Computing supply and demand elasticities via formulae or via simulation

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Abstract: Shows how supply and demand elasticities may be estimated (a) by formulae involving elasticity parameters and shares; or (b) by using the SAGEM program. A modified ORANI-G is used as an example model.

Computer files relating to this document will be found at:

http://www.copsmodels.com/archivep.htm#tpmh0155

Contents

1. Introduction

Partial equilibrium models often incorporate explicit supply and demand elasticities as parameters. For a CGE model these elasticities are implicit; they depend on the functional forms of production and demand systems, the linkages between sectors; and on the closure. Below we describe a way to estimate these implicit supply and demand elasticities using the ORANI-G model. This allows us to compare the implicit CGE elasticities with other estimates of supply and demand elasticities.

Previous related work includes O'Toole and Matthews (2002), and chapter 5 of the GTAP book (Hertel [ed], 1997).

This document goes with a ZIP of computer files, which should be unpacked into a new empty folder. To run the examples, some version of GEMPACK needs to be installed.

1.1. A simplified example

The diagram below illustrates a supply-demand system for some commodity. If the demand curve moves right (but supply curve is stationary), leading to observed percent changes p in price and q in quantity, we could deduce the supply elasticity to be q/p.

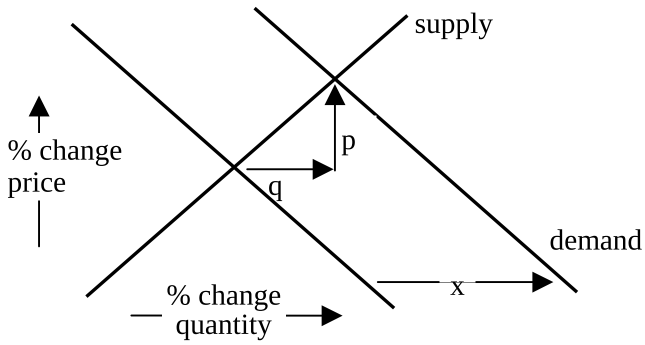

Figure 1: Measuring supply elasticity

Similarly if the supply curve moved right (with stationary demand curve), leading to different percent changes p and q, we could deduce the demand elasticity to be -q/p[1].

Using the idea of a wedge between supply and demand curves (eg, a tax wedge) we can deduce both supply and demand elasticities at once. Suitable TABLO code is shown overleaf (the supplied file SIMPLE.TAB). Notice that supply and demand are linked by the last two equations, but the market for, say, good 1, is quite independent of the market for good 2.

[1] Here we define the demand elasticity to be a positive number: the decline in quantity due to a 1% rise in price.

```
Coefficient (all,c,COM) SUPELS(c) # supply elasticity #;
Update (change)(all,c,COM) SUPELS(c)=0;
Read SUPELS from file BASEDATA header "ESUP";
Variable
(all,c,COM) qsup(c) # supply quantity #;
(all,c,COM) psup(c) # supply price #;
Equation  E_qsup   (all,c,COM) qsup(c) = SUPELS(c)*psup(c);
! Demand curves !
Coefficient (all,c,COM) DEMELS(c) # demand elasticity #;
Update (change)(all,c,COM) DEMELS(c)=0;
Read DEMELS from file BASEDATA header "EDEM";
Variable
(all,c,COM) qdem(c) # demand quantity #;
(all,c,COM) pdem(c) # demand price #;
Equation  E_qdem   (all,c,COM) qdem(c) = -DEMELS(c)*pdem(c);
! Demand-Supply links !
Variable
(all,c,COM) tp(c) # price wedge #;
(all,c,COM) fx(c) # quantity wedge #;
Equation E_psup  (all,c,COM) qdem(c) =  fx(c) + qsup(c);
Equation E_pdem  (all,c,COM) pdem(c) =  tp(c) + psup(c);
```

Suppose fx and tp are exogenous, and we shock tp by 1% for all goods. Then, using simulation results we can deduce the elasticity values as:

```
SUPELS(c) = qsup(c)/psup(c);
DEMELS(c) = -qdem(c)/pdem(c);
```

Equally we could shock fx (the quantity wedge) to obtain similar results.

1.2. Running the above example

Open a command prompt (aka DOS box) in the Simple subfolder and type

```
runsimple.bat
```

This should produce results files elastp.har (using tp shocks) and elastq.har (using fx shocks). In each case the original supply and demand elasticities (read from file simple.har) can be deduced from simulation results. However, if one of the demand or supply elasticities is zero, the fx shocks give better estimates. We can see the reason by referring to Figure 1: the tp shocks correspond to a vertical (price) shift which has no effect on a vertical supply or demand schedule.

In the more realistic ORANI-G example described later, we observe that supply curves tend to be rather flat, while demand curves are fairly steep. Also, movement of demand curves induces small movements in supply curves (and vice versa). Hence it turns out to be best to use price wedges to estimate demand elasticities, and quantity wedges to estimate supply elasticities.

1.3. Understanding RunSimple.BAT

It is worthwhile to thoroughly understand the parts of RunSimple.BAT before going on to the more complicated ORANI-G example:

- The model equations (shown above) are in file SIMPLE.TAB.

The first GEMSIM run:

```
gemsim -cmf simple.cmf
```

- establishes the model closure.
- saves Equations and LU4 files which are later used by SAGEM. The LU4 file contains the elasticity of each endogenous variable to each exogenous variable.
- includes an irrelevant shock.

The command

```
sagem -cmf sagemp.cmf
```

runs 10 separate Johansen simulations (corresponding to 10 separate tp shocks). The 10 solutions appear as separate columns in solution file sagemp.sl4.

The command

```
sltoht -sti sagemp.sti
```

converts sagemp.sl4 into sagemp.sol, a HAR file that can be read by a TABLO program.

The command

```
gemsim -cmf calcp.cmf
```

uses results from sagemp.sol to compute supply and demand elasticities, and compares these to the original values stored in simple.har. Results from calcp.cmf are in elastp.har. There we can see that we can exactly recover the original supply and demand elasticities from the results of the SAGEM simulations [unless one of the original supply and demand elasticities was zero].

2. Computing elasticities with ORANI-G

We now apply (and extend) the method presented above to a real-world CGE model, ORANI-G. Unlike the SIMPLE model, in ORANI-G commodity markets are inter-dependent: the price of grain affects the price of bread.

Compared to the stylized SIMPLE model, ORANI-G presents several complications that require decisions:

- Supply and demand elasticities relate quantities to prices, but which prices? Purchasers' prices differ from basic (ex-factory) prices because of taxes and retail/transport margins. We decide to use basic prices to compute both supply and demand elasticities.
- The supply-demand diagram of Figure 1 makes no distinction between industries and commodities. In ORANI-G each industry could produce several commodities and each commodity might be produced by several industries. We decided to compare supply and demand elasticities in commodity (rather than industry) dimension.
- CGE elasticities of course depend on the closure. We compute results for two familiar closures: short-run and long-run.

A fairly standard version of ORANIG.TAB is used (although the regional extension has been removed). To assist with our elasticity computations we added various sections (which can be found by searching for 'addelast'). The additions are of two types:

- Traditionally ORANIG.TAB has included formulae to express industry short-run supply elasticities as a function of factor shares and primary-factor substitution elasticities. We added code to also compute long-run supply elasticities and to convert industry supply elasticities into commodity supply elasticities (using MAKE shares). We added more code to compute demand elasticities w.r.t. basic prices as a market-share weighted average of agent-specific demand elasticities w.r.t. purchasers' prices. We can compare these formulae-based elasticities to simulated CGE elasticities.
- We added exogenous price and quantity 'wedge' variables of commodity dimension to key market-clearing and price-transmission equations, making more explicit the distinction between demand and supply for both quantities and prices. These additions are shown in the excerpt below:

```
!addelast!
Equation
 E_pq1  # Each industry gets the same price for a given commodity #
 (all,c,COM)(all,i,IND) pq1(c,i) = psup(c);
 E_psup  # price transmission #
 (all,c,COM) p0com(c) = psup(c) + fp0com(c);
 E_xsup # Total output of commodities (as simple addition) #
 (all,c,COM) xsup(c)  = sum{i,IND, [MAKE(c,i)/MAKE_I(c)]*q1(c,i)};
 E_x0com # Demand=Supply for commodities #
 (all,c,COM) x0com(c) = xsup(c) + fx0com(c);
```

Thus we can compute CGE supply and demand elasticities by shocking either fx0com or fp0com. As mentioned above, we found it more practical to compute supply elasticities using fx0com (quantity) shocks, and demand elasticities using fp0com (price) shocks.[2]

2.1. Formula-based elasticities

We can inspect equations of the model and use formulae for own-price coefficients to produce estimates of supply and demand elasticities. Such formulae incorporate assumptions, such as:

- for demand elasticities, only one price varies.
- for supply elasticities, some inputs are fixed, whilst the rest are in elastic supply.

We may need to combine and manipulate model equations. It's important to note that these formula-based elasticities are model-specific. For example, in ORANI-G only households substitute between commodities, while in the similar TERM model industries can also do this. This would affect demand elasticity formulae.

2.1.1. Formula-based demand elasticities

The total market demand elasticity for some commodity, E, is a market-share weighted average of user-specific demand elasticities E_k:

$$E = \sum_k S_k E_k \qquad \text{where } S_k = \text{sales share user k}$$

But here we wish E to be an elasticity w.r.t. basic prices while the E_k are usually elasticities w.r.t. purchasers' prices. To accommodate this we need to add a term:

$$E = \sum_k S_k H_k E_k \qquad \text{where } H_k = \text{share of basic in purchasers price}$$

Values for E_k derive from 3 sources:

- for the export user, E_k is just the export demand elasticity.
- for other users, E_k derives mainly from the import substitution effect:

  $E_k = S_m\sigma$ where S_m = import share and σ is the 'Armington' elasticity.
- for the supernumerary (luxury) part of household use, there is in addition a unitary elasticity of substitution between commodities.

The actual formulae may be seen by searching through ORANIG.TAB for 'compute demand elasticities'. The formulae have been arranged to show each user's contribution to the total elasticity.

In these formulae we make no distinction between short-run and long-run demand elasticities.

2.1.2. Formula-based supply elasticities

Appendix J of the ORANI-G document (Horridge, 2000) shows how the industry supply elasticity, R, may be estimated from the CES primary factor demand equations, giving:

$$R = \sigma S_V/(S_F H_F)$$

where σ is the primary factor substitution elasticity, S_F is the share within primary factor costs of fixed primary factors (eg, land or capital), S_V = 1-S_F, and H_F is share of primary factors in total cost. We assume (inaccurately) for these estimates that intermediate inputs are available elastically.

In most shortrun closures, the fixed factors are capital and land, while labour is assumed be in elastic supply. In most longrun closures, only land is fixed, so for industries that use no land R would be infinite (we set values at 999).

An additional complication is that we wish to compute commodity (rather than industry) supply elasticities. We need to take two mechanisms into account:

[2] The model's definitions of expenditure and income GDP do not include the effects of shocking either fp0com or fx0com, so that wedge shocks will cause reported expenditure and income GDP results to diverge.

- Imagine that the electricity commodity is supplied half by the coal-fired-electricity industry (R=0.1) and half by the gas-fired-electricity industry (R=0.3). Then the commodity supply elasticity is 0.2 (the MAKE-based average).
- Imagine that the Agriculture industry uses only land (ie, R=0) to produce Animals and Crops via a CET function. Even though R=0, if the Crops price goes up, the Crops part of output would rise, giving a supply elasticity of σS_a where σ is the CET substitution elasticity and S_a is the share of Animals in output.

See the formulae for SRSUPPELASTC and LRSUPPELASTC in ORANIG.TAB.

3. Running the batch script to compute shortrun CGE elasticities

To compute the shortrun ORANI-G elasticities, from the command line, type

```
RunExm.bat
```

The most important lines of this file are shown below:

```
REM  Batch script to compute ORANI-G elasticities

REM  create GSS,GST files for both ORANIG and CALC
tablo -pgs oranig >tb1.log
tablo -pgs calc >tb2.log

REM  create EQ4, LU4 files for ORANIG
gemsim -cmf oranigsr.cmf  >run1.log
REM Above, use oraniglr.cmf for long-run elasticities

REM  do a SAGEM simulation for each price shock
sagem -cmf sagemp.cmf >sagemp.log
REM  convert SL4 file to SOL HAR file
sltoht -sti sagemp.sti >sltohtp.log

REM  do a SAGEM simulation for each quantity shock
sagem -cmf sagemq.cmf >sagemq.log
sltoht -sti sagemq.sti >sltohtq.log

REM  compute supply elasticities from quantity shock results
REM  and demand elasticities from price shock results
gemsim -cmf calc.cmf
if errorlevel 1 goto err
```

As indicated above, you could compute long-run elasticities by using oraniglr.cmf at the ORANIG stage. The database used, BASEDATA.HAR, is an Australian dataset used in training courses: it distinguishes 37 commodities produced by 35 industries (there is agricultural multiproduction).

The steps listed above follow those in the SIMPLE example. Again, the first GEMSIM run:

```
gemsim -cmf oranigsr.cmf
```

- establishes the model closure.
- saves Equations and LU4 files which are later used by SAGEM. The LU4 file contains the elasticity of each endogenous variable to each exogenous variable.
- includes an irrelevant shock.
- produces a useful SUMMARY file.

The SUMMARY file contains values for the formulae-based elasticities and useful related coefficients. For the supply elasticities the useful headers are:

Header	Dimension	Coefficient	Name
FACT	IND*FAC	FACTOR	Primary Factor Costs
SRSE	IND	SRSUPPELASTI	Short-run supply elasticity (ind)
LRSE	IND	LRSUPPELASTI	Longrun supply elasticity (ind)
SRSC	COM	SRSUPPELASTC	Short-run supply elasticity (com)
LRSC	COM	LRSUPPELASTC	Long-run supply elasticity (com)
SOWN	COM	OWNSHARE	Share of com c output used in making c
MAKE	COM*IND	MAKE	Multiproduct Matrix

For the demand elasticities they are:

Header	Dimension	Coefficient	Name
MSHR	COM	IMPSHR	Share of imports in local market
DEME	COM*SALECAT2	DEMELS	Per user demand elasticity w.r.t to basic price
DEMC	COM*SALECAT2	DEMELS	User components of DEMELS
SSHR	COM*SALECAT2	SALESSHR	User sales shares
EDEM	COM	TOTDEMELS	Total demand elasticity w.r.t to basic price

The command

```
sagem -cmf sagemp.cmf
```

runs 37 separate Johansen simulations (corresponding to price wedge shocks). The 37 solutions appear as separate columns in solution file sagemp.sl4. The command

```
sltoht -sti sagemp.sti
```

converts sagemp.sl4 into sagemp.sol, a HAR file that can be read by a TABLO program.

Similarly the commands

```
sagem -cmf sagemq.cmf
sltoht -sti sagemq.sti
```

runs 37 separate Johansen simulations (corresponding to quantity wedge shocks), the results of which (after sltoht) appear as as separate columns in HAR file sagemq.sol.

The second GEMSIM run

```
gemsim -cmf calc.cmf
```

uses results from sagemq.sol to compute CGE supply elasticities and from sagemp.sol to calculate CGE demand elasticities[3]. In each case the CGE elasticities are computed as the ratio of quantity to price changes. CALC.TAB presents these CGE elasticities at header ELST[4] in ELAST.HAR alongside the formulae-based elasticities described previously. Some shortrun results are shown in Table 1 below, where the columns are as follows:

- SRSE the formula-based shortrun supply elasticity
- SUPPLY the CGE shortrun supply elasticity
- EDEM the formula-based demand elasticity
- DEMAND the CGE shortrun demand elasticity

In general the CGE supply elasticities are closely correlated to (but less than) their formula-based counterparts. The CGE numbers are lower because thay take into account the actual supply elasticities of intermediate inputs (the formulae assume these to be infinite). The scatterplot of Figure 2 shows how

[3] Although not shown in the text, RunExm.bat also includes CALC runs to compute both supply and demand CGE elasticities using only quantity shocks, or only price shocks.

[4] The other headers PSUP, QSUP, PDEM and QDEM may be used to compute cross-elasticities, if desired.

close the correlation is. For two outliers the CGE elasticities are significantly less than their formula-based counterparts. These are:

- MeatDairy, which depends on agricultural inputs with low supply elasticity.
- GovAdminDfnc, which is very labour-intensive, so the formula-based elasticity is much too high.

As seen in Figure 3, the CGE demand elasticities are also closely correlated to their formula-based counterparts, but this time the CGE elasticities tend to be the higher of the two. To understand why, consider demand for BeefCattle, which faces little import competition, and sells almost wholly to the MeatDairy industry. The formula-based elasticity is thus very low (it assumes that output of MeatDairy is fixed). In reality a rise in BeefCattle prices also raises the price of MeatDairy so reducing MeatDairy output and MeatDairy demand for BeefCattle. The CGE elasticity takes this into account and so is higher than the formula-based estimate.

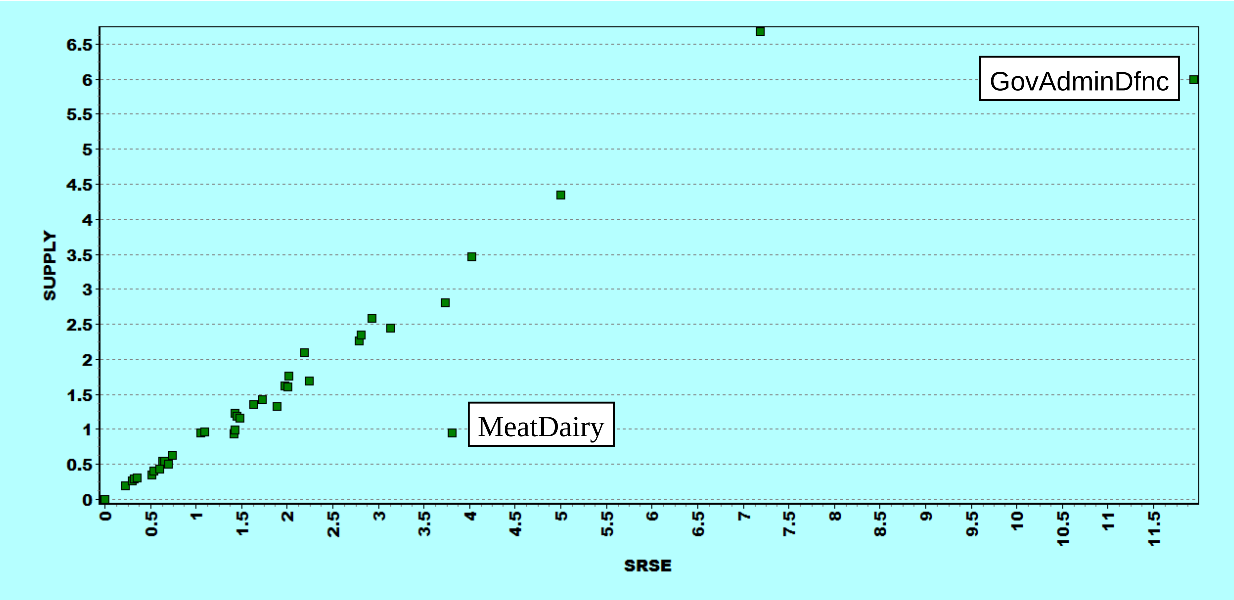

Figure 2. CGE (SUPPLY) versus formula-based (SRSE) short-run supply elasticities

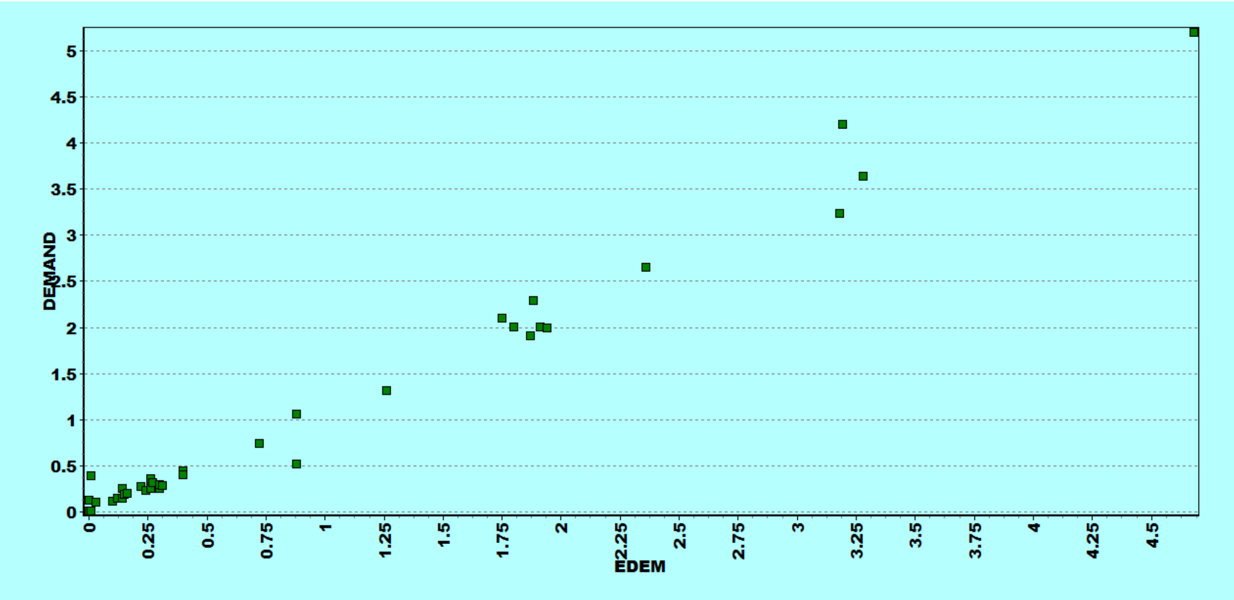

Figure 3. CGE (DEMAND) versus formula-based (EDEM) short-run demand elasticities

Table 1: CGE and formula-based short-run supply and demand elasticities

ELASTS	SRSE	SUPPLY	EDEM	DEMAND
1 WoolMutton	0.60	0.43	0.72	0.74
2 GrainsHay	0.51	0.35	3.18	3.24
3 BeefCattle	0.54	0.40	0.01	0.39
4 OtherAgric	0.35	0.31	0.88	1.06
5 ForestFish	2.19	2.10	1.26	1.32
6 Mining	0.30	0.26	4.68	5.20
7 MeatDairy	3.81	0.95	3.28	3.64
8 OthFoodProds	2.01	1.61	1.80	2.01
9 DrinksSmokes	1.05	0.95	0.26	0.30
10 Textiles	2.24	1.69	1.88	2.29
11 ClothingFtw	4.02	3.47	1.94	2.00
12 WoodProds	3.13	2.44	0.26	0.36
13 PaperPrint	1.45	1.19	0.27	0.32
14 Petrol_CoalP	0.32	0.30	0.14	0.25
15 Chemicals	1.48	1.16	1.75	2.10
16 RubberPlastc	2.02	1.76	0.40	0.44
17 NonMetlMinrl	1.43	1.23	0.10	0.12
18 BasicMetals	1.89	1.33	3.19	4.20
19 FabMetalPrd	2.79	2.26	0.22	0.28
20 TransportEqp	1.97	1.62	2.36	2.65
21 OthMachnEqp	1.73	1.43	1.91	2.01
22 MiscManuf	2.93	2.58	0.40	0.40
23 ElecGasWater	0.23	0.20	0.14	0.15
24 Construction	1.09	0.96	0.00	0.01
25 Trade	1.42	0.93	0.00	0.13
26 Repairs	0.65	0.55	0.26	0.25
27 Hotel_Cafe	2.81	2.35	0.31	0.29
28 Transport	0.74	0.63	1.87	1.91
29 CommunicSrvc	0.63	0.54	0.16	0.20
30 FinanceInsur	0.70	0.51	0.15	0.19
31 OwnerDwellng	0.00	0.00	0.88	0.52
32 PropBusSrvc	1.43	0.99	0.03	0.11
33 GovAdminDfnc	11.94	5.99	0.01	0.01
34 Education	7.19	6.68	0.12	0.15
35 HealthCommun	3.73	2.81	0.30	0.25
36 CultuRecreat	1.63	1.36	0.30	0.30
37 OtherService	5.00	4.35	0.24	0.23

4. Longrun CGE elasticities

Using a different closure file (oraniglr.cmf), we can again run RunExm.bat to compute longrun CGE elasticities. In the longrun closure non-land-using industries have no fixed factor, so that supply elasticities are almost infinite: results are unsatisfactory. Demand elasticities are very similar to the shortrun closure, but generally higher — see Table 2 below. Again the explanation is derived demand. In the shortrun, a rise in the BeefCattle price is partially offset by a fall in MeatDairy capital rentals; this does not occur in the longrun (rates of return are fixed), so that MeatDairy prices rise more, and MeatDairy (and thus BeefCattle) sales fall more.

Table 2: Shortrun and longrun CGE demand elasticities

ELASTS	Shortrun	Longrun
1 WoolMutton	0.74	0.81
2 GrainsHay	3.24	3.31
3 BeefCattle	0.39	0.82
4 OtherAgric	1.06	1.19
5 ForestFish	1.32	1.31
6 Mining	5.20	4.99
7 MeatDairy	3.64	3.64
8 OthFoodProds	2.01	2.02
9 DrinksSmokes	0.30	0.29
10 Textiles	2.29	2.33
11 ClothingFtw	2.00	2.00
12 WoodProds	0.36	0.39
13 PaperPrint	0.32	0.32
14 Petrol_CoalP	0.25	0.23
15 Chemicals	2.10	2.09
16 RubberPlastc	0.44	0.44
17 NonMetlMinrl	0.12	0.14
18 BasicMetals	4.20	4.18
19 FabMetalPrd	0.28	0.30
20 TransportEqp	2.65	2.79
21 OthMachnEqp	2.01	2.04
22 MiscManuf	0.40	0.41
23 ElecGasWater	0.15	0.17
24 Construction	0.01	0.30
25 Trade	0.13	0.22
26 Repairs	0.25	0.26
27 Hotel_Cafe	0.29	0.30
28 Transport	1.91	1.87
29 CommunicSrvc	0.20	0.20
30 FinanceInsur	0.19	0.20
31 OwnerDwellng	0.52	0.75
32 PropBusSrvc	0.11	0.14
33 GovAdminDfnc	0.01	0.01
34 Education	0.15	0.16
35 HealthCommun	0.25	0.27
36 CultuRecreat	0.30	0.31
37 OtherService	0.23	0.23

5. Additional Chinese example

The China subfolder of the supplied zip archive repeats the Australian ORANI-G example above, but with a 149-sector Chinese database (national.har). In this case the MAKE matrix is diagonal: industries and commodities are the same. The results are similar to those described above; except that agricultural supply elasticities are higher. This is because farm land and capital shares in the Chinese database are smaller than in the Australian — perhaps because some Chinese farm value-added was reported as labour earnings rather than as land or capital rentals[5].

6. Extending the idea to other models

The methodology described could be applied to other static, single-region CGE models (although different model equations might lead to different formulae for the formula-based elasticity estimates). For multi-regional or multi-period models we might need to extend or sharpen our elasticity definitions. For example, in a multi-regional model we might define elasticities at the national or at the regional level. At the regional level we could define elasticities as the quantity responses to a local or alternatively to a uniform national price shock. Our choice would depend on how we wished to use the elasticities.

For multi-period models a shock in period t will have an effect in period t and in later periods. Perhaps longrun elasticities could be defined as the response in period t+20 to a shock in t.

7. Conclusion

We have described how ORANI-G may be slightly modified to enable the computing of CGE demand and supply elasticities.Using SAGEM, the computation is very efficient and quick. The CGE elasticities, which allow for inter-industry effects, are closely correlated to formula-based elasticity estimates (based on shares and parameters) which do not. We explain the numerical differences between the two types of elasticity estimate.

We may wish to compare the implicit, CGE, elasticities revealed by our model to other (perhaps partial-equilibrium) estimates, or even to adjust the model so that it implies CGE elasticities that match external estimates. Understanding formula-based elasticity estimates would help us do this.

8. References

Hertel, Lanclos,Pearson and Swaminathan (1997), Aggregation and computation of equilibrium elasticities; Chapter 5 in: Global Trade Analysis: Modeling and Applications, Hertel (ed), Cambridge University Press.

Horridge (2000), ORANI-G: A General Equilibrium Model of the Australian Economy, CoPS/IMPACT Working Paper Number OP-93, at www.copsmodels.com/elecpapr/op-93.htm

O'Toole and Matthews (2002), General Equilibrium, Partial Equilibrium and the Partial Derivative: Elasticities in a CGE model, Paper presented to the 2002 EcoMod International Conference on Global Modeling, Brussels.

[5] In the Australian database, labour in agriculture is around 20% of primary factor costs; in the Chinese database the corresponding labour share is 95%.