General Equilibrium, Partial Equilibrium and the Partial Derivative: Elasticities in a CGE model

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Abstract

In this paper we discuss the concepts of general equilibrium elasticity, partial equilibrium elasticity and partial derivative elasticity, particularly in relation to the confusion between the latter two concepts in some of the general equilibrium literature. In order to elucidate the distinction between partial equilibrium and partial derivative elasticity, we decompose the general equilibrium output elasticities via a Leontief inverse and the general equilibrium price elasticities via the Ghoshian inverse.

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1 Introduction

In analysing a set of policy changes, economists can choose between partial and general equilibrium methodologies. The partial equilibrium methodology concentrates on a particular subsection of the economy, with all other variables being treated as exogenous to the model. Given this concentration of resources, it is usually possible to model the particular industry/commodity chosen in much greater detail and with much greater care than is the case with general equilibrium models. On the other hand, general equilibrium models attempt to describe the entire economic system, capturing not only the direct impact of (say) a policy shock on the relevant market, but also the impact on other areas of the economy and feedback effects from these to the original market.

One of the obvious payoffs of having operational forms of both partial and general equilibrium methodologies is in our ability to compare the results. The principal difficulty with a direct comparison between the two is that it can be difficult to exactly match the features of the relatively intricate partial equilibrium model with those of its more cumbersome general equilibrium cousin. As a compromise, we can decompose the general equilibrium results of a simulation into its determinants in order to isolate the partial equilibrium effect, and thus answer numerous questions such as the importance of feedback effects in the overall results and the primary sources of these feedbacks. Such a strategy has a long tradition in input-output analysis, whereby model results are frequently presented as multiplier values which can be applied to partial results to measure economy wide effects.

One approach to decomposing general equilibrium model results has been to run a general equilibrium model while exogenously setting all but one of the prices. An example of this can be seen in Staehr (1999) and Pohl-Nielson (2000). We argue in this paper that this methodology is incorrect. By imposing zero price changes we are not recreating a partial equilibrium world. Instead, we are creating a very special world which insists that even though changes have taken place in one area of the economy, quantity flows must change such that there are no price changes elsewhere in the economy. We argue that this mistake derives from the confusion between the ideas of a partial equilibrium elasticity and a partial derivative elasticity.

Therefore, we clearly distinguish the concepts partial equilibrium and partial derivative elasticity, and show in simple algebra how one is formed from the other. This paper has an additional innovation of breaking down both quantity and price elasticities. Staehr (1998), for example, only breaks down quantity elasticities into their partial components, while for price elasticities he uses general equilibrium values rather than decomposing them to their partial equilibrium components. We do this via the Ghoshian inverse. In discussing general equilibrium we will be referring to a typical one-country model, while for ease of exposition we will assume that the partial equilibrium model we are replicating models agriculture, with non-agriculture covering all industries and commodities not included in the partial equilibrium analysis.

This paper is constructed as follows. In section 2 we define what we mean by the terms general equilibrium and partial equilibrium elasticity, and clearly distinguish them from the partial derivative elasticity. In section 3 we introduce the IMAGE model, and provide a discussion of which elements of a general equilibrium model are replicated in a typical partial equilibrium model. In section 4 we work through the correct approach to replicate general equilibrium and partial equilibrium results from the underlying partial derivative elasticities. Finally, section 5 concludes.

2 General Equilibrium, Partial Equilibrium and the Partial Derivative Elasticity

The three terms that we must be entirely comfortable with before starting are those of general equilibrium, partial equilibrium and partial derivative elasticity. The term *general equilibrium quantity elasticity* has the obvious interpretation as the total percentage change in Q_i due to a 1% change in price j, including all feedback effects:

$$\varepsilon_{ij}^{GE} \equiv \frac{d \ln Q_i}{d \ln P_j} \forall i, j = 1,...., N. \qquad (1)$$

Therefore, the quantity change reflects the total impact of the change in price, including the change in household demand, the change in intermediate demand due to competition with the imported variant, changes in investment demand, changes in government demand and changes in intermediate demand which result from the changes in each of the final demand categories. The quantity change also reflects changes in household income and the impact of any government budget or balance of trade constraints. The general equilibrium result can be easily read off the computer output of the CGE model in question.

Staehr (1998) defines the *partial equilibrium quantity elasticity* as the isolated (or partial) effect on the equilibrium quantity of product *i* of an increase in the price of product *j* if we keep all other prices in the model constant. This, however, is a mistake. What he is describing instead is the commonly used but entirely mathematical construct of the partial derivative elasticity of output for good *j* due to a one percent change in a price *p*. If we consider an input-output table, quantity changes in (say) beef can cause quantity changes in other areas of the economy (such as demand for other meats, fertilisers, electricity, machinery, etc.) which in turn can have a further second round impact on the demand for cattle, without any change in price, as long as factors are perfectly elastic in supply and mobile across sectors. A partial derivative elasticity produces a vector of quantity outputs that is consistent with all prices bar one being fixed. It is, therefore, very much a general equilibrium idea. This latter point does not seem to be widely understood – the partial derivative elasticity within a general equilibrium model is different from the partial derivative elasticity within a partial equilibrium model. A partial equilibrium elasticity on the other hand only *ignores* many of the prices in the model, but does not assume that they necessarily are zero.

The confusion arises perhaps because if a partial equilibrium result is being replicated in a general equilibrium model, the easiest route is to insist on all other prices being zero, in an effort to restrict feedback effects on the sub-sector of interest. The point is that this is not the end of the story if we wish to produce a partial equilibrium, as this produces a partial derivative elasticity, and further steps must be taken to convert the results into a partial equilibrium elasticity.

Consider figure 1, which shows the routes by which price and quantity flow from the sector of interest (in this case assumed to be agriculture) to the rest of the economy. In figure 1a, shows this for general equilibrium, figure 1b shows it for the partial derivative elasticity, in figure 1c it is shown for what actually occurs in real partial equilibrium models while in figure 1d it is shown for our strategy for modelling partial equilibrium via a general equilibrium model.

In a general equilibrium solution, the initial impulse from agriculture will travel along A via price and quantity effects, alter the rest of the economy, and then travel back along B to cause a further change in agriculture. This will continue *ad infinitum,* though at ever decreasing magnitudes. Let us assume that there is only one 'circuit', namely, an initial impulse from agriculture which alters the economy in some way, which then travels back to influence agriculture again. For a partial derivative elasticity, all prices bar one are held constant, with quantities being allowed to adjust to ensure this occurs, even when feedback effects are accounted for. These adjustments in quantities in other sectors would, in a general equilibrium model, also influence the activity level in agriculture, thus implying that we cannot treat the resulting change in the output of Q_i as a partial equilibrium change.

In a real partial equilibrium model as shown in 1c, we 'allow' prices other than the one of interest to alter the rest of the economy, but we do not know, or do not care, how they do so. For example, we might be aware that a change in agricultural output will have an effect on (say) the output price of some non-agricultural products. Nowhere in the model do we deny or try to tie these non-agricultural price changes to zero, we simply do not consider them, nor any feedback effects they may have. The final diagram shows how we should try correctly to replicate a partial equilibrium model via a general equilibrium model. What we must do is to 'smother' route B, i.e. to stop the changes both in terms of price and quantity in the rest of the economy rebounding back and having an impact on the sector which caused the initial impulse, in this case agriculture. We do this by smothering price changes via route A, even though this is technically not consistent with a partial equilibrium model which knows nothing of the rest of the economy, and certainly does not assume that all other prices are constant. The difference between partial derivative elasticity (figure 1b) and partial equilibrium (figure 1d) is that with the latter we must also make sure to adequately smother quantity feedback effects via route B. In GE models, this is typically done by turning off market clearing equations.

Figure 1a: General Equilibrium

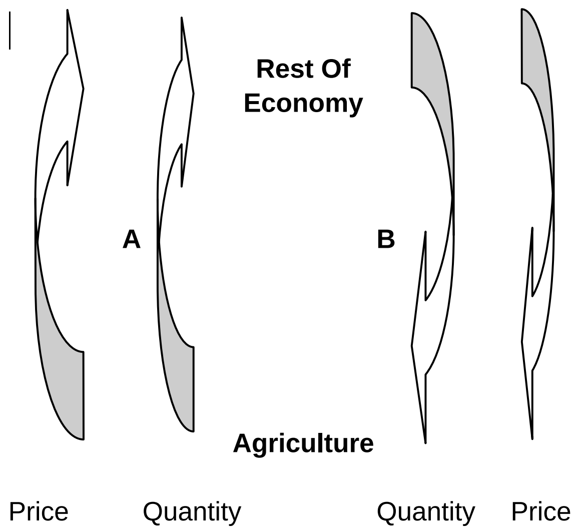


Figure 1b: Partial derivative elasticity

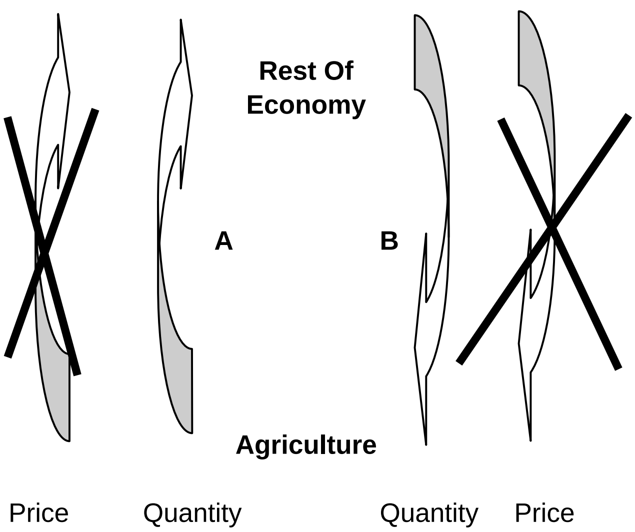


Figure 1c: Partial Equilibrium
What is actually measured in a PE model

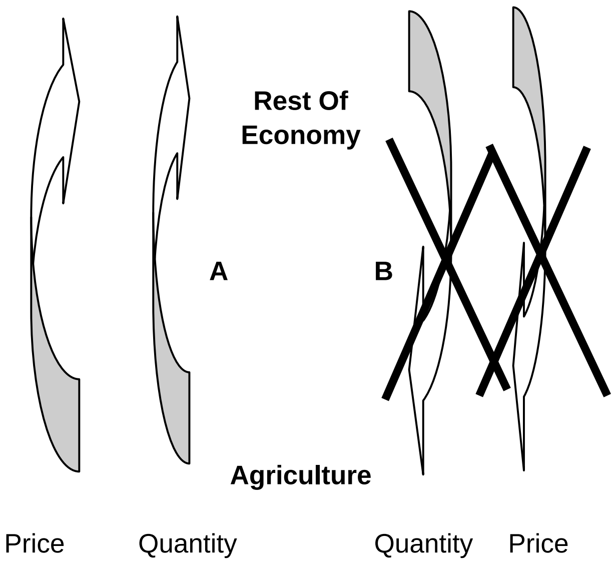
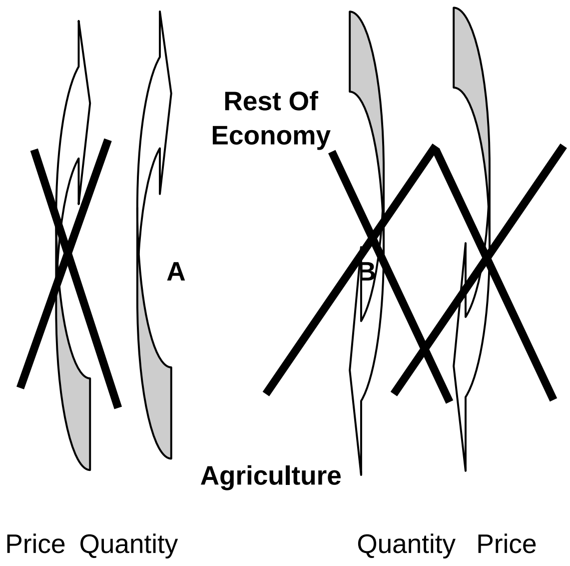


Figure 1d: Partial Equilibrium
Correct Strategy in GE models to estimate PE



In summary, if we smother price movements along route A, we are actually calculating a *partial derivative elasticity* which allows quantity effects to flow to the rest of the economy and rebound back. This is clearly not appropriate if our aim is to replicate partial equilibrium. A number of authors have made this error. By fixing prices but not quantities what they are in effect doing is creating a special 'general equilibrium' economy whereby quantity moves to offset pressures on prices due to the fact that prices are fixed. To replicate a partial equilibrium model in a general equilibrium context, therefore, we must go further than the partial derivative elasticity which only restricts prices. We must also restrict quantity flows to ensure they cannot rebound on the market of interest. In the discussion below we outline the assumptions required to accurately reproduce a partial equilibrium result within a general equilibrium model.

3 The IMA*GE* Model

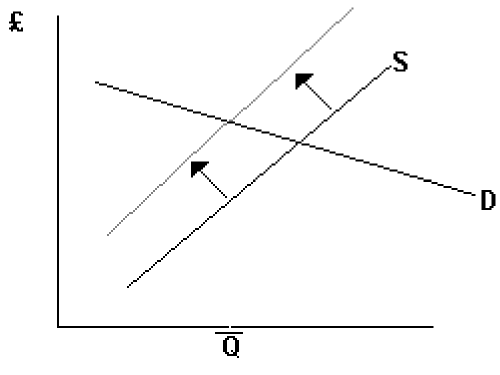
To illustrate the methodology employed, we deconstruct the results of a simulation run on the IMA*GE* model, a CGE model of Ireland.

The IMA*GE* model is based on the widely known ORANI model (Dixon et al. 1982) of the Australian economy which has been used extensively for policy analysis in Australia for nearly two decades. The model has a theoretical structure that is typical of many CGE models. It is a static model, as it does not have any mechanism for the accumulation of capital. It is based entirely on the assumption of perfect competition, with no individual buyer or seller being able to influence price. Demand and supply equations are derived from the solution of optimisation problems (e.g. profit or utility maximization) for private sector agents. The model allows for multiple household types, export destinations, land types and labour occupations. It also incorporates an explicit treatment of government revenue and expenditure. For further details see O'Toole and Matthews (2002a) and O'Toole and Matthews (2002b).

The model distinguishes 34 industries, the first eight of which relate to farm level production, and a further 6 of which relate to food processing, resulting in the 14 industries on which the partial equilibrium model is built. There are two sources of commodities, namely domestic and overseas. There are nine occupational groups and three household types, namely urban, rural farm, and rural non-farm. The model potentially allows every industry to produce several commodities by using domestic or imported intermediates and a primary factor composite consisting of land, labour and capital. This would suggest a very large and complex system that would be extremely difficult to calibrate. To keep the model to a manageable size, we assume, firstly, that each industry only produces one good and secondly, that input-output separability holds.

To illustrate the procedures presented, we investigate the impact on the output of the cattle sector of a 1% increase in the price of cattle in Ireland resulting from a negative technology shock to that sector. The way the shock is imposed is illustrated in figure 2. The supply curve shifts leftwards due to a deterioration in technology that results in a rise in the price of cattle of exactly 1%. Therefore we are shifting the supply curve and moving along the demand curve. A possible explanation for the deterioration in technology is poor weather. A justifiable alternative would have been to shift the demand curve and move along the supply curve. We report the general equilibrium elasticities obtained by calculating the partial derivative quantity elasticities and the total derivative price elasticities as could be obtained in IMA*GE*, that is, the change in the output of cattle that results from the price change of 1%. We then report results from a partial equilibrium model that only takes account of induced changes in the food industry and other agricultural industries of the initial price shock to the cattle sector.

Figure 2:
Imposing a 1% rise in Cattle Price via a
Productivity Shock



Great care must be taken in constructing the closure which is used to mimic a partial equilibrium model in a general equilibrium setting. A partial equilibrium model can be very complex in that its results can factor in the effects on a number of the industrial sectors which have key up-stream or down-stream linkages with the industry in question. The results obtained in any particular counterfactual are specific to the way in which the partial equilibrium counterfactual is defined.

The assumptions that are assumed to underlie this fuller partial equilibrium model are listed below. In the decomposition example it has not been possible to follow these rules exactly, though deviations are discussed in full in the relevant sections. The treatment suggested below is similar to that of Bautista et al (1998) and Hertel (1997, p 30).

Assumption 1: Price changes for non-agricultural commodities are exogenously set at zero.

Assumption 2: Factor prices changes for non-agricultural industries are exogenously set at zero, therefore there can be no 'rebound' effect whereby the initial agricultural shock impacts on non-agricultural factor prices which in turn disturb the agricultural factor markets.

Assumption 3: Increased/decreased non-agricultural output can have no impact on demand for agricultural commodities as an intermediate input.

Assumption 4: Potential macro feedback effects arising from the agricultural shock due to the balance of payments constraint, household budget changes etc. are not reflected in the partial equilibrium model.

The first two assumptions relate to the price interactions between agricultural and non-agricultural commodities. The first assumption states that the price of non-agricultural commodities is not influenced by any other price changes within the model. So, for example, the price of catering services is not influenced by changes in the price of agricultural products, even if agricultural products are an important input into catering services. The second assumption relates to factor markets, and says that agricultural factor markets are not influenced by changes in non-agricultural factor markets. The third assumption states that a change in output of non-agricultural products has no influence on the aggregate demand for agricultural products. So a doubling of catering output has no influence on demand for agricultural products, even if a sizeable share of existing catering inputs comes from agriculture. Finally, the last assumption states that 'budget' constraints on agents are ignored. For example, a partial equilibrium model will not allow for an endogenous change in consumer income to influence the price of cattle. In the general equilibrium model, the adverse effect on consumer income of the negative supply shock in the cattle sector is factored into the overall results.

4 Reconciling Elasticities

Given all of the above, the remainder of the paper details a methodology to deconstruct the general equilibrium elasticities produced by a CGE model into the partial derivative elasticities, and further shows how a partial equilibrium result can be reconstructed from these partial derivative elasticities. This is necessarily a complex procedure because, as discussed above, a partial derivative elasticity is very much a general equilibrium concept.

The logic behind the approach chosen can be explained as follows. Following Staehr (1998), we will take the example of the elasticity of a quantity (output of a farm level product) with respect to price. The total derivative of output of the first commodity Q_1 can be expressed as a weighted sum of the derivatives of Q_1 with respect to each of the price variables P_1, P_2,…, P_n and the level of income Y.

$$dQ_1 = \frac{\partial Q_1}{\partial P_1} dP_1 + \frac{\partial Q_1}{\partial P_2} dP_2 + \ldots + \frac{\partial Q_1}{\partial P_n} dP_n + \frac{\partial Q_1}{\partial Y} dY \tag{2}$$

Defining $v_i = \ln V_i$, we write:

$$\frac{dq_1}{dp_1}^{GE} = \sum_{j=1}^{n} \frac{\partial q_1}{\partial p_j}^{GE} \frac{dp_j}{dp_1}^{GE} + \frac{\partial q_1}{\partial y} \frac{dy}{dp_1}^{GE} \tag{3}$$

Further, given:

$$\frac{\partial q_1}{\partial p_j}^{GE} = \frac{\partial q_1}{\partial p_j}^{PE} + \left(\frac{\partial q_1}{\partial p_j}^{GE} - \frac{\partial q_1}{\partial p_j}^{PE} \right) \text{and } \frac{dp_1}{dp_j}^{GE} = \frac{dp_1}{dp_j}^{PE} + \left(\frac{dp_1}{dp_j}^{GE} - \frac{dp_1}{dp_j}^{PE} \right)$$

we can expand out expression (3) above very easily to identify the partial equilibrium components. Further, rearranging so that the first *m* industries are agriculture, with the remaining *n-m* industries comprising the non-agricultural economy gives:

$$\frac{dq_1}{dp_1}^{GE} = \sum_{j=1}^{m} \frac{\partial q_1}{\partial p_j}^{PE} \frac{dp_j}{dp_1}^{PE} + \sum_{j=1}^{n} \left(\frac{\partial q_1}{\partial p_j}^{GE} \right) \left(\frac{dp_j}{dp_1}^{GE} - \frac{dp_j}{dp_1}^{PE} \right) + \sum_{k=1}^{m} \left(\frac{\partial q_1}{\partial p_k}^{GE} - \frac{\partial q_1}{\partial p_k}^{PE} \right) \left(\frac{dp_k}{dp_1}^{PE} \right) + \frac{\partial q_1}{\partial y} \frac{dy}{dp_1}^{GE} \qquad (4)$$

General Equilibrium

Partial Equilibrium

Therefore the process of obtaining a partial equilibrium elasticity involves calculating the partial derivative elasticity of output in industry 1 with respect to price changes in agricultural industries and then calculating the total derivative, partial equilibrium, price changes dp_j / dp_1^{PE}. We do this by calculating the partial equilibrium term independently using matrix methods which are outlined below. The remainder, i.e. the difference between the general equilibrium result as calculated in the model and the partial equilibrium elasticity consists of the three other terms on the right hand side of equation (4). The second term adjusts for the fact that the impact on output in agricultural industries is different when the full general equilibrium price changes for agricultural products are used, rather than the partial equilibrium agricultural price changes. The third term adjusts for the effect on changes in the prices of all nonagricultural commodities on the general equilibrium versus the partial equilibrium output in industry 1, while the last term adjusts for the household income effect.

In terms of the four assumptions required to identify a partial equilibrium elasticity set out in section 3, assumptions 1 and 2 are satisfied on the basis that the first term on the right hand side of equation (4) is only calculated on the basis of partial equilibrium prices. The third assumption is met, as can be seen from the third term on the right hand side of the equation which is the term for changes in non-agricultural output impacting on agricultural output at partial equilibrium prices. The final assumption is captured in the last term on the right hand side of equation 4.

While in a general equilibrium framework we cannot distinguish between demand and supply elasticities. The above expression suggests two closely related concepts – the quantity elasticity and the price elasticity. The former is the rate of change of a quantity with respect to a change in price, while the former is the rate of change of a price with respect to a change in another price. In section 4.1 below we will calculate the $\partial q_1 / \partial p_j$ terms, while in section 4.2 below we will work back from $dp_j / dp_1{}^{GE}$ to calculate $dp_j / dp_1{}^{PE}$. In section 4.3 we bring these together to calculate the first term on the right hand side of equation (4).

The partial equilibrium price elasticities, being based on the Ghoshian inverse, assume perfect substitutability among all inputs, with outputs being perfectly complementary. This is the exact opposite of the Leontief production function where all inputs are essential and used in fixed proportions, while outputs are perfectly substitutable (Oosterhaven, 1989). Therefore, by using the methodology outlined below, in calculating the $\partial q_1 / \partial p_j$ terms we are effectively making an assumption of perfectly elastic factor supply, as production is factor specific and must use these factors in a fixed proportion. In calculating the dp_j / dp_1 terms we are assuming that demand is perfectly elastic, and soaks up any supply produced.

Therefore, the Leontief approach as employed is based on the presumption that a partial equilibrium modeler assumes that whatever demand is created for output can be met by additional supply. The modeler calculates prices based on the assumption that the country/region in question is a price taker in output markets, and that supply can be soaked up by additional demand. Therefore the Ghoshian model would not be an appropriate mechanism for subdividing price if a large amount of output went to the domestic market, as it is unreasonable to assume that a doubling of (say) cattle production in Ireland can be accommodated for by a doubling of consumer demand for beef. However, as long as Irish beef has a small share in international markets, this can be supported.

4.1 Calculation of the Quantity Partial Derivative Elasticities $\partial q_1 / \partial p_j^{\ PE}$ and $\partial q_1 / \partial p_j^{\ GE}$

The partial derivative elasticities for both partial and general equilibrium (or $\partial q_1 / \partial p_j^{\ PE}$ and $\partial q_1 / \partial p_j^{\ GE}$ respectively) represent the change in output of commodity 1 due to a change in the price of commodity *j* taking only repercussions within the agricultural sector into account and taking the economy wide repercussions into account respectively. This will give us the second column of figures as shown in table 1 below, and can be seen as the first element in the first term on the right hand side of equation (4). We first of all calculate direct changes in demand for commodity 1, which in the IMAGE model is the cattle sector, and then calculate changes in intermediate output demand for cattle due to this final demand increase. Intermediate demand for the domestic variant can change for two reasons. Firstly, final demand can change, which will have knock on effects for intermediate demand. We refer to this as the intermediate output effect. Secondly, for a given level of final demand, intermediate supply can substitute between the domestic and imported variants as price changes. We refer to this as the intermediate price effect.

The calculation of the direct change in demand for good *i* due to a change in price p_i (adjusted to allow for margin price) is straightforward for exports as we have an explicit demand curve. For household, investment and intermediate substitution demand there are explicit elasticities for the CES functions which govern the substitutability between the domestic and imported variant, so calculating the change in demand due to a change in price (with import prices kept constant) is a straightforward matter. Each of these changes is a function of the proportion of the domestic variant relative to the imported variant in each use, and the value of the elasticity of substitution.

The next step is to convert this vector of final demands for each product *i* into demand for cattle through inter-industry linkages. Both are easily calculated as the $(1, i)^{th}$ element of the appropriate Leontief inverse matrix which measures the total impact of a change of 1 unit in final demand of good *i* on the output of cattle. Note, the Leontief inverse calculates the amount of increased output of good 1 due to a one unit increase in final demand for good *i*. Therefore we have to convert these results into partial derivative elasticities by an appropriate scaling before using them in connection with the rest of the model. In the GE case, the appropriate Leontief model is the 34 * 34 $(I-A)^{-1}$ matrix. In the PE case, the appropriate Leontief model is the 14 * 14 $(I-A)^{-1}$ matrix consisting only of the agricultural industries.

In the case in hand of a rise in the cattle price by 1% arising from a negative supply-side shock, the direct effect leads to a fall in exports and household consumption, while those industries that use cattle in intermediate production, mainly meat processors, will start to source more cattle from overseas that have become relatively cheaper. All of these result in a fall in demand for the domestic variant. The indirect effect arises from the fact that as demand for cattle as a final product falls (because of the fall in exports, household demand, direct intermediate demand etc), so will the intermediate demand for cattle, to the extent that cattle are required in their own production or through other input-output linkages. As mentioned earlier, this process generates the values of the general equilibrium and partial equilibrium partial output derivative elasticities under the assumption of perfectly elastic factor supply.

4.2 Calculation of the Total Price Derivatives $dp_j / dp_1{}^{PE}$ and $dp_j / dp_1{}^{GE}$

The price elasticity measures the percentage change in one equilibrium price given a one percent change in another equilibrium price. Ultimately, all changes in prices are a weighted average of changes in the price of value added and changes in the price of imports. Given that all import prices in the model are exogenous and that all direct subsidy and tax rates remain unchanged, it follows that all changes in prices are a weighted average of changes in factor prices. To determine these weights we observe that:

$$\frac{dp_i}{dp_1} = \sum_{j=1}^{n} \frac{\partial p_i}{\partial p_j} \frac{dp_j}{dp_1} + \sum_{f=1}^{m} \frac{\partial p_i}{\partial p_{prim}} \frac{dp_{prim}}{dp_1} \tag{5}$$

$$\frac{dp_i}{dp_1} = S_{ij} \frac{dp_j}{dp_1} + T_{prim} \frac{dp_{prim}}{dp_1} \tag{6}$$

where S_{ij} is the n*n matrix where the ij^{th} element represents the share of good *j* in the production of industry *i*. This is known as the Ghoshian inverse after Ghosh (1958), and its use as a price model was suggested by Oosterhaven (1989). Similarly, T_{prim} represents the share of primary factors in the production of industry *i*. Rearranging (2) gives:

$$\frac{dp_i}{dp_1} = \left(I - S_{ij}\right)^{-1} T_{prim} \frac{dp_{prim}}{dp_1} \tag{7}$$

Therefore, applying the primary factor price changes as observed from the model gives us the commodity price changes that can also be observed from the model by way of a check. Given the link between factor prices and commodity prices, it now remains to divide up factor prices into those that would be observed in the partial equilibrium model and those that would not. This way, we can determine what the partial equilibrium prices would be.

In summary, the strategy to distinguish between general and partial equilibrium price elasticities is as follows: We have deconstructed the commodity price changes into the primary factor changes underlying them. We will now identify the primary factor price changes that could, arguably, be derived from a partial equilibrium model. Then, using the methodology derived above, we work backwards to calculate the commodity price changes consistent with the primary factor price changes. Note that we do not use the same weights when working backwards. Instead of the 34*34 $(I\text{-}S_{ij})^{-1}$ matrix, we calculate the corresponding 14*14 inverse matrix of agricultural industries/commodities.

The breakdown of the factor prices actually employed is as follows. Firstly, in a partial equilibrium model, any upward or downward pressure on factor prices due to changes in activity levels in the non-agricultural sectors is not accounted for. In other words, the 'equilibrium' reached in partial equilibrium does not cater for endogenous changes in any prices other than for agricultural products, though they can be exogenously imposed. This still leaves the problem of determining which changes in the price of agricultural products can be assumed to have arisen out of a partial equilibrium model. We will assume that all agricultural capital price changes and all land price changes are incorporated into the partial equilibrium analysis. This intuitively appealing division contains a slight deceit – it in essence assumes that the entire change in agricultural capital prices and in land prices was due to 'within agriculture' considerations. In fact, some of the change in price of agricultural capital and land is due to feedbacks from non-agricultural industries. This is ignored. We assume that all non-agricultural capital price changes and all labour price changes are not incorporated into the partial equilibrium analysis. Given a fixed labour supply, a change in price of one percent is likely to have an impact on the aggregate wage rate, an effect we assume is not captured in partial equilibrium analysis. A similar argument goes for non-agricultural capital. Through this method we now have a vector of price changes representing the partial equilibrium price elasticities.

4.3 Constructing the Partial Equilibrium Elasticities

Combining the partial equilibrium, quantity vector elasticities with the total derivative, partial equilibrium, price vector elasticities gives all the relevant components to calculate the first term of equation (4) above.

The general and partial output and price elasticities which when multiplied sum to the aggregate elasticity are shown below in table 1.

Table 1: Changes in Sectoral Output and Prices due to 1% Change in Cattle Price in Both Partial and General Equilibrium

	Output Elasticities		Price Elasticities		Contribution to Aggregate Elasticity	
	GE	PE	GE	PE	GE	PE
1 Cattle	**-0.286928**	**-0.286868**	**1.00000**	**0.995153**	**-0.286928**	**-0.285478**
2 Milk	-0.000075	-0.000065	0.00751	0.004193	-0.000001	0.000000
3 Sheep+Wool	-0.000053	-0.000046	0.01725	0.013695	-0.000001	-0.000001
4 Pigs,Poul.Hors	-0.000961	-0.000926	0.01978	0.014115	-0.000019	-0.000013
5 Wheat,Bar.Oats	-0.000020	0.000000	0.00892	0.006332	0.000000	0.000000
6 Fruit + Veg	-0.000016	0.000000	0.01239	0.011019	0.000000	0.000000
7 Root+Green	-0.000020	0.000000	0.01129	0.005950	0.000000	0.000000
8 Other Crops	-0.000004	0.000000	0.00932	0.005937	0.000000	0.000000
9 Meat	-1.323469	-1.322974	0.71543	0.705970	-0.946848	-0.933980
10 Milk Prods.	-0.002337	-0.001750	0.01099	0.001682	-0.000026	-0.000003
11 Farm Anim.Feed	-0.002006	-0.001945	0.02833	0.022108	-0.000057	-0.000043
12 Other Food nes	-0.017411	-0.016059	0.01638	0.001961	-0.000285	-0.000031
13 Sheep Meat	-0.016741	-0.016664	0.07630	0.070304	-0.001277	-0.001172
14 Other Meat	-0.031533	-0.031395	0.07577	0.068402	-0.002389	-0.002148
15 Forestry	-0.000014	0	-0.03997	0	0.000001	0
16 Fishing	-0.000030	0	0.00524	0	0.000000	0
17 Petrol.+Coal	-0.000028	0	0.00444	0	0.000000	0
18 Elec.,Gas,Wat	-0.000228	0	0.01543	0	-0.000004	0
19 Non-Met.Min.	-0.000308	0	0.01646	0	-0.000005	0
20 Chemicals	-0.001054	0	0.01194	0	-0.000013	0
21 Metal,Eng.,Veh.	-0.003600	0	0.00962	0	-0.000035	0
22 Beverag.+Tobac	-0.000999	0	0.01352	0	-0.000014	0
23 Textil.Cloth.Lea.	-0.017005	0	0.02965	0	-0.000504	0
24 Wood+Paper	-0.001421	0	0.01630	0	-0.000023	0
25 Rubb.Plast.,O.M	-0.000341	0	0.01396	0	-0.000005	0
26 Construction	0.000000	0	0.01618	0	0.000000	0
27 Trade Marg.+Rep	-0.000168	0	0.01815	0	-0.000003	0
28 Lodging+Cater.	-0.000810	0	0.01866	0	-0.000015	0
29 Transport	-0.004054	0	0.01622	0	-0.000066	0
30 Communications	-0.000654	0	0.01670	0	-0.000011	0
31 Credit+Insur.	-0.000473	0	0.01554	0	-0.000007	0
32 Other Mkt.Serv.	-0.009780	0	0.01450	0	-0.000142	0
33 Non Market Services	-0.006173	0	0.02886	0	-0.000178	0
34 Dwellings	-0.000495	0	0.09694	0	-0.000048	0
Aggregate Elasticity ignoring agents budget constraints:					**-1.23890**	**-1.22287**

For example, consider the fourth row, *pigs poultry and horses*. Consider first the general equilibrium elasticities. The quantity elasticity (-0.000961) represents the change in demand for cattle due to a 1% change in the price of pigs, poultry and horses, all other prices remaining unchanged. The price elasticity (0.01978) represents the change in the price of pigs, poultry and horses due to the full general equilibrium simulation in which cattle price is shocked by 1%. Therefore the product of the two terms, (-0.00019) is the change in demand for cattle due to the change in price of *pigs, poultry and horses* induced in a general equilibrium framework by the change in price of cattle by 1%. The values of the output and price derivative elasticities for many of the non agricultural industries are small, so their contribution to the aggregate elasticities is also small. These reflect the third term of the decomposition equation (4) as shown above. The cause of the difference in the aggregate general equilibrium and partial equilibrium results is due instead to the second term of the decomposition equation (4).

5 Conclusion

In this paper we have discussed the nature of general equilibrium elasticities, partial equilibrium elasticities and partial derivative elasticities. In particular, we have discussed the confusion between the two latter terms, which results in incorrect estimates of partial equilibrium results for the purpose of comparison with general equilibrium results. In particular, we have argued that the partial derivative elasticity differs between partial equilibrium and general equilibrium. The *partial derivative quantity elasticity* is the isolated (or partial) effect on the equilibrium quantity of product *i* of an increase in the price of product j if we keep all other prices in the model constant. It is, therefore, very much a general equilibrium idea. The partial derivative elasticity within a general equilibrium model is different from the partial derivative elasticity within a partial equilibrium model. A *partial equilibrium elasticity* on the other hand only *ignores* many of the prices in the model, but does not assume that they necessarily are zero. To help distinguish between partial equilibrium and partial derivative elasticities, we decompose the general equilibrium output elasticities via a Leontief inverse and the general equilibrium price elasticities via the Ghoshian inverse.

In summary, the correct method for isolating partial equilibrium results in a CGE model is as follows. Firstly, we identify those *m* sectors in the CGE model which we define as our partial equilibrium counterfactual. Secondly, the quantity effect on industry *j* of a 1% increase in the price of output of industry *j* has to be calculated.

This involves a straight forward application of the Leontief inverse, taking care to allow for substitution with imports. We firstly calculate all the final demand effects (including changes in competitiveness with imports), and use the reduced m*m Leontief inverse to calculate the economy wide effects. The price effects can be broken down in a similar fashion using the m*m Ghoshian inverse to translate induced changes in factor prices calculated from the full Ghoshian inverse to their partial equilibrium version.

The specific results in terms of shocking the cattle price in the IMA*GE* model by 1% were calculated by way of example. The values of the output and price derivative elasticities for many of the non agricultural industries proved to be very small, so their contribution to the aggregate elasticities (which was formed as a product of the two) was also small. These reflect the third term of the decomposition equation as shown above. The cause of the difference in the aggregate general equilibrium and partial equilibrium results is due mainly to the second term of the decomposition of the decomposition equation.

What are our expectations of the magnitude of partial equilibrium versus general equilibrium results more generally? There are two sources of general equilibrium feedback that are likely to impact on agricultural production. The first is simply the impact of inter-industry connections that can be captured by an input output model. The second is the impact of changes in the economy wide price of scarce commodities such as workers, capital or land, as well as changes in macro variables such as nominal consumption and investment.

The magnitude of the first is likely to be shaped as an inverted-U with respect to the share of industries under examination in total output. In other words, a small industry comprising 1% of total output will have relatively small knock on effects on other industries, while a large industry with a 99% share of total output will have most of the inter-industry linkages already internalised, so the ‘general equilibrium’ inter-industry linkages remaining will be minor. Therefore the importance of inter-industry linkages is likely to be at a maximum for those industries that comprise around 50% of total output, all other things being equal. Note that all things are unlikely to be equal. We are likely to choose related industries to construct our partial equilibrium results. In the example in this paper, we defined the partial equilibrium model to include all sectors in the wider food industry. This gives rise to result that cattle changes have relatively limited impact on the non-agricultural sectors left out of the PE simulation. While the impact on the own industry output of a change in its price is unambiguously negative, the fact that final demand changes for other commodities can be positive or negative means that the sign of the total impact is ambiguous.

The magnitude of the factor price and macro variable effects is likely to be small when shocks are applied to industries that comprise a small share of total output, and likely to be large for industries that comprise a large share of total output. Therefore the combined effect of the two effects is indeterminate. It is likely to be larger for industries up to 50% of total output, and thereafter it might rise or fall, depending on which effect dominates.

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