TABLO coding CES production function nests using a mapping

Ilan Salbe

22 September 2021

Contents

1 TABLO coding CES production function nests using a mapping 3
2 The Methodology 3
2.1 Derive an inputs-to-nests matrix equivalent to the mapping 4
2.2 Find the sub-products' shares and price 4
2.3 Find the CES equations 5
3 Where more than one production function is needed 5

Tables

Table 1 Mapping from inputs to sub-products 3
Table 2 Matrix equivalent to mapping 4

Figures

Figure 1 GTAP-E Production Function 7

1 TABLO coding CES production function nests using a mapping

Constant elasticity of substitution (CES) production function nests provide a flexible mechanism for modelling the substitution of inputs in response to price changes in those inputs. GEMPACK coding of multi-level nests isn't complex, however it is repetitive, time-consuming and the resulting nest structure is not easily deciphered. This note presents a simple and transparent method for coding production function nests. The method works well where one production function applies to all activities (industries). If more than one function is needed, for example one each for primary, manufacturing, administrative and 'other' activities, then some repetitive coding is still required. This repetition could be avoided if the equivalent of a matrix to vector mapping function could be constructed with GEMPACK. The method described here extends to constant elasticity of transformation (CET) nests.

At the heart of the coding methodology is the understanding that <u>any nesting structure</u> can be represented by a mapping from the set of *'all commodities, endowments and sub-products*[1]*'* to the set of '*all sub-products'*. The GTAP-E model production function nesting structure, as depicted in Figure 1 (on page 7), is used for illustration with Table 1 providing its mapping. In what follows (also in the attached sample TABLO code) the mapping function is named $INTONEST_i$.

Table 1 Mapping from inputs to sub-products

INPUTS	T0 Sub-Products NEST
COM_1-COM_{n-5}	TOP
crude oil	non-coal energy sub-product
gas	non-coal energy sub-product
petr. products	non-coal energy sub-product
coal	non-electric. energy sub-product
electricity	energy sub-product
Land	endowment-energy sub-product
Labour	endowment-energy sub-product
Capital	capital-energy sub-product
endowment energy sub-product (EES)	TOP
capital-energy sub-product (KES)	endowment-energy sub-product
energy sub-product (ES)	capital-energy sub-product
non-electric. energy sub-product (NEES)	energy sub-product
non-coal energy sub-product (NCES)	non-electric. energy sub-product

With a mapping established, all that remains are some relatively simple steps to derive input shares and the CES equations. The coding of these steps can be reused for different production function structures. The attached TABLO file contains a cut-down version of the code.

2 The Methodology

[1] A sub-product is the aggregate of the inputs that go into it

There is more than one way to bake cake and use leads to refinement. What follows is the current recipe.

2.1 Derive an inputs-to-nests matrix equivalent to the mapping

The matrix version of the mapping is used to derive the price of sub-products. The matrix takes the form:

$$\begin{aligned} NESTMATRIX_{n,i} &= 0 \; if \; input \; i \; isn't \; an \; input \; to \; sub-product \; n \\ &= 1 \; if \; input \; i \; is \; a \; direct \; input \; to \; sub-product \; n \\ &= \; k + \; 1 \; if \; input \; i \; is \; a \; indirect \; input, having \; being \; a \; direct \; or \; indirect \\ & \quad \; input \; to \; k \; sub-products \end{aligned}$$

The nest matrix for the nest of Figure 1 (or its equivalent in Table 1) is given in Table 2. The TABLO code for deriving $NESTMATRIX_{n,i}$ from the mapping $INTONEST_i$ appears in the attached.

Table 2 Matrix equivalent to mapping

Sub-product	COM1	COM2	....	COMn-5	Crude oil	gas	Petr. Products	coal	Electricity	Land	Labour	Capital	EES	KES	ES	NEES	NCES
EES					5	5	5	4	3	1	1	2		1	2	3	4
KES					4	4	4	3	2			1			1	2	3
ES					3	3	3	2	1							1	2
NEES					2	2	2	1									1
NCES					1	1	1										
TOP	1	1	1	1	6	6	6	5	4	2	2	3	1	2	3	4	5

2.2 Find the sub-products' shares and price

The total expenditure on sub-products can be discerned from the matrix's rows. Taking the ES sub-product as an example, the firm expenditure on the ES sub-product equals the sum of expenditure on each of the commodities and endowments that (directly or indirectly) make up ES, that is *crude oil*, *gas*, *petroleum products* and *electricity* (no endowments in this case). Expenditure is calculated in this manner for all sub-products. Expenditure shares on the direct-inputs of a sub-products ('1' in the matrix) are then the expenditure on those direct-inputs divided by the total expenditure on the sub-product. This leads to the following shares-formula and the equation for the price of sub-product:

$$\begin{aligned} NESTSHARES_{n,i,a,r} = 0 + if[&NESTMATRIX_{n,i} = 1, \\ & if(i \; in \; DEMD, VFP_{i,a,r)}) + \\ & if(i \; in \; NESTS, NESTSUM_{i,a,r})]/NESTSUMS(n,a,r) \\ ] \end{aligned}$$

$$\begin{aligned} pf_{n,a,r} = if\{&n <> top, \\ & [\textstyle\sum_{c \in COMM} NESTSHARES_{n,c,a,r} * (pfa_{c,a,r} - afa_{c,a,r})] + \\ & [\textstyle\sum_{e \in ENDW} NESTSHARES_{n,c,a,r} * (pfe_{e,a,r} - afa_{e,a,r})] + \\ & [\textstyle\sum_{nn \in NESTS} NESTSHARES_{n,c,a,r} * (pf_{e,a,r})] + \\ \} \; \mathbf{+} & \\ if\{&n <> top, \end{aligned}$$

$po_{a,r}$

}

$po_{a,r}$, being the output price of an activity product, is derived in a zero profits equation located in another (original) part of the code.

2.3 Find the CES equations

With $INTONEST_i$ mapping the set of inputs, 'ENDW+COMM+NESTS', to the set of sub-products 'NESTS', the CES equations for commodities, endowments and sub-products are respectively:

$$
\begin{aligned}
qfa_{i,a,r} = & -afa_{i,a,r} + \\
& if(INTONEST_i <> \text{top}, \\
& \quad qf_{INTONEST_i,a,r} - ELASNEST_{INTONEST_i,a,r} * \left[pfa_{i,a,r} - afa_{i,a,r} - pf_{INTONEST_i,a,r}\right] + \\
& if[INTONEST_i = top, \\
& \quad qo_{a,r} - ao_{a,r} - ELASNEST_{INTONEST_i,a,r} * [pfa_{i,a,r} - ao_{a,r} - afa_{i,a,r} - po_{a,r}] \\
& \qquad i \in COMM, a \epsilon ACTS, r \in REG
\end{aligned}
$$

$$
\begin{aligned}
qfe_{i,a,r} = & -afe_{i,a,r} + \\
& if(INTONEST_i <> \text{top}, \\
& \quad qf_{INTONEST_i,a,r} - ELASNEST_{INTONEST_i,a,r} * \left[pfe_{i,a,r} - afe_{i,a,r} - pf_{INTONEST_i,a,r}\right] + \\
& if[INTONEST_i = top, \\
& \quad qo_{a,r} - ao_{a,r} - ELASNEST_{INTONEST_i,a,r} * [pfe_{i,a,r} - afe_{i,a,r} - ao_{a,r} - po_{a,r}] \\
& \qquad i \in ENDW, a \epsilon ACTS, r \in REG
\end{aligned}
$$

$$
\begin{aligned}
qfe_{i,a,r} = & \\
& if(INTONEST_i <> \text{top}, \\
& \quad qf_{INTONEST_i,a,r} - ELASNEST_{INTONEST_i,a,r} * \left[pfe_{i,a,r} - afe_{i,a,r} - pf_{INTONEST_i,a,r}\right] + \\
& if[INTONEST_i = top, \\
& \quad qo_{a,r} - ao_{a,r} - ELASNEST_{INTONEST_i,a,r} * [pfe_{i,a,r} - ao_{a,r} - po_{a,r}] \\
& \qquad i \in NESTS, a \epsilon ACTS, r \in REG
\end{aligned}
$$

with $qo_{a,r}$ falling out of the model's zero profits and market clearing equations.

3 Where more than one production function is needed

The GTAP-E model has one production function which applies to all activities. Elasticities can vary across activities, so having a single production function is not overly restrictive, certainly for the sorts of applications for which GTAP-E is used. However, there may be applications where it is desirable that different industry groups have their own production functions.

In terms of augmenting the mapping depicted in Table 1, what is required is (or the equivalent of):

- a 'T0 Sub-Products NEST' column for each production function;
- a 'ACTS2PFN' mapping from the set 'ACTS' (industries) to the set 'NESTS (sub-products)'; and
- a matrix to vector mapping function '$INTONEST_{i,ACTSPFN_a}$' that would place any combination of 'input i and activity a' into the appropriate production function sub-product.

GEMPACK only allows vector to vector mapping. Section 10.1.7 (Multidimensional sets and tuples) of the GEMPACK manual indicates that equivalent functionality is generally available using GEMPACK multi-dimensional sets and projection mappings. Some experiments were carried out to replicate

$\boldsymbol{INTONEST_{i,ACTS2PFN_a}}$ functionality using these but without success? A work around would be simply to duplicate the code that works for one production for each additional production function – messy, though still far less messy than existing coding practices. Other possibilities may exist, including an entirely different methodology.

```
                                              qo(j,r)  [ao(j,r)]     TOTAL OUTPUT NEST
                                              /\
                                             /  \  <----- CES
                                            /    \
                                           /      \
                                          /        \
                            endowment-energy       non-energy inputs
           Val.-add.-en. nest   /| \                            /\
               CES ----->      / |  \                          /  \  <------ CES
                              /  |   \                        /    \
                             /   |    \                      /      \
                            /    |     \                    /        \
                         Land  Labor  K-E subproduct      Domestic     Foreign
                                        /\
                                       /  \
                                      /    \              Capital-energy nest
                                     /      \   <--------- CES (subst. el. ELFKEN)
                                    /        \
                                   /          \
                                  /            \
                                 /              \
                                K              energy subproduct
                                                      /\        Energy nest
                                                     /  \ <--- CES (subst. el. ELFENY)
                                                    /    \
                                                   /      \
                                                  /        \
                                                 /          \
                                             electricity  non-electr. energy
                                                                  /\
               Non-electricity energy nest                       /  \
               CES (subst. el. ELFNELY)  ---------------->      /    \
                                                               /      \
                                                              /        \
                                                   non-coal energy   coal
                                                         /\
               Non-coal energy nest    ------------>    /  \
               CES (subst. el. ELFNCOAL)               /    \
                                                      /      \
                                                     /        \
                                                    /          \
                                          crude oil, gas, petr. products
```

Figure 1 GTAP-E Production Function