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Derivation of offer functions, equation (T1)

by

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(We introduce changes in tastes, B_i and B_{ij}, and changes in efficiency, A_i and A_{ij})

Consider the following optimization problem:

Choose L_{ij}, i = 1, …, m; j = 1, 2

to maximize $$\mathrm{CES}_i\left[B_i \mathrm{CES}_j\left(B_{ij}W_{ij}L_{ij}\right)\right] \tag{A1}$$

subject to $$\sum_i A_i \sum_j A_{ij}L_{ij} = N \quad . \tag{A2}$$

This problem relates in an obvious way to problem (2.1) to (2.3) specified for the behaviour of people in group q (we suppress the q identifier). L_{ij} is the offer by people in group q to activity i of status j. For employment activities (jobs) j refers to award and non-award. For non-employment activities (short-term and long-term unemployment), we do not require a j subscript. However, it is easier to do the algebra assuming that there is award and non-award unemployment. Subsequently, we can assume that group q's preferences are such that they choose only one type of unemployment, say award unemployment.

Problem (A1) – (A2) can be solved in two stages.

Stage 1. For each i we choose L_{ij}, j = 1, 2

to maximize $$\mathrm{CES}_j\left(B_{ij}W_{ij}L_{ij}\right) = \left[\sum_j\left(B_{ij}W_{ij}L_{ij}\right)^{-\rho}\right]^{-1/\rho} \tag{A3}$$

subject to $$\sum_j A_{ij}L_{ij} = L_i \quad . \tag{A4}$$

ρ is a parameter of the CES utility function and L_i is the sum over statuses of the optimal offers (adjusted for efficiency) by q to activity i. B_{ij} allows for changes in tastes: if B_{ij} increases, then group q derives increased utility from a dollar earned in activity ij. A_{ij} allows for changes in efficiency: if A_{ij} decreases, then group q can deliver an increased number of units of labour of type ij without reducing the number of units that it delivers to any other activity.

We rewrite (A3) – (A4) as: choose L_{ij}

$$\text{to maximize } CES_j\left(\frac{B_{ij}}{A_{ij}} W_{ij} A_{ij} L_{ij}\right) = \left[\sum\nolimits_j \left(\frac{B_{ij}}{A_{ij}} W_{ij} (A_{ij} L_{ij})\right)^{-\rho}\right]^{-1/\rho} \qquad (A5)$$

$$\text{subject to } \sum_j (A_{ij} L_{ij}) = L_i \quad . \qquad (A6)$$

Applying Lagrangian methods to (A3) – (A4) gives

$$A_{ij} L_{ij} = L_i * \left(\frac{B_{ij}}{A_{ij}} W_{ij}\right)^{-\rho/(1+\rho)} \Big/ \sum\nolimits_k \left(\frac{B_{ik}}{A_{ik}} W_{ik}\right)^{-\rho/(1+\rho)} \text{, for } i = 1, \ldots, m;\ j = 1, 2 \ . \qquad (A7)$$

Stage 2. We choose L_i, i = 1, …, m

$$\text{to maximize } CES_i\left(B_i W_i L_i\right) = \left[\sum\nolimits_i \left(\frac{B_i}{A_i} W_i (A_i L_i)\right)^{-\gamma}\right]^{-1/\gamma} \qquad (A8)$$

$$\text{subject to } \sum_i A_i L_i = N \quad . \qquad (A9)$$

W_i is the wage that can be earned by people in group q from activity i. It is the value of the objective function in problem (A5) – (A6) when L_i = 1. That is,

$$W_i = \left[\sum\nolimits_k \left(\frac{B_{ik}}{A_{ik}} W_{ik}\right)^{-\rho/(1+\rho)}\right]^{-(1+\rho)/\rho} \text{, for } i = 1, \ldots, m \ . \qquad (A10)$$

(A10) can be derived by setting L_i = 1 in (A7) and the substituting into (A5).

Problem (A8) – (A9) gives

$$A_i L_i = N * \left(\frac{B_i}{A_i} W_i\right\}^{-\gamma/(1+\gamma)} \Big/ \sum\nolimits_h \left(\frac{B_h}{A_h} W_h\right\}^{-\gamma/(1+\gamma)} \text{, for } i = 1, \ldots, m \ . \qquad (A11)$$

Together, (A7), (A10) and (A11) give offer functions of the form (T1) in our AER paper of 2003.

Comments

(1) Calibration We assume that in the initial situation all of the A_{ij}s and A_is have the value 1. Units can be defined so that all wage rates are initially one. Then, as mentioned in the text, for given values of the L_{ij}'s and the substitution parameters ρ and γ, we can generate the initial values for all the B_{ij}s. Notice that the B's can be normalized to initially satisfy $\sum_j B_{ij} = 1$ and $B_i = 1$.

(2) Interpretation of substitution parameters This is easier in the context of percentage change equations than levels equations. Percentage change versions of (A7), (A10) and (A11) are

$$a_{ij} + l_{ij} = l_i + (\sigma - 1) * \left[b_{ij} - a_{ij} + w_{ij} - \sum_k (b_{ij} - a_{ij} + w_{ij}) * S_{ik}\right], \text{ i= 1, …, m, j = 1, 2,} \quad \text{(A12)}$$

$$w_i = \sum_k (b_{ik} - a_{ik} + w_{ik}) * S_{ik}, \text{ i= 1, …, m,} \quad \text{(A13)}$$

and

$$a_i + l_i = n + (\phi - 1) * \left[(b_i - a_i + w_i) - \sum_h (b_h - a_h + w_h) * S_h\right], \text{ i= 1, …, m.} \quad \text{(A14)}$$

In these equations:

S_{ik} is the share of $A_{ij}L_{ij}$ in L_i;
S_h is the share of $A_h L_h$ in N;
σ = 1/(1+ρ) is the elasticity of substitution for group q between a dollar earned in an award activity and in the corresponding non-award activity; and
ϕ = 1/(1+γ) is the elasticity of substitution for group q between a dollar earned in any activity and in any other activity.

Because we think that workers are not very concerned about whether their jobs are award or non-award, we adopted a high value, 5, for σ. On the other hand, we assume that workers are cautious about moving between activities. Consequently, we adopted a lower value for ϕ than for σ. In our main simulation ϕ = 3. In a sensitivity simulation ϕ = 2.1.

(3) Labour-supply elasticity Ignore the occupation dimension.

An employed worker has two choices: to supply to an employment activity or to supply to short-term unemployment. The elasticity of labour supply from employed workers, that is the percentage change in offers from these workers to employment for a one per cent increase in the wage from employment relative to the wage from unemployment is $(\phi - 1) * S^{emp}_{unemp}$. For employed workers we assume that S^{emp}_{unemp} is small, 0.005. Thus with ϕ = 3, the elasticity of labour supply for employed workers is 0.01.

A short-term unemployed worker has two choices: to supply to an employment activity or to supply to long-term unemployment. The elasticity of labour supply from this group of workers, that is their percentage change in offers to employment for a one per cent increase in the wage from employment relative to the wage from unemployment is $(\phi - 1) * S_{unemp}^{short}$. For short-term unemployed workers we assume that S_{unemp}^{short} is 0.25. Thus with ϕ = 3, the elasticity of labour supply for short-term unemployed workers is 0.5.

A long-term unemployed worker has two choices: to supply to an employment activity or to supply to long-term unemployment. The elasticity of labour supply from this group of workers, that is their percentage change in offers to employment for a one per cent increase in the wage from employment relative to the wage from unemployment is $(\phi - 1) * S_{unemp}^{long}$. For long-term unemployed workers we assume that S_{unemp}^{long} is 0.5. Thus with ϕ = 3, the elasticity of labour supply for long-term unemployed workers is 1.0.

The elasticity of supply of labour (η) for the entire labour force is a weighted average of the supply elasticities for the employed, the short-term unemployed and the long-term unemployed. Thus, in our central simulation

$$\eta = (100/117)*0.005 + (6/117)*0.5 + (11/117)*1.0 = 0.124 \quad .$$

(We assume that the shares of the employed and the short- and long-term unemployed in the labour force are 100/117, 6/117 and 11/117).

In the sensitivity simulation where we assume that ϕ = 2.1,

$$\eta = (100/117)*0.0028 + (6/117)*0.275 + (11/117)*0.55 = 0.068 \; .$$