THE UNIVERSITY OF
NEW SOUTH WALES

A New and Problematic Axiom for Price and Quantity Indexes

Robert Hill

2002/03

SCHOOL OF ECONOMICS

DISCUSSION PAPER

ISSN 1323-8949
ISBN 0 7334 1935

A New and Problematic Axiom for Price and Quantity Indexes

Robert J. Hill*

School of Economics

University of New South Wales

Sydney 2052, Australia

Tel. 61-2-9385-3076, Fax. 61-2-9313-6337

E-Mail. r.hill@unsw.edu.au

May 20, 2002

A new axiom for price and quantity indexes is proposed. This axiom requires that, if prices (quantities) of all goods and services rise more in region A than in region B, then the price (quantity) index for region A must exceed that for region B. Although this axiom seems quite weak and at the same time fundamental, it is shown that all the main formulae violate it. The implications of this finding for the index number literature are explored. (*JEL* C43, E31, O47)

KEYWORDS: Price Index; Quantity Index; Impossibility Theorem; Fox Paradox

$*$ The author would like to thank Robert Summers for stimulating his interest in this topic during a conversation at a recent conference.

1. Introduction

Price and quantity indexes are widely used in Economics, especially for the measurement of inflation and growth, for efficiency comparisons across firms, and for comparisons of living standards and the cost of living across countries and regions. A number of alternative formulae have been proposed in the index number literature for making bilateral spatial or temporal comparisons. The evolution of this literature is surveyed in Diewert (1993). Over the last one hundred years, the formulae that have received the most attention are Laspeyres, Paasche, Edgeworth, Walsh, Fisher, Törnqvist and Vartia.[1] Irving Fisher (1922) considered a far larger selection of bilateral index number formulae. However, the formulae listed above are the ones that to varying degrees have stood the test of time.

There has been much debate in the index number literature on the appropriate choice of formula. Two main schools of thought have emerged on this issue. The axiomatic approach proposes a set of axioms that a price (quantity) index should satisfy. It then discriminates between competing price (quantity) index formulae on the basis of the axioms each formula satisfies and fails. It is worth noting that no formula satisfies all of the axioms. The definitive reference on the axiomatic approach is Fisher (1922). The Fisher index emerged as Fisher's preferred formula, and was subsequently named after him, even though it was originally proposed by Bowley in 1899 (see Diewert (1993)). More recent contributions to the literature include Eichhorn and Voeller (1976) and Balk (1995).[2]

The economic approach, by contrast, is firmly grounded in a utility maximization setting. It assumes that observed quantities are the utility maximizing choices of agents faced by prevailing prices. The object of measurement according to the economic ap-

[1] All these formulae are defined later in the paper.

[2] The axiomatic approach has also been applied to multilateral methods which require comparisons between three or more regions to be transitive (see Balk (1996), Diewert (1999), van Veelen (2002) and Armstrong (2003)).

proach is the cost-of-living index which measures the change, from one period or region to the next, in the minimum cost of reaching a given level of utility. The problem for the economic approach is how to compute the cost-of-living index. Under the assumption of utility maximizing behavior, Diewert (1976) showed that certain observable price indexes exactly equal the cost-of-living index if the underlying cost function has a particular functional form. For example, the Törnqvist price index is exact for the translog cost function. Diewert reasoned that we should prefer those price indexes that are exact for *flexible* cost functions (i.e., cost functions that can approximate an arbitrary linearly homogeneous cost function to the second order). He defined such price indexes as *superlative*. Superlative quantity indexes are defined in an analogous manner.

Diewert's superlative index number methodology seems to provide the perfect solution to the index number problem. Superlative indexes are firmly grounded in economic theory (i.e., they approximate to the second order the underlying cost-of-living index), and they are functions of observable prices and quantities, and hence are easy to compute. In consequence, a clear consensus has emerged in the index number literature that inflation and growth should be measured using a superlative index number formula (see, for example, Triplett, 1996). As a result, the Bureau of Economic Analysis in the United States recently switched from Paasche to Fisher (a superlative index) in its computation of the GDP deflator (see Landefeld and Parker, 1997).

However, Diewert (1976) also showed that there are an infinite number of superlative index numbers. Of the formulae mentioned above, the only ones that are superlative are Walsh, Fisher and Törnqvist. These three formulae tend to approximate each other closely in practice (although it should be noted that not all superlative indexes approximate each other closely – see Hill, 2002). So the current consensus is that one of these three formulae should be used, and that in practice the choice between them is of little consequence.

This paper proposes a new axiom that, at first glance, seems very weak and which at the same time seems fundamental. The axiom is problematic because it turns out that all the formulae discussed above violate it, and moreover all but one of the known superlative formulae also violate it. The exception is a “bad” superlative index that one would never want to use in practice. The new axiom builds on a paradoxical result observed by Fox (1999). In the context of efficiency comparisons between firms based on actual and minimal feasible cost estimates, Fox noticed that it is possible for a multi-product firm to be more efficient at producing each product than any other firm, yet it may not be the most efficient firm overall. This result has since become known as the “Fox paradox” in the efficiency and productivity analysis literature (see Färe and Grosskopf (2000)). It is shown here that this paradox is rather more pervasive than previously realized. Using the new axiom, an impossibility theorem is derived. Its implications for the construction of price and quantity indexes are discussed at the end of the paper. Although the impossibility theorem does not necessarily change the consensus, it is nevertheless important that researchers in the field and users of index numbers should be aware that the formulae they are using violate this axiom.

2. Methodology and Notation

This paper will be motivated in the context of a comparison of the inflation or growth rates of two regions (or countries). However, the comparison could equally well be between two firms. The two regions are denoted by A and B. Let $i = 1, \ldots, N$ index the basket of goods and services over which the price and quantity indexes are defined. The price (quantity) of good i in time period t in regions A and B is denoted by p_{ti}^A and p_{ti}^B (q_{ti}^A and q_{ti}^B), respectively. It is assumed that $p_{ti}^A, p_{ti}^B, q_{ti}^A, q_{ti}^B > 0 \;\; \forall i, t$. Also, let s_{ti}^j denote the share of total expenditure in time period t on good i in region j.

$$s_{ti}^j = \frac{p_{ti}^j q_{ti}^j}{\sum_{k=1}^N p_{tk}^j q_{tk}^j}$$

(i) Superlative Indexes

Diewert (1976) defines a quantity (price) index as *superlative* if it is *exact* for a *flexible* aggregator (unit cost) function. The concepts of *exactness* and *flexibility* are defined as follows: an aggregator (unit cost) function is *flexible* if it can provide a second order approximation to an arbitrary linearly homogeneous aggregator (unit cost) function. A quantity index Q_{12} is *exact* for the aggregator function $f(q)$ if utility (profit) maximization implies that

$$\frac{f(q_2)}{f(q_1)} = Q_{12},$$

where $f(q_2)/f(q_1)$ is an implicit Konüs (1924) quantity index. Similarly, a price index P_{12} is *exact* for the unit cost function $c(p)$ if utility (profit) maximization implies that

$$\frac{c(p_2)}{c(p_1)} = P_{12},$$

where $c(p_2)/c(p_1)$ is a Konüs (1924) cost-of-living index.

Diewert then shows that all quadratic-mean-of-order-r quantity indexes and price indexes are superlative.[3] These indexes, which differ by the parameter $r \in (-\infty, +\infty)$, are defined as follows:

$$Q_{12}^{r,j} = \frac{\left(\sum_{i=1}^{N} s_{1i}^j (q_{2i}^j / q_{1i}^j)^{r/2}\right)^{1/r}}{\left(\sum_{i=1}^{N} s_{2i}^j (q_{2i}^j / q_{1i}^j)^{-r/2}\right)^{1/r}} \qquad r \neq 0, \qquad Q_{12}^{0,j} = \prod_{i=1}^{N} \left[\left(\frac{q_{2i}^j}{q_{1i}^j} \right)^{\frac{s_{1i}^j + s_{2i}^j}{2}} \right], \tag{1}$$

$$P_{12}^{r,j} = \frac{\left(\sum_{i=1}^{N} s_{1i}^j (p_{2i}^j / p_{1i}^j)^{r/2}\right)^{1/r}}{\left(\sum_{i=1}^{N} s_{2i}^j (p_{2i}^j / p_{1i}^j)^{-r/2}\right)^{1/r}} \qquad r \neq 0, \qquad P_{12}^{0,j} = \prod_{i=1}^{N} \left[\left(\frac{p_{2i}^j}{p_{1i}^j} \right)^{\frac{s_{1i}^j + s_{2i}^j}{2}} \right]. \tag{2}$$

[3] The quadratic-mean-of-order-r quantity index is exact for the quadratic-mean-of-order-r aggregator function, which is itself flexible. Likewise, the quadratic-mean-of-order-r price index is exact for the quadratic-mean-of-order-r unit cost function, which is also flexible. See Diewert (1976).

A second family of superlative quantity indexes and price indexes can be derived implicitly as follows:

$$\tilde{Q}_{12}^{r,j} = \frac{1}{P_{12}^{r,j}} \frac{\sum_{i=1}^{N} p_{2i} q_{2i}}{\sum_{i=1}^{N} p_{1i} q_{1i}}, \tag{3}$$

$$\tilde{P}_{12}^{r,j} = \frac{1}{Q_{12}^{r,j}} \frac{\sum_{i=1}^{N} p_{2i} q_{2i}}{\sum_{i=1}^{N} p_{1i} q_{1i}}. \tag{4}$$

The attraction of superlative index numbers is that they approximate to the second order the underlying cost-of-living index, while at the same time being easy to compute since they are functions of observable prices and quantities. It is not clear that economic theory can be used to discriminate between different values of r, since the quadratic-mean-of-order-r aggregator function is flexible for all values of r. However, for certain values of r the superlative formula simplifies in an intuitively appealing manner. For this reason attention has focused primarily on three superlative formulae. These are the Törnqvist ($r = 0$), Walsh ($r = 1$ in the implicit superlative formula) and Fisher ($r = 2$) indexes defined below:

$$\text{Törnqvist}: \quad P_{12}^{0,j} = \prod_{i=1}^{N} \left(\frac{p_{2i}^j}{p_{1i}^j} \right)^{\frac{s_{1i}^j + s_{2i}^j}{2}}, \quad Q_{12}^{0,j} = \prod_{i=1}^{N} \left(\frac{q_{2i}^j}{q_{1i}^j} \right)^{\frac{s_{1i}^j + s_{2i}^j}{2}}, \tag{5}$$

$$\text{Walsh}: \quad \tilde{P}_{12}^{1,j} = \frac{\sum_{i=1}^{N} p_{2i}^j \sqrt{q_{1i}^j q_{2i}^j}}{\sum_{i=1}^{N} p_{1i}^j \sqrt{q_{1i}^j q_{2i}^j}}, \quad \tilde{Q}_{12}^{1,j} = \frac{\sum_{i=1}^{N} \sqrt{p_{1i}^j p_{2i}^j} q_{2i}^j}{\sum_{i=1}^{N} \sqrt{p_{1i}^j p_{2i}^j} q_{1i}^j}, \tag{6}$$

$$\text{Fisher}: \quad P_{12}^{2,j} = \left(\frac{\sum_{i=1}^{N} p_{2i}^j q_{2i}^j}{\sum_{i=1}^{N} p_{1i}^j q_{2i}^j} \frac{\sum_{i=1}^{N} p_{2i}^j q_{1i}^j}{\sum_{i=1}^{N} p_{1i}^j q_{1i}^j} \right)^{1/2}, \quad Q_{12}^{2,j} = \left(\frac{\sum_{i=1}^{N} p_{2i}^j q_{2i}^j}{\sum_{i=1}^{N} p_{2i}^j q_{1i}^j} \frac{\sum_{i=1}^{N} p_{1i}^j q_{2i}^j}{\sum_{i=1}^{N} p_{1i}^j q_{1i}^j} \right)^{1/2}. \tag{7}$$

(ii) The Vartia Index

Vartia (1974, 1976) proposed the following variant on the Törnqvist index. Its properties are also discussed by Sato (1976) and Diewert (1978).

$$\ln P_{12}^j = \sum_{i=1}^{N} \left\{ \left[\frac{L(p_{1i}^j q_{1i}^j, p_{2i}^j q_{2i}^j)}{L(\sum_{k=1}^{N} p_{1k}^j q_{1k}^j, \sum_{k=1}^{N} p_{2k}^j q_{2k}^j)} \right] \ln \left(\frac{p_{2i}^j}{p_{1i}^j} \right) \right\}, \tag{8}$$

$$\ln Q_{12}^j = \sum_{i=1}^{N} \left\{ \left[\frac{L(p_{1i}^j q_{1i}^j, p_{2i}^j q_{2i}^j)}{L(\sum_{k=1}^{N} p_{1k}^j q_{1k}^j, \sum_{k=1}^{N} p_{2k}^j q_{2k}^j)} \right] \ln \left(\frac{q_{2i}^j}{q_{1i}^j} \right) \right\}, \tag{9}$$

where L is a logarithmic mean function which is defined as follows:

$$L(a,b) = \frac{a-b}{\ln a - \ln b} \quad \text{for} \quad a,b > 0, \quad a \neq b \quad \text{and} \quad L(a,a) = a.$$

Unlike Törnqvist, Vartia is not superlative (although it is pseudo-superlative – see Diewert, 1978). However, Vartia does have the advantage of satisfying the strong factor reversal test, i.e., if P_{12}^j and Q_{12}^j are computed using the Vartia formula, then

$$P_{12}^j Q_{12}^j = \frac{\sum_{i=1}^N p_{2i} q_{2i}}{\sum_{i=1}^N p_{1i} q_{1i}}.$$

By contrast, the Fisher index is the only known superlative index that satisfies the strong factor reversal test (i.e., $P_{12}^{r,j} = \tilde{P}_{12}^{r,j}$ and $Q_{12}^{r,j} = \tilde{Q}_{12}^{r,j}$ only if $r = 2$).

(iii) Pure Price Indexes

Diewert (2002) defines a price index as *pure* if it can be written in the following form:

$$P_{12}^j = \frac{\sum_{i=1}^N p_{2i}^j q_{Xi}^j}{\sum_{i=1}^N p_{1i}^j q_{Xi}^j}, \tag{10}$$

where q_X^j denotes a reference quantity vector that may equal q_1^j, q_2^j or some combination of q_1^j and q_2^j. It is assumed that $q_{Xi}^j > 0 \; \forall i$. Four pure price indexes, in particular, have received attention in the index number literature. If $q_X^j = q_1^j$, then (10) reduces to a Laspeyres price index. Conversely, if $q_X^j = q_2^j$, then (10) reduces to a Paasche price index. If $q_{Xi}^j = (q_{1i}^j + q_{2i}^j)/2$, then we obtain the Edgeworth price index. If $q_{Xi}^j = \sqrt{q_{1i}^j q_{2i}^j}$, then we obtain the Walsh price index, $\tilde{P}_{12}^{1,j}$. This is the only known superlative price index that is also pure.

Corresponding quantity indexes can be derived implicitly from pure price indexes using the factor reversal test. The implicit indexes derived from the Laspeyres, Paasche, Edgeworth and Walsh price indexes, respectively, are the Paasche, Laspeyres, implicit Edgeworth and implicit Walsh quantity indexes.

(iv) Pure Quantity Indexes

A quantity index is *pure* if it can be written in the following form:

$$Q_{12}^{j} = \frac{\sum_{i=1}^{N} p_{Xi}^{j} q_{2i}^{j}}{\sum_{i=1}^{N} p_{Xi}^{j} q_{1i}^{j}}, \tag{11}$$

where p_X^j denotes a reference price vector that may equal p_1^j, p_2^j or some combination of p_1^j and p_2^j. It is assumed that $p_{Xi}^j > 0\ \forall i$. If $p_X^j = p_1^j$, then (11) reduces to a Laspeyres quantity index. Conversely, if $p_X^j = p_2^j$, then (11) reduces to a Paasche quantity index. If $q_{Xi}^j = \sqrt{q_{1i}^j q_{2i}^j}$, then we obtain the Walsh quantity index, $\tilde{Q}_{12}^{1,j}$. This is the only known superlative quantity index that is also pure.

The implicit indexes derived from the Laspeyres, Paasche and Walsh quantity indexes, respectively, are the Paasche, Laspeyres and implicit Walsh price indexes.

3. An Impossibility Theorem

Here we define three axioms relating to price indexes. Analogous axioms could be defined for quantity indexes.

$$A1. \quad \left(\frac{p_{2i}^A}{p_{1i}^A}\right) > \left(\frac{p_{2i}^B}{p_{1i}^B}\right) \quad \text{for } i = 1, \ldots, N \quad \Rightarrow P_{12}^A > P_{12}^B$$

$$A2. \quad \lim_{s_{1i}^j \to 1, s_{2i}^j \to 1} P_{12}^j = \left(\frac{p_{2i}^j}{p_{1i}^j}\right) \quad \text{for } i = 1, \ldots, N$$

$A3.$ P_{12}^j is a continuous function of p_{ti}^j, q_{ti}^j for $t = 1, 2$ and $i = 1, \ldots, N$.

The axiom $A1$ states that if the price relative in region A is higher than in region B for every single commodity, then the price index for region A should exceed that for region B. The axiom $A2$ states that, in the limit, as the expenditure share of commodity i in region j tends to one in both periods 1 and 2, the price index of region j should equal the price relative of commodity i. The axiom $A3$ is self-explanatory.

Theorem. There does not exist a price index formula $P_{12}^j : R_{++}^{4N} \to R_+$ that satisfies $A1$, $A2$ and $A3$ for all possible price and quantity vectors p_t^j, $q_t^j \gg 0$, where $j = A, B$ and $t = 1, 2$.

Proof. Suppose there exists a price index formula P_{12}^j that satisfies A1, A2 and A3. Suppose further that $p_{2i}^A/p_{1i}^A > p_{2i}^B/p_{1i}^B \ \forall i$. It follows from $A1$ that $P_{12}^A > P_{12}^B$. We now order the N commodities as follows:

$$\frac{p_{21}^A}{p_{11}^A} = \min_{i=1,\ldots,N} \left(\frac{p_{2i}^A}{p_{1i}^A}\right),$$
$$\frac{p_{2N}^B}{p_{1N}^B} = \max_{i=1,\ldots,N} \left(\frac{p_{2i}^B}{p_{1i}^B}\right).$$

Suppose also that $p_{21}^A/p_{11}^A < p_{2N}^B/p_{1N}^B$. Now it follows from $A2$ that

$$\lim_{s_{11}^A \to 1, s_{21}^A \to 1} P_{12}^A = \frac{p_{21}^A}{p_{11}^A} < \frac{p_{2N}^B}{p_{1N}^B} = \lim_{s_{1N}^B \to 1, s_{2N}^B \to 1} P_{12}^B.$$

Since $p_{ti}^A, p_{ti}^B, q_{ti}^A, q_{ti}^B > 0 \ \forall i, t$, the expenditure shares s_{11}^A, s_{21}^A, s_{1N}^B and s_{2N}^B must all be strictly less than one. However, it follows from the continuity axiom, $A3$, that by making the expenditure shares s_{11}^A, s_{21}^A, s_{1N}^B and s_{2N}^B sufficiently large, the price index formulae, P_{12}^A and P_{12}^B, can be brought arbitrarily close to the price relatives p_{21}^A/p_{11}^A and p_{2N}^B/p_{1N}^B, respectively. Thereby $A1$ is violated, contradicting the original assumption. *Q.E.D.*

4. Implications of the Theorem

(i) Laspeyres, Paasche, Vartia and Superlative Indexes

The Laspeyres price index formula can be expressed as a weighted arithmetic mean of the price relatives, with the weights given by the expenditure shares of period 1.

$$P_{12}^j = \sum_{i=1}^{N} s_{1i} \left(\frac{p_{2i}^j}{p_{1i}^j}\right) \tag{12}$$

Similarly, the Paasche price index formula can be expressed as a weighted harmonic mean of the price relatives, with the weights given by the expenditure shares of period 2.

$$P_{12}^{j} = \left[\sum_{i=1}^{N} s_{2i} \left(\frac{p_{2i}^{j}}{p_{1i}^{j}} \right)^{-1} \right]^{-1} \tag{13}$$

It is easily verified from (12), (13) and (8), respectively, that Laspeyres, Paasche and Vartia price indexes satisfy $A2$ and $A3$. Similarly, from (1), (2), (3) and (4) it is clear that, for $|r| < \infty$, P_{12}^{r} and $\tilde{P}_{12}^{r}$ also satisfy $A2$ and $A3$. It follows, therefore, from the impossibility theorem that these price index formulae all violate $A1$.

Using the limiting properties of mean value functions (see Hardy, Littlewood and Pólya, 1934), it can be shown that, in the limit as $|r|$ tends to infinity, the superlative price index depends only on the smallest and largest price relative.[4]

$$\lim_{r \to -\infty} P_{12}^{r,j} = \lim_{r \to +\infty} P_{12}^{r,j} = \sqrt{\min_{i=1,\ldots,N} \left(\frac{p_{2i}^{j}}{p_{1i}^{j}} \right) \max_{i=1,\ldots,N} \left(\frac{p_{2i}^{j}}{p_{1i}^{j}} \right)} \tag{14}$$

It follows immediately from (14) that $\lim_{|r| \to \infty} P_{12}^{r,j}$ satisfies $A1$ and $A3$, but violates $A2$.

(ii) Pure Price and Quantity Indexes

Although Laspeyres and Paasche price indexes satisfy $A2$, pure price indexes, in general, do not. Nevertheless, these indexes still violate $A1$. The reason for this becomes clear if the formula for a pure price index is expressed as a weighted arithmetic mean of the price relatives:

$$P_{12}^{j} = \sum_{i=1}^{N} s_{1Xi}^{j} \left(\frac{p_{2i}^{j}}{p_{1i}^{j}} \right), \tag{15}$$

where

$$s_{1Xi}^{j} = \frac{p_{1i}^{j} q_{Xi}^{j}}{\sum_{k=1}^{N} p_{1k}^{j} q_{Xk}^{j}}.$$

It follows immediately from (15) that pure price indexes satisfy the following condition:

$$\lim_{s_{1Xi}^{j} \to 1} P_{12}^{j} = \left(\frac{p_{2i}^{j}}{p_{1i}^{j}} \right) \quad \forall i. \tag{16}$$

[4] This result was derived by Allen and Diewert (1981, footnote 13).

Since the assumption that $s^j_{1Xi} \rightarrow 1$ for commodity i does not in itself impose any constraint on the price relatives, a similar example to the one used to prove the theorem can be used to show that, by satisfying the condition in (16), a pure price index must violate $A1$ for any allowable reference quantity vector $q_X \gg 0$.

Pure quantity indexes likewise fail the quantity index equivalent of $A1$. To see why it is useful to write the formula for a pure quantity index as follows:

$$Q^j_{12} = \sum_{i=1}^{N} s^j_{X1i} \left(\frac{q^j_{2i}}{q^j_{1i}} \right), \qquad (17)$$

where

$$s^j_{X1i} = \frac{p^j_{Xi} q^j_{1i}}{\sum_{k=1}^{N} p^j_{Xk} q^j_{1k}}.$$

It follows from (17) that pure quantity indexes satisfy the following condition:

$$\lim_{s^j_{X1i} \rightarrow 1} Q^j_{12} = \left(\frac{q^j_{2i}}{q^j_{1i}} \right) \quad \forall i. \qquad (18)$$

Again, it can be shown that, by satisfying this condition, a pure quantity index must violate the quantity index equivalent of $A1$ for any allowable reference price vector $p_X \gg 0$.

Even if regions A and B use the same reference quantity (price) vector, pure price (quantity) indexes will still violate $A1$ (or its quantity index equivalent). This is because the expenditure share vectors s^j_{1X} and s^j_{X1} will, in general, still differ across regions.

5. Conclusion

We have shown that all the main bilateral price index formulae, including all but one of the known superlative indexes, violate axiom $A1$. This result is rather disconcerting given that this axiom seems so fundamental. No consolation can be derived from the one superlative price index that satisfies $A1$. The exception is $\lim_{|r| \rightarrow \infty} P^r_{12}$. It is hard to defend using this price index formula since it depends only on the smallest and largest price relatives (see equation (14)). Even pure price and quantity indexes that use the same reference quantity or price vector for all regions fail $A1$.

The only way to ensure that a price index satisfies $A1$ is for it to depend only on the price vectors of the two periods being compared. Diewert (1995) refers to such indexes as *elementary* indexes. They are also sometimes referred to as *unweighted* indexes, since they do not weight the price relatives by their corresponding expenditure shares. It should be noted that $\lim_{|r|\to\infty} P_{12}^r$ is an example (admittedly a rather unusual one) of an elementary index. The literature on elementary price indexes is surveyed in Diewert (1995). However, it is hard to justify the use of elementary price indexes except in cases where expenditure shares are unavailable, as is the case at the most disaggregated level in the consumer price index (CPI). Therefore, desirable as the axiom $A1$ may be, the price that must be paid to satisfy it is too high.

An alternative approach might be to abandon price and quantity indexes in favor of price and quantity indicators. The former and latter decompose into price and quantity components a value *ratio* and value *difference*, respectively. The literature on price and quantity indicators was developed in the 1920s by Bennet and Montgomery, but was then largely ignored until the 1990s when it was revived by Diewert (1998) and Balk, Färe and Grosskopf (2000). Price and quantity indicators are particularly useful to accounting theory for the analysis of budgeting performance, and for consumer surplus analysis (see Diewert (1998)). Färe and Grosskopf (2000) argue, in the context of the efficiency and productivity analysis literature, that the Fox paradox can be avoided by using a *difference* rather than *ratio* measure of efficiency. However, in our context, indicators are no better than indexes. For example, consider a Laspeyres-type price indicator, denoted here by $\hat{P}_{12}$. It is related to a Laspeyres price index, denoted by P_{12}, as follows:

$$\hat{P}_{12} = \sum_{i=1}^{N} p_{2i}q_{1i} - \sum_{i=1}^{N} p_{1i}q_{1i} = \frac{\sum_{i=1}^{N} p_{2i}q_{1i}}{\sum_{i=1}^{N} p_{1i}q_{1i}} - 1 = P_{12} - 1. \tag{19}$$

It is clear from (19) that $\hat{P}_{12}$ will fail the indicator equivalent of $A1$. It remains to be seen whether all the main indicators fail the indicator equivalents of $A1$.

In conclusion, it is important that researchers and users should be aware of the fact that all the major price index formulae violate $A1$. Nevertheless, this does not mean that the whole literature on weighted index numbers is flawed. On closer inspection, it is not clear that we should even desire price indexes to satisfy $A1$. This is because not all price changes are equally important. A consumer is much more concerned about an increase in the price of a good that constitutes a large share of her total expenditure than over an increase in the price of a good that she rarely purchases (and when purchased constitutes a small share of her total expenditure). Once one accepts that price indexes should incorporate this fact, then it follows immediately that we cannot expect a price index to always satisfy $A1$.

References

Allen, R. C., and W. E. Diewert (1981), "Direct Versus Implicit Superlative Index Number Formulae," *Review of Economics and Statistics* 63(3), 430-435.

Armstrong, K. G. (2003), "A Restricted Domain Multilateral Test Approach to the Theory of International Comparisons," *International Economic Review*, forthcoming.

Balk, B. M. (1996), "A Comparison of Ten Methods for Multilateral International Price and Volume Comparison," *Journal of Official Statistics* 12, 199-222.

Balk, B. M. (1995), "Axiomatic Price Index Theory: A Survey," *International Statistical Review* 63, 69-93.

Balk, B. M., R. Färe and S. Grosskopf (2000), "Economic Price and Quantity Indicators," Paper presented at Economic Measurement Group Workshop, School of Economics, University of New South Wales, March 2001.

Diewert, W. E., (2002), "The Consumer Price Index and Index Number Purpose," *Journal of Economic and Social Measurement*, forthcoming.

Diewert, W. E. (1999), "Axiomatic and Economic Approaches to International Comparisons," in *International and Interarea Comparisons of Income, Output and Prices*, ed. by A. Heston and R. E. Lipsey. Chicago: University of Chicago Press, 13-87.

Diewert, W. E. (1998), "Index Number Theory using Differences instead of Ratios," Discussion Paper 98-10, Department of Economics, University of British Columbia.

Diewert, W. E., (1995), "Axiomatic and Economic Approaches to Elementary Price Indexes," Discussion Paper 95-01, Department of Economics, University of British Columbia.

Diewert, W. E. (1993), "The Early History of Price Index Research," in W. E. Diewert and A. O. Nakamura (ed.), *Essays in Index Number Theory, Volume 1*, 33-66. North-Holland: Amsterdam.

Diewert, W. E. (1978), "Superlative Index Numbers and Consistency in Aggregation," *Econometrica* 46, 883-900.

Diewert, W. E. (1976), "Exact and Superlative Index Numbers," *Journal of Econometrics* 4, 115-145.

Eichhorn, W. and J. Voeller (1976), *Theory of the Price Index*, Lecture Notes in Economics and Mathematical Systems, Vol. 140, Berlin: Springer-Verlag.

Färe, R. and S. Grosskopf (2000), "Outfoxing a Paradox," *Economics Letters* 69, 159-163.

Fisher, I. (1922), *The Making of Index Numbers*, Houghton-Mifflin, Boston.

Fox, K. J. (1999), "Efficiency at Different Levels of Aggregation: Public versus Private Sector Firms," *Economics Letters* 65, 173-176.

Hardy, G., J. E., Littlewood and G. Pólya (1934), *Inequalities*, (Cambridge: Cambridge University Press).

Hill, R. J. (2002), “Superlative Index Numbers: Not All of Them Are Super,” Mimeo.

Konüs, A. A. (1924), “The Problem of the True Index of the Cost of Living,” translated in *Econometrica 7*, (1939), 10-29.

Landefeld, J. S. and R. P. Parker (1997), “BEA’S Chain Indexes, Time Series, and Measures of Long-Term Economic Growth,” *Survey of Current Business* May, 58-68.

Sato, K. (1976), “The Ideal Log-Change Index Number,” *Review of Economics and Statistics* 58, 223-228.

Triplett, J. E., (1996), “The Importance of Using Superlative Index Numbers,” Paper prepared for CSO Meeting on Chain Indexes for GDP, London, 26 April, 1996.

van Veelen, M. (2002), “An Impossibility Theorem Concerning Multilateral International Comparison of Volumes,” *Econometrica* 70(1), 369-375.

Vartia, Y. O. (1976), “Ideal Log-Change Index Numbers,” *Scandinavian Journal of Statistics* 3, 121-126.

Vartia, Y. O. (1974), *Relative Changes and Economic Indices*, Licensiate Thesis in Statistics, University of Helsinki.