

s_1 : proportion of total demand for non-sex-specific adult time demanded for home duties supplied by the father ;

$s_2 \equiv (1-s_1)$: proportion of total demand for non-sex-specific adult time demanded for home duties supplied by the mother .

Then, by definition

$$(5.8) \qquad \gamma^F_{L.} \equiv \gamma^H_{L1} + \gamma^H_{L2} + s_1 \gamma^F_{L..} + s_2 \gamma^F_{L..} \quad .$$

The distribution $\{s_1,s_2\}$ of the non-specific home work load $\gamma^F_{L..}$ will be endogenized below as part of the household's utility maximization.

Now let γ^F_{L1} and γ^F_{L2} be the jointly agreed minimal needs of genuine leisure for husband and wife respectively. Then the family management's perceptions of broadly-defined basic needs of 'leisure' by the husband and wife respectively, are

$$(5.9) \qquad \gamma^F_{n+2} \equiv \gamma_{L1} = \gamma^F_{L1} + s_1 \gamma^F_{L..} + \gamma^H_{L1} \quad ,$$

and

$$(5.10) \qquad \gamma^F_{n+3} \equiv \gamma_{L2} = \gamma^F_{L2} + s_2 \gamma^F_{L..} + \gamma^H_{L2} \quad ,$$

in which s_1 and s_2 are yet to be determined. In (5.10), we have adopted a notation which identifies the wife's 'leisure' as the $(n+3)^{nd}$ good in the system; the $(n+2)^{nd}$ good is husband's 'leisure'.

If γ^F_{L1}, $\gamma^F_{L..}$, and γ^F_{L2} are linear in the number of children in different age groups, then

$$(5.11) \qquad \gamma^F_{n+2} \equiv \gamma_{L1} = \tilde{a}_{L1} + \sum_{m=1}^{4} \tilde{b}_{L1m} \tilde{M}_m + s_1\tilde{c}_{L1} + s_1 \sum_{m=1}^{4} \tilde{d}_{11m} \tilde{M}_m \quad ,$$

and

$$(5.12) \qquad \gamma^F_{n+3} \equiv \gamma_{L2} = \tilde{a}_{L2} + \sum_{m=1}^{4} \tilde{b}_{L2m} \tilde{M}_m + s_2\tilde{c}_{L1} + s_2 \sum_{m=1}^{4} \tilde{d}_{L2m} \tilde{M}_m \quad ,$$

in which the $\tilde{a}$'s, $\tilde{b}$'s, $\tilde{c}$'s and $\tilde{d}$'s are parameters.

Marginal Preferences

The 'basic needs' of the family for the $n+3$ goods in the system have now been formulated. The remaining Klein-Rubin parameters are the marginal preferences β_i in the expression

$$(5.13) \qquad U^F = \sum_{i=1}^{n+3} \beta_i \, \ell n(x_i - \gamma^F_i) \quad .$$

Different authors have treated the β_i's differently in the demographic context. Kakwani, for instance, in developing equivalence scales treated the β_i as invariant to demographic characteristics (including family size);[1] Williams, on the other hand, allowed the β_i to vary

1. N.C. Kakwani, "Household Composition and Measurement of Income Inequality and Poverty with Application to Australian Data", University of New South Wales, School of Economics, Discussion Paper No. 19 (June 1976), pp. 34 (mimeo).

with age of household head and with family size.[1] No clear functional relationship between the β_i's and family size and only a limited relationship between the β_i's and the age of household head emerged from Williams' work; provisionally, therefore, we opt to treat the β_i's as demographically invariant. With this simplification the family utility function is

$$(5.14) \qquad U^F = \sum_{i=1}^{n+1} \beta_i \, \ell n(x_i - \gamma_i^F) + \beta_{n+2} \, \ell n(L_1 - \gamma_{L1}^F - s_1 \gamma_{L..}^F - \gamma_{L1}^H) + \beta_{n+3} \, \ell n(L_2 - \gamma_{L2}^F - (1-s_1)\, \gamma_{L..}^F - \gamma_{L2}^H) \quad ,$$

where $L_1 \equiv x_{n+2}$ and $L_2 \equiv x_{n+3}$ are husband's and wife's 'leisure' respectively.

Family Budget Constraint

The family budget constraint now becomes a function with two arguments : H_1 (husband's hours of work) and H_2 (wife's hours of work). Gross before tax earnings would be an additive separable function in H_1 and H_2 ; net disposable income after tax cannot be obtained so simply because of interdependence of tax liabilities between husband and wife. Under Australian tax practice, these interdependencies are caused by changes in allowable concessional deductions in respect of dependants which result from the entry of a second breadwinner into the workforce. Firstly, the wife herself by taking a job will reduce

1. Ross Williams, "Engel Curves and Demand Systems : Demographic Effects on Consumption Patterns in Australia", Impact of Demographic Change on Industry Structure in Australia, Preliminary Working Paper No. SP-07, Industries Assistance Commission, Melbourne, January, 1977 (mimeo), pp. 35.

or cancel her husband's former concessional deduction in respect of her dependancy on him; secondly, it will always pay the family to deduct other concessional allowances from the taxable income of the earner facing the higher marginal tax rate. The latter can, of course, change, depending upon the respective number of hours worked by the husband and wife. Finally, eligibility for social service payments may vary according to the workforce participation and earnings of the two parties. With the exception of any effects that differing levels of disposable income might bring about by altering the level of the HAVE subsidy, we neglect the last of these points in the analysis which follows.

The after-tax disposable earnings function will take the form,

$$(5.15) \qquad \tilde{G}(H_1, H_2) = g_1(H_1) + g_2(H_2) - \tilde{T}(H_1, H_2, \alpha) \quad ,$$

in which g_1 and g_2 are gross (before-tax) earnings schedules, and $\tilde{T}$ is tax liability, which is a function of the respective hours worked, and the family's non-labour income, α .

The budget constraint analagous to (3.11) can now be written :

$$(5.16) \qquad (1 - k)[\tilde{G}(H_1, H_2) + \alpha] + (b\pi S + \rho) = \sum_{i=1}^{n} p_i x_i + \pi S \quad .$$

Lagrangean

Analogous to (3.13) is the following Lagrangean for the family utility maximization problem :

(5.17) $$\tilde{F}(\tilde{\lambda}, x_1,\ldots,x_n; S; L_1, L_2)$$

$$= \sum_{i=1}^{n} \beta_i \, \ell n(x_i - \gamma_i^F) + \beta_S \, \ell n(S - \gamma_S^F) + \beta_{n+2} \, \ell n(L_1 - \gamma_{L1}^F - s_1 \gamma_{L..}^F - \gamma_{L1}^H)$$

$$+ \beta_{n+3} \, \ell n(L_2 - \gamma_{L2}^F - (1 - s_1) \gamma_{L..}^F - \gamma_{L2}^H)$$

$$+ \tilde{\lambda}\left\{(1 - k) \left[\tilde{G}(T - L_1, T - L_2) + \alpha\right] + a(b\pi S + \rho) - \sum_{i=1}^{n} p_i x_i - \pi S\right\} .$$

First Order Conditions

If the earnings/hours schedule, $\tilde{G}$, is globally differentiable in hours worked, H_1 and H_2, then the first order conditions for family utility maximization are :

(5.18) $$\frac{\partial \tilde{F}}{\partial x_i} = 0 = \frac{\beta_i}{(x_i - \gamma_i)} - \tilde{\lambda} p_i \qquad (i=1,\ldots,n) \; ;$$

(5.19) $$\frac{\partial \tilde{F}}{\partial S} = 0 = \frac{\beta_S}{(S - \gamma_S)} - \tilde{\lambda}\pi(1 - ab) \quad ;$$

$$(5.20) \quad \frac{\partial \tilde{F}}{\partial s_1} = 0 = \frac{-\beta_{n+2}\,\gamma^F_{L..}}{\left[L_1 - \gamma^F_{L1} - s_1\,\gamma^F_{L..} - \gamma^H_{L1}\right]}$$

$$+ \frac{\beta_{n+3}\,\gamma^F_{L..}}{\left[L_2 - \gamma^F_{L2} - (1 - s_1)\,\gamma^F_{L..} - \gamma^H_{L2}\right]} \quad ;$$

$$(5.21) \quad \frac{\partial \tilde{F}}{\partial L_j} = 0 = \frac{\beta_{n+1+j}}{\left[L_j - \gamma^F_{Lj} - s_j\,\gamma^F_{L..} - \gamma^H_{Lj}\right]}$$

$$- \tilde{\lambda}(1 - k)\,\frac{\partial \tilde{G}}{\partial H_j} \qquad (j=1,2) \quad .$$

Manipulation of (5.20) reveals that the solution value of s_1 -- the share of non-sex-specific work at home done by the male adult -- is linear in L_1 and L_2 . To keep the notation simple, write

$$(5.22) \quad \begin{cases} s_1 = A_1 + B_{11}\,L_1 + B_{12}\,L_2 \quad , \\ s_2 = A_2 + B_{21}\,L_1 + B_{22}\,L_2 \quad , \end{cases}$$

in which the coefficients $\{A_i;\ B_{ij};\ i,j=1,2\}$ are functions of utility parameters $\{\beta_{n+1+j}\ ;\ \gamma^F_{Lj}\ ,\ \gamma^H_{Lj}\ ,\ \gamma^F_{L..}\ ;\ j=1,2\}$ only. The signs of B_{11} and B_{22} are unambiguously positive : an increase in the time spent in "leisure" by either adult partner implies taking an increased share of the burden of non-sex-specific work at home.[1]

Now rearrange (5.18), (5.19) and (5.21) so that only betas are on the left, and sum over all $(n+3)$ goods in the system. As in (3.19) this leads to an expression for the marginal utility of money, namely, in the present case

$$(5.23) \qquad \tilde{\lambda} = 1/\left\{ a\rho + (1-k)\left[\tilde{G}(H_1, H_2) + \sum_{j=1}^{2} L_j \frac{\partial \tilde{G}}{\partial H_j} + \alpha\right] - \sum_{j=1}^{n} p_j \gamma_j - \pi(1-ab)\gamma_S - (1-k)\sum_{j=1}^{2} \Gamma_j^F \frac{\partial \tilde{G}}{\partial H_j} \right\}$$

in which

$$(5.24) \qquad \Gamma_j^F = \gamma_{Lj}^F + s_j \gamma_{L..}^F + \gamma_{Lj}^H \qquad (j=1,2) \quad .$$

As before, the denominator on the right of (5.23) can be interpreted as the surplus of "full income" over (family) "basic needs", where leisure has been valued at the marginal wage (after correction for the subsidy-induced distortion).

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1. In a fully egalitarian marriage, presumably, $\beta_{n+2} = \beta_{n+3}$, $\gamma_{L1}^F = \gamma_{L2}^F$ and $\gamma_{L1}^H = \gamma_{L2}^H$. In this case, if earnings opportunities were such as to result in $L_1 = L_2$, then the algebra implies $s_1 = s_2 = ½$, as expected.

Substitution from (5.22) into (5.24), and thence into (5.23), gives an expression in which the variables s_1 and s_2 no longer appear; namely

$$(5.25) \qquad \tilde{\lambda} = 1/\left\{ a\rho + (1-k)\left[\tilde{G}(H_1, H_2) + \sum_{j=1}^{2} L_j - \gamma^F_{L..} A_j \frac{\partial\tilde{G}}{\partial H_j} + \alpha\right] - \sum_{j=1}^{n} p_j \gamma_j - \pi(1-ab)\gamma_S - (1-k)\sum_{j=1}^{2} \Gamma^F_{j.} \frac{\partial\tilde{G}}{\partial H_j}\right\} ,$$

in which

$$(5.26) \qquad \Gamma^F_{j.} = \gamma^F_{Lj} + \gamma^H_{Lj} \qquad (j=1,2)$$

(the sex-specific parts of Γ^F_j) , and

$$(5.26a) \qquad A_1 = \frac{\left(\gamma^F_{L2}\beta_{n+2} - \gamma^F_{L1}\beta_{n+3}\right) + \left(\gamma^H_{L2}\beta_{n+2} - \gamma^H_{L1}\beta_{n+3}\right) + \beta_{n+2}\gamma^F_{L..}}{\gamma^F_{L..}\left(\beta_{n+2} + \beta_{n+3}\right)} ;$$

while

$$A_2 = 1 - A_1 .$$

For notational convenience, rewrite (5.25) as

$$(5.27) \qquad \tilde{\lambda} = 1/(\tilde{Z} - \tilde{P}'\tilde{\Gamma}) ,$$

in which $\tilde{Z}$ is the residue from "full income" after deducting the imputed cost, reckoned at (subsidy-distorted) marginal wage rates, of non-sex-specific work in the home; and in whcich

$$(5.28) \qquad \Gamma = \left\{\gamma_1,\ldots,\gamma_n;\ \gamma_S;\ \Gamma^F_{1.},\ \Gamma^F_{2.}\right\}' ,$$

and where $\tilde{P}$ is the vector of appropriate prices and shadow prices (after distortion by the subsidy).

Demand Functions

The demand functions are found by back substitution of $\tilde{\lambda}$ from (5.27) into (5.18), (5.19) and (5.21) :

$$(5.29) \qquad x_i = \gamma_i + \frac{\beta_i}{p_i}(\tilde{Z} - \tilde{P}'\tilde{\Gamma}) \qquad (i=1,\ldots,n) \ ;$$

$$(5.30) \qquad S = \gamma_S + \frac{\beta_S}{\pi(1 - ab)} (\tilde{Z} - \tilde{P}'\tilde{\Gamma}) \ ;$$

$$(5.31) \qquad L_j = \Gamma_{j.} + \gamma^F_{L..}\left\{A_j + \sum_{\ell=1}^{2} B_{j\ell} L_\ell\right\} + \frac{\beta_{n+1+j}}{(1 - k)\frac{\partial\tilde{G}}{\partial H_j}} (\tilde{Z} - \tilde{P}'\tilde{\Gamma}) \qquad (j=1,2)^1 .$$

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1. This expression cannot be simplified by treating it as a system of two linear equations in L_1 and L_2, or as three linear equations in L_1, L_2 and s (where the equation for s comes from (5.20)). This is because the relevant coefficient matrices are singular.

This development is very close to that of Section 3. If we place a subscript on L and H (to indicate whether husband or wife), and reinterpret ψ and ψ' wherever they occur as $\frac{\partial \tilde{G}}{\partial H_j}$ and $\frac{\partial^2 \tilde{G}}{\partial (H_j)^2}$, most of the results in Section 3 require only slight modifications. The amount of algebraic manipulation involved, however, is formidable, and hopefully will be reported in a later paper.

If interdependencies in tax liability are ignored, the results in Tables 3 and 4 can be used in the context of a family where both parents work. The relevant utility parameters, of course, are those of the family, and the relevant income is family income. Substitutions from (5.6) or (5.7) into (5.29), (5.30) and (5.31) can be used to make explicit the dependence of commodity and "leisure" demands upon family size.

6. CONCLUSION

A commodity specific subsidy of the general type being considered for use in the HAVE scheme modifies income and substitution effects in a fairly straightforward way. The resultant demand systems have been derived above as a modification of the linear expenditure system, LES.

Panel data on consumption patterns and work behaviour of families are not available in Australia. In order to estimate the effects of a housing subsidy using the model of Section 3 it will be necessary in the experimental phase to obtain panel information from families receiving a subsidy and also from a control group not receiving the subsidy. The type of data required can be inferred from Sections 3 and 5; it may be useful, however, to list briefly the main areas that ideally should be covered :

(1) Demographic data on each member of family (age, sex, marital status).

(2) Socio-economic data : education, employment status, occupation, industry, etc.

(3) Location data : residence and work.

(4) Housing data : type of housing and tenure (rented/owned, number of rooms, etc.; appraised market rental value of space occupied).

(5) Ownership of durables (list types, how old, owned outright or on HP).

(6) Consumption expenditure classified by commodity groups.

(7) Non-labour income data : source of income and amount.

(8) Labour income and hours data : standard hours of work and amount received for standard hours, overtime hours and payment; tax deductions and tax liability; availability of overtime; details of award (if any) under which employed; moonlighting.

It is recommended that questionnaire designs be based, as far as practicable, on those for similar ABS surveys. This will have the advantage of avoiding pitfalls uncovered in ABS pretesting and as well ensure that the data collected can be compared with those from ABS sources.

The consideration of overtime options at penalty rates of pay leads to a treatment of the marginal wage rate (which is the shadow price of leisure) as endogenous. This greatly enriches the set of possible supply responses in the number of hours worked. In the traditional analysis, on the other hand, the marginal wage rate is exogenous. Given an additive separable utility function, this assumption results in the income effects of changes in the wage rate swamping the (oppositely signed) substitution effects. This leads to the famous "backward bending" labour supply curve.

When the marginal wage is treated as endogenous, however, the relative strengths of the income and substitution effects depend on the final (i.e. exogenous) source of the wage change. This leads to a great variety of possible responses. If, for example, standard hours are reduced but the basic hourly rate of pay and the steepness of the progression in the overtime pay scale are unchanged, under plausible assumptions the typical worker would opt to work longer hours than previously. This is because the income effect of the change is more than offset by the higher shadow cost of leisure. This and other results have important implications for the effect on incentives of different taxation policies.

To estimate the supply of hours response of a typical worker, the following data are sufficient :

(i) the number of hours he currently works, the availability of overtime, and the conditions on which it is available;

(ii) the personal income tax schedule;

(iii) the size of non-labour income;

(iv) the marginal leisure preference (β_L) ;

(v) the average propensity to save (s) ;

and

(vi) Frisch's parameter ω (the elasticity with respect to total expenditure of the marginal utility of total expenditure).

Data on (iv) can be obtained from econometric demand studies; a great deal of empirical evidence on (vi) is already available in the literature.[1]

1. See especially Lluch, Powell and Williams, op. cit., Ch. 4.

The remaining data hopefully can be obtained from surveys. To determine the aggregate supply response it is necessary to determine the frequency distribution of positions taken by individuals on the various earnings/ hours offer curves available in the market. The different positions opted for will depend on the distribution of family utility parameters; the latter in turn will be influenced by household demography. At each point on each offer curve, a particular supply elasticity with respect to any particular exogenous change will apply. By appropriate weighting and addition of these individual responses, the aggregate supply response may be estimated.

The shadow valuation of time spent in the home by women depends first on the family's utility pay-off from proper care of its younger members and second on the opportunity cost of the wife's potential role in the labour market. The former is related to the age and number of children. An approach towards incorporating their influence on observed workforce behaviour by husband and wife has been sketched; elaboration of these ideas hopefully will be taken up in a later paper.

APPENDIX A

The Marginal Wage Schedule

In Section 4 it was mentioned that global differentiability can be preserved by choosing a somewhat complicated functional form for the marginal wage schedule. Such a form is considered below.[1]

$$\text{(A.1)} \qquad \psi(H) = \sqrt{(\varepsilon\mu_0)^2 + \left[\frac{\mu_1}{2}(H - \mu_2)\right]^2} + \frac{\mu_1}{2}(H - \mu_2) + \mu_0$$

When the parameter ε is chosen sufficiently small, (A.1) is well approximated by

$$\text{(A.1')} \qquad \psi(H) = \left|\frac{\mu_1}{2}(H - \mu_2)\right| + \frac{\mu_1}{2}(H - \mu_2) + \mu_0 \quad ,$$

The chosen form (A.1) of $\psi(H)$ reflects the assumption that the marginal hourly wage remains constant as H increases from 0 to standard hours μ_2 , and then rises continuously over the relevant range of variation in hours worked.[2] The parameters can be interpreted as in (4.1) and (4.2) :

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1. In equation (A.1), $\sqrt{\quad}$ means the positive square root.
2. The consumption is the same as that in Section 4.

μ_0 is the standard hourly wage rate for work done up to the standard number of hours per week, μ_2 ; and μ_1 is the parameter reflecting the steepness of the overtime progression. As we have pointed out above, by choosing a very small value of the approximation parameter ε , we could make the hourly wage between 0 and μ_2 hours per week virtually constant. The hourly wage rises continuously for work in excess of μ_2 hours per week.

The total earnings schedule $G(H)$ is obtained by integrating (A.1) :

$$\text{(A.2)} \qquad G(H) = \int_0^H \left(\sqrt{(\varepsilon\mu_0)^2 + \left[\frac{\mu_1}{2}(h - \mu_2)\right]^2} + \frac{\mu_1}{2}(h - \mu_2) + \mu_0 \right) dh .$$

The exact form of the solution to (A.2) is :

$$\text{(A.2a)} \qquad G(H) = \frac{\mu_1}{2}\left\{ \frac{H - \mu_2}{2} \sqrt{a^2 + (H - \mu_2)^2} + \frac{\mu_2}{2} \sqrt{a^2 + \mu_2^2} \right.$$

$$+ \frac{a^2}{2} \ln\left(H - \mu_2 + \sqrt{a^2 + (H - \mu_2)^2}\right)$$

$$\left. - \frac{a^2}{2} \ln\left(- \mu_2 + \sqrt{a^2 + \mu_2^2}\right)\right\} + \frac{\mu_1}{4}(H - \mu_2)^2$$

$$- \frac{\mu_1 \mu_2^2}{4} + \mu_0 H ,$$

in which

(A.2b) $$a = 2\varepsilon\mu_0/\mu_1 \quad .$$

Equation (A.2a) may be written

(A.3) $$G(H) = A + B + C \quad ,$$

in which

(A.3a) $$A = \frac{\mu_1}{2}\left\{\frac{H-\mu_2}{2}\sqrt{a^2 + (H-\mu_2)^2} + \frac{\mu_2}{2}\sqrt{a^2+\mu_2^2}\right\} \quad ,$$

(A.3b) $$B = \frac{\mu_1}{2}\,\frac{a^2}{2}\left\{\ell n\left(H - \mu_2 + \sqrt{a^2 + (H-\mu_2)^2}\right) - \ell n\left(-\mu_2 + \sqrt{a^2+\mu_2^2}\right)\right\} \quad ,$$

and

(A.3c) $$C = \frac{\mu_1}{4}\left[(H-\mu_2)^2 - \mu_2^2\right] + \mu_0 H \quad .$$

Since, irrespective of the values of H and μ_2, B vanishes in the limit as ε^2 approaches zero, for small ε we obtain a good approximation using

(A.4) $$G(H) \doteq A + C \quad .$$

Putting a^2 (viz., ε) to zero in A and C within (A.4) yields an expression for $G(H)$ at $H = \mu_2$. For $H < \mu_2$ and $H > \mu_2$ the expressions for $G(H)$ will be approximately the same as Section 4.

<u>Wage Elasticity of Hours Worked at $H = \mu_2$</u>

In Table 2 a general formula is given for the elasticity of labour hours supplied with respect to parameters of $G(H)$:

(A.5)
$$\frac{\partial \ell n H}{\partial \ell n |\mu_j|} = \frac{\mu_j \left[\frac{\Theta+G}{\psi}\frac{\partial\psi}{\partial\mu_j} - \frac{\partial G}{\partial\mu_j}\right]}{H\left[\frac{\psi}{\beta_L} - \frac{\psi'}{\psi}(\Theta+G)\right]} .$$

In order to derive the expressions in (A.5), in addition to $G(H)$ and $\psi(H)$ we require $\psi'(H)$, as well as the following :

$$\frac{\partial G}{\partial\mu_j} \quad \text{and} \quad \frac{\partial\psi}{\partial\mu_j} \qquad (j = 0, 1 \text{ and } 2) .$$

Differentiating (A.1) with respect to H we get

(A.6)
$$\psi'(H) = \frac{\mu_1^2 (H - \mu_2)}{4\sqrt{(\varepsilon\mu_0)^2 + \left(\frac{\mu_1}{2}(H - \mu_2)\right)^2}} + \frac{\mu_1}{2} .$$

and, provided ε is kept finite, when $H = \mu_2$,

(A.7)
$$\psi'(H) = \frac{\mu_1}{2} .$$

The above expressions and the others required for (A.5) are summarized in Table A.1 for $H = \mu_2$. (In deriving the terms, we have differentiated the exact function with respect to each parameter holding H and all other parameters constant and then assumed that terms containing ε^2 may be neglected.)

Table A.1 : Expressions for Various Terms in the Elasticity Formulae of $H = \mu_2$

Term	Expression
$\psi(H)$	$(1+\varepsilon)\ \mu_0$
$G(H)$	$\mu_0 H\ (= \mu_0\ \mu_2)$
$\psi'(H)$	$\frac{\mu_1}{2}$
$\frac{\partial\psi}{\partial\mu_0}$	1
$\frac{\partial\psi}{\partial\mu_1}$	0
$\frac{\partial\psi}{\partial\mu_2}$	$-\frac{\mu_1}{2}$
$\frac{\partial G}{\partial\mu_0}$	$H\ (= \mu_2)$
$\frac{\partial G}{\partial\mu_1}$	0
$\frac{\partial G}{\partial\mu_2}$	μ_0

When these expressions are substituted in (A.5), it can be observed that the elasticity of hours supplied with respect to μ_1 will be zero. Elasticities with respect to μ_0 and μ_2 will contain the arbitrarily chosen constant ε . Therefore the numerical values will also contain ε . In other words, the elasticities of hours supplied with respect to μ_0 and μ_2 at $H = \mu_2$, cannot be estimated as they depend on an arbitrary number.