

# LABOUR AND MIGRATION POLICY IN SOUTH AFRICA

by

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*With special thanks to*  
*Jessika Andreina*



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# CONTENTS

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TABLE OF CONTENTS .....	ii
LIST OF TABLES .....	iv
LIST OF FIGURES .....	v
SUMMARY .....	vi
STATEMENT OF AUTHORSHIP .....	viii
ACKNOWLEDGMENTS .....	ix
 CHAPTER 1	 INTRODUCTION
1.1	Labour and Migration ..... 1   2
1.4	Outline of Study ..... 1   5
1.3	Aim and Contribution of Study ..... 1   7
1.4	Literature Review ..... 1   9
 CHAPTER 2	 MODEL THEORY
2.1	Introduction ..... 2   2
2.2	Theoretical Structure of the ZAR-M Model ..... 2   3
2.3	Capital Accumulation Mechanism in ZAR-M ..... 2   15
2.4	Labour Market Mechanism in ZAR-M ..... 2   20
2.5	Concluding Remarks ..... 2   41
 CHAPTER 3	 MODEL DATABASE
3.1	Introduction ..... 3   2
3.2	Input-Output Database ..... 3   3
3.3	Labour Market Database ..... 3   8
3.4	Concluding Remarks ..... 3   24
 CHAPTER 4	 MODEL CLOSURES
4.1	Introduction ..... 4   2
4.2	A Stylized CGE Model ..... 4   3
4.3	Developing the Decomposition Closure ..... 4   7
4.4	Developing the Forecast Closure ..... 4   11
4.5	Developing the Policy Closure ..... 4   20
4.6	Concluding Remarks ..... 4   24



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CHAPTER 5	SIMULATIONS	
5.1	Introduction .....	5   2
5.2	Baseline Forecast .....	5   4
5.3	Policy Simulations .....	5   21
5.4	Concluding Remarks .....	5   48
CHAPTER 6	CONCLUSION	
6.1	Overview of Study .....	6   2
6.2	Main Findings .....	6   3
6.3	Future Research .....	6   4
BIBLIOGRAPHY AND REFERENCES .....		7   2
APPENDIX A	THE PERCENTAGE-CHANGE APPROACH	
A.1	Introduction .....	A   2
A.2	Linearisation Error .....	A   4
A.3	The Cobb-Douglas Function .....	A   9
A.4	The CES Function .....	A   13
A.5	The Klein-Rubin Function .....	A   18
APPENDIX B	MODEL CLOSURES	
B.1	Exogenous Variables in the Decomposition Closure .....	B   2
B.2	Swap Statements to Generate the Forecast Closure .....	B   4
B.3	Exogenous Variables in the Forecast Closure .....	B   5
B.4	Swap Statements to Generate the Policy Closure .....	B   6
B.5	Exogenous Variables in the Policy Closure .....	B   7
APPENDIX C	SIMULATION RESULTS .....	C   2

# LIST OF TABLES

---

TABLE 2A	<i>BOTE-M: Stylized Representation of ZAR-M</i>
TABLE 2B	<i>Description of Variables in BOTE-M</i>
TABLE 2C	<i>Capital Accumulation Mechanism</i>
TABLE 2D	<i>Capital-Supply Function</i>
TABLE 2E	<i>Notation in the Exposition of the Capital Accumulation Mechanism</i>
TABLE 2F	<i>Actual and Expected Rates of Return</i>
TABLE 2G	<i>Labour Market Mechanism</i>
TABLE 2H	<i>Notation in the Exposition of the Labour Market Mechanism</i>
TABLE 2I	<i>Flows from Start-of-Year Categories to End-of-Year Activities</i>
TABLE 3A	<i>Extended Labour Force with Legal Status by Labour Market Function and Industry</i>
TABLE 3B	<i>Extended Labour Force with Illegal Status by Labour Market Function and Industry</i>
TABLE 3C	<i>Legal Labour Flows from Start-of-Year Categories to End-of-Year Activities</i>
TABLE 3D	<i>Illegal Labour Flows from Start-of-Year Categories to End-of-Year Activities</i>
TABLE 4A	<i>A Stylized CGE Model</i>
TABLE 4B	<i>Marginal Products of Capital and Labour</i>
TABLE 4C	<i>Decomposition Closure for the Stylized Model</i>
TABLE 4D	<i>Capital Accumulation Mechanism in ZAR-M</i>
TABLE 4E	<i>Swap Statements to Produce Forecast Closure (Step 1)</i>
TABLE 4F	<i>Swap Statements to Produce Forecast Closure (Step 2)</i>
TABLE 4G	<i>Swap Statements to Produce Forecast Closure (Step 3)</i>
TABLE 4H	<i>Forecast Closure for the Stylized Model</i>
TABLE 4I	<i>Swap Statements to Produce Policy Closure</i>
TABLE 4J	<i>Policy Closure for the Stylized Model</i>
TABLE 5A	<i>Exogenously Forecast Variables</i>
TABLE 5B	<i>Baseline Forecast for Selected Macroeconomic Variables</i>
TABLE 5C	<i>Baseline Forecast for Selected Labour Market Variables (Stocks)</i>
TABLE 5D	<i>Baseline Forecast for Selected Labour Market Variables (Flows)</i>

# LIST OF FIGURES

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FIGURE 2A *Structure of Production for a Representative Industry*

FIGURE 2B *Expected Rates of Return*

FIGURE 2C *Labour Market Dynamics*

FIGURE 3A *Stylized Representation of a CGE Model Database*

FIGURE 4A *Demand and Supply of Exports*

FIGURE 5A *Employment of Illegal Workers (Scenario 1)*

FIGURE 5B *Flows of Illegal Migrants (Scenario 1)*

FIGURE 5C *GDP, Capital and Labour (Scenario 1)*

FIGURE 5D *Expenditure Aggregates (Scenario 1)*

FIGURE 5E *Selected Industry Outputs (Scenario 1)*

FIGURE 5F *Employment of Legal Workers (Scenario 1)*

FIGURE 5G *Employment and Real Wages (Scenario 1)*

FIGURE 5I *Flows of Illegal Migrants (Scenario 2)*

FIGURE 5J *GDP, Capital and Labour (Scenario 2)*

FIGURE 5K *Expenditure Aggregates (Scenario 2)*

FIGURE 5L *Selected Industry Outputs (Scenario 2)*

FIGURE 5M *Employment of Legal Workers (Scenario 2)*

FIGURE 5N *Employment and Real Wages (Scenario 2)*

FIGURE 5P *Sensitivity Analysis*

# SUMMARY

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This thesis concerns labour and migration in South Africa. We use a dynamic computable general equilibrium model to analyse the effects of a policy-induced cut to illegal immigration on the local economy. We run the same policy simulation under two distinct modelling scenarios. The first scenario portrays typical labour market conditions and behaviour associated with economies that tend to operate at or near full employment. Simulation results under *Scenario 1* indicate that a cut in low-skilled immigration would detract from the long-term welfare of local workers by generating a deterioration in the occupation-mix of their employment. With less low-skilled immigrants, native workers would slip down the skills and earnings ladder. This finding supports modelling evidence for the United States presented in Dixon *et al* (2011).

However, the South African labour market can hardly be described as ‘typical’. The impact of high unemployment and a legal minimum wage at the lower end of South Africa’s labour market stands in contrast to the economy implicitly described under the assumptions of *Scenario 1*. The second scenario recognises this contradiction between our modelling assumptions in *Scenario 1* and available evidence for the South African labour market. A new wage adjustment process assumed for *Scenario 2* accounts for the vast surplus of low-skilled legal workers available in the local market.

Simulation results under *Scenario 2* indicate that a reduction in illegal immigration would benefit legal residents. The favourable result in *Scenario 2* relative to that in *Scenario 1* is driven by two factors. The first is a much larger gain in legal employment at the lower end of the market. With illegal labour becoming scarcer, employers shift some of their demand for labour towards legal workers. Under *Scenario 2*, very little wage pressure arises from this increased demand for legal workers, resulting in a significant upsurge in their employment levels relative to that in *Scenario 1*.

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The second factor is that the occupation-mix effect is considerably less negative in *Scenario 2* than in *Scenario 1*. In the absence of wage increases in low-skilled occupations, *Scenario 1* does not generate a shift in legal work preferences towards these occupations. By presenting both modelling scenarios we are able to better understand and explain our simulation results. We are also able to clearly contrast our set of results against those in the Dixon *et al* (2011) study of the United States, which used a similar methodology. The importance of applying suitable labour market mechanisms when modelling the South African economy is highlighted.

Two main contributions emerge from this thesis. The first relates to the policy simulations. Detailed analysis of the impact of reduced illegal immigration on the South African economy provides policymakers and researchers with fresh evidence based on a state-of-the-art methodology. The second relates to the economic model. In developing the theory and database of the model used in this study we establish a flexible analytical framework to evaluate many other topical issues in South Africa. The detailed and economy-wide nature of the model is well-suited to not only conducting policy analysis, but also forecasting and historical analysis.

# STATEMENT OF AUTHORSHIP

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This thesis contains no material that has been accepted for the award of any other degree or diploma in any university or equivalent institution, and to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

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Heinrich R. Bohlmann

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*Soli Deo Gloria*



# CHAPTER 1

## INTRODUCTION

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## 1.1 Labour and Migration

More people live outside their country of birth today than at any time in history (UNDESA, 2009a; UNDP, 2009). The economist, John Kenneth Galbraith famously described migration as ‘the oldest action against poverty’ (Galbraith, 1979). Indeed, in both theory and practice, economic factors have formed the cornerstone of migration decisions (Borjas, 1994; UNDESA, 2004; Adepoju, 2006). As a mobile factor of production, labour is always seeking to move to where it is best employed. However, various constraints prevent this from occurring on a global scale. Virtually every government in the world today has policies and legislation in place that limits the flow of labour, especially lower-skilled labour, into their country (UNDESA, 2007). For many countries the impacts of such policies are far reaching, but as of yet, not well understood or analysed in a general equilibrium environment.

The economic impact of migration varies substantially according to time, place and economic conditions (Borjas, 1994). In the wake of the 2008 Global Financial Crisis, the total number of unemployed persons worldwide rose sharply (IMF, 2010; UNDESA, 2010). Migration patterns too were altered in many countries (PHC, 2010). To date, economic and labour market recovery has been slow and is forecast to remain sluggish for some time (IMF, 2011). The fragile recovery process following the Financial Crisis has also been dealt further blows as various natural disasters across the globe and recent incidents of political instability across the Middle East and North Africa have caused more distress and uncertainty.

For the working age population of any country the issue of employment is viewed in a most serious light. The bleak job market prospects that have faced many workers during this period have once again placed the spotlight firmly on immigration policy and the issue of illegal immigration.<sup>1</sup> Inevitably, unemployed local residents at the lower end of the market often point the finger at illegal migrants as the reason for their misfortune. Many countries,

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<sup>1</sup> Throughout this study the term ‘illegal’ will be used to describe any form of unauthorised or undocumented immigration.

especially those who receive high numbers of immigrants, have reviewed their migration policy and border protection measures as a result. In only a small number of cases have detailed economic analysis been undertaken to properly understand the impact of such proposed changes to their labour market or economy in general. Landmark studies by Dixon & Rimmer (2009) and Dixon *et al* (2011) are some of the finest works in this regard. In these studies the authors use a large-scale computable general equilibrium (CGE) model to measure the economic benefits of immigration reform and quantify the effects of a policy-induced cut to illegal immigration in the United States, respectively. A more in-depth look at these studies is contained in the literature review section.

In few regions across the world is migration policy of such critical importance as Southern Africa. The southern tip of the African continent hosts a group of countries with vastly differing levels of economic fortune and political stability. These circumstances have contributed greatly to the unabated flow of migrants, both legal and illegal, witnessed throughout the region. As may be expected, South Africa's economic strength and strategic location in the region has made it an attractive option for potential migrants.

Despite being the region's most vibrant economy, labour market problems have continued to blight South Africa's own progress and development. High structural unemployment, no doubt a legacy of Apartheid, has often been cited as the country's most pressing issue. Furthermore, strict immigration policies and criteria have limited the number of legal entrants, but done little to prevent an influx of illegal immigrants over the years. Combined with other issues such as skilled emigration, the labour market challenges facing the South African government are clear to see.

This study focuses on illegal immigration to South Africa and the impact of successfully implementing policy changes to reduce the inflow of such migrants. Recognising the benefits of the CGE approach, we use the same methodology as in the above mentioned Dixon & Rimmer studies to measure the potential effects of this policy change on the local economy. In conducting this exercise, we introduce the ZAR-M economic model of South

Africa. Based on the MONASH model developed by Dixon & Rimmer (2002), ZAR-M is a large-scale and detailed CGE model of the South African economy. The model's core database for the base year 2006 is documented in Bohlmann (2010). These references provide a full description of the model's theoretical structure and basic data elements. Readers needn't have an intimate knowledge of CGE modelling though. The stylized 'back-of-the-envelope' model presented in the following chapter combined with our description of the ZAR-M labour market theory and data is sufficient for understanding all the relevant CGE mechanisms in this study. In the following section we provide more insight to the problem of illegal immigration from a South African perspective. The remainder of this chapter briefly outlines the contributions of this thesis and reviews some of the most relevant literature.

### *South African context*

The economic landscape in South Africa is as diverse as its people. The Rainbow Nation is the largest economy on the African continent. Abundant supplies of natural resources combined with well-developed infrastructure in its major cities have been outstanding features of the economy for many decades. It produces a wide variety of goods and services for both the domestic and export markets. Since the first democratic elections in 1994 economic growth has been strong. This is largely attributable to improvements in total factor productivity and increased openness to trade and capital flows (Du Plessis & Smit, 2007). Disciplined monetary and fiscal policy has also been complementary in creating a suitable environment for growth and development. However, all is not well. Many socio-economic problems still plague the country. Foremost is overall unemployment which has remained above 30 per cent contributing to widespread poverty and inequality (OECD, 2008; StatsSA, 2009).<sup>2</sup>

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<sup>2</sup> Overall unemployment here refers to the expanded definition of unemployment which includes both the officially unemployed and discouraged work-seekers.

Unfortunately, many other African countries face even more trying circumstances. Relatively poor economic and political conditions persist in some of South Africa's closest neighbours (UNECA, 2010). Workers from these countries often apply to enter South Africa in search of a better life. As explained by the Department of Home Affairs, South Africa can only accommodate a certain number of immigrants. With a vast reserve of unskilled and semi-skilled labour, the government's position is that no one in these lower-skill categories will normally be accepted as an immigrant worker in South Africa. Motivated by the gap in potential earnings and relative employment conditions, many desperate workers who fail to meet legal immigration requirements continue to look for jobs in South Africa. This has led to large flows of illegal immigrants into South Africa.

The issue of illegal immigration in South Africa has been at the forefront of social and political debate for many years. Xenophobic attacks on foreigners since the economic downturn in 2008 have highlighted both the migrants' vulnerability in society and the frustration of local residents in being unable to find work (M&G Online, 2010). Poor service delivery and policy implementation has also come under the spotlight. Reducing the flow of illegal immigrants has been a stated policy objective of the South African government for a number of years (UNDESA, 2007). The Immigration Amendment Bill, recently passed by South Africa's National Assembly, appears to further deepen the government's stance on the matter. Given the intricate nature of the problem, finding a way to assess both the anticipated and unanticipated effects of successfully implementing such a policy is important. As a response, this thesis aims to inform policy discussions by providing an analytical framework and detailed simulation of the economic impacts of a reduction in illegal immigration to South Africa.

## **1.2 Outline of Study**

This study contains six chapters and three appendices. The current chapter introduces the topic under investigation, details the aim and contribution of the study and briefly reviews

some of the most recent literature relevant to the study of migration. The final chapter summarises the study's findings and provides some concluding remarks. Chapters 2 to 5 are the core chapters detailing the various modelling aspects of this study on labour and migration in South Africa. Appendices A to C provide information supplemental to specific sections or topics in the main chapters.

Chapter 2 documents the ZAR-M theoretical specification. The first sections give a general overview of the CGE methodology and ZAR-M model's theoretical structure. To assist readers in understanding the main macroeconomic relationships and basic functioning of the model, a stylized representation of ZAR-M is then discussed. This is followed by detailed descriptions of the model's capital and labour market mechanisms. The labour market mechanism is of particular importance in this study. All the labour market equations are listed and explained in the final sections of this chapter.

Chapter 3 documents the ZAR-M database. After introducing the general features of the core input-output database, we provide a summary of the most important data elements describing the macro economy in the base year 2006. This is followed by a section detailing the model's labour market data. The ZAR-M labour database contains a high level of detail, allowing workers to be modelled according to labour market function, birthplace and legal status. Particular attention is paid to the initial flow of labour from start-of-year categories to end-of-year activities.

Chapter 4 describes the standard model closures available to MONASH-style dynamic CGE modellers. Due to the large number of variables and equations in the full ZAR-M model, a stylized CGE model is used to explain the nature of each model closure. The long-run decomposition, and the necessary swaps required to produce the baseline forecast and policy simulation closures, are explained.



Chapter 5 contains all the modelling results and policy analysis relevant to this study. We first simulate a plausible baseline forecast for the South African economy up to 2020. After motivating the policy shocks, we run the policy simulation under two different modelling scenarios. The results of the two scenarios are then compared and contrasted. After a comprehensive analysis and explanation of the simulation results, some policy implications and concluding remarks round off the chapter.

### 1.3 Aim and Contribution of Study

The aim of this study is to provide policymakers with a detailed general equilibrium analysis of the impact of immigration reform on the South African economy. More specifically, we investigate the impact of a policy-induced cut to employment of illegal immigrants in South Africa. This is achieved by simulating a reduction in the preferences of foreign-born workers with illegal status for moving to and earning money in South Africa. A change in such supply-side preferences may be brought on in a number of ways. The successful implementation of policies that increase border security around South Africa or improve economic and political stability in neighbouring countries are considered here. The design and calibration of the policy simulation is discussed in the early part of Chapter 5.

We evaluate the economic consequences of this policy-induced cut to illegal immigration using ZAR-M, a dynamic computable general equilibrium (CGE) model of South Africa. In achieving this outcome, a number of noteworthy contributions are made. Starting from a standard MONASH-style dynamic CGE model specification described in Dixon & Rimmer (2002), ZAR-M is produced by adding and altering numerous equations in the model code. These changes allow us to produce an economic model more reflective of the South African environment. Following Dixon & Rimmer (2003) and Dixon *et al* (2011), we also specify equations facilitating the flow of migrants, distinguished by birthplace and legal status, in the local labour market. The model theory is discussed in Chapter 2.

Our next contribution is to construct a suitable database for South Africa that includes a detailed treatment of labour and migration flows in the model's base year. To be validated and considered useful, theories need be combined with data in order to generate results. Our database makes it possible to quantitatively analyse the policy's impact within the model's general equilibrium environment. The ZAR-M model's theoretical specification and database also builds a framework allowing us to investigate related labour and migration issues in South Africa such as skilled emigration. The model database is discussed in Chapter 3.

Our final contribution is the actual policy simulation and interpretation of results. This provides policymakers with new evidence and analysis to consider, based on a state-of-the-art methodology. After developing suitable model closures and constructing a plausible baseline forecast, we carefully interpret the economic impacts of the policy-induced cut to illegal immigration as deviations away from the unperturbed baseline. In the process, we develop two distinctive modelling scenarios for the policy simulation. This allows us to better understand and explain the role of illegal migrants, as well as the policy's impact, in the local labour market. The simulations are discussed and analysed in Chapter 5.

As the first study utilising a dynamic CGE model – specifically tailored to include detailed migration flows – to address the issue of illegal immigration in South Africa, we hope that interested readers will find our analysis both informative and thorough. Our attempt at modelling this controversial policy issue purely aims to provide fresh evidence and analysis to policymakers. We do not aim to pass any moral judgment or question the plight of distressed workers who seek employment opportunities illegally. Legitimate asylum seekers are also not considered here. Taking these points into consideration, we believe that this study has the potential to make a valuable contribution to the migration literature in South Africa. In the following section we briefly review some of the most recent literature relevant to our study of labour and migration.

## 1.4 Literature Review

A large literature exists on theories explaining international migration and related issues such as remittances and human development (Massey *et al*, 1993; Borjas, 1994; UNDESA, 2004; Page & Plaza, 2006, UNDP, 2009). There is also a growing literature of empirical studies measuring issues such as assimilation, selection and earnings of immigrants, changes in cohort quality, the general impact and performance of migrants in the labour market and immigration reform (Borjas, 2001; Borjas, 2003; Borjas *et al*, 2008; Chiswick & Miller, 2008; Chortareas *et al*, 2008; Dustmann *et al*, 2008; Ruhs, 2008; Dixon & Rimmer, 2009; Hanson, 2009; Nickell, 2009; Orrenius & Zavodny, 2009; Dixon *et al*, 2011; Ottaviano & Peri, 2011). Of further importance to this study is literature that has developed a suitable methodology for measuring the economy-wide impact of changes to migration policy. In this regard, we found the CGE approach used in Dixon *et al* (2011) the most compelling. For the purpose of this study, we will restrict our literature review to issues of illegal immigration and methodology.

Studies on illegal immigration in South Africa have been severely hampered by a lack of quality data. Even more than a decade ago, estimates of the number of illegal migrants in South Africa ranged between 2 and 5 million (Solomon, 2000). Statistics South Africa only provides data on documented migration. In 2009, the South African government revealed to parliament that they simply do not know how many illegal immigrants were residing in the country. They were also unable to provide any current estimate of the number of illegal or undocumented migrants, stating that ‘obtaining such a figure was difficult’. Their position is understandable, as producing a reliable estimate would indeed not only be difficult, but also very costly. Based on deportation numbers and general observation, recent estimates from agencies such as the South African Police Service suggest the total number of illegals may even exceed 5 million.

Reliable data do exist on the number of asylum seekers though. These figures help provide some insight into factors driving illegal immigration to South Africa. In 2009, South Africa was the country that received the largest number of asylum applications in the world. As a clear indication of the increasing turmoil in some of its neighbouring countries, asylum applications jumped from 28,000 in 2005 to a staggering 220,000 in 2009 (UNHCR, 2010). In 2008, the South African government granted official refugee status to 10,000 out of a reported 110,000 applicants. By the end of 2009 local NGOs estimated a backlog of around 400,000 asylum applications. If we were to assume around one in three unsuccessful asylum seekers were to escape deportation and remain in the country illegally, this would equate to a substantial inflow of illegal immigrants from this group alone. By combining all available information in the literature as well as various reports in the popular media, we conservatively estimated the number of illegal migrants active in the local labour market in 2006 at just above 2 million workers. This number excludes those outside the working age population and the not economically active. As noted in the outline, more details concerning the data used in this study can be found in Chapter 3.

In contrast to the lack of data on the number of illegal migrants in South Africa, statistical agencies in countries such as the United States or United Kingdom have regularly provided such estimates. Adding to their comprehensive datasets on documented migration, these estimates have allowed researchers to investigate migration related topics in far greater depth. Most empirical studies concerning immigration have used econometric techniques to measure various labour market impacts and trends. Some interesting and at times controversial analysis has emerged from these studies. Much of the current disagreement between academics relates to the impact of immigrants on the labour market prospects of natives. Of perhaps greater significance is the criticism against the partial equilibrium approach found in many econometric models that attempt to measure the impact of migrants on the native economy. As noted by authors of these models themselves, a general equilibrium approach would be a more appropriate way to measure the impact of migration on both labour and capital markets whilst capturing any terms-of-trade effects that may arise.

For the United States, recent studies by Borjas (2003), Hanson (2006), Massey (2007), Borjas *et al* (2008), Dixon & Rimmer (2009), Dixon *et al* (2011) and Ottaviano & Peri (2011) have made notable contributions to the immigration literature. These studies also highlight the lack of consensus that remains in the literature. The Borjas camp has constantly found the impact of immigration to have adverse effects on native employment opportunities and wage levels (Borjas, 2003; Borjas *et al*, 2008). Their disagreement with the findings in, for example, Ottaviano & Peri (2011) stem mainly from different interpretations of the substitutability between immigrant and natives workers. Whereas the Borjas studies doesn't reject the hypothesis that immigrants and natives are perfect substitutes, the Ottaviano & Peri model explicitly allow for a small but significant degree of imperfect substitution between immigrants and natives of similar education and experience. In our view however, the efforts in both these and other studies are eclipsed by the methodology and theory applied in the Dixon & Rimmer studies to immigration issues in the United States.

The modelling approach and theory contained in Dixon *et al* (2011) was the most advanced we have seen in a study dealing with migration. Using a detailed and dynamic CGE or economy-wide model of the United States, aptly named USAGE-M<sup>3</sup>, the authors were able to carefully trace and explain the impacts of changes to immigration policy on the local economy. The model's labour market mechanism was key to facilitating the detailed policy analysis. The model distinguished workers by labour function, birthplace and legal status. It also allowed for varying degrees of imperfect substitution between native and foreign-born workers, and workers with legal and illegal status, across more than 50 occupation types. Equilibrium rates of unemployment were assumed in the model's behaviour, suggesting an economy at or near full-employment. The assumed wage-adjustment process is compatible with search-theoretic models. This shows that reductions in labour supply, and resulting reductions in the unemployment rate, generate decreases in the value of having a job relative to the value of not having a job, thereby emboldening workers to demand higher wage rates

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<sup>3</sup> USAGE-M is a variant of the USAGE model that specifically allows for migration flows. USAGE was developed by the Centre of Policy Studies in collaboration with the U.S. International Trade Commission. The theoretical structure of USAGE is similar to that of the MONASH model of Australia documented in Dixon & Rimmer (2002).

(Bohringer *et al*, 2005; Rogerson *et al*, 2005). It is also compatible with efficiency-wage theory, where employers offer wage rates that optimise workers effort per dollar of wage cost. The theory suggests that the effort-optimising wage rate rises when there is a decrease in labour supply, and as a result, a temporary decrease in unemployment (Layard *et al*, 1994; Borjas, 2009). The well-documented nature and wide range of applications to which the USAGE and other MONASH-style models have been applied to helped make it a convincing choice as starting point for our own ZAR-M model.<sup>4</sup>

Relatively few studies have focused specifically on illegal immigration. As noted before, empirical studies in South Africa have virtually been ruled out due to a lack of reliable time-series data. The findings in the Dixon & Rimmer studies support the view that illegal migrants do more good than harm to native workers in the United States. The so-called ‘occupation-mix’ effect is significant in determining this result. Here, illegal migrant workers are shown to effectively push native workers up the earnings ladder by filling a higher proportion of low-paying and typically low-skilled jobs. With a policy-induced cut to the employment of illegal migrants, relatively more of these low-paying jobs are taken up by native workers over time. The smaller economy and adverse change in the skill or occupation-mix of the native population, and therefore their compensation, was shown to outweigh the slight gains in their employment levels at the lower-end of the market with the introduction of such a policy.

In studying the validity of the Dixon & Rimmer labour market mechanism and model behaviour to the South African situation, we found one significant incompatibility. Given the nature of the local labour market, we could not accept the assumption of equilibrium rates of unemployment in the wage adjustment process for South Africa. We address this in *Scenario 2* of our policy simulations and clearly highlight the implications of this change to the ZAR-M model.

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<sup>4</sup> See Dixon & Rimmer (2010a) and Dixon *et al* (2012) for an overview of how the MONASH style of modelling has evolved and been applied since the 1970s.

As noted before, there exists a large literature on regional and international migration topics from both economists and non-economists. This section clearly does not intend to review the full breadth of the migration literature. Our overview of the main topics in the literature is quickly followed by a discussion more relevant to issues of illegal immigration in South Africa and finding a suitable methodology to measure the economy-wide impact of changes to its migration policy. We continue our investigation into labour and migration policy in South Africa by describing the theoretical structure of the ZAR-M model in Chapter 2.





# CHAPTER 2

## MODEL THEORY

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## 2.1 Introduction

Computable general equilibrium (CGE) models such as the well-documented comparative-static ORANI (Dixon *et al.*, 1982) or its dynamic successor MONASH (Dixon & Rimmer, 2002) belong to the Walras-Johansen class of economy-wide models that provide industry-level disaggregation in a quantitative description of the whole economy. These models typically postulate neo-classical production functions and price-responsive demand functions, linked around an input-output matrix in a general equilibrium model that endogenously determines quantities and prices. In contrast to a partial equilibrium analysis, general equilibrium models are therefore able to account for all the linkages between sectors and agents of an economy.

Building on the pioneering work of Johansen (1960), MONASH-type models have the added advantage of having incorporated several extensions to the initial framework.<sup>1</sup> The inclusion of Armington's imperfect substitution specification between similar domestically produced and imported goods along with downward-sloping export demand curves effectively overcame the flip-flop problem related to trade modelling. Greater flexibility in the choice of model closure, the ability to deal with large dimensions, a detailed treatment of margin costs, and the elimination of linearisation error within the Johansen framework have been equally important. Accessibility to CGE modelling has also been enhanced through the development of tailored software packages such as GEMPACK and GAMS (Horridge & Pearson, 2011).<sup>2</sup>

Four basic tasks distinguish CGE based analysis (Adams, 2005). The first is theoretical derivation and description of the model, which incorporates the formal theory and linearisation of equations. Following the approach of Johansen (1960), linearisation of the system of equations simplifies both the implementation and interpretation of complex

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<sup>1</sup> See Dixon & Rimmer (2010) for a detailed account of Leif Johansen's contribution to the field of CGE modelling and advances that have been made since his seminal publication in 1960.

<sup>2</sup> GEMPACK is a suite of software applications designed specifically for general equilibrium modelling within the Johansen framework. It was developed at the Centre of Policy Studies and documented in Harrison & Pearson (1996). The General Algebraic Modeling System (GAMS) was initially developed at the World Bank. It is maintained by the GAMS Development Corporation and documented online at [www.gams.com/docs](http://www.gams.com/docs).

functional forms such as the CES or CRESH in the model.<sup>3</sup> Second is calibration, which incorporates the construction of a balanced database and evaluation of coefficients. The choice of appropriate behavioural parameters is critical in this regard as they determine the responsiveness of agents to relative price and income changes. Third is solving the model using a suitable closure. The model closure is important as it determines the economic environment under which a simulation is run. The system of linear equations that make up the model is then solved via a series of matrix manipulations.<sup>4</sup> The final task involves proper interpretation of simulation results, drawing only on values given in the database, the underlying theory or model closure.

In this chapter we describe the theoretical structure of the ZAR-M model. To give readers an intuitive understanding of the overall structure of the model, a stylized representation of the ZAR-M model in functional form is first set out. This is followed by a detailed description of the capital and labour market mechanisms in the ZAR-M model. The remaining tasks involving database construction, model closures, and interpretation of simulation results are discussed in subsequent chapters.

## 2.2 Theoretical Structure of the ZAR-M Model

### *Overview of ZAR-M theory*

The ZAR-M model used in this study is a recursive-dynamic CGE model of the South African economy. Its theoretical structure closely follows that of the MONASH model of Australia developed by Dixon & Rimmer (2002). To facilitate the analysis of migration flows it also incorporates a detailed labour market specification similar to that introduced in the USAGE-M model (Dixon *et al*, 2011). The model is solved using GEMPACK and

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<sup>3</sup> See Hanoch (1971) for the derivation of the CRESH functional form and Dixon *et al* (1992:126–128) for an example on how to linearise the CRESH function for use within the Johansen framework. An example of linearising the Cobb-Douglas and CES functional forms is included in the Appendix.

<sup>4</sup> See Appendix A for a brief introduction to writing equations in linearised form and solving the system of equations via matrix inversion.

implemented on a database representing the South African economy for the base year 2006. The database incorporates multi-product industries, multi-industry commodities, and different occupational groups. Final users of commodities include investors, households, government and exporters. To simplify our analysis, ZAR-M is modelled with a single representative household and central government in this study. A description of the database is included in Chapter 3.

The linearised system of equations that make up ZAR-M describes the theory underlying the behaviour of participants in the economy. It contains equations describing i) industry demands for primary factors and intermediate inputs; ii) final household, investment, government and foreign demand for commodities; iii) pricing in the economy which sets pure profits from all activities to zero; iv) market clearing equations for various primary factors and commodities; and v) miscellaneous or definitional items such as GDP, aggregate employment and the consumer price index.

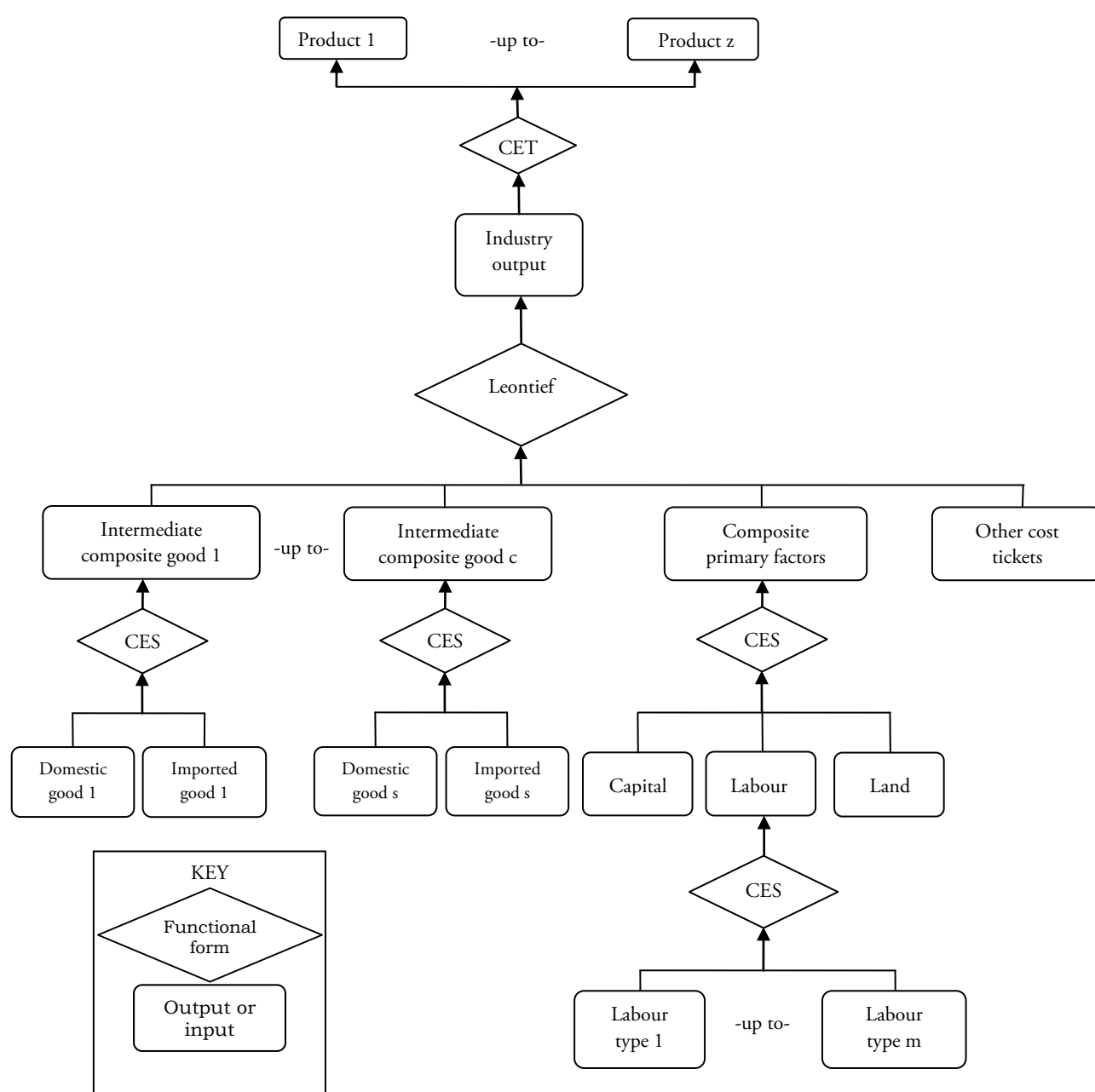
ZAR-M allows each industry to produce several commodities, using as inputs combinations of domestic and imported commodities, different types of labour, capital and land. The multi-input, multi-output production specification is kept manageable by a series of separability assumptions, illustrated by the nesting shown in Figure 2A. This nested production structure reduces the number of estimated parameters required by the model. Optimising equations determining the commodity composition of industry output are derived subject to a CET function, while functions determining industry inputs are determined by a series of nests. At the top level, intermediate commodity composites, a primary-factor composite, and other cost tickets<sup>5</sup> are combined using a Leontief or fixed proportions production function. Consequently, they are all demanded in direct proportion to industry output or activity. Each commodity composite is a CES function of a domestic good and its imported equivalent. This incorporates Armington's assumption of imperfect substitutability for goods by place of production (Armington, 1969). The primary-factor composite is a CES aggregate of composite labour, capital and land, with composite labour

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<sup>5</sup> Other cost tickets are a useful device for allowing for costs of holding liquidity, costs of holding inventories and other miscellaneous production costs (Dixon *et al*, 1982:70).

itself a CES aggregate of different labour types. In ZAR-M, the labour composite incorporates labour sourced from different skill categories, birthplaces and legal status, allowing price-induced substitution between these characteristics. Although all industries share this common production structure, input proportions and behavioural parameters may vary between industries.

FIGURE 2A Structure of Production for a Representative Industry



Demand and supply equations for industries and households are derived from the solutions to the optimisation problems which are assumed to underlie the behaviour of private sector agents in conventional neo-classical microeconomics. Each industry minimises cost subject to given input prices and a constant returns to scale production function. Households maximise a Klein-Rubin utility function subject to their budget constraint. Units of new industry-specific capital are determined as cost-minimising combinations of domestic and imported commodities. Imperfect substitutability between sources of commodities is modelled using the Armington CES assumptions. The export demand for any local commodity is inversely related to its foreign-currency price. The price of imports is exogenously determined, consistent with the assumption of South Africa being a small open economy. Government consumption and the details of direct and indirect taxation are also recognised in the model. Markets are assumed to be competitive which implies that zero pure profits are captured in any sector or activity.

The dynamic elements of ZAR-M allow for inter-temporal links describing i) physical capital accumulation; ii) financial asset/liability accumulation; and iii) lagged adjustment processes for labour. Capital accumulation is specified separately for each industry, and linked to industry-specific net investment. Investment in each industry is positively related to its expected rate of return on capital. An industry's end-of-year capital stock is therefore calculated as the sum of start-of-year capital stock plus investments during the year minus depreciation. End-of-year  $t$  capital stock then determines start-of-year  $t+1$  capital stock. Thus, investment this period only affects capital stock in the next period. A similar mechanism for financial asset/liability accumulation is specified. Adjustments to the national net foreign liability position of households are related to the annual investment/savings imbalance, revaluations of assets and liabilities, and remittance flows during the year. Changes in the public sector debt are related to the public sector deficit incurred during the year. In policy simulations, the labour market follows a lagged adjustment path where wage rates are allowed to respond over time to gaps between demand and supply for labour.

Containing thousands of lines of computer code describing many equations and variables, the ZAR-M model is too large to be fully documented in a study of this nature. In Table 2A we give a stylized ‘back-of-the-envelope’ representation of the theoretical structure of the model. This allows us to describe the most important macroeconomic relationships and basic functioning of the model without encumbering the reader with too much detail. A complete exposition and discussion of the capital and labour market theory in ZAR-M is provided in the following section.

### *Stylized representation of ZAR-M*

Equations (E2.1–E2.21) describe the key macroeconomic relationships in ZAR-M. This system of equations contained in Table 2A will hereafter be referred to as the BOTE-M model. Miniature models such as BOTE-M have become a popular method for presenting and explaining the core elements in CGE models. Its simple and compact nature is also useful when interpreting simulation results produced by the full model.

In BOTE-M, equations (E2.1–E2.16) describe variables within any given year of a dynamic simulation. Equations (E2.17–E2.19) describe how key stock variables move through time, and hold between any two successive years of a dynamic simulation. Equations (E2.20–E2.21) describe the real wage adjustment mechanism applicable to policy simulations.

Equation (E2.1) is the well-known identity describing real gross domestic product (GDP) from the expenditure side. In South Africa, private households contribute around 60 per cent to GDP, with investment and government expenditure each contributing roughly 20 per cent. The balance of trade ( $X-M$ ) typically shows a deficit. Equation (E2.2) describes an economy-wide constant returns to scale production function, relating real GDP from the supply side to inputs of capital, labour, and primary-factor augmenting technical change. In South Africa, both compensation of employees and gross operating surplus contribute around 50 per cent of GDP at factor cost.

TABLE 2A *BOTE-M: Stylized Representation of ZAR-M*


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GDP	= C + I + <b>G</b> + (X-M)	(E2.1)
GDP	= <b>A</b> *f( <b>K</b> , D)	(E2.2)
C	= <b>APC</b> *HINC	(E2.3)
HINC	= GDP*f(TofT)*(1-TQ) – ( <b>BTRW</b> .D)*TL – ( <b>NFLH</b> *R)	(E2.4)
GINC	= GDP*f(TofT)*TQ + ( <b>BTRW</b> .D)*TL – <b>NFLG</b> *R	(E2.5)
GNDI	= HINC + GINC	(E2.6)
M	= f(GDP, TofT, <b>TWS</b> )	(E2.7)
TofT	= PX/ <b>PM</b>	(E2.8)
PX	= f(X, <b>F_X</b> )	(E2.9)
PY	= f( <b>CPI</b> , TofT)	(E2.10)
I/K	= R_IK	(E2.11)
I	= f(RoR, <b>F_I</b> )	(E2.12)
RoR	= f( <b>K</b> /L, TofT, <b>A</b> )	(E2.13)
<b>BTRW</b>	= f( <b>K</b> /L, TofT, <b>A</b> )	(E2.14)
ATRW	= <b>BTRW</b> *(1-TL)	(E2.15)
D	= L – <b>U</b>	(E2.16)
$\Delta K$	= I – <b>DEP</b> * <b>K</b>	(E2.17)
$\Delta \text{NFLG}$	= <b>G</b> – GINC	(E2.18)
$\Delta \text{NFLH}$	= I – (1- <b>APC</b> )*HINC + <b>REM</b>	(E2.19)
$\Delta \text{ATRW}$	= f(D, L)	(E2.20)
L	= f(ATRW, $\Delta \text{L\_PREF}$ )	(E2.21)

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TABLE 2B *Description of Variables in BOTE-M*


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A	Primary-factor augmenting technical change
APC	Average propensity to consume
ATRW , BTRW	After-tax real wage , Before-tax real wage
C	Real private household expenditure
CPI	Consumer price index
D , L , U	Labour demand , Labour supply , Unemployment
DEP	Depreciation rate
F_I , F_X	Shift in investment , Shift in export demand schedule
G	Real government expenditure
GDP	Real gross domestic product
GINC , HINC	Real government income , Real household income
GNDI	Gross national disposable income
I	Real investment expenditure
K	Capital stock
MPK , RoR	Marginal product of capital , Rate of return on capital
NFLG	Real net foreign liabilities of government
NFLH	Real net foreign liabilities of households
PX , PM	Foreign-currency export price , Foreign-currency import price
PY	GDP deflator
R	Interest rate on net foreign liabilities
REM	Net outward remittance flows
R_IK	Investment/Capital ratio
TL , TQ	Labour tax rate , Production tax rate
ToFT	Terms of trade
TWS	Cost-neutral import/domestic preference twist
X , M	Export volumes , Import volumes
$\Delta K$	Change in capital stock between years $t$ and $t+1$
$\Delta NFLG$	Change in net foreign liabilities of government between years $t$ and $t+1$
$\Delta NFLH$	Change in net foreign liabilities of households between years $t$ and $t+1$
$\Delta ATRW$	Change in after-tax real wage between years $t$ and $t+1$
$\Delta L\_PREF$	Change in labour supply preferences between years $t$ and $t+1$

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Equation (E2.3) relates private household expenditure ( $C$ ) to household disposable income via the average propensity to consume. Equations (E2.4) and (E2.5) define real household disposable income ( $HINC$ ) and government revenue ( $GINC$ ), respectively. The first term in both these equations describe the value of real GDP which could be expressed as  $[(GDP \cdot PY)/CPI]$ .  $PY$  measures the average price level of GDP, and as such contains no information on import prices.  $PY$  does however incorporate prices of domestically-produced exports. In contrast, the  $CPI$  contains no information on export prices, but does include import prices. We can therefore interpret  $[PY/CPI]$  as a function of the terms of trade and write this component as  $GDP \cdot f(TofT)$ .  $HINC$  is defined as total income available for household expenditure after taking into account tax and net foreign liability payments.  $GINC$  is defined as the sum of all production and labour taxes collected minus any interest payments by government on its foreign liabilities. Equation (E2.6) confirms that (E2.4) and (E2.5) exhaust all claims on gross national disposable income ( $GNDI$ ).

Equation (2.7) relates imports ( $M$ ) to the level of GDP, the terms of trade, and an import/domestic preferences twist variable. The terms of trade ( $TofT$ ) is defined in (E2.8) as the foreign-currency price of domestically produced exports relative to the price of imports. Commodity exports in ZAR-M are inversely related to foreign-currency prices via constant elasticity demand functions. This is summarised by (E2.9) which relates the foreign-currency price of exports ( $P_X$ ) to the volume of exports and an export-demand shift variable. This is consistent with the assumption of South Africa being an open economy facing downward-sloping demand curves for its exports. This allows us to incorporate appropriate export-demand elasticities for South African commodities in ZAR-M. Import prices are exogenous as South Africa is considered a price-taker in the global import market.

In our determination of (E2.10) we write the percentage-change of the economy-wide output price as  $P_Y = [S_A P_A + S_X (P_X - P_M)]$ . Following the notation employed by many MONASH-style CGE modellers,  $S_A$  and  $S_X$  reflects the share of absorption and exports in the economy, while  $P_Y$ ,  $P_A$ ,  $P_X$  and  $P_M$  represent the percentage-change in the price of their respective upper-case variables. For BOTE-M we assume that the price of absorption is

reflected in the CPI, and that trade is balanced. From here, we are able to write equation (E2.10) which relates the price of economy-wide output (PY) to the consumer price index and terms of trade. PY may also be interpreted as the GDP deflator.

Equation (E2.11) defines the investment-capital ratio ( $R_{IK}$ ) whilst equation (E2.12) relates investment expenditure (I) to the rate of return on capital (RoR) and an investment-demand shift variable. In (E2.11),  $R_{IK}$  may also be used to determine the gross capital growth rate.

Since the production function in (E2.2) is constant returns to scale, marginal product functions are homogenous of degree zero and can thus be expressed as functions of the capital-labour (K/L) ratio and technical change (A). In our description of the capital and labour markets, we recognise that the marginal product of capital (MPK) is negatively related to the K/L ratio and the marginal product of labour (MPL) positively related to the K/L ratio. In determining (E2.13) we assume that the rate of return on capital (RoR) can be expressed as  $[Q/PI]$  with Q the factor payment to capital and PI the price index for new investments. We then assume Q is determined by the value of the marginal product of capital, written as  $[MPK*PY]$ . With MPK a function of the K/L ratio and technical change, and  $[PY/PI]$  a function of the terms of trade, we are able to summarise this relationship through equation (E2.13).

In determining (E2.14) we assume that the before-tax real wage of consumers (BTRW) can be expressed as  $[W/CPI]$  with W the factor payment to labour and CPI the consumer price index. We assume W is determined by the value of the marginal product of labour, written as  $[MPL*PY]$ . In similar fashion to (E2.13), we are then able to write equation (E2.14) linking the BTRW to the K/L ratio, technical change and terms of trade effect. Equation (E2.15) defines the after-tax real wage (ATRW) and equation (E2.16) allows for unemployment (U).

Equations (E2.17–E2.19) relate movements in three key stock variables to relevant flow variables. Equation (E2.17) shows that changes in capital stock ( $\Delta K$ ) is calculated as the sum of new capital investments minus depreciation of old capital stock. Equation (E2.18) relates changes in government's net foreign liability position ( $\Delta NFLG$ ) to the public sector deficit incurred during the year. Equation (E2.19) relates changes in the net foreign liability position of households ( $\Delta NFLH$ ) to the excess of investment over savings and net outward remittance flows from migrants.

Equations (E2.20) and (E2.21) capture the real wage adjustment and labour supply mechanism in policy simulations. Where a policy has elevated labour demand relative to labour supply, the after-tax real wage ( $\Delta ATRW$ ) will increase over time relative to its baseline value. Under this specification, the local labour market does not clear in the short-run perturbed scenario. An appropriate parameter in the wage adjustment equation governs the lagged wage response to gaps between labour demand and supply. Labour demand ( $D$ ) is determined as a function of the before-tax real wage in (E2.14) and labour supply ( $L$ ) is determined in (E2.21) as a function of the after-tax real wage and any change in preferences of workers for offering their services to a local activity.

To complete our description of the BOTE-M model we have to consider an appropriate closure for the system of equations in Table 2A. In doing so we must distinguish between equations that describe economic relationships within any given year (E2.1–E2.16), equations that describe movements in stock variables between years (E2.17–E2.19) and equations describing the real wage adjustment mechanism (E2.20–E2.21). In our exposition of BOTE-M we consider a typical short-run recursive-dynamic modelling environment. The development and interpretation of different model closures will be discussed more thoroughly in Chapter 4.

Within any given year,  $K$ ,  $NFLH$  and  $NFLG$  can be considered exogenous, with movements between years dependent on their respective flow variables. Similarly, our ‘sticky’ real wage adjustment mechanism allows us to effectively treat  $BTRW$ , and therefore  $ATRW$ , as fixed within any given year, with movement between years dependent on the interaction between labour demand and supply. Reflecting changes in policy or economic conditions that are considered extraneous to the model we also set  $DEP$ ,  $REM$  and  $\Delta L\_PREF$  exogenous.

Recognising that (E2.17–E2.21) governs dynamics across years, our task of finding a suitable model closure narrows to evaluating (E2.1–E2.16). We assume that the labour market clears within this set of equations and that the unemployment level is fixed or exogenous within any given year. These 16 equations contain 31 unknown variables. As the number of endogenous variables must correspond to the number of equations, 15 variables must therefore be treated as exogenous in order to close the model. In Table 2A we provide a quick reference to our choice of model closure by highlighting exogenous variables in bold. For these equations, a conventional year-on-year short-run closure would have  $GDP$ ,  $C$ ,  $I$ ,  $X$ ,  $M$ ,  $D$ ,  $L$ ,  $ATRW$ ,  $GINC$ ,  $GNDI$ ,  $HINC$ ,  $TofT$ ,  $PX$ ,  $PY$ ,  $RoR$  and  $R\_IK$  determined endogenously, given exogenous values for  $A$ ,  $APC$ ,  $G$ ,  $K$ ,  $BTRW$ ,  $TQ$ ,  $TL$ ,  $NFLG$ ,  $NFLH$ ,  $R$ ,  $PM$ ,  $U$ ,  $F\_X$ ,  $F\_I$  and  $TWS$ .

Under this closure, each equation can be readily associated with the determination of a specific endogenous variable. This also allows us to follow the GEMPACK convention of naming each equation for the variable it is considered to endogenously determine. For large models such as ZAR-M this is particularly useful as it allows the software to find an initial automatic closure by treating all those variables named by an equation as endogenous, and setting all remaining variables as exogenous. This saves the user a great amount of time as most of the numerous technical change and shift variables, all naturally exogenous, would already be chosen and listed as such.

The short-run closure applied to (E2.1–E2.16) via our choice of exogenous variables reflects the standard macro assumptions of ‘sticky’ real wages and fixed capital stocks in primary-factor markets. Although ATRW is endogenous in our model closure, it can effectively be seen as fixed within any given year since both BTRW and TL are exogenous in the short-run. To simplify our analysis, we assume that labour markets clear within this set of equations. Hence, with BTRW and K fixed, and A, TQ and TL also exogenous, (E2.14) can be identified with the determination of D. Since U is exogenous, this allows (E2.16) to determine L. With K and A exogenous, (E2.2) then determines GDP. With GDP now determined, (E2.4–E2.6) calculates GNDI and its distribution between HINC and GINC. With HINC determined by (E2.4) and APC exogenous, (E2.3) determines C. Ignoring any movements in the ToFT, (E2.7) determines M with GDP already determined and TWS exogenous. With L determined by (E2.14), and K and A exogenous, (E2.13) determines the RoR via the marginal product of capital. This determines I via (E2.12), which allows R<sub>IK</sub> to be calculated via (E2.11). With GDP, C, I, G and M explained, (E2.1) determines X. With PM exogenous, this determines PX and ToFT via (E2.8) and (E2.9), respectively. With BTRW and TL fixed in the short-run, (E2.15) simply determines ATRW. To allow the absolute price level to be determined, the CPI acts as the numeraire in our system of equations. With the CPI exogenous and ToFT already determined, (E2.10) determines PY.

This exposition of the BOTE-M model describes the key macroeconomic relationships in ZAR-M under a typical short-run recursive-dynamic environment, with every equation linked to the determination of a specific endogenous variable. BOTE-M is useful as a quick reference to ZAR-M and providing insight for the interpretation of simulation results. In the following sections we include a more complete description of the capital and labour market mechanisms in the actual ZAR-M model. These markets are critical in the determination of the supply-side of the model. The labour market description in particular will enhance our understanding of the model within the context of the simulations conducted in the study.

## 2.3 Capital Accumulation Mechanism in ZAR-M

Recursive-dynamic modelling requires that an economy's capital stock be allowed to adjust over time according to the level of net investment. Given an initial level of capital  $[K_t(j)]$  and a mechanism for determining investment  $[I_t(j)]$ , equation (E2.22) can be used to trace out the path of industry  $j$ 's capital stock. The rate of growth in industry  $j$ 's capital stock  $[K\_GR_t(j)]$  is defined by (E2.23) and linked to expected rates of return  $[ERoR_t(j)]$  in (E2.24). Equation (2.24) defines an investment-supply curve showing that the rate of return investors require depends on the rate of growth in industry  $j$ 's capital stock. This physical capital accumulation mechanism in ZAR-M is summarised in Table 2C below.

TABLE 2C *Capital Accumulation Mechanism*

$K_{t+1}(j) = K_t(j) * [1 - DEP_t(j)] + I_t(j)$	(E2.22)
$K\_GR_t(j) = \frac{K_{t+1}(j)}{K_t(j)} - 1$	(E2.23)
$ERoR_t(j) = \psi [K\_GR_t(j)]$	(E2.24)

Equation (E2.22) in Table 2C shows that the amount of capital available for use in industry  $j$  at the end of year  $t$  is calculated as start-of-year  $t$  capital stock minus depreciation, plus new capital investments during year  $t$ . End-of-year  $t$  capital stock then determines start-of-year  $t+1$  capital stock. In ZAR-M, the capital-supply function for industry  $j$ , equivalent to  $\psi$  in (E2.24), describes the relationship between  $j$ 's expected rate of return and the proportionate growth in  $j$ 's capital stock between successive years. A complete exposition of the capital-supply function and determination of rates of return for industry  $j$  is given in Table 2D.

TABLE 2D *Capital-Supply Function*

$$ERoR_t(j) = \{RoRN_t(j) + F\_ERoR_t(j) + F\_ERoR_t\} + [1/C_t(j)] * \\ \left[ \ln\{K\_GR_t(j) - K\_GR\_MIN(j)\} - \ln\{K\_GR\_MAX(j) - K\_GR_t(j)\} \right. \\ \left. - \ln\{TREND\_K(j) - K\_GR\_MIN(j)\} + \ln\{K\_GR\_MAX(j) - TREND\_K(j)\} \right] \quad (E2.25)$$

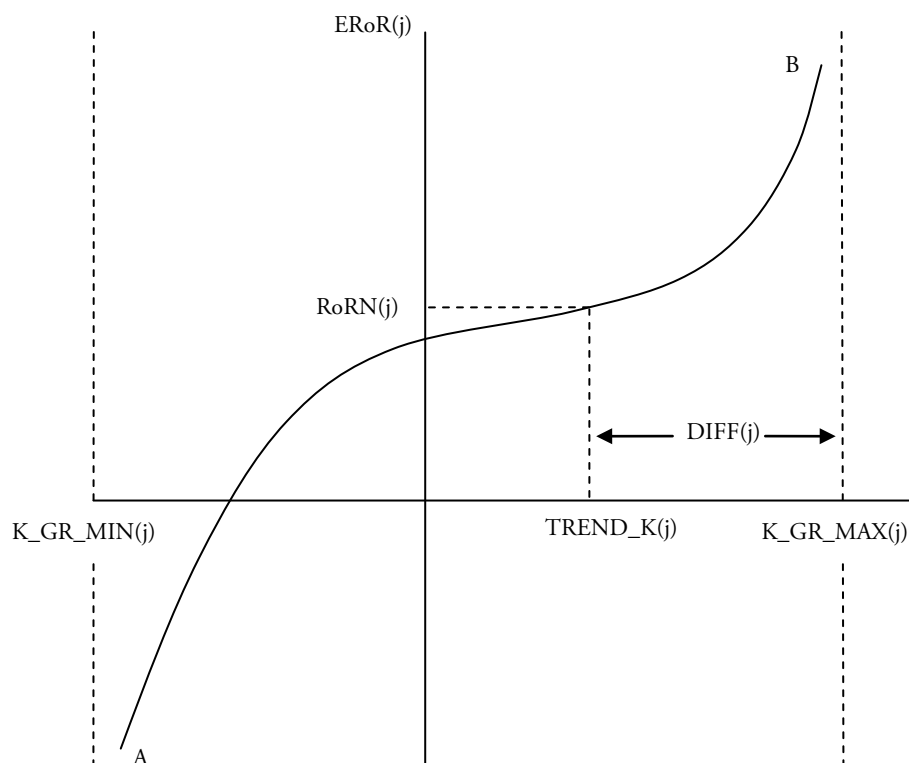
$$C_t(j) = \left[ \frac{\partial ERoR_t(j)}{\partial K\_GR_t(j)} \Big|_{K\_GR_t(j)=TREND\_K_t(j)} \right]^{-1} * \\ \left[ \frac{K\_GR\_MAX_t(j) - K\_GR\_MIN_t(j)}{\{K\_GR\_MAX_t(j) - TREND\_K_j(j)\} \{TREND\_K_t(j) - K\_GR\_MIN_t(j)\}} \right] \quad (E2.26)$$

$$\left[ \frac{\partial ERoR_t(j)}{\partial K\_GR_t(j)} \Big|_{K\_GR_t(j)=TREND\_K_t(j)} \right]^{-1} = S\_KGR\_ERoR_t \quad (E2.27)$$

$$del\_error_t(j) = [1/C_t(j)] * [1/\{K\_GR_t(j) - K\_GR\_MIN_t(j)\} \\ + 1/\{K\_GR\_MAX_t(j) - K\_GR_t(j)\}] * del\_k\_gr_t(j) \quad (E2.28) \\ + d\_f\_error_t(j) + d\_f\_error_t$$

Under the expected rates of return specification in equation (E2.25) we can interpret the equilibrium expected rate of return in industry  $j$  as an inverse logistic function of the proportionate growth in  $j$ 's capital stock. This relationship is illustrated by the AB curve in Figure 2B. With the  $F\_ERoR$  terms exogenous and set to zero, it shows that if industry  $j$  is to attract sufficient investment in year  $t$  to achieve a capital growth rate of  $TREND\_K_t(j)$ , it must have an  $ERoR_t(j)$  equal to  $RoRN_t(j)$ . An increase (or decrease) in industry  $j$ 's capital growth beyond its  $TREND\_K_t(j)$  must therefore be accompanied by an  $ERoR_t(j)$  in excess of (or below) its  $RoRN_t(j)$  level.



FIGURE 2B *Expected Rates of Return*

Source: Adapted from Dixon & Rimmer (2002:191)

Evaluation of the parameter  $C_t(j)$  becomes important in simulations where (E2.25) play an active role. In choosing appropriate values for  $C_t(j)$  we note its derivation from (E2.25) in equation (E2.26). We can therefore estimate  $C_t(j)$  if we assign a value to the reciprocal of the slope of the AB curve in Figure 2B in the region of  $K_{GR_t}(j) = TREND\_K_t(j)$ . Since no reliable data exists which will enable us to complete this task, we follow a similar approach to that adopted in the MONASH model. An average value over all industries for the sensitivity of capital growth to variations in expected rates of return is used as a proxy, denoted by the term  $S\_KGR\_ERoR_t$  in (E2.27), and based on estimates used in Dixon & Rimmer (2002) and Roos (2011). For readers familiar with GEMPACK, equation (E2.28) shows the linearised form of (E2.25) as it appears in the actual ZAR-M model code.

TABLE 2E *Notation in the Exposition of the Capital Accumulation Mechanism*


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$K_t(j)$	Capital stock available for use in industry $j$ at the start of year $t$
$K_{t+1}(j)$	Capital stock available for use in industry $j$ at the end of year $t$ , or equivalently, at the start of year $t+1$
$I_t(j)$	New investment in industry $j$ during year $t$
$DEP_t(j)$	Rate of depreciation on capital stock of industry $j$
$ERoR_t(j)$	Expected rate of return on capital of industry $j$
$RoRN_t(j)$	Historically normal rate of return on capital of industry $j$
$F\_ERoR_t(j)$	Industry-specific vertical shift in the capital-supply curve
$F\_ERoR_t$	Uniform vertical shift in the capital-supply curve
$K\_GR_t(j)$	Rate of growth in the capital stock of industry $j$ during year $t$
$TREND\_K_t(j)$	Historically normal rate of growth in the capital stock of industry $j$
$K\_GR\_MIN_t(j)$	Minimum possible rate of growth in the capital stock of industry $j$ typically set at the negative of the rate of depreciation in industry $j$
$K\_GR\_MAX_t(j)$	Maximum feasible rate of growth in the capital stock of industry $j$ defined as $TREND\_K_t(j)$ plus $DIFF_t(j)$ with $DIFF_t(j)$ set at a value of 0.08 to prevent large simulated capital growth rates
$C_t(j)$	Positive parameter that controls the sensitivity of industry $j$ 's capital growth to variations in its expected rate of return
$S\_KGR\_ERoR_t$	Estimate of the average value over all industries of the sensitivity of capital growth to variations in expected rates of return
$ARoR_t(j)$	Actual rate of return on capital for industry $j$
$NPV_t(j)$	Net present value of purchasing a unit of capital for use in industry $j$ in year $t$
$PINV_t(j)$	Cost of buying or constructing a new unit of capital for use in industry $j$ in year $t$
$PCAP_t(j)$	Rental rate on industry $j$ 's capital in year $t$ , that is, the user cost of a unit of capital in year $t$
$R_t$	Nominal rate of interest for all industries in year $t$
$INF_t$	Rate of inflation in year $t$

---

In Table 2F we briefly describe how actual rates of return in ZAR-M are determined and used to form expected values. To find the actual rates of return for industries we start by defining the net present value of purchasing a unit of capital for use in industry  $j$  as the discounted rental value of an extra unit of capital in year  $t+1$ , plus the discounted re-sale value of the asset in year  $t+1$  taking into account depreciation, minus the immediate outlay cost of the new capital. This relationship, ignoring the role of taxes, is given by (E2.29). To derive a rate of return formula we divide both sides of (E2.29) by  $PINV_t(j)$ , as shown in (E2.30). This yields the expression for actual rates of return on capital for industry  $j$  as shown in (E2.31). As noted, the determination of capital growth and investment in ZAR-M requires use of expected rather than actual rates of return.  $ERoR_t(j)$  is subsequently formed by generating expectations held in year  $t$  about  $ARoR_t(j)$ . Under static expectations this is achieved by assuming that rental rates and asset prices will increase by the current rate of inflation as shown in (E2.32) and (E2.33) (Dixon & Rimmer, 2002:190-194).

TABLE 2F *Actual and Expected Rates of Return*

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$$NPV_t(j) = \frac{PCAP_{t+1}(j)}{1 + R_t} + \frac{PINV_{t+1}(j)[1 - DEP_t(j)]}{1 + R_t} - PINV_t(j) \quad (E2.29)$$

$$\frac{NPV_t(j)}{PINV_t(j)} = \frac{PCAP_{t+1}(j)}{(1 + R_t) * PINV_t(j)} + \frac{PINV_{t+1}(j)[1 - DEP_t(j)]}{(1 + R_t) * PINV_t(j)} - \frac{PINV_t(j)}{PINV_t(j)} \quad (E2.30)$$

$$ARoR_t(j) = \frac{PCAP_{t+1}(j)}{(1 + R_t) * PINV_t(j)} + \frac{PINV_{t+1}(j)[1 - DEP_t(j)]}{(1 + R_t) * PINV_t(j)} - 1 \quad (E2.31)$$

$$PCAP_{t+1}(j) = PCAP_t(j) * [1 + INF_t] \quad (E2.32)$$

$$PINV_{t+1}(j) = PINV_t(j) * [1 + INF_t] \quad (E2.33)$$


---

## 2.4 Labour Market Mechanism in ZAR-M

Given the nature of this study, the labour market mechanism in ZAR-M represents the most important part of the model enabling us to conduct simulations on migration policy in South Africa. The labour market theory in ZAR-M closely follows that of USAGE-M developed in Dixon *et al* (2011), with added detail to facilitate the analysis of skilled migrants. The six key ingredients in the labour market specification of ZAR-M are i) the division of the labour force into categories at the start of each year reflecting labour force functions of people in the previous year; ii) the identification of labour force activities, that is, what people do during the year; iii) the determination of labour supply from each category to each activity; iv) the determination of labour demand in employment activities; v) the specification of wage adjustment processes reflecting labour demand and supply; and vi) the determination of everyone's activity, that is, who finds employment and what happens to those who do not.

The dynamics of the labour market mechanism is broadly illustrated by Figure 2C. Categories (*cat*) and activities (*act*) are defined by birthplace (*b*), legal status (*s*) and labour force function (*f*) of workers. The activities that people in a given category undertake each year are determined mainly by their willingness to offer their services to that activity, relative to offers from people in other categories, and by employers' demand for the services of that activity. Table 2G summarises the system of equations associated with the theory of the labour market mechanism in ZAR-M for quick reference. In the following section each theme in the labour market mechanism will be discussed in detail. Equations contained in Table 2G will be repeated within the discussion of these themes.

FIGURE 2C Labour Market Dynamics

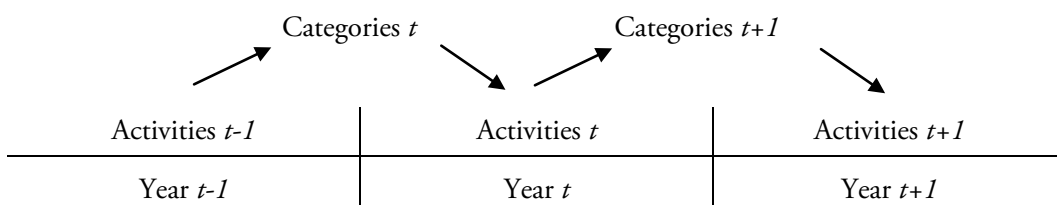


TABLE 2G Labour Market Mechanism

Number of people in each category at the beginning of year  $t$

$$CAT_t(b, s, f) = ACT_{t-1}(b, s, f) * 0.99 \quad \text{for all } b, s \text{ and } f \neq \text{new}$$

$$CAT_t(b, s, 'new') = \text{exogenous} \quad \text{for all } b, s$$

Labour supply by people from categories to activities

$$L_t(cat; act) = CAT_t(cat) * \psi_L [PREF_t(cat; act), ATRW_t(act)]$$

$$L_t(act) = \sum_{cat} L_t(cat; act) \quad \text{for all categories } cat \text{ \& activities } act$$

Labour demand and employment

$$D_t(b, s, o, j) = D_t(j) * \psi_D [BTRW_t(bb, ss, oo) \quad \forall bb, ss \text{ and local occupations } oo] \\ \text{for all } b, s, \text{ local occupations } o \text{ and industries } j$$

$$D_t(j) = Z_t(j) * \psi_D [BTRW_t(j), K_t(j), A_t(j)] \quad \text{for all local industries } j$$

Relationship between after-tax and before-tax real wages

$$ATRW_t(b, s, o) = BTRW_t(b, s, o) * [1 - TL_t(b, s)] \quad \text{for all } b, s, \text{ local occupations } o$$

$$ATRW_t(b, s, u) = BTRW_t^{ave}(b, s) * F_t(b, s) \quad \text{for all } b, s, \text{ unemployment } u$$

Wage adjustment mechanism

$$\left( \frac{ATRW_t^{policy}(b, s, o)}{ATRW_t^{base}(b, s, o)} \right) - \left( \frac{ATRW_{t-1}^{policy}(b, s, o)}{ATRW_{t-1}^{base}(b, s, o)} \right) = \beta(s, o) \left( \frac{D_t^{policy}(b, s, o)}{D_t^{base}(b, s, o)} - \frac{L_t^{policy}(b, s, o)}{L_t^{base}(b, s, o)} \right) \\ \text{for all } b, s, \text{ local occupations } o$$

Vacancies and movement into employment activities

$$V_t(act) = E_t(act) - H_t[act; act] \quad \text{for all local employment activities } act$$

$$H_t(cat; act) = V_t(act) * \left[ \frac{L_t(cat; act)}{\sum_{v \neq act} L_t(v; act)} \right] \quad \begin{array}{l} \text{for all categories } cat \neq \text{activities } act \text{ \&} \\ \text{for all local employment activities } act \end{array}$$

TABLE 2G Labour Market Mechanism (continued)

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Vacancies and movement into employment activities (continued)

$$H_t(cat; cat) = CAT_t(cat) - \sum_{act \neq cat} H_t(cat; act) \quad \text{for all employment categories } cat$$

Movements into unemployment activities and migrant destinations

$$H_t(cat; u) = \begin{cases} L_t(cat; u) + [\mu(cat) * CAT_t(cat)] & \text{for } S \text{ unemployment activity } u \\ 0 & \text{for } L \text{ unemployment activity } u \end{cases}$$

for all local employment categories  $cat$

$$H_t(cat; u) = \begin{cases} 0 & \text{for } S \text{ unemployment activity } u \\ CAT_t(cat) - \sum_{act \in eact} H_t(cat; act) & \text{for } L \text{ unemployment activity } u \end{cases}$$

for all local unemployment  $cat$  & all unemployment activities  $u$

$$H_t(cat; u) = \begin{cases} CAT_t(cat) - \sum_{act \in leact} H_t(cat; act) & \text{for } cat \text{ dom, leg \& } S \text{ unemp act } u \\ CAT_t(cat) - \sum_{act \in leact} H_t(cat; act) & \text{for } cat \text{ fgn, leg \& auk migrant act } u \\ CAT_t(cat) - \sum_{act \in leact} H_t(cat; act) & \text{for } cat \text{ fgn, ill \& zmn migrant act } u \\ 0 & \text{otherwise} \end{cases}$$

for all new  $cat$  & all unemployment or migrant activities  $u$

$$H_t(cat; u) = \begin{cases} L_t(cat; u) & \text{for all local } cat \\ CAT_t(cat) - \sum_{act \in leact} H_t(cat; act) & \text{for all foreign } cat \end{cases}$$

for all non-new  $cat$  & migrant activities  $u$

$$\sum_{cat} H_t(cat; act) = E_t(act)$$

for all local unemployment & migrant activities  $act$

---

TABLE 2H    *Notation in the Exposition of the Labour Market Mechanism*

## Categories and activities

$CAT_t(b, s, f)$	Number of people at the start of year $t$ who are from birthplace $b$ , have status $s$ , and who performed labour force function $f$ in year $t-1$
$CAT_t(b, s, 'new')$	Number of people at the start of year $t$ who are from birthplace $b$ , have status $s$ , and were not in the extended labour force in year $t-1$
$ACT_{t-1}(b, s, f)$	Number of people in labour force activity $(b, s, f)$ during year $t-1$

## Labour supply and labour demand

$L_t(cat; act)$	Labour supply of people in category $cat$ to activity $act$ during year $t$ with both $cat$ and $act$ described by the dimensions $(b, s, f)$
$PREF_t(cat; act)$	Variable reflecting the labour supply preferences of people in category $cat$ for working in activity $act$ during year $t$
$D_t(b, s, o, j)$	Demand for labour inputs by industry $j$ for employment activity $o$ with $(b, s)$ characteristics
$D_t(j)$	Total labour input to industry $j$
$\psi$	Homothetic function
$Z_t(j)$	Activity of industry $j$
$BTRW_t(j)$	Overall before-tax real wage to industries
$K_t(j)$	Capital stock of industry $j$
$A_t(j)$	Other variables that influence demand for labour in industry $j$
$BTRW_t(b, s, o)$	Before-tax real wage of workers in employment activity $o$ with $(b, s)$ characteristics
$ATRW_t(b, s, o)$	After-tax real wage of workers in employment activity $o$ with $(b, s)$ characteristics
$ATRW_t(b, s, u)$	After-tax real wage received by labour in unemployment activity $u$ representing some form of social security payment
$TL_t(b, s)$	Labour tax rate applying to all $(b, s)$ workers in employment activity $o$
$F_t(b, s)$	Fraction of $BTRW_t^{ave}(b, s)$ received in unemployment activity $u$

TABLE 2H Notation in the Exposition of the Labour Market Mechanism (continued)

## Wage adjustment mechanism

$L_t(b, s, o)$	Total labour supply by $(b, s)$ workers to employment activity $o$
$D_t(b, s, o)$	Total labour demand for $(b, s)$ workers in employment activity $o$
$ATRW_t^{base}(b, s, o)$	Base or forecast value of $ATRW_t(b, s, o)$
$BTRW_t^{base}(b, s, o)$	Base or forecast value of $BTRW_t(b, s, o)$
$L_t^{base}(b, s, o)$	Base or forecast value of $L_t(b, s, o)$
$D_t^{base}(b, s, o)$	Base or forecast value of $D_t(b, s, o)$
$\beta$	Positive parameter that controls the response of wage rates to gaps between labour demand and supply

## Vacancies and movement into activities

$E_t(act)$	Total employment in activity $act$
$V_t(act)$	Vacancies in employment activity $act$
$H_t[c; a]$	Actual flow of people from category $cat$ to activity $act$ during year $t$
$\mu$	Fraction of people in employment category $cat$ at the start of year $t$ who become involuntarily unemployed during year $t$

## Description of set elements

$b$	Denotes birthplace of workers with set elements distinguishing domestic ( <i>dom</i> ) and foreign ( <i>fgn</i> )
$s$	Denotes status of workers with set elements distinguishing legal ( <i>leg</i> ) and illegal ( <i>ill</i> )
$f$	Denotes labour force functions of workers with set elements distinguishing employment in 11 local occupations, two foreign occupations ( <i>auk</i> & <i>zmn</i> ), two types of unemployment ( <i>S</i> & <i>L</i> ) and new entrants at the start of the year
$j$	Denotes local industries with set elements distinguishing 27 different multi-product industries



### *Establishing labour categories and activities*

Our first task in the labour market specification is the division of labour into identifiable categories at the start of each year reflecting activities of people in the previous year. At the start of each year everyone in the extended labour force, including new entrants, offers their services to an activity. Depending on various labour supply and demand factors, workers then flow into a particular activity during the year that determines their end-of-year position. At the end of each year it is assumed that one per cent of people in every activity drop out of the labour force through either death or retirement. New entrants to the labour force are exogenously specified at the start of each year to correspond with population growth rates representative of each region in the model. As indicated by Figure 2C, this dynamic process is repeated every year.

Start-of-year categories and end-of-year activities are defined over multiple dimensions that describe the birthplace (*b*), status (*s*) and labour market function (*f*) of workers. Each set within the (*b, s, f*) dimensions contain elements detailing the characteristics of workers in the extended labour force. We distinguish between i) domestic (*dom*) and foreign (*fgn*) born; ii) legal (*leg*) and illegal (*ill*) status; and iii) labour force functions that include employment in three local skilled occupations, eight local lower skilled occupations, short-term (*S*) and long-term (*L*) unemployment, a foreign skilled (*auk*) and foreign lower-skilled (*zmn*) occupation for migrants, and new entrants (*new*) to the labour market. The choice of local occupation groups is based on the classification used in labour force surveys conducted by Statistics South Africa (StatsSA, 2007; 2009). In the local job market, skilled occupation groups include ‘legislators, senior officials and managers’ (*lsm*), ‘professionals’ (*prf*) and ‘technical and associate professionals’ (*tch*). Lower-skilled occupation groups include ‘clerks’ (*clk*), ‘service and sales workers’ (*srv*), ‘agriculture and fishery workers’ (*sag*), ‘craft and related trades workers’ (*crf*), ‘plant and machine operators and assemblers’ (*opr*), ‘elementary workers’ (*elt*), ‘domestic workers’ (*dwk*) and ‘other or unspecified workers’ (*usf*).

Within the context of the labour market description, ‘local’ refers to South Africa, and ‘foreign’ to countries outside South Africa. The foreign occupation ‘*auk*’ represents employment of skilled workers outside South Africa. All workers in the skilled migrant category are assumed to have legal status and are therefore treated as potential skilled immigrants to South Africa. The domestic-born in *auk* represent the number of South African skilled emigrants in foreign occupations. The foreign occupation ‘*zmn*’ represents employment of foreign-born lower-skilled workers outside South Africa. All workers in the lower-skilled migrant category are assumed to have illegal status and are therefore treated as potential low skilled and illegal immigrants to South Africa. In the application of this study we assume that no worker who was born domestically can have illegal status. We also do not allow any transition in the birthplace or status characteristics of workers between periods in this study.

Thus, in modelling the flow of workers from categories to activities during the year we identify 16 start-of-year and 15 end-of-year labour market functions, two birthplace and two legal status dimensions. Labour market functions include the 11 local employment occupations, two types of unemployment, two types of foreign employment for migrants and new entrants to the labour market at the start of each year. Figure 2D, presented later in this chapter, gives a summary of the different labour flows possible between start-of-year categories and end-of-year activities in the ZAR-M framework.

The link between the number of people in different activities in year  $t-1$  and the number of people in each category at the start of year  $t$  is represented by the upward-sloping arrows in Figure 2C and specified by equations (E2.34–E2.35).

---


$$CAT_t(b, s, f) = ACT_{t-1}(b, s, f) * 0.99 \quad \text{for all } b, s \text{ and } f \neq \text{new} \quad (\text{E2.34})$$

$$CAT_t(b, s, 'new') = \text{exogenous} \quad \text{for all } b, s \quad (\text{E2.35})$$


---

### *Labour supply from categories to activities*

In the determination of labour supply from each category to each activity, we impose a number of assumptions and restrictions on the movement of labour within ZAR-M. We ensure that workers in a category with birthplace  $b$  and status  $s$  make offers only to activities with these characteristics. For example, people in the domestic-born / legal status / clerks worker ( $dom, leg, clk$ ) category can offer only to activities with the domestic-born ( $dom$ ) and legal status ( $leg$ ) characteristics. We also assume that the vast majority of workers in a particular employment category will offer to work in the same employment activity during the year. Reflecting the broad divisions in skill levels across the labour force, we also impose restrictions on the ability of workers to offer across different occupation groups via the theoretical specification and initial settings in the database of ZAR-M. Within the dimension of labour market functions ( $f$ ) we distinguish between ‘skilled’ and ‘lower-skilled’ employment activity sets. Skilled activities include employment in three local occupations ( $lsm, prf, tch$ ) and one foreign occupation ( $auk$ ). Lower-skilled activities include employment in eight local occupations ( $clk, srv, sag, crf, opr, elt, dwk, usf$ ) and one foreign lower-skilled occupation ( $zmn$ ). Workers employed in any of the eight local or one foreign lower-skilled categories are effectively only allowed to offer their services within these same nine employment activities. Alternatively, they may also offer to become voluntarily unemployed. Workers who are in either of the unemployment categories ( $S, L$ ) at the start of the year may offer to any employment activity or choose to remain unemployed. Initial settings in the database for ZAR-M assign weights to the share of skilled and lower-skilled workers in unemployment to ensure realistic flows from these categories into activities of different skill levels.

In ZAR-M workers make labour supply decisions based on relative wages between occupations they are allowed to offer to, and their preferences for earning money in a particular activity. Labour supply functions postulate that at the start of year  $t$ , people in category  $cat$  decide their offers to activity  $act$  during year  $t$  by solving an optimisation problem whereby they choose:

---


$$L_t(cat; act) \quad \text{for all } act(b, s, f)$$

to maximise

$$\psi_L [ATRW_t(act) * L_t(cat; act) \quad \forall \quad act(b, s, l)] \quad (E2.36)$$

subject to

$$\sum_{act} L_t(cat; act) = CAT_t(cat) \quad (E2.37)$$

where

$\psi_L$  is a homothetic utility function

---

In equations (E2.36) and (E2.37) workers in category  $cat$  treat money earned in different activities as imperfect substitutes. This is a convenient and flexible specification through which we can allow the supply of labour to shift between activities in response to after-tax payments received. By specifying a separate utility function for each  $cat$ , we ensure that each category makes offers to activities that are compatible with the category's birthplace, legal status and occupational characteristics. In ZAR-M,  $\psi_L$  has the CES form:

---

$$\psi_L = \left[ \sum_{act} \left[ PREF_t(cat; act) * ATRW_t(act) * L_t(cat; act) \right]^{\frac{\eta}{1+\eta}} \right]^{\frac{1+\eta}{\eta}} \quad (E2.38)$$

where

$\eta$  is a parameter reflecting the ease with which people feel they can shift between activities

---

The  $PREF_t(cat; act)$ 's play two important roles in the analysis of the labour market. The first is via their initial settings, that is, the values assigned to them in the base year data derived from the strengths of initial labour offers. For example, by setting  $PREF_{2006}(cat; act)$  at relatively high values when  $cat$  and  $act$  agree in their  $(b, s)$  characteristics and have a

labour force function referring to the same occupation, we ensure that most people employed in occupation  $o$  in year  $t-1$  offer to continue to work in  $o$  in year  $t$ . We are also able to restrict certain offers by setting  $PREF_{2006}(cat; act)$  at zero. This allows us to, for example, ensure that no person in any of the other or unskilled employment categories in year  $t-1$  is able to offer to work in a skilled employment activity in year  $t$ . Similarly, we ensure that no person can stay in short-run unemployment in successive years or move from long-run unemployment back to short-run unemployment. By setting  $PREF_{2006}(cat; act)$  at an appropriately large value where both  $cat$  and  $act$  have the functional characteristic of unemployment, we introduce a discouraged-worker or hysteresis effect for the unemployed.

The second role of the  $PREF_t(cat; act)$ 's is to introduce shocks in the policy simulation runs. By exogenously changing the preference of workers towards or away from a particular activity, we are able to simulate the impact of various labour market policies in ZAR-M. Examples may include policy interventions that increase border security or penalties on employers of illegal workers that will reduce the preference of foreign-illegal workers for moving to South Africa. These policies and their application via  $PREF_t(cat; act)$  will be discussed more thoroughly in Chapter 5.

Under (E2.38), the optimisation problem in (E2.36–E2.37) generates labour-supply functions of the form:

---


$$L_t(cat; act) = CAT_t(cat) * \left[ \frac{[PREF_t(cat; act) * ATRW_t(act)]^\eta}{\sum_q [PREF_t(cat; q) * ATRW_t(q)]^\eta} \right] \quad (E2.39)$$

*for all cat & act*

with total supply of labour to activity  $act$  obtained as

$$L_t(act) = \sum_{cat} L_t(cat; act) \quad \text{for all cat \& act} \quad (E2.40)$$


---

The use of the parameter  $\eta$  in (E2.38) is a minor variation of the standard CES functional form. Interpreting its role within the labour market mechanism is illuminated when writing the labour-supply function in (E2.39) in linearised or percentage-change form.<sup>6</sup> Readers familiar with GEMPACK will immediately recognise the linearised CES form of (E2.41) with lower-case symbols representing percentage changes in the variables denoted by their corresponding upper-case symbols. From here we can easily interpret  $\eta$  as the substitution elasticity of workers in making their labour supply decisions.

---


$$l_t(cat; act) = cat_t(cat) + \eta^* [atrw_t(act) - atrw_t^{ave}(cat)] + \eta^* [pref_t(cat; act) - pref_t^{ave}(cat)]$$

(E2.41)

---

Equation (E2.41) implies that workers in category  $cat$  will switch their offers towards activity  $act$ , with  $cat \neq act$ , if the after-tax real wage rate in activity  $act$  rises relative to an average of the wage rates across all the activities in which category  $cat$  workers could participate. With  $\eta$  typically set at around 1.5, we assume that the number of workers who wish to change jobs, in particular potential immigrants, are quite sensitive to changes in relative wages. However, an increase in  $ATRW_t(act)$  does not have much effect on  $L_t(act; act)$ . Since a major part of labour supply from category  $cat$  to any employment activity  $act$  is from incumbents, that is,  $L_t(act; act)$  is a large fraction of  $L_t(act)$ , the term  $[atrw_t(act) - atrw_t^{ave}(cat)]$  is always close to zero. Effectively, the labour-supply elasticity is therefore relatively low. Policy changes affecting the labour supply preferences of workers via  $pref_t(cat; act)$  may be similarly interpreted.

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<sup>6</sup> The parameter  $\eta$  in (E2.38) is equivalent to the parameter  $\sigma - 1$  used in standard CES notation. This alternative implementation is required because of the appearance of the price term  $ATRW$  in the utility maximisation problem in (E2.36 – E2.37).

### *Labour demand and employment*

In determining labour demand and employment within ZAR-M, we first determine the amount of industry-level labour inputs. We specify labour inputs employed by local industry  $j$  along conventional CGE lines as a function of the industry's overall activity level  $Z_t(j)$ , overall before-tax real wage  $BTRW_t(j)$ , capital stock  $K_t(j)$ , and other variables such as technical change  $A_t(j)$ . The overall before-tax real wage rate to industry  $j$  is determined as an average of the wage rates applying to all types of labour employed by the industry. These industry-level outcomes are represented via (E2.42) and (E2.43).

---


$$D_t^1(j) = Z_t(j) * \psi^1[BTRW_t(j), K_t(j), A_t(j)] \quad (\text{E2.42})$$

$$BTRW_t^1(j) = \psi^1[BTRW_t(b, s, o) \forall \text{ local occupations } o] \quad (\text{E2.43})$$


---

Within local industry  $j$ 's labour input, the demand for labour by birthplace  $b$ , status  $s$  and local occupation  $o$  is determined by solving a nested CES cost minimisation problem of the form (E2.44–E2.47). Appropriate substitution elasticities between different occupation types, place of birth and legal status of workers are implemented via this functional form. Industry  $j$  therefore satisfies its labour requirements by choosing:

---


$$D_t(b, s, o, j), D_t^3(s, o, j), \text{ and } D_t^2(o, j)$$

to minimise

$$\sum_{b, s, o} BTRW_t(b, s, o) * D_t(b, s, o, j) \quad (\text{E2.44})$$

subject to

$$D_t^1(j) = CES_o[D_t^2(o, j)] \quad (\text{E2.45})$$

$$D_t^2(o, j) = CES_s \left[ D_t^3(s, o, j) \right] \quad (\text{E2.46})$$

$$D_t^3(s, o, j) = CES_b \left[ D_t(b, s, o, j) \right] \quad (\text{E2.47})$$

Under the CES functional form, the optimisation problem in (E2.44–E2.47) generates nested labour-demand functions for all  $(b, s)$  workers in local occupations  $o$  by industry  $j$  of the form:

$$D_t(b, s, o, j) = D_t(j) * \psi_D \left[ BTRW_t(bb, ss, oo) \quad \forall \text{ } bb, ss \text{ and local occupations } oo \right] \quad (\text{E2.48})$$

which can be expressed in linearised or percentage-change form as

$$\begin{aligned} d_t(b, s, o, j) = & d_t(j) - \sigma_o * [btrw_t(o, j) - btrw_t(j)] \\ & + \sigma_s * [btrw_t(s, o, j) - btrw_t(o, j)] \\ & + \sigma_b * [btrw_t(o, b, s) - btrw_t(s, o, j)] \end{aligned} \quad (\text{E2.49})$$

with total demand for  $(b, s)$  labour in local occupation  $o$  obtained as

$$D_t(b, s, o) = \sum_j D_t(b, s, o, j) \quad (\text{E2.50})$$

Once again, readers familiar with GEMPACK will recognise the nested CES form of (E2.49) with lower-case symbols representing percentage changes in the variables denoted by their corresponding upper-case symbols. The implementation of the various substitution elasticities, represented by the  $\sigma$ 's, is also clearly illustrated via (E2.49). Due to the lack of reliable data on the estimation of these  $\sigma$  parameters for South Africa, these values typically represent judgments in ZAR-M. However, evidence for other countries do suggest that substitution possibilities between occupations,  $\sigma_o$ , are quite low (typically between 0.3 and



0.5) in comparison to the substitution possibilities between status,  $\sigma_s$ , and birthplace,  $\sigma_B$  (typically between 4 and 7) (Dixon *et al*, 2011). These and other data issues will be discussed more thoroughly in Chapter 3. In equation (E2.51) we show that employment of all  $(b, s)$  workers in local occupation  $o$  is determined by the corresponding demand.

---


$$E_t(b, s, o) = D_t(b, s, o) \quad (\text{E2.51})$$


---

### *Wage adjustment mechanism*

Following the wage adjustment process described for policy simulations in USAGE-M, we assume that after-tax real wages for all local occupations  $o$  in ZAR-M adjusts according to equation (E2.52).

---


$$\left( \frac{ATRW_t(b, s, o)}{ATRW_t^{base}(b, s, o)} \right) - \left( \frac{ATRW_{t-1}(b, s, o)}{ATRW_{t-1}^{base}(b, s, o)} \right) = \beta(s, o) \left( \frac{D_t(b, s, o)}{D_t^{base}(b, s, o)} - \frac{L_t(b, s, o)}{L_t^{base}(b, s, o)} \right) \quad (\text{E2.52})$$


---

In the baseline forecast, after-tax real wages  $(ATRW_t^{base})$  and labour demand  $(D_t^{base})$  are typically set according to available projections. Labour supply  $(L_t^{base})$  evolves naturally according to initial settings in the base year data, and the exogenous addition of new entrants. In policy simulations, these variables are typically set as endogenous. This allows us to evaluate the impact of the policy under consideration on these variables. Equation (E2.52) implies that if a policy causes the demand for  $(b, s, o)$  labour to increase relative to its supply, then after-tax real wages between years  $t-1$  and  $t$  would increase relative to its baseline values. Similarly, if a policy causes the supply of labour to decrease relative to demand, after-tax real

wages may be expected to rise for the particular type of labour under investigation. The positive parameter  $\beta(s, o)$  reflects the speed of adjustment in the labour market, that is, it controls the response of after-tax real wage rates to gaps between labour demand and supply. In ZAR-M, equation (E2.52) therefore has the role of determining after-tax real wage rates for local occupations. Before-tax real wage rates for local occupations  $o$  and unemployment activities  $u$  are then determined by applying the appropriate labour tax rates and social benefits received according to (E2.53–E2.54).

---


$$ATRW_t(b, s, o) = BTRW_t(b, s, o) * [1 - TL_t(b, s)] \quad (\text{E2.53})$$

$$ATRW_t(b, s, u) = BTRW_t^{ave}(b, s) * F_t(b, s) \quad (\text{E2.54})$$


---

### *Vacancies and actual labour flows into activities*

Our final task in the labour market specification is the determination of everyone's activity during year  $t$ , that is, who finds employment and what happens to those who do not. This specification is required because under equation (E2.52), markets for local occupations in ZAR-M do not clear. That is, labour demand and labour supply are not equal in the short-run. The link between workers in category *cat* at the start of year  $t$  and activities *act* during year  $t$  is represented by the downward-sloping arrows in Figure 2C and specified by equations (E2.55–E2.62). Table 2I serves as a useful reference in keeping track of different labour flows from start-of-year categories to end-of-year activities in the economy. Flows shown in Table 2I are aggregated over all birthplaces and legal status of workers. Areas in which no number appears represent flows which are either not allowed or deemed possible within the context of this study and the ZAR-M theoretical specification and database. Such non-permitted flows are typically modelled by equations setting the particular flow equal to zero.

TABLE 2I      *Flows from Start-of-Year Categories to End-of-Year Activities*

<div> <div>Activities end-of-yr</div> <div>Categories st-of-yr</div> </div>		Skilled Local Employment <i>for all (b,s)</i>			Lower Skilled Local Employment <i>for all (b,s)</i>								Local Unemployment <i>for all (b,s)</i>		Foreign Migrants <i>for all (b,s)</i>	
		<i>lsm</i>	<i>prf</i>	<i>tch</i>	<i>c</i>	<i>s</i>	<i>s</i>	<i>c</i>	<i>o</i>	<i>e</i>	<i>d</i>	<i>u</i>	<i>short</i>	<i>long</i>	<i>zmn</i>	<i>auk</i>
Skilled Local Employment <i>for all (b,s)</i>	<i>lsm</i>	1	1	1	-	-	-	-	-	-	-	-	2	-	-	5
	<i>prf</i>	1	1	1	-	-	-	-	-	-	-	-	2	-	-	5
	<i>tch</i>	1	1	1	-	-	-	-	-	-	-	-	2	-	-	5
Lower Skilled Local Employment <i>for all (b,s)</i>	<i>clk</i>	-	-	-	1	1	1	1	1	1	1	1	2	-	5	-
	<i>srv</i>	-	-	-	1	1	1	1	1	1	1	1	2	-	5	-
	<i>sag</i>	-	-	-	1	1	1	1	1	1	1	1	2	-	5	-
	<i>crf</i>	-	-	-	1	1	1	1	1	1	1	1	2	-	5	-
	<i>opr</i>	-	-	-	1	1	1	1	1	1	1	1	2	-	5	-
	<i>elt</i>	-	-	-	1	1	1	1	1	1	1	1	2	-	5	-
	<i>dwk</i>	-	-	-	1	1	1	1	1	1	1	1	2	-	5	-
	<i>usf</i>	-	-	-	1	1	1	1	1	1	1	1	2	-	5	-
Local Unemployment <i>for all (b,s)</i>	<i>short</i>	1	1	1	1	1	1	1	1	1	1	1	-	3	5	5
	<i>long</i>	1	1	1	1	1	1	1	1	1	1	1	-	3	5	5
Foreign Migrants <i>for all (b,s)</i>	<i>zmn</i>	-	-	-	1	1	1	1	1	1	1	1	-	-	5	-
	<i>auk</i>	1	1	1	-	-	-	-	-	-	-	-	-	-	-	5
New Entrants <i>for all (b,s)</i>	<i>new</i>	1	1	1	1	1	1	1	1	1	1	1	4	-	4	4

Each numbered area in Table 2I indicates a particular type of labour flow in ZAR-M. The equations (E2.55–E2.62) governing these flows from start-of-year categories to end-of-year activities are explained in the sections hereafter. Features of the actual data contained within Table 2I are discussed in Chapter 3.

**Area 1** in Table 2I represents flows from all categories  $cat$  to local employment activities  $act$  that may occur during the year. These flows into local employment activities are described by equations (E2.55–E2.57).

---


$$V_t(act) = E_t(act) - H_t[act; act] \quad (E2.55)$$

$$H_t(cat; act) = V_t(act) * \left[ \frac{L_t(cat; act)}{\sum_{v \neq act} L_t(v; act)} \right] \quad \text{for } cat \neq act \quad (E2.56)$$

$$H_t(cat; cat) = CAT_t(cat) - \sum_{act \neq cat} H_t(cat; act) \quad (E2.57)$$


---

In (E2.55) we define vacancies in local employment activity  $act$  in year  $t$  as employment in the particular activity during year  $t$  minus the number of jobs filled by incumbents in that activity. Incumbents refer to locally employed workers whose start-of-year category and end-of-year activity ( $b, s, o$ ) characteristics, did not change. In (E2.56) we model the flow of non-incumbents to local employment activity  $act$  as being proportional to the vacancies in that activity and the share of category  $cat$  in the supply of labour to activity  $act$  from workers outside category  $act$ . In modelling (E2.56) we assume that there is always competition for jobs, that is, we assume the number of people from outside category  $act$  who plan to work in employment-activity  $act$  is greater or equal to the number of vacancies in  $act$ . This ensures that  $H_t(cat; act)$  will be less than or equal to  $L_t(cat; act)$  for all categories  $cat \neq act$  and that  $V_t(act)$  will not become negative. Incumbents from employment-category  $cat$  who remain in activity  $cat$  are defined in (E2.57) as the number of workers in category  $cat$  minus the number who move out of activity  $cat$  during the year. Workers in employment-category  $cat$  who planned to work in a different activity  $cat \neq act$  but who are unable to move to  $act$  due to insufficient vacancies simply remain in  $act$ .

In the application of ZAR-M to this study, we distinguish between ‘skilled’ and ‘lower-skilled’ local occupations, and ‘skilled’ and ‘lower-skilled’ migrant regions. We do not allow any offers or actual flows across these different groups. The different skill groups are modelled separately with similar equations to (E2.55–E2.57) specified for each broad skill group. If workers in any of the lower-skilled employment categories wish to become employed in any of the skilled activities, they have to move into unemployment first. In subsequent years these workers may then be able to offer from one of the unemployment categories to one of the skilled-employment activities in ZAR-M. The time spent in unemployment would then represent the time required by these workers to gain the necessary skills via training or further study to fulfil the requirements of a skilled position.

**Area 2** in Table 2I represents flows from all local employment categories  $cat$  to local unemployment activities  $u$ . These flows into short-run unemployment are described by equation (E2.58). It is not possible in ZAR-M to flow from an employment category directly into long-run unemployment.

---


$$H_t(cat; u) = L_t(cat; u) + [\mu(cat) * CAT_t(cat)] \quad (E2.58)$$


---

The number of workers who flow into short-run unemployment comprise of two parts. The first is voluntary moves denoted by  $L_t(cat; u)$ . Involuntary moves, that is, workers who get sacked, are determined as a fraction,  $\mu(cat)$ , of the number of workers in the particular category. When the conditions that generate positive values for  $V_t(act)$  hold,  $\mu(cat)$  is typically set at an exogenously given minimum value appropriate for each  $(b, s, o)$ . However, it is possible that (E2.58) in conjunction with (E2.56) may give values for  $H_t(cat; cat)$  in (E2.57) that exceed  $E_t(act)$ . Such a scenario would generate negative values for  $V_t(act)$  in (E2.55). To avoid this situation we then allow  $\mu(cat)$  to become endogenous and increase

to a level which ensures that  $V_t(act)$  is at least zero. When  $\mu(cat)$  moves above its minimum value, then there are involuntary flows from employment-category  $cat$  to unemployment caused by an overall shortage of jobs.

**Area 3** in Table 2I represents flows from all local unemployment categories  $cat$  to local unemployment activities  $u$ . These flows into long-run unemployment are described by equation (E2.59). It is not possible in ZAR-M to remain in short-run unemployment in successive years or flow from long-run unemployment back to short-run unemployment.

---


$$H_t(cat; u) = CAT_t(cat) - \sum_{act \in eact} H_t(cat; act) \quad (E2.59)$$


---

The number of unemployed workers who flow into long-term unemployment is the sum of those workers in both short-term and long-term unemployed categories who fail to obtain a job in any employment activity during the year. In the implementation of (E2.59), we allow a large share of foreign-born workers to flow back to their respective countries of origin should they fail to obtain local employment in successive years.

**Area 4** in Table 2I represents flows from new entrant category  $cat$  to all local unemployment or migrant activities  $u$ . These flows into unemployment and migrant activities are described by equation (E2.60). It is not possible in ZAR-M for new entrants to flow directly into long-run unemployment. We do not allow foreign-born new entrants who fail to obtain a job in any local employment activity to flow into local unemployment. Movement across any broad skill classification by foreign-born new entrants are also not allowed.

---


$$H_t(cat; u) = \begin{cases} CAT_t(cat) - \sum_{act \in leact} H_t(cat; act) & \text{for } cat \text{ dom, leg \& } S \text{ unemp act } u \\ CAT_t(cat) - \sum_{act \in leact} H_t(cat; act) & \text{for } cat \text{ fgn, leg \& auk migrant act } u \\ CAT_t(cat) - \sum_{act \in leact} H_t(cat; act) & \text{for } cat \text{ fgn, ill \& zmn migrant act } u \end{cases}$$

(E2.60)

---

The number of domestically-born new entrants who flow into short-term unemployment is the sum these new entrant category workers who fail to obtain a job in any local employment-activity *act*. Similarly, foreign-born new entrants who flow into their respective migrant activities are those who fail to obtain a job in any local employment-activity *act*. As may be expected, the vast majority of foreign-born new entrants end up flowing into foreign activities. Given the restrictions imposed in the implementation of (E2.60), we model these particular flows using three separate equations.

**Area 5** in Table 2I represents flows from all non-new categories *cat* to all foreign migrant activities *u*. These flows into migrant activities are described by equation (E2.61). As indicated by the zeros in Table 2I, we maintain the assumption that no flows across broad skill groups, that is, skilled and low skilled occupations, are possible.

---


$$H_t(cat; u) = \begin{cases} L_t(cat; u) & \text{for all local cat} \\ CAT_t(cat) - \sum_{act \in leact} H_t(cat; act) & \text{for all foreign cat} \end{cases}$$

(E2.61)

---

The number of workers who flow from any local category into any foreign migrant activity is assumed to be voluntary. That is, all offers to activities outside South Africa are effectively accepted within the broad skill classification. Via the initial settings in the database and theoretical specification of ZAR-M, these flows typically represent the natural flow of foreign migrants who return home to their country of birth, or in this application, the migrant destination compatible with status and skill classification. These flows also capture the emigration of domestically-born skilled workers to the foreign occupation *aus*. Since flows from employment in a migrant category to local unemployment are not allowed, the flow of migrant workers to migrant activities are simply determined as the number of workers in the foreign migrant category at the start of the year minus the number that obtain jobs in local South African employment activities during the year.

#### *Completing the link between categories and activities*

To complete the link from categories at the start of year  $t$  to activities during year  $t$  we include equation (E2.62). A similar equation for all local employment activities is not required since it is implied by equations (E2.55) and (E2.56).

---


$$\sum_{cat} H_t(cat; act) = E_t(act) \quad \text{for all local unemployment \& migrant activities } act$$

(E2.62)

---

The system of equations (E2.34–E2.62) described in this section along with Table 2I detail the labour market mechanism in ZAR-M. Calibration of initial settings on both the demand and supply-side of the labour market is essential to ensure the proper functioning of this mechanism in a recursive-dynamic framework as outlined in Figure 2C. These and other complementary data issues will be discussed more thoroughly in Chapter 3.



## 2.5 Concluding Remarks

In this chapter we described the theoretical structure of the ZAR-M model. ZAR-M is a recursive-dynamic CGE model of the South African economy whose theoretical foundations are based on the MONASH (Dixon & Rimmer, 2002) and USAGE-M (Dixon *et al*, 2011) models. The accompanying database to ZAR-M in this study represents the South African economy for the base year 2006. To provide readers with a general overview and intuitive understanding of the model, we first set out a stylized representation of the model in Table 2A named BOTE-M. Within this framework all the key macroeconomic relationships in ZAR-M were described. In subsequent sections we gave a detailed exposition and description of the capital and labour market mechanisms in the full ZAR-M model.

The capital accumulation mechanism in ZAR-M forms an integral part of the dynamics of the model. Based on the standard MONASH specification in Dixon & Rimmer (2002:190-194) and outlined in Table 2C, 2D and 2F, it shows that additions to the capital stock for local industries in future periods are a function of net investment in the current period. Investment within this framework is specified as a function of expected rates of return on capital for each industry.

The theoretical specification of the labour market mechanism in ZAR-M closely follows that of USAGE-M developed in Dixon *et al* (2011), with added detail to facilitate the analysis of skilled migrants to and from South Africa. This specification, summarised in Table 2G, represents a leap in the ability of CGE models to deal with migrant flows in a recursive-dynamic framework. Six key ingredients in the labour market mechanism are identified and described according to equations (E2.34–E2.62). These include i) the division of the labour force into categories at the start of each year reflecting labour force functions of people in the previous year; ii) the identification of labour force activities, that is, what people do during the year; iii) the determination of labour supply from each category to each activity; iv) the determination of labour demand in employment activities; v) the specification of wage

adjustment processes reflecting labour demand and supply; and vi) the determination of everyone's activity, that is, who finds employment and what happens to those who do not.

Proper functioning of the labour market mechanism in ZAR-M relies heavily on the calibration of initial settings for both supply and demand for different types of labour. The careful construction of initial flows from activities in year  $t-1$  to categories in year  $t$  and choice of parameters in the determination of these flows are therefore essential. A description of the model's database highlighting these labour and migration elements is given in Chapter 3.





# CHAPTER 3

## MODEL DATABASE

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### 3.1 Introduction

Computable or applied general equilibrium models require data to produce quantitative results. The structure of a typical CGE model database is based upon a country's input-output data for a given year. The theory of the model is then essentially a set of equations that describe how the cells of the input-output database move through time and move in response to given policy shocks.

One of the distinguishing features of the Johansen computational framework is its ability to cope with many, highly disaggregated, dimensions. Modellers are therefore able to conduct simulations across multiple industries, commodities, occupations or household types. For example, the USAGE model of the United States can be run with up to 500 industries and 700 occupation types across 51 regions.<sup>1</sup> Credibility enhancing detail is easily added within this framework. For example, purchaser's prices are usually disaggregated to reflect the contribution of basic prices, margin costs and taxes in each commodity.

With the GEMPACK software package, the model theory or TABLO code is implemented on a database, typically written in header array files containing all the necessary input data.<sup>2</sup> For single-country models such as ZAR-M, these data files combine to give a snapshot of the country's economic structure in a particular year, referred to as the model's base year. A typical CGE model's input-output database can be set out in three parts: an absorption matrix; a joint-production matrix; and a vector of import duties. Additional data may also be required depending on the type of CGE model and its application. For example, detailed labour market databases were required for ZAR-M to model migrant flows in South Africa. The construction of these labour databases is discussed later in this chapter.

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<sup>1</sup> USAGE is a large-scale dynamic CGE model of the United States developed by the Centre of Policy Studies in collaboration with the U.S. International Trade Commission. The theoretical structure of USAGE is similar to that of the MONASH model of Australia (Dixon & Rimmer, 2002).

<sup>2</sup> GEMPACK is a suite of software applications developed specifically for general equilibrium modelling within the Johansen framework. The model theory is written in TABLO code using the TABmate application. Data files are prepared in the suite's ViewHAR application and read from header array files.

The base year chosen in this study is 2006. It represents a year in which South Africa's most important economic indicators remained relatively stable throughout. The main data sources used to produce the ZAR-M database for the base year were the supply and use tables (StatsSA, 2008; 2010d), social accounting matrix (StatsSA, 2010a), national accounts and balance of payments data (SARB, 2007; 2010) and labour force survey data (StatsSA, 2007; 2009). Guidelines from the United Nations Handbook of National Accounting (UNDESA, 1999) were also followed to construct parts of the database according to the required dimensions. The information contained within supply and use tables encapsulates that found in input-output tables produced in other countries. The final database also includes parameter values for various price and substitution elasticities used in the model. These values are informed by econometric studies where available. The core ZAR-M database and parameter values are documented in Bohlmann (2010) and downloadable from the Centre of Policy Studies website. We briefly highlight some of its most important numbers in the following section.

### 3.2 Input-Output Database

#### *The structure of a stylized CGE model database*

Figure 3A is a schematic representation of a typical CGE model's core input-output database, consisting of an absorption matrix, joint-production matrix and vector of import duties. It reveals the basic structure of the model's database which may be disaggregated along many factors of production and final users, with multiple dimensions possible within each. We use the GEMPACK naming system, made popular in CGE models such as ORANI-G and MONASH, to label the main data cells within the absorption matrix.<sup>3</sup>

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<sup>3</sup> See Harrison & Pearson (1996) or Horridge (2000) for more details on the GEMPACK naming system. Each cell in the illustrative absorption matrix contains the name of a corresponding data matrix.

FIGURE 3A *Stylized Representation of a CGE Model Database*

			<i>Absorption Matrix</i>					
			1	2	3	4	5	6
			Producers	Investors	Household	Export	Government	Inventories
			← I →	← I →	← H →	← 1 →	← 1 →	← 1 →
Size								
1	Basic Flows	↑ C×S ↓	V1BAS	V2BAS	V3BAS	V4BAS	V5BAS	V6BAS
2	Margins	↑ C×S×M ↓	V1MAR	V2MAR	V3MAR	V4MAR	V5MAR	n/a
3	Taxes	↑ C×S ↓	V1TAX	V2TAX	V3TAX	V4TAX	V5TAX	n/a
4	Labour	↑ O ↓	V1LAB	<p> C = Number of commodities  I = Number of industries  S = Source (domestic, imported)  M = Number of commodities used as margins  O = Number of occupation types  H = Number of household types </p>				
5	Capital	↑ 1 ↓	V1CAP					
6	Land	↑ 1 ↓	V1LND					
7	Production Taxes	↑ 1 ↓	V1PTX					
8	Other Cost Tickets	↑ 1 ↓	V1OCT					

<i>Joint Production Matrix</i>	
Size	← I →
↑ C ↓	MAKE

<i>Tariffs</i>	
Size	← 1 →
↑ C ↓	V0TAR

Source: Adapted from Horridge (2000)



Columns 1 to 6 in the absorption matrix indicate users or demanders of commodities in the economy. They include i) producers, ii) investors, iii) households, iv) exporters, v) the government, and vi) change in inventories. Local producers and investors are divided across  $I$  industries, whilst households are divided across  $H$  household types. Each of the  $C$  commodity types identified in the model can be obtained locally or imported from overseas, indicated by its source dimension  $S$ . The model also distinguishes between  $M$  domestically produced goods used as margin services which are required to transfer commodities from their sources to their users. The sum of basic flows, margins and taxes are equal to flows valued in purchaser's prices. Commodity taxes are payable on goods and services purchased and explicitly accounted for in the model.

As well as intermediate inputs, current production requires inputs of primary factors. These include i) labour divided across  $O$  occupation groups, ii) fixed capital and iii) agricultural land. Production taxes include output taxes or subsidies that are not user-specific. Other cost tickets allow for miscellaneous taxes and fees charged on firms. The two external matrices in Figure 3A provide additional information on the structure of the economy. In principle, each of the  $I$  industries is capable of producing any of the  $C$  commodities. The joint production or MAKE matrix shows the value of output of each commodity by each industry. Tariffs on imports are assumed to be levied at rates which vary by commodity but not by user. The revenue obtained is represented by the tariff vector  $V0TAR$ .

Each cell in the illustrative absorption matrix in Figure 3A contains the name of a corresponding data matrix. For example,  $V3MAR$  is a 4-dimensional array showing the cost of  $M$  margins services on the flows of  $C$  goods, from source  $S$ , to each of the  $H$  household types. As noted before, credibility enhancing detail can easily be added within this framework.  $V1LAB$  is typically only a 2-dimensional array showing the value of labour inputs in occupation  $O$  for industry  $I$ . In ZAR-M however, the  $V1LAB$  matrix is further disaggregated to show the birthplace,  $B$ , and legal status,  $S$ , characteristics of wages earned by workers. That is,  $V1LAB$  is a 4-dimensional array in ZAR-M showing the compensation of employees by industry  $I$  according to their occupation  $O$ , birthplace  $B$  and legal status  $S$ .

characteristics. In some applications, modellers may also choose to sum data across certain dimensions. For example, labour and household source data were aggregated across all racial groups and income categories in constructing this database.

### *Key aggregates in the core ZAR-M database*

The ZAR-M model is implemented on a database representing the South African economy for the base year 2006. It features 27 industries and commodities, 11 local occupation groups and a single representative household. The core database, parameter values and data sources are documented in Bohlmann (2010) and downloadable from the Centre of Policy Studies website.

The main data sources used to construct the model's basic input-output structure were the 2006 supply and use tables (StatsSA, 2008) and the 2005 social accounting matrix (StatsSA, 2010a). Using Figure 3A as our guide, we describe the salient features and key aggregates of the economy in 2006 in this section. Readers should note that the sum of basic flows (BAS), margins (MAR) and taxes (TAX) for users, represented in columns 1 to 6 of the absorption matrix, adds up to their purchaser's price values (PUR).

- Gross domestic product (GDP) at market prices was R1,741 billion in 2006. From the supply side, we find this number by adding the values of labour inputs (V1LAB), capital rentals derived from gross operating surplus data (V1CAP), land rentals (V1LND), net production taxes (V1PTX), all indirect taxes (TAX) and tariffs (V0TAR) in the absorption matrix. From the demand side, we find GDP at market prices by adding the purchaser's values (VPUR) of all final users (columns 2 to 6) in the economy, minus the value of imports by final users. This is equivalent to the well-known GDP identity where  $GDP = C + I + G + (X - M)$ .
- Total intermediate use at purchaser's prices was R2,055 billion in 2006. We find this number by summing the values for producers (V1PUR) over all dimensions.

- Total final use or demand at purchaser's prices was R2,314 billion in 2006. We find this number by summing the purchaser's values for investors (V2PUR), households (V3PUR), exports (V4PUR), government (V5PUR) and inventories (V6PUR) over all dimensions.
- Components of final demand at purchaser's prices include household consumption of R1,088 billion, investment of R324 billion, government expenditure of R338 billion, exports of R515 billion and a combined change in inventories and residual value worth R47 billion.
- The value of the joint production matrix or MAKE matrix was R3,599 billion in 2006. This reflects the value of all domestically produced basic flows plus margins. We find these numbers by summing values for domestically sourced basic flows (BAS) and margins (MAR), respectively, over all users.
- The total value of margins used in 2006 was R363 billion, whilst taxes less subsidies on products in 2006 was R197 billion. We find these numbers by summing values for margins used (MAR) and taxes (TAX), respectively, over all users.
- The value of labour inputs was R755 billion in 2006. We find this number by summing the wage bill matrix (V1LAB) over all dimensions.
- Gross operating surplus was R758 billion in 2006. From this amount we allocate R732 billion to capital rentals (V1CAP) over all industries and R26 billion to land rentals (V1LND) over all industries using land for productive purposes. The total value of capital stock (VCAP) in the economy was estimated at R3,803 billion. This figure was calibrated to produce sensible positive values for gross capital growth (V2PUR/VCAP), net capital growth allowing for depreciation and rates of return (V1CAP/VCAP) across all industries.
- Net production taxes were R29,951 million in 2006. We find this number by summing the production taxes matrix (V1PTX) over all industries.
- Total exports in 2006 was worth R515,794 million, whilst total imports in 2006 was worth R573,495 million, resulting in a trade deficit of R57,701 million.
- The average gold price was \$605 per ounce in 2006, whilst the average ZAR/USD exchange rate was R6.78. The consumer price index rose by 4.6 per cent in 2006.

In this particular study, labour market data in ZAR-M is fully disaggregated to reflect the role of migrants in South Africa. Labour data, including employment, wage bill and migrant flows, are modelled with a labour force function, industry, birthplace and legal status dimension. A number of research reports and statistics were considered in producing these highly detailed matrices for the base year. The construction of these labour data elements will be discussed more thoroughly in the next section.

### 3.3 Labour Market Database

#### *Basic dimensions to the ZAR-M labour database*

The ZAR-M database contains detailed labour market data to describe the characteristics of workers and facilitate the dynamic modelling of migrant flows in South Africa. Following the theoretical specification of the ZAR-M model described in the previous chapter, base year data concerning the value of labour inputs and numbers of employed and unemployed are specified with birthplace (*b*) and legal status (*s*) dimensions on top of the usual occupation (*o*) and industry (*i*) dimensions. Initial data for labour flows between start-of-year categories and end-of-year activities are specified across labour force function (*f*), which includes all occupation groups as a subset, birthplace (*b*) and legal status (*s*) dimensions.

The labour force in ZAR-M takes on an expanded definition. It includes workers who are employed in any local occupation, the recently unemployed and discouraged work-seekers. The ZAR-M labour database also includes potential legal and illegal migrants working outside South Africa. Local employment data is specified for 11 occupation groups across 27 local industries. The choice of occupation groups in ZAR-M is based on the classification used in labour force surveys conducted by Statistics South Africa. It is split between three skilled and eight lower-skilled occupation groups. Skilled occupation groups include ‘legislators, senior officials and managers’ (*lsm*), ‘professionals’ (*prf*) and ‘technical and associate professionals’ (*tch*). Lower-skilled occupation groups include ‘clerks’ (*clk*), ‘service

and sales workers' (*srv*), 'agriculture and fishery workers' (*sag*), 'craft and related trades workers' (*crf*), 'plant and machine operators and assemblers' (*opr*), 'elementary workers' (*elt*), 'domestic workers' (*dwk*) and 'other or unspecified workers' (*usf*). Local employment data by occupation is then split according to the 27 industry classification in the 2006 supply and use tables (StatsSA, 2008).

Those in the labour force who are not employed at the start of the base year are classified as either short-term (*short*) or long-term (*long*) unemployed. New entrants (*new*) to the labour market are exogenously specified via a start-of-year category. To facilitate the flow of skilled and lower-skilled migrants, two foreign occupations groups are specified. The first of these foreign occupations, labelled *zmn* in the model database, represent a pool of potential low-skilled illegal migrants from countries outside South Africa. Illegal migrants returning home also flow into this occupation group. The second foreign occupations, labelled *auk* in the model database, represent both a destination for potential skilled legal emigrants from South Africa and a source of potential skilled immigrants to South Africa.

In modelling the initial offers and actual flow of workers from categories to activities in ZAR-M we therefore identify 16 start-of-year and 15 end-of-year labour market functions, two birthplace and two legal status dimensions. Labour market functions include the 11 local employment occupations, two types of unemployment, two types of foreign employment for migrants and new entrants to the labour market at the start of each year. Categories and activities are the same with the exception of new entrants (*new*), which is specified as a start-of-year category only. All workers in the ZAR-M labour database are modelled with a birthplace (*domestic* or *foreign*) and legal status (*legal* or *illegal*) dimension.

TABLE 3A *Extended Labour Force with Legal Status by Labour Market Function and Industry (thousands of people)*

<i>Labour Matrix</i>	agric	coal	gold	othmining	foodbev	textfoot	woodpaper	petrochem	glassnm	metmach	electrical	radioinstr	transequip	othmanuf	electricity	water	construc	trade	hotels	transser	communic	financial	estate	business	gengov	healthsoc	otherserv	Ind28	Ind29	noind
lsm	44	2	3	4	15	6	15	29	3	18	5	1	14	7	5	2	42	203	15	33	23	122	8	43	139	8	63	0	0	0
prf	14	2	2	4	5	2	8	15	1	10	3	2	4	4	3	1	20	64	2	10	11	86	4	59	92	39	135	0	0	0
tch	31	3	6	8	15	10	22	38	3	40	5	4	18	10	15	3	43	245	12	45	33	179	34	69	194	30	114	0	0	0
clk	31	3	3	6	13	4	15	27	2	25	6	2	12	7	6	1	33	315	13	40	32	197	9	53	298	27	70	0	0	0
srv	26	2	4	6	24	3	10	26	2	23	4	2	20	5	4	1	22	302	42	32	15	48	3	107	749	26	79	0	0	0
sag	210	1	2	2	18	1	4	4	---	1	1	---	1	2	---	1	3	21	3	1	2	2	---	6	18	4	21	0	0	0
crf	66	28	50	47	30	17	49	48	16	105	15	6	31	23	34	1	334	429	7	37	11	21	5	42	132	5	80	0	0	0
opr	77	18	35	45	40	18	25	72	6	62	8	3	39	12	14	3	48	169	8	142	17	9	4	31	104	6	58	0	0	0
elt	335	10	18	32	73	14	25	54	7	50	9	5	33	11	8	4	328	760	46	50	24	29	7	47	339	16	131	0	0	0
dwk	78	2	4	3	10	5	7	14	2	15	1	1	5	3	2	1	19	86	9	11	4	11	1	25	141	19	263	0	0	0
usf	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	---	---	---	1	---	---	1	---	5	0	0	0
short	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	914
long	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6,502
zmn	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
auk	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10,003	0
new	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	900

--- indicates a number smaller than one thousand

TABLE 3B *Extended Labour Force with Illegal Status by Labour Market Function and Industry (thousands of people)*

<i>Labour Matrix</i>	agric	coal	gold	othmining	foodbev	textfoot	woodpaper	petrochem	glassnm	metmach	electrical	radiotinstr	transequip	othmanuf	electricity	water	construc	trade	hotels	transser	communic	financial	estate	business	gengov	healthsoc	otherserv	Ind28	Ind29	noind
lsm	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
prf	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
tch	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
clk	2	---	---	---	1	---	1	2	---	1	---	---	1	---	---	---	2	18	1	2	2	11	---	3	---	2	4	0	0	0
srv	4	---	1	1	4	1	2	4	---	4	1	---	3	1	1	---	4	47	6	5	2	8	---	17	---	4	12	0	0	0
sag	84	---	1	---	7	---	1	1	---	1	---	---	---	---	---	---	1	8	1	---	1	1	---	2	---	1	8	0	0	0
crf	15	6	11	10	7	4	11	11	4	24	3	1	7	5	8	---	74	95	2	8	2	5	1	9	---	1	18	0	0	0
opr	8	2	3	5	4	2	3	7	1	6	1	---	4	1	1	---	5	17	1	14	2	1	---	3	---	1	6	0	0	0
elt	81	4	9	10	18	3	6	13	2	12	2	1	8	3	2	1	89	194	11	12	6	7	2	11	---	4	32	0	0	0
dwk	25	---	1	1	3	2	2	4	1	4	---	---	2	1	1	---	6	27	3	3	1	3	---	8	---	6	82	0	0	0
usf	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	1	0	0	0
short	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	158
long	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	257
zmn	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9,920	0	0
auk	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
new	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	400

--- indicates a number smaller than one thousand

TABLE 3C Legal Labour Flows from Start-of-Year Categories to End-of-Year Activities (thousands)

<div>Activities end-of-yr</div> <div>Categories st-of-yr</div>		Skilled Local Employment for all (b)			Lower Skilled Local Employment for all (b)								Local Unemployment for all (b)		Foreign Migrants for all (b)	
		lsm	prf	tch	clk	srv	sag	crf	opr	elt	dwk	usf	short	long	zmn	auk
Skilled Local Employment for all (b)	lsm	791	3	6	0	0	0	0	0	0	0	0	36	0	0	13
	prf	3	545	4	0	0	0	0	0	0	0	0	25	0	0	8
	tch	6	4	1,115	0	0	0	0	0	0	0	0	51	0	0	21
Lower Skilled Local Employment for all (b)	clk	0	0	0	1,143	2	---	2	1	2	---	---	54	0	0	0
	srv	0	0	0	2	1,373	---	2	1	3	1	---	65	0	0	0
	sag	0	0	0	---	---	301	---	---	---	---	---	14	0	0	0
	crf	0	0	0	2	2	---	1,525	1	3	1	---	72	0	0	0
	opr	0	0	0	1	1	---	1	986	2	---	---	46	0	0	0
	elt	0	0	0	2	3	1	4	2	2,220	2	---	105	0	0	0
	dwk	0	0	0	---	1	---	1	---	1	655	---	31	0	0	0
	usf	0	0	0	---	---	---	---	---	---	---	10	---	0	0	0
Local Unemployment for all (b)	short	22	15	34	17	21	4	23	15	34	10	---	0	1,251	0	4
	long	35	23	52	62	73	16	82	52	120	35	---	0	5,252	0	6
Foreign Migrants for all (b)	zmn	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	auk	7	5	10	0	0	0	0	0	0	0	0	0	0	0	9,570
New Entrants for all (b)	new	8	6	12	7	8	2	9	6	13	4	---	410	0	0	415

--- indicates a number smaller than one thousand



TABLE 3D *Illegal Labour Flows from Start-of-Year Categories to End-of-Year Activities (thousands)*

<div> <div>Activities end-of-yr</div> <div>Categories st-of-yr</div> </div>		Skilled Local Employment for all (b)			Lower Skilled Local Employment for all (b)								Local Unemployment for all (b)		Foreign Migrants for all (b)	
		lsm	prf	tch	clk	srv	sag	crf	opr	elt	dwk	usf	short	long	zmn	auk
Skilled Local Employment for all (b)	lsm	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	prf	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	tch	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Lower Skilled Local Employment for all (b)	clk	0	0	0	61	2	---	2	1	2	---	---	6	0	1	0
	srv	0	0	0	2	201	---	2	1	3	1	---	22	0	4	0
	sag	0	0	0	---	---	110	---	---	---	---	---	12	0	2	0
	crf	0	0	0	2	2	---	315	1	3	1	---	34	0	6	0
	opr	0	0	0	1	1	---	1	92	2	---	---	10	0	2	0
	elt	0	0	0	2	3	1	4	2	500	2	---	53	0	10	0
	dwk	0	0	0	---	1	---	1	---	1	190	---	20	0	4	0
	usf	0	0	0	---	---	---	---	---	---	---	2	---	0	---	0
Local Unemployment for all (b)	short	0	0	0	---	1	---	1	---	2	1	---	0	66	16	0
	long	0	0	0	---	2	1	3	1	4	2	---	0	191	32	0
Foreign Migrants for all (b)	zmn	0	0	0	9	29	16	46	13	72	27	---	0	0	9,453	0
	auk	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
New Entrants for all (b)	new	0	0	0	---	2	1	3	1	4	2	---	0	0	389	0

--- indicates a number smaller than one thousand

In setting up the extended labour market data by industry, not-employed and foreign migrant workers were allocated to three ‘phantom industries’. Local workers who are unemployed were allocated to an industry labelled ‘*noind*’, referring to the fact that they are not employed in any industry. Workers in the foreign migrant occupations *zmn* and *auk* were allocated to industries labelled ‘*ind28*’ and ‘*ind29*’, respectively, in the database. All other locally employed workers were allocated to one of the 27 local industries.

Various subset combinations may be used in the model to specify certain data matrices. For example, the wage bill matrix (V1LAB) in ZAR-M is a 4-dimensional array. Its labour function dimension contains only the 11 local occupation groups and its industry dimension only the 27 local industries. It also includes birthplace and legal status dimensions. That is, the size of the V1LAB matrix in ZAR-M is  $11 \times 27 \times 2 \times 2$ , containing 1188 data entries. The extended labour force matrix (LABOUR) in ZAR-M is also a 4-dimensional array, but is defined over the full range of elements in each set. The initial LABOUR matrix gives a snapshot of every worker’s characteristics at the start of the base year, defined across all labour function, industry, birthplace and legal status set elements. The size of the LABOUR matrix is therefore  $16 \times 30 \times 2 \times 2$ , containing 1920 data entries. An example of the LABOUR matrix for workers with legal and illegal status, aggregated over all birthplaces, is shown in Table 3A and 3B, respectively. That is, Table 3A shows the number of workers, aggregated over all birthplaces, with legal status employed in each industry, according to all start-of-year labour force function categories. Table 3B shows the same, but for workers with illegal status in ZAR-M. We can only show two dimensions of these 4-dimensional arrays at any one time. Fortunately, GEMPACK allows easy manipulation in viewing these large data matrices through its ViewHAR application.

Both the initial labour offers (OFFERS) and actual flows (FLOWS) matrices are also 4-dimensional arrays, but of slightly different dimensions. As noted before, the OFFERS matrix describes from which labour market function category to which labour market function activity, workers of all birthplace and legal status characteristics in the model offer their services to. The FLOWS matrix describes to which activity workers actually moved

into during the year, that is, whose offers were successful and what happened to those whose bids were unsuccessful. The OFFERS and FLOWS matrices in ZAR-M are defined over all 16 start-of-year categories, two birthplaces, two legal status and 15 end-of-year activities. Since we assume, in this application of ZAR-M, that workers do not change their birthplace or legal status, the size of these matrices reduce to  $16 \times 2 \times 2 \times 15$ , containing 960 data entries each. An example of the FLOWS matrix for each legal status, aggregated over all birthplaces (*b*), is shown in Table 3C and 3D. In the next section we provide more detail on how these data matrices for the base year were constructed.

### *Building an appropriate labour market database for ZAR-M*

Original data from the September 2006 labour force survey (StatsSA, 2007; 2009) and 2005 social accounting matrix (StatsSA, 2010a) provided the building blocks to constructing the labour market database for ZAR-M. As a first step, local wage bill and employment matrices for 2006, with occupation and industry dimensions, were compiled from these data sets. To set up the extended labour force database, information on the officially unemployed and discouraged work-seekers was also taken from these sources.

Motivated by the model's detailed theoretical specification, there were a number of challenging tasks in constructing an appropriate database incorporating international migrants. The most obvious of these was that no reliable data on the current stock or annual flows of migrants in South Africa exist in the detail required by the dynamic ZAR-M model. The local wage bill, employment and unemployment source data also contains no detail on the birthplace and legal status characteristics of workers. To overcome these data challenges, a number of additional research reports, statistics and available databases were consulted. These included Solomon (2000), StatsSA (2005; 2010b), Walmsley *et al* (2007), UNDESA (2009a), UNDP (2009), UNHCR (2010) and M&G Online (2010). Reports from local

authorities concerning the number of deportations, applications for asylum and evidence that has emerged after the recent spate of xenophobic attacks were also considered.

In adding the required birthplace and legal status data dimensions to the original wage bill and employment matrices, the share of labour inputs contributed to the local economy was effectively divided into three distinct population groups. The largest of these are the domestically-born legal residents, representing the typical South African citizen employed by a local industry in a particular occupation. The second group is the foreign-born legal residents, representing skilled immigrants employed by local industries in South Africa. The third group is the foreign-born illegal residents, representing the lower-skilled illegal immigrants employed by local industries in South Africa. We do not consider the fourth possible combination, domestically-born illegal residents, as we assume that all persons born in South Africa have legal status.

Due to the lack of reliable data, splitting the original occupation by industry wage bill and employment data into these three population groups required a number of assumptions. One of the guidelines used to estimate the number and share of undocumented illegal workers in each occupation and industry was the relative size of informal sector employment. Occupation groups such as ‘agriculture and fishery’ (*sag*) has a relatively large share of workers employed in the informal sector. It therefore had a higher percentage of ‘foreign-illegal’ workers allocated within it than, for example, ‘clerks’ (*clk*) which had a much smaller share of workers informally employed. Other assumptions included no ‘foreign-illegal’ contributions allocated to any of the three skilled occupation groups (*lsm*, *prf*, *tch*) and only very small ‘foreign-legal’ contributions allocated to the eight lower-skilled occupation groups (*clk*, *srv*, *sag*, *crf*, *opr*, *elt*, *dwk*, *usf*). For the wage bill matrix allocation, we assumed a lower average wage for illegal workers compared to legal workers in the same occupation. This incorporates the idea that some illegal workers may have chosen to undercut competing legal

workers demanding the legal minimum wage from employers. Alternatively, it could also represent lower productivity of illegal workers as a direct result of their status by having to avoid local authorities that might threaten them with deportation.

With base year employment and wage bill data appropriately set up, the next step was to expand the labour force matrix to include those not-employed in any of the 27 local industries at the start of the base year. Data for the officially unemployed and discouraged work-seekers were both included in the ZAR-M labour market database.<sup>4</sup> Since unemployed workers in ZAR-M are classified as either short-term or long-term unemployed, numbers in the initial database had to be adjusted to reflect this change in labour market category definition. Around 20 per cent of the total unemployed were classified as short-term and the remaining 80 per cent as long-term. The short-term unemployed are defined as those persons who have been unemployed for one year or less, whilst the long-term unemployed in ZAR-M are defined as those unemployed for longer than one year. As previously described, those who are short-term unemployed in year  $t$  and fail to find a job in the following period will be classified as long-term unemployed in year  $t+1$ . Long-term unemployed persons who again fail to find work simply remain classified as long-term unemployed. The high number of discouraged work-seekers in South Africa, combined with the sluggish rate of employment growth explains the high percentage of workers in the long-term unemployed category.

New entrants to the labour market are exogenous specified at the start of each year for all three birthplace and legal status combinations. For the ‘domestic-legal’ group, the number of new entrants reflects population growth figures for South Africa. For the ‘foreign-legal’ and ‘foreign-illegal’ groups, new entrant numbers reflect population growth rates in line with

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<sup>4</sup> According to Statistics South Africa’s official definition, ‘unemployed’ persons are those aged 15–65 who did not have a job or business in the seven days prior to the labour force survey but had looked for work or taken steps to start a business in the four weeks prior to the interview and were available to take up work within two weeks of the interview. ‘Discouraged work-seekers’ are persons who want to work and are available to work but who say that they are not actively looking for work.

countries whose labour migration characteristics match those described in ZAR-M for ‘*auk*’ and ‘*zmn*’ occupation groups, respectively. Naturally, the vast majority of foreign-born new entrants with legal status flow into the foreign ‘*auk*’ occupation group, whilst those with illegal status flow mainly into the foreign ‘*zmn*’ occupation group. All start-of-year new entrants are initially allocated to the phantom industry ‘*noind*’. As dictated by the model’s equations, the difference between the number of foreign-born new entrants in start-of-year categories and the number of new entrants that actually flow into their respective foreign activities each year equals the number of immigrants to South Africa from these ‘new entrant’ categories. To complete our start-of-year labour database for ZAR-M we also had to model potential migrants of working age outside South Africa. Workers in the ‘*zmn*’ occupation group employed by the ‘*ind28*’ industry, represent potential lower-skilled immigrants to South Africa. Workers in the ‘*auk*’ group employed by the foreign ‘*ind29*’ industry represent potential skilled immigrants. The start-of-year numbers in each of these groups were calibrated to produce sensible labour flows in line with our modelling assumptions and available data.

In the ZAR-M labour database, start-of-year  $t$  category data reflect end-of-year  $t-1$  activity data. New entrants are exogenously added at the start of each year and deaths and retirements are accounted for between periods. Once a start-of-year  $t$  database is constructed according to labour market function, birthplace and legal status, an end of year  $t-1$  database is easily derived. The next step is then to construct an appropriate initial labour offers (OFFERS) and actual flows (FLOWS) matrix reflecting labour supply and labour demand in the economy’s base year. The OFFERS matrix describes the supply of labour from categories to activities. Typically, most workers will supply their labour to employment activities within their broad skill group, with only a small percentage of people offering to remain or become voluntarily unemployed. The FLOWS matrix describes the actual flow of workers from categories to activities, reflecting the demand for labour. The outcome of the FLOWS

matrix in the base year should reflect the actual state of the labour force in the economy at the end of 2006.

Using Tables 3A to 3D as our guide, we conclude this section by describing the key features of the extended ZAR-M labour market as modelled in the base year below.

- The labour force matrix gives a snapshot of the extended labour force by function, birthplace, legal status and industry in the base year.
- Labour force functions include employment in 11 local occupation groups, short and long-term unemployment, two foreign migrant occupations and new entrants to the extended labour market. Industries include 27 local and three phantom industries capturing both employed and not-employed workers in the extended labour force.
- There were 13.3 million people employed by local industries in the base year. From this number, 11.6 million were estimated to have legal status and 1.7 million illegal status. Aggregate employment figures were adjusted upwards from the labour force survey estimates as an undercounting of illegal workers, especially in the informal sector, was assumed.
- There were 7.8 million people unemployed in the base year. From this number, 1.1 million were allocated to short-term unemployment and the remaining 6.7 million to long-term unemployment. Aggregate unemployment figures were also adjusted upward slightly from the labour force survey estimates.
- The foreign migrant occupations represent a pool of workers viewed as potential skilled (*auk*) or lower-skilled (*zmn*) immigrants. These occupations also serve as a destination for foreign-born migrants in South Africa returning home. The *auk* occupation also serves as a destination for skilled emigrants leaving South Africa.

- The initial stock of workers in *auk* also includes around 600,000 South African born workers. Numbers for the initial pool of workers in the *auk* and *zmn* occupation groups, and foreign-born new entrants were calibrated to maintain sensible relationships in terms of labour force stocks and flows in ZAR-M.
- The initial offers matrix (OFFERS) describes labour supply preferences from start-of-year categories to end-of-year activities of all workers in the extended labour force. The actual flows matrix (FLOWS) describes to which activity workers actually moved into during the year, reflecting labour demand conditions.
- Start-of-year categories and end-of-year activities are specified by labour force function, birthplace and legal status dimensions. We do not allow workers to change their birthplace or legal status in this application of ZAR-M. Workers in local or foreign employment categories are also not allowed to offer to any employment activities outside their broad skill group.
- The vast majority of workers in local and foreign employment categories offer to remain in their respective employment activities. That is, a large share of labour offers from employment categories to employment activities in Figure 3D are diagonal.
- Assumptions regarding the share of non-diagonal offers from local employment categories differ between the three population groups. In order to avoid detection, it was assumed that illegal workers are relatively more averse towards changing their occupation.
- There are relatively more non-diagonal offers originating from the *zmn* foreign migrant category towards local employment activities, highlighting the desire of workers in this group to find jobs in South Africa.



- Offers from employment categories to unemployment activities reflect workers who wish to become voluntarily unemployed. All such offers are accepted and initial data is calibrated to show sensible flows in this regard.
- Workers in unemployment categories offer mostly to employment activities. Those who offer to remain in unemployment represent discouraged work-seekers.
- All offers from workers in local categories to foreign activities are accepted. Initial data on the outflow of these workers is set up to reflect realistic numbers of illegal migrants returning home and skilled local workers emigrating from South Africa.
- New entrants are exogenously specified according to each of the three population groups and calibrated to fit their respective labour force growth trends. The vast majority of new entrants offer to activities closest to their population characteristics. That is, domestic-born new entrants with legal status offer mainly to the 11 local employment activities, foreign-born new entrants with legal status offer mainly to the migrant activity *auk*, and foreign-born new entrants with illegal status offer mainly to the migrant activity *zmn*.
- The actual flows matrix in the base year is set up to replicate actual labour force outcomes. That is, employment, unemployment and migration flows into activities should be reflective of available labour force data for 2006.
- In constructing the FLOWS matrix we directly account for a number of labour market details. These include foreign-born illegal immigrant flows in both directions; local and foreign-born skilled migrant flows in both directions; the dismissal of workers from local occupations; discouraged work-seekers; high rates of unemployment in the local labour force; voluntary unemployment incorporating those undergoing further training; and control over which type of occupations workers are able to move between based on their skill and population characteristics.

- We model a net inflow of lower-skilled foreign-born illegal workers of around 150,000 in the base year. This consists of an inflow of 225,000 foreign-born illegal workers, that is, flows from the *zmn* category to any local occupation activity; and an outflow of 75,000 foreign-born illegal workers returning home, that is, flows from any local occupation category to the *zmn* activity.
- We model a net outflow of skilled domestic-born workers of around 50,000 in the base year. The largest contribution to outflows of domestic-born workers is from the ‘new entrants’ category. This is consistent with reports of new matriculants and graduates going overseas for a period of time after completing their studies. It is also indicative of the relatively poor job prospects many new entrants to the labour market in South Africa face. With a total of 60,000 domestic-born workers emigrating to *auk*, an inflow of close to 10,000 workers returning home from the foreign occupation *auk* is modelled for the base year.
- We assume a high rate of return migration for foreign-born workers who fall into unemployment. That is, a relatively large share of foreign-born workers in local unemployment categories moves back to either *auk* or *zmn*, depending on their legal status.
- We assume that between four and six per cent of domestic-born local workers are dismissed every year, that is, workers who involuntarily flow from an employment category to an unemployment activity. A higher rate of dismissal is assumed for illegal workers as they lack any formal protection. To reflect the difficulties in recruiting skilled foreign-born workers, we assume a lower dismissal rate for this group.

- Initial base year data for the OFFERS and FLOWS matrices are essential to the functioning of the labour market mechanism in ZAR-M. As described in the theoretical specification of the model, the success of any labour offer hinges on a number of factors. These include the strength of the offer, that is, the number of offers from one group relative to another and the number of vacancies in each occupation influenced by dismissals, employment growth and non-diagonal offers. The initial OFFERS matrix in the labour database must be set up and calibrated alongside the initial FLOWS matrix to ensure sensible outcomes with regard to these labour market characteristics.
- The database includes labour demand elasticities concerning the substitutability of legal and illegal workers, and local and foreign-born workers for a given change in their relative wage. A higher elasticity value is chosen for birthplaces ( $\sigma_B$ ) relative to legal status ( $\sigma_S$ ) in the labour demand equation (E2.49). This reflects the ability of employers to differentiate more easily between a worker's citizenship than their legal status.

### 3.4 Concluding Remarks

In this chapter we described the database of the ZAR-M model. The base year data used to construct the model's database provides a snapshot of the economy in 2006. In the first section we set out the basic input-output structure of a CGE model database and described the key aggregates for the South African economy in the base year. Figure 3A schematically represents the input-output structure of the model database. The following section provided a detailed account of the structure, data and assumptions in the labour market database used in this application of ZAR-M. Of great importance was the modelling of migration flows. The calibration of initial labour offers and actual flows matrices was critical to ensuring sensible labour market outcomes. This included initial immigration and emigration flows. All labour data was disaggregated to show the birthplace and legal status of workers in the model's extended labour force.

The level of detail in the ZAR-M model required a wide range of data sources to be consulted. However, due to a lack of reliable immigration and emigration data, a number of assumptions were necessary to correctly model the initial flow of workers by labour force function, birthplace and legal status dimensions. The key features and aggregates of the labour market database were subsequently discussed. As an extension to the base year data described in this chapter, the baseline forecast described in Chapter 5 should be consulted as it contains further insights to the data presented here.





# CHAPTER 4

## MODEL CLOSURES

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## 4.1 Introduction

The choice of model closure is one of the most important aspects in economy-wide modelling. For detailed computable general equilibrium (CGE) models such as ZAR-M or USAGE-M the number of equations ( $m$ ) and variables ( $n$ ) can be very large. CGE models often contain many more variables than equations. To close the model and compute a solution,  $(n-m)$  variables must therefore be treated as exogenous. Alternatively stated, the number of endogenous variables must equal the number of equations for the model closure to be valid. The selection of the  $(n-m)$  exogenous variables is largely user-determined, but should be chosen to best describe the economic environment for which the simulation is run. CGE models usually contain many non-linear equations and relationships. Following Johansen (1960), we simplify the implementation of our model by linearising the system of equations and writing the variables in terms of changes or percentage changes.<sup>1</sup> This linear system is then solved by matrix manipulations, generously computed by the software package GEMPACK, giving the effects on the  $(m)$  endogenous variables of changes in the  $(n-m)$  exogenous variables.

The ability to have flexible model closures was one of the most useful extensions to Johansen's early framework. Led by the efforts of modellers at the IMPACT Project<sup>2</sup> in the mid 1970s, the closure flexibility initially described for the comparative-static ORANI model in Dixon *et al* (1982) was a significant step in allowing more practical and policy-oriented CGE modelling work. The development of modelling software packages such as GEMPACK (Harrison & Pearson, 1996) further improved the ease with which model closures could be found and altered. The idea of flexible closures was fully extended in the dynamic MONASH model with the development of four basic model closures (Dixon & Rimmer, 2002:233-277). These closures each had a distinct application in mind. Starting from a typical one-period long-run decomposition closure, swaps were applied between

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<sup>1</sup> See Appendix for a brief introduction to writing equations in percentage-change form or Dixon & Rimmer (2002) for a more detailed explanation on the percentage-change approach to solving CGE models.

<sup>2</sup> The IMPACT Project was initially set up by the Australian Government and tasked with building various policy-oriented economic models, most notably the ORANI model. More information about the IMPACT Project can be found on the Centre of Policy Studies and IMPACT Project website at [www.monash.edu.au/policy](http://www.monash.edu.au/policy)



endogenous and exogenous variables to find closures suitable for historical analysis, forecasting, and policy simulations.

In this chapter we use a stylized CGE model to illustrate and explain the main features of the different closures used in this study. We start from a typical long-run closure and then apply various swaps to find our forecast and policy closures. A complete exposition of each closure for the full ZAR-M model is included in the Appendix.

## 4.2 A Stylized CGE Model

To gain a better understanding of the main features of each model closure, we follow the approach of Dixon & Rimmer (2002) by first developing a stylized ‘back-of-the-envelope’ version of a typical CGE model as reference. This model, described in Table 4A, is a simplified version of BOTE-M described in Chapter 2. It captures the main macroeconomic relationships in ZAR-M and is useful in producing readily interpretable results without losing any broad detail. To understand the finer features of the full model it remains necessary to consider aspects of the ZAR-M database and micro theory.

TABLE 4A *A Stylized CGE Model*

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GDP	= C + I + G + (X–M)	(E4.1)
GDP	= A*f(K, L)	(E4.2)
C + G	= APC*GDP	(E4.3)
C / G	= R_CG	(E4.4)
M	= f(GDP, ToFT, TWS)	(E4.5)
ToFT	= f(X, F_X)	(E4.6)
I	= f(RoR, F_I)	(E4.7)
R_IK	= I / K	(E4.8)
CPI	= f(PY, ToFT)	(E4.9)
RoR	= f(K/L, ToFT, A)	(E4.10)
RW	= f(K/L, ToFT, A)	(E4.11)

In the stylized model, (E4.1) is the well-known identity for real gross domestic product (GDP) from the expenditure side. In South Africa, the size of private household consumption (C) relative to GDP is around 60 per cent. Investment (I) and government expenditure (G) each contribute roughly 20 per cent to GDP. South Africa's balance of trade (X-M) typically shows a small deficit. Equation (E4.2) describes the production function for real GDP from the supply side, relating total output in the economy to inputs of capital (K) and labour (L), and a primary-factor productivity or technical change term (A). In South Africa, both compensation of employees and gross operating surplus contribute around 50 percent of GDP at factor cost. For the stylized model it is assumed that all sectors are competitive and all markets clear.

Equation (E4.3) relates the sum of private (C) and public (G) consumption to GDP via the average propensity to consume (APC), whilst equation (E4.4) defines the ratio of private to public consumption (R\_CG). Equation (E4.5) relates imports (M) positively to the level of GDP, the terms of trade (ToFT), and an import/domestic preference twist variable (TWS). In our stylized model we make the simplifying assumption that there is only one imported good and one domestically produced good which is both consumed locally and exported. Ignoring tariffs and other taxes, the import-determining price can then be represented by the ToFT, that is, the price of the domestically produced (and also exported) good relative to the price of the imported good. Equation (E4.6) relates the ToFT to the volume of exports (X) and an export demand shift variable (F\_X). Similar to the MONASH model of Australia, this is consistent with the assumption of South Africa being an open economy facing downward-sloping demand curves for its exports, with F\_X representing movements in those demand curves. With regard to imports, South Africa is considered a price-taker. Equation (E4.7) relates investment expenditure (I) to the rate of return on capital (RoR) and a demand shift variable (F\_I) whilst equation (E4.8) defines the investment-capital ratio (R\_IK). Equation (E4.9) relates the price of domestically consumed goods (CPI) to the price of the domestically produced good (PY) and the terms of trade effect (ToFT).

In our description of the capital and labour markets, we recognise that the marginal product of capital (MPK) is negatively related to the K/L ratio and the marginal product of labour (MPL) positively related to the K/L ratio. In determining (E4.10) we assume that the rate of return on capital (RoR) can be expressed as  $[Q/PI]$  with  $Q$  the factor payment to capital and  $PI$  the price index for new investments. We then assume  $Q$  is determined by the value of the marginal product of capital, written as  $[MPK \cdot PY]$ . With  $MPK$  a function of the K/L ratio and technical change ( $A$ ), and  $[PY/PI]$  a function of the terms of trade (ToFT), we are able to summarise this relationship through equation (E4.10). In determining (E4.11) we assume that the real wage (RW) can be expressed as  $[W/CPI]$  with  $W$  the factor payment to labour and  $CPI$  the consumer price index. We then assume  $W$  is determined by the value of the marginal product of labour, written as  $[MPL \cdot PY]$ . In similar fashion to (E4.10), we are then able to write equation (E4.11) linking the RW to the K/L ratio, technical change ( $A$ ) and terms of trade (ToFT) effect.

In deriving (E4.10) and (E4.11) we assume that domestic production is via a constant returns-to-scale production function of capital and labour inputs, and that the costs of employing capital and labour equal the values of the marginal products of capital and labour, respectively. This enables us to derive the ‘back-of-the-envelope’ equations used in Table 4B to better interpret movements in the capital and labour markets, and subsequently the K/L ratio.<sup>3</sup>

TABLE 4B *Marginal Products of Capital and Labour*

$$\frac{\partial GDP}{\partial K} = MPK \approx \frac{1}{A} * \frac{Q}{P_y} \quad \text{or equivalently} \quad \frac{\partial GDP}{\partial K} = MPK \approx \frac{1}{A} * \frac{Q}{P_i} * \frac{P_i}{P_y} \quad (E4.12)$$

$$\frac{\partial GDP}{\partial L} = MPL \approx \frac{1}{A} * \frac{W}{P_y} \quad \text{or equivalently} \quad \frac{\partial GDP}{\partial L} = MPL \approx \frac{1}{A} * \frac{W}{P_c} * \frac{P_c}{P_y} \quad (E4.13)$$

<sup>3</sup> The two ‘back-of-the-envelope’ equations in (E4.12) and (E4.13) are easily derived by maximising economy-wide profits,  $P_Y \cdot Y - (W \cdot L + Q \cdot K)$ , subject to a Cobb-Douglas production function where  $Y = A[L^\beta \cdot K^{(1-\beta)}]$

In the capital (E4.12) and labour (E4.13) equations,  $Q$  and  $W$  are factor payments to capital and labour;  $P_i$  and  $P_c$  the price indexes for new investment and consumption goods;  $P_y$  the price index for domestically produced goods; and  $MPK$  and  $MPL$  the marginal products of capital and labour, respectively. In this exposition, the term  $P_c$  is equivalent to the CPI.

In equation (E4.12), the  $(Q/P_y)$  term can be split into two components or effects to enhance our analysis. The first term,  $(Q/P_i)$ , can be interpreted as the rate of return on capital (RoR). The second term,  $(P_i/P_y)$ , similar to the  $(P_c/P_y)$  term in (E4.13), can be interpreted as a decreasing function of the terms of trade (ToFT). This is because both  $P_i$  and  $P_c$  include imports but not exports, whilst  $P_y$  includes exports but not imports. The terms of trade effect is especially important in economies for which  $X$  and  $M$  are relatively large values. In equation (E4.13), the  $(W/P_y)$  term can be interpreted as the real producer wage or cost of employing a unit of labour. To enhance our analysis, we again split this term into two effects. The first term,  $(W/P_c)$ , can be interpreted as the real consumer wage, and the second,  $(P_c/P_y)$ , as the terms of trade effect.

Recognising that the marginal product of capital ( $MPK$ ) is negatively related to the capital-labour ratio ( $K/L$ ), equation (E4.12) can be used to show that the  $K/L$  ratio in (E4.10) is negatively related to the RoR, and positively related to the ToFT and  $A$ . That is, as the relative amount of capital in the economy increases, and the  $MPK$  falls, we can expect a decline in the rate of return on capital investments. Similarly, with the marginal product of labour ( $MPL$ ) positively related to the  $K/L$  ratio, equation (E4.13) can be used to show that the  $K/L$  ratio in (E4.11) is positively related to the  $RW$ . Alternatively, we can think of the labour-capital ratio ( $L/K$ ) as being negatively related to the  $RW$ .

Despite its simplicity, the stylized model outlined in Table 4A captures the main mechanisms operating in the ZAR-M model. This allows us to develop and explain the different closure possibilities of the model on a macroeconomic level, without encumbering the reader with too much detail. The stylized model is also useful as a quick reference to the full model and providing insight for the interpretation of simulation results. Where results

from the full model differ from those suggested by the stylized model, we have to consider the finer details of the actual model theory and database. In the following sections we use the stylized model as our main reference for developing and interpreting the features of each model closure used within the context of this study.

### 4.3 Developing the Decomposition Closure

The decomposition closure can best be described as a standard one-period long-run closure.<sup>4</sup> Although not explicitly used in this study, the decomposition closure serves as a good base from where to develop other closures, such as the forecast and policy closure. By applying a number of standard macroeconomic assumptions, we are able to find a suitable long-run decomposition closure for the stylized model described in Table 4A.

From the supply-side of the economy, described by (E4.2), we can assume aggregate labour (L) fixed at its full employment level. This is consistent with the concept of a fixed non-accelerating inflation rate of unemployment (NAIRU) in the long-run. Real wages (RW) are considered flexible in the long-run as remuneration packages are periodically renegotiated. By absorbing any demand-side pressure via changes in real wages, the labour market is allowed to clear. Capital stock (K) is endogenous in the long-run to reflect changes in net investment. Rates of return on capital investments (RoR) are assumed to be exogenous. Many factors that influence RoR in the long-run, such as interest rates and risk premiums, are relatively stable over time and unlikely to be affected by policy decisions. Technical change (A) is assumed to be exogenous in the long-run. We also expect the K/L ratio to stabilise at a level consistent with the economy's steady-state level of growth in the long-run.

With GDP now determined from the income side, we can use (E4.1) to analyse movements from the expenditure side of the economy. As suggested by macroeconomic evidence, we can expect private household consumption (C) to closely follow GDP in the long-run. Similarly,

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<sup>4</sup> See Dixon & Rimmer (2002) for a complete description of the decomposition closure with illustrative examples of its practical application.

we can expect government (G) and import (M) demands to follow household consumption. This implies that both the average propensity to consume (APC) and the ratio of private to public consumption ( $R_{CG}$ ) should be exogenous in the long-run. Investment (I) is endogenous via (E4.7), but bound to the gross capital growth trend in the base data, that is  $(I_{base}/K_{base})$ , since rates of return (RoR) are assumed to be constant in the long-run. This allows the overall investment to capital ratio ( $R_{IK}$ ) to be determined via (E4.8). In (E5.1), the level of exports (X) is then determined as a residual to balance GDP from the expenditure side with GDP from the income side.

To accommodate the required move in X, the terms of trade (ToT) capturing export prices, is allowed to adjust. The consumer price index (CPI) is exogenous and acts as the numeraire in our stylized model. The numeraire determines the absolute price level in the economy.<sup>5</sup> Other variables such as the import/domestic preference twist (TWS) and demand shift variables ( $F_X$ ,  $F_I$ ) are assumed to be exogenous in the long-run. In the context of equations (E4.12) and (E4.13) we can interpret any technical change affecting the marginal product of capital, to be reflected in a change to K with the RoR fixed in the long-run. Technical change affecting the marginal product of labour will be absorbed through a change to the real wage in the long-run under full employment.

The choice of endogenous and exogenous variables in the decomposition closure for the stylized model can therefore be outlined as in Table 4C. Under this closure, each equation of the stylized model can be readily associated with the determination of a specific endogenous variable. With RoR and L fixed in the long-run, and A also exogenous, (E4.10) and (E4.11) can be identified with the determination of K and RW. With RoR fixed,  $F_I$  exogenous and K already determined, this allows equations (E4.7) and (E4.8) to determine I and  $R_{IK}$ , respectively. With L and A exogenous, (E4.2) then determines GDP. C and G is determined via (E4.3) and (E4.4) given a fixed APC and  $R_{CG}$ . Ignoring any movements in ToT, (E4.5) determines M with GDP already determined and TWS exogenous. With GDP, C, I, G and M explained, equation (E4.1) determines X, which then allows (E4.6) to determine

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<sup>5</sup> To determine the absolute price level in the economy requires one nominal price variable to be set as exogenous. Another popular choice as numeraire in CGE modelling is the nominal exchange rate.

ToftT. With the CPI set as the numeraire and ToftT already determined, equation (E4.9) determines PY. Readers may also note that the number of variables listed as endogenous in Table 4C correspond to the number of equations in Table 4A.

For the full ZAR-M model, GEMPACK may be used at first to generate a valid automatic closure by listing as exogenous all those variables not directly described or matched by an equation in the model code. When following the GEMPACK naming convention, each equation is typically named for the variable it is considered to endogenously determine. Although slightly unorthodox for a model in which the system of equations is solved simultaneously, this naming convention allows the software to find an automatic closure by treating all those variables named by an equation as endogenous, and setting all remaining variables as exogenous. Accordingly, the choice of  $(n-m)$  exogenous variables correspond very closely to the structure of the model. From the automatic closure, a small number of swap statements may be required to produce a model closure of sensible economic meaning, such as the long-run decomposition closure for the stylized model described in Table 4C. The main advantage to using the automatic closure as a starting point for developing any further model closures is that it saves the user a great amount of time as most of the numerous technical change, preference, and shift variables, all naturally exogenous, would already be chosen and listed as such.

After developing a valid and sensible model closure, users are encouraged to alter their closure through swap statements. Swap statements in the model closure file are a useful way of keeping track of any changes to the economic environment or assumptions imposed on the model. This involves a simple statement below the initial closure specification instructing GEMPACK to swap one variable for another on the exogenous list. With the decomposition closure as our starting point, we use this approach to develop a baseline forecast closure for our stylized model in the next section.

*TABLE 4C    Decomposition Closure for the Stylized Model*

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Endogenous variables

- i)      gross domestic product (GDP)
- ii)     real wages (RW)
- iii)    capital stock (K)
- iv)    household consumption (C)
- v)     investment expenditure (I)
- vi)    government expenditure (G)
- vii)    imports (M)
- viii)   exports (X)
- ix)    terms of trade (ToT)
- x)     ratio of investment to capital stock (R\_IK)
- xi)    domestic price index (PY)

Exogenous variables

- i)      employment (L)
  - ii)     rates of return (RoR)
  - iii)    average propensity to consume (APC)
  - iv)    investment demand shifter (F\_I)
  - v)     ratio of private to public consumption (R\_CG)
  - vi)    import/domestic preference twist (TWS)
  - vii)    technical change (A)
  - viii)   export demand shifter (F\_X)
  - ix)    consumer price index (CPI)
-



#### 4.4 Developing the Forecast Closure

The forecast closure is used in simulations to produce a believable business-as-usual or baseline picture of the future evolution of the economy. One of the main purposes of CGE models is to provide projections of the impact of economic policy changes on a wide variety of economic variables. To accomplish this, a baseline forecast scenario incorporating available forecast data is first simulated. A perturbed scenario incorporating the relevant policy shocks is then run and compared to the outcome of the baseline scenario, with deviations usually reported as percentage changes. When conducting policy simulations stretching over multiple periods, the importance of a suitable baseline forecast becomes apparent. The effects of policies imposed on economies with structures likely to be relevant in the future are often different from the effects imposed on economies with the structures of the present.

For the baseline forecast closure, we try to exogenise everything that we think we know about the future. The choice of  $(n-m)$  exogenous variables is therefore usually based on the availability of reliable macroeconomic forecast data, with little regard to causation. In the application of the forecast simulation we exogenise many naturally endogenous variables such as the components of GDP from the expenditure side, the consumer price index, population growth, etc. to take advantage of forecasts prepared by various analysts and macroeconomic specialists. To allow these variables to be exogenous requires us to endogenise many naturally exogenous variables such as the average propensity to consume, the positions of foreign demand curves, the positions of domestic export supply curves, etc. Some technical change and preference variables may also be exogenous and given shocks that are informed by trends derived from historical simulations or available time-series data.

Baseline forecast simulations for recursive-dynamic CGE models are performed as a sequence of annual solutions. The forecast closure is therefore short-run in nature. Because we are dealing with annual solutions, start-of-year  $t+1$  stock variables such as capital or net foreign liabilities are completely determined by end-of-year  $t$  stock variables in the baseline

solution. While these start-of-year stock variables can be thought of as exogenous in the computation for any year  $t$ , they should be thought of as endogenous for the sequence of annual solutions which make up the forecast simulation. For example, changes in net investment together with changes in capital stocks in year  $t$  determine the change in end-of-year capital stocks for year  $t$ , which subsequently determine the change in start-of-year capital stocks for year  $t+1$ . In ZAR-M, this capital accumulation mechanism can be expressed as follows:

TABLE 4D    *Capital Accumulation Mechanism in ZAR-M*

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$$K_{t+1} = K_t * [1 - DEP_t] + I_t \quad (E4.14)$$

with

$$I_t = f(ERoR_t) \quad (E4.15)$$


---

As shown in equation (E4.14), end-of-year capital stock ( $K_{t+1}$ ) is calculated as the net value of start-of-year capital stock ( $K_t$ ), taking into account the rate of depreciation ( $DEP_t$ ) on old capital stock, plus new capital investments made during the year ( $I_t$ ). In equation (E4.15) we relate investment decisions to expected rates of return ( $ERoR$ ). Thus, investment this period only affects the growth rate of capital in the next period. This ‘gestation’ period for new investments allows us to keep start-of-year capital stock exogenous and end-of-year capital stock endogenous when developing year-on-year short-run closures for the ZAR-M model.

In terms of the closure for the stylized model, we have to create a suitable short-run forecast environment and exogenise the macro variables for which reliable annual forecast data typically exists. This requires a number of swaps to our current decomposition closure. The first step involves moving from a one-period long-run closure to a short-run closure. With regard to the factor market, we assume real wages ( $RW$ ) to be ‘sticky’ and aggregate labour ( $L$ ) to be flexible in the short-run. Aggregate capital stock ( $K$ ) is fixed in short-run with rates

of return (RoR) allowed to adjust. The following swaps to our model closure described earlier in Table 4C are therefore required:

*TABLE 4E Swap Statements to Produce Forecast Closure (Step 1)*

---

<i>Previously exogenous variables</i>	<i>Newly exogenous variables</i>
employment (L) .....	real wages (RW)
rates of return (RoR) .....	capital stock (K)

---

The second step involves incorporating available macroeconomic forecast data for household consumption (C), investment expenditure (I), government expenditure (G) and imports (M) into our baseline projection. Most of these macro variables that determine GDP from the expenditure side in (E4.1) are typically forecast by specialist agencies, allowing us to integrate these values into our projection. As noted before, in order for these variables to become exogenous and adopt their forecasted values, we have to endogenise some naturally exogenous variables. In this step we endogenise the average propensity to consume (APC), investment demand shifter (F\_I), ratio of private to public consumption (R\_CG) and the import/domestic preference twist (TWS) variables via the system of equations in (E4.1–E4.11). The following additional swaps to our model closure described earlier in Table 4C should therefore be made:

*TABLE 4F Swap Statements to Produce Forecast Closure (Step 2)*

---

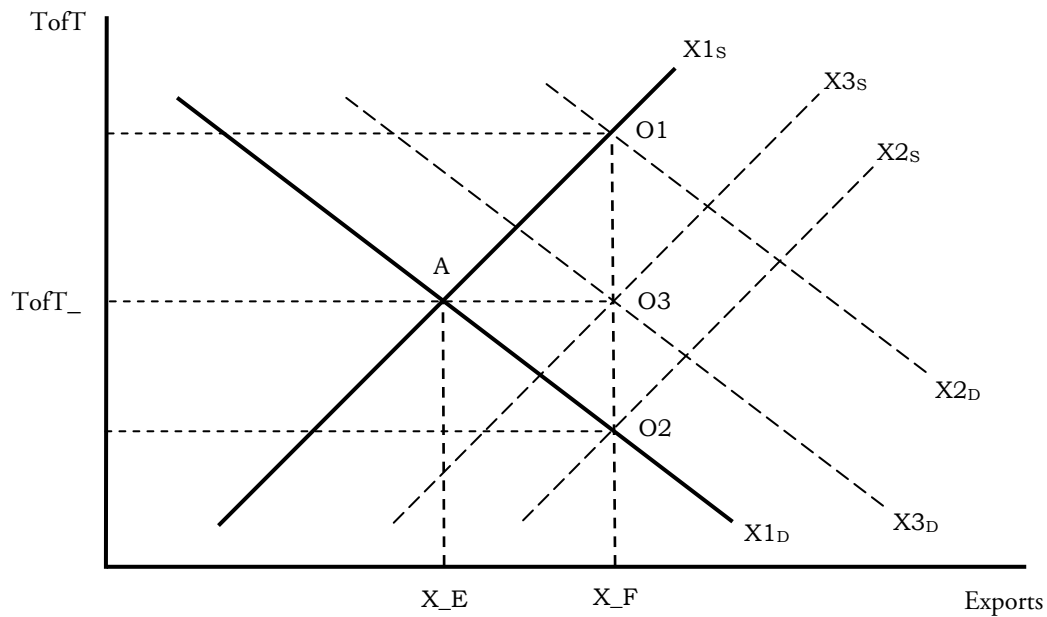
<i>Previously exogenous variables</i>	<i>Newly exogenous variables</i>
average propensity to consume (APC) .....	household consumption (C)
investment demand shifter (F_I) .....	investment expenditure (I)
ratio of private to public consumption (R_CG) .....	government expenditure (G)
import/domestic preference twist (TWS) .....	imports (M)

---

The third and final step in producing our forecast closure for the stylized model involves incorporating available forecast data for exports ( $X$ ). In the first step, we exogenised real wages ( $RW$ ) and capital stock ( $K$ ). This allowed us to create a suitable short-run environment and include any reliable forecast data on these variables. In the second step, we exogenised  $C$ ,  $I$ ,  $G$  and  $M$  from the expenditure-side of the model to further accommodate available forecast data on these variables. To also exogenise exports ( $X$ ), we must determine which variable to endogenise.

Figure 4A illustrates demand and supply curves for exports as functions of the terms of trade ( $TofT$ ), with the current market clearing position indicated by point A. This shows an equilibrium export quantity of  $X\_E$  and export price of  $TofT\_E$ . From here we can achieve a desired level of exports at  $X\_F$  by shifting either the export demand curve from ( $X1_D$ ) to ( $X2_D$ ) giving point O1, or the export supply curve from ( $X1_S$ ) to ( $X2_S$ ) giving point O2.

FIGURE 4A Demand and Supply of Exports



Generating these shifts requires us to define and understand the corresponding demand and supply curves in Figure 4A. The nature of the downward-sloping export demand curve is clear, it is simply (E4.6) from Table 4A. In this equation, demand for domestically-produced exports is expressed as  $[X_D = f(TofT, F - X)]$ . Defining the upward-sloping export supply curve is more complicated. To explain the nature of this supply curve, we start by writing the identity contained in (E4.1) as  $[X = GDP - (C + I + G - M)]$ . With  $C$ ,  $I$ ,  $G$  and  $M$  exogenous, this equation expresses  $X$  as a function of  $GDP$ . From equation (E4.2) we know that  $GDP$  is determined via the production function as  $[GDP = A * f(K, L)]$  with labour ( $L$ ) determined via equation (E4.11) as  $[L = f(RW, K, ToFT, A)]$ . Substituting this expression into (E4.2), we find  $[GDP = f(RW, K, ToFT, A)]$ , which allows us to write exports ( $X$ ) in (E4.1) as  $[X_S = f(RW, K, ToFT, A) - (C + I + G - M)]$ . For given values of  $RW$ ,  $K$  and  $A$ , this defines the export supply ( $X_S$ ) curve in Figure 4A that traces out the relationship between  $X$  and  $ToFT$ .

However, we have yet to prove that this expression for the  $X_S$  curve has a positive slope. Following our earlier discussion of equation (E4.13) in Table 4B, we recognise that  $\left[ \frac{\partial L}{\partial ToFT} \right] \geq 0$ . That is, holding  $RW$ ,  $K$  and  $A$  constant, an increase in the price of exports would lead to an increase in employment in the short-run. We derive this result via the terms of trade effect in equation (E4.13). Since  $P_y$  does not include import prices, and  $P_c$  does not include export prices, any movement in  $\left[ \frac{P_c}{P_y} \right]$  must therefore be a function of the terms of trade. An increase in export prices, reflected via  $P_y$ , will then lead to a reduction in  $\left[ \frac{P_c}{P_y} \right]$ . With real wages  $\left[ \frac{W}{P_c} \right]$  and technical change  $\left[ \frac{1}{A} \right]$  fixed, the fall in  $\left[ \frac{P_c}{P_y} \right]$  will lead to a fall in the marginal product of labour ( $MPL$ ) according to (E4.13).

If we assume optimising conditions are to be maintained, any fall in the  $MPL$  in the short-run must then be accompanied by an increase in  $L$  with  $RW$  fixed. This confirms that

$\left[ \frac{\partial L}{\partial TofT} \right] \geq 0$  in the short-run. An increase in  $L$  will then contribute to an increase in

$GDP$  via (E4.2). For given values of  $(C + I + G - M)$ , this will also increase  $GDP$  in (E4.1), and as a result  $X$ . According to the relationships determined in this exposition, it

confirms that our earlier expression for export supply,

$[X_s = f(RW, K, TofT, A) - (C + I + G - M)]$ , has a positive slope.

Having established the underlying nature of the downward-sloping export demand and upward-sloping export supply curve, we return to Figure 4A. We now know how to shift both curves within the framework of equations (E4.1–E4.11). From the export demand side, we can allow  $X_D$  to move by endogenising the export demand shifter  $(F - X)$ . The export demand shifter captures the willingness to pay of foreigners for domestically-produced exports. This setting will accommodate a shift from  $X1_D$  to  $X2_D$ , indicated by point O1 in Figure 4A. The model then determines the market clearing  $TofT$  consistent with this point and desired export quantity of  $X_F$ . From the supply side, we can allow  $X_s$  to move by introducing movements in  $[f(RW, K, TofT, A)]$ . With  $RW$  and  $K$  fixed, this leaves endogenisation of  $A$  as our only legitimate option. This setting will accommodate a shift from  $X1_s$  to  $X2_s$ , indicated by point O2 in Figure 4A. The model then again determines the market clearing  $TofT$  consistent with this point and  $X_F$ .

As suggested by Figure 4A, either option would require a relatively large change in the price of exports or  $TofT$ . The practical results produced by both these options, that is, moving to either point O1 or point O2 in our example from Figure 4A, is not considered satisfactory within our application of the forecast simulation in ZAR-M.

A third alternative for imposing an exogenous forecast value for exports must therefore be considered. It is often the case with variables such as the  $TofT$  that we do not have sufficient data to make reliable forecasts. However, historical evidence suggests that movements in the  $TofT$  are relatively stable over the long-term. To prevent large changes in the  $TofT$ , and maintain our exogenous setting of exports at  $X_F$ , we should therefore combine both options described earlier. That is, we must endogenise both technical change ( $A$ ) and the export-demand shifter ( $F_X$ ). This will allow us to exogenise the quantity of exports at  $X_E$  and fix the  $TofT$  at  $TofT_E$ . The assumption of no change in the  $TofT$  over the simulation period can in itself be considered as a forecast, albeit a rather conservative one. To complete the move from our model closure described earlier in Table 4C to the forecast closure summarised in Table 4H, the following additional swaps should be made:

*TABLE 4G Swap Statements to Produce Forecast Closure (Step 3)*

<i>Previously exogenous variables</i>	<i>Newly exogenous variables</i>
technical change (A) .....	exports (X)
export demand shifter ( $F_X$ ) .....	terms of trade ( $TofT$ )

After step 3 we now have virtually all macroeconomic variables for which forecast data usually exist chosen as exogenous. In the full ZAR-M model we have many more variables and therefore flexibility in how we apply our swaps and which variables we can set as exogenous. It is often found that with the introduction of labour/capital preference twist and other shift variables in the full model, we can also exogenise employment, even with real wages exogenous, to incorporate available forecast and trend data.

Care should always be taken in developing these closures as not any combination of  $(n-m)$  variables will be legitimate. For example, at least one monetary variable should be included on the exogenous list in order to determine the absolute price level in the model. In terms of the forecast closure for ZAR-M, we also have to be aware of the potential over-determination of GDP when incorporating exogenous forecasts for employment (L). With the exogenisation of C, I, G, X and M, GDP from the expenditure side is tied down. If technical change (A) were also entirely exogenous, then GDP from the supply side would also be tied down by the exogenous setting of aggregate employment (L) and capital (K). In ZAR-M then, to allow GDP from the supply side to adjust to equal GDP from the expenditure side, overall primary-factor augmenting technical change must be endogenised in the forecast closure.

Given the proposed swaps to the model closure, the choice of endogenous and exogenous variables in the forecast closure for the stylized model can therefore be outlined as in Table 4H. A complete exposition of the swap statements required to move from the decomposition closure to the forecast closure in ZAR-M is included in the Appendix. Details concerning the actual forecast values and baseline forecast results for this study are given in Chapter 5.



TABLE 4H    *Forecast Closure for the Stylized Model*


---

Endogenous variables

- i)      gross domestic product (GDP)
- ii)     employment (L)
- iii)    rates of return (RoR)
- iv)    average propensity to consume (APC)
- v)     investment demand shifter (F\_I)
- vi)    ratio of private to public consumption (R\_CG)
- vii)   import/domestic preference twist (TWS)
- viii)  technical change (A)
- ix)    export demand shifter (F\_X)
- x)     ratio of investment to capital stock (R\_IK)
- xi)    domestic price index (PY)

## Exogenous variables

- i)      real wages (RW)
  - ii)     capital stock (K)
  - iii)    household consumption (C)
  - iv)    investment expenditure (I)
  - v)     government expenditure (G)
  - vi)    imports (M)
  - vii)   exports (X)
  - viii)  terms of trade (ToT)
  - ix)    consumer price index (CPI)
-

## 4.5 Developing the Policy Closure

The policy closure is used in simulations analysing the impact of an exogenous shock to the economy, beyond the natural baseline scenario. Policy simulations are performed as a sequence of annual solutions in recursive-dynamic models. As a result, the closure reflects many short-run features. The policy closure is more orthodox than the forecast closure used to construct a baseline for the economy. In this closure, naturally endogenous variables such as the macro components of GDP are again set to be endogenous. The impact on macro variables is usually of particular interest to policymakers and must therefore be allowed to respond to the policy change under consideration. ZAR-M also has the ability to produce disaggregated results on an industry level, giving an indication of the possible winners and losers of a proposed policy. From the supply side of the economy, employment is set as endogenous, with real wages ‘sticky’ in the short-run. Variables usually unaffected by policy decisions in the short-run are set as exogenous. Correspondingly, naturally exogenous variables such as the positions of foreign demand curves or the average propensity to consume are set as such, and take on values generated in the baseline forecast. That is, there will be no deviation between the policy and forecast simulation values for these variables.

In the full ZAR-M model we often impose budget neutrality for policy simulations involving government, that is, we allow no change to public sector debt as a result of the policy. To achieve this result we let consumption taxes adjust to compensate for the change in government finances.

In terms of the closure for the stylized model, our choice of  $(n-m)$  exogenous variables is similar to that of the decomposition closure. The only difference lies in the short-run nature of the policy closure versus the long-run nature of the decomposition closure. The more orthodox nature of the policy closure, as compared to the forecast closure, is also evident.

Variables which are exogenous in the policy run will take on values imposed or generated in the forecast simulation. Policy shocks imposed on any of these exogenous variables reflect changes away from their forecasted values and generate changes in the model's endogenous variables away from their respective forecasted values. This deviation away from the baseline forecast may then be interpreted as the impact of the particular policy under consideration. Typically, the choice of variables required to carry the policy shocks is unique to every study.

From the expenditure side of the economy, we endogenise household consumption (C) and government expenditure (G) via the consumption function described in (E4.3) and (E4.4). Investment (I) is endogenised via (E4.7). With the balance of trade (X–M) endogenous, we exogenise the export demand shifter (F<sub>X</sub>) and import/domestic preference twist variable (TWS). With exports (X) determined via (E4.1), the ToT is endogenised via (E4.6).

With the policy closure applied within a short-run environment, we exogenise start-of-year capital stock and activate the 'sticky' wage adjustment mechanism. This allows us to endogenise aggregate employment (L) and rates of return on capital (RoR) via (E4.10) and (E4.11). Technical change (A) is usually set as exogenous in policy simulations. Under these assumptions, technical change and the real wage rate together determine the rate of return on capital via the factor-price frontier. The factor-price frontier is the relationship between the marginal product of labour (MPL), which is tied down by the real wage (RW), and the marginal product of capital (MPK), which determines the rate of return on capital (RoR). This relationship is determined by the equations and assumptions underlying (E4.2) and (E4.10–E4.13) described earlier. With labour being paid according to its marginal product, employment is determined by the real wage rate. Capital and technical change is then determined via the marginal-product function for labour. For a general understanding of the policy closure, we can therefore think of capital stock (K), real wages (RW), and technical change (A) as being exogenous in our stylized model.

We have now established a number of possible swaps that will allow us to move from a forecast closure to a typical short-run oriented policy closure. The following swaps from our stylized model closure in Table 4H should therefore be made:

*TABLE 4I Swap Statements to Produce Policy Closure*

---

<i>Previously exogenous variables</i>	<i>Newly exogenous variables</i>
household consumption (C) .....	average propensity to consume (APC)
investment expenditure (I) .....	investment demand shifter (F_I)
government expenditure (G) .....	ratio of private to public consumption (R_CG)
imports (M) .....	import/domestic preference twist (TWS)
exports (X) .....	technical change (A)
terms of trade (TofT) .....	export demand shifter (F_X)

---

Given the proposed swaps to the model closure, the choice of endogenous and exogenous variables in the policy closure for the stylized model can be outlined as in Table 4J. A complete exposition of the swap statements required to move from the forecast closure to the policy closure in ZAR-M is included in the Appendix. Details concerning the actual policy shocks and simulation results for this study are given in Chapter 5.

TABLE 4J      *Policy Closure for the Stylized Model*


---

Endogenous variables

- i)      gross domestic product (GDP)
- ii)     employment (L)
- iii)    rates of return (RoR)
- iv)    household consumption (C)
- v)     investment expenditure (I)
- vi)    government expenditure (G)
- vii)   imports (M)
- viii)  exports (X)
- ix)    terms of trade (ToT)
- x)     ratio of investment to capital stock (R<sub>IK</sub>)
- xi)    domestic price index (PY)

## Exogenous variables

- i)      real wages (RW)
  - ii)     capital stock (K)
  - iii)    average propensity to consume (APC)
  - iv)    investment demand shifter (F<sub>I</sub>)
  - v)     ratio of private to public consumption (R<sub>CG</sub>)
  - vi)    import/domestic preference twist (TWS)
  - vii)   technical change (A)
  - viii)  export demand shifter (F<sub>X</sub>)
  - ix)    consumer price index (CPI)
-

## 4.6 Concluding Remarks

In this chapter we described the basic features of forecast and policy closures using a stylized CGE model as our reference. By starting from a typical long-run decomposition closure and making carefully considered swaps to our choice of  $(n-m)$  exogenous variables, we were able to derive closures for both the baseline forecast and policy simulations. In choosing the  $(n-m)$  variables for the forecast closure, we include a variable on the exogenous list only if we know its value, irrespective of how we think that value is determined. The policy closure has a more orthodox specification of  $(n-m)$  variables where we only include a variable on the exogenous list if we believe its value can be determined independently of any other variable. As with the decomposition closure, endogenous variables in the policy closure usually relate to those described by an equation in the model code.

The different notions of exogeneity in decomposition and policy closures on the one hand compared with forecast closures on the other reflect different objectives. In forecast simulations we use all available data and information about the future to generate a believable baseline for the economy. In policy simulations we attempt to explain the effects of specific shocks to the economy. For this purpose we need to make realistic assumptions about which variables are likely to be unaffected and which will be responsive to the shocks under consideration (Dixon & Rimmer, 2002:276).

With the Johansen/Euler type solution strategy implemented through GEMPACK, moving between different types of model closures has been made easy. Once a sensible model closure has been developed, alterations to the choice of  $(n-m)$  exogenous variables can be made through swap statements. However, care should always be taken in maintaining the conditions for a legitimate closure. Although closures may be developed for specific simulation purposes, the economic meaning of the assumptions imposed via the closure should always be interpretable. Using a stylized model for this purpose is well advised.

It is important to acknowledge the difference in describing a closure based on the stylized model in Table 4A, which can be thought of as a small comparative-static CGE model, and a closure based on the detailed recursive-dynamic ZAR-M model used in this study. In ZAR-M we have to consider the various adjustment mechanisms and stock/flow relationships of a recursive-dynamic model. Additional cost-neutral preference twists and shift variables also allow greater flexibility in the set up of our model closures. However, the underlying nature of each closure remains the same, regardless of which model we use as reference. A complete exposition of each closure for the ZAR-M model is included in Appendix B.





# CHAPTER 5

## SIMULATIONS

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## 5.1 Introduction

The main purpose of the ZAR-M model is to provide projections of the effects of economic policy changes on a wide variety of economic variables. In this chapter we focus on analysing the impact of policies related to labour and migration issues in South Africa, in particular that of illegal immigration. Adding context to the recent immigration debate in South Africa is the fact that these movements have occurred during times where the overall unemployment rate has been consistently above 30 per cent (OECD, 2008).<sup>1</sup>

After producing a plausible baseline forecast of the South African economy up to 2020, the policy simulations conducted in this study measure the impact of successfully implementing the government's stated migration policy objective of reducing illegal immigration (UNDESA, 2007). The study by Dixon *et al* (2011) plays a major role in the modelling conducted in this chapter. As documented in the literature review, Dixon *et al* (2011) found that a reduction in illegal immigration for the United States harms the overall welfare and job prospects of legal residents in the long-run. As part of our investigation, we run ZAR-M under two different scenarios or sets of assumptions. The *first* uses labour market and wage adjustment assumptions equivalent to that in the Dixon *et al* study. This includes the modelling of upward-sloping labour supply curves and equilibrium rates of unemployment. The unsatisfactory results generated in the context of the South African labour market pushed us to develop a second set of assumptions. The *second* set of assumptions simulates a more realistic picture of the South African labour market at its lower end. Importantly, we consider high unemployment rates among low-skilled workers and a legal minimum wage in the wage adjustment process under the second set of assumptions.

The policy simulation considers a reduction in illegal immigration via a reduction in the preferences of foreign-born workers with illegal status for moving to and earning money in South Africa. A change in such supply-side preferences may be brought on in a number of ways. Since we do not consider the direct cost of implementing the policy in this study, the

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<sup>1</sup> Overall unemployment here refers to the expanded definition of unemployment which includes both the officially unemployed and discouraged work-seekers.

choice of policy to achieve the result is somewhat arbitrary providing that the shock is transmitted via a reduction in the supply of illegal migrants. What we have in mind here is the introduction of policies that increase border security around South Africa or improve economic and political stability in neighbouring countries. Anecdotal evidence suggests that both these factors have contributed significantly to the large inflow of illegal migrants to South Africa.

In order to measure the long-term impact of a policy change within a recursive-dynamic framework, a baseline forecast scenario incorporating available forecast data must first be simulated. The baseline reflects the changes expected to occur in the economy, over a given period of time, if the policy under consideration was not implemented. The choice of baseline forecast can affect the results of the policy simulation and should therefore be carefully considered (Dixon & Rimmer, 2002). The perturbed scenario incorporating the relevant policy shocks is then run and compared to the outcome of the business-as-usual baseline scenario, with deviations usually reported as percentage changes.

In this chapter we use the ZAR-M model to conduct the baseline forecast and relevant policy simulations. ZAR-M is solved using the GEMPACK software package developed by Harrison & Pearson (1996).<sup>2</sup> In the next section we first compose a baseline forecast for the South African economy. The most important macroeconomic and labour market forecast results are reported and briefly discussed. We then conduct the policy simulations. Policy results are reported on both a macro and industry level to provide additional insight on their potential impacts. Labour market impacts are fully disaggregated to take advantage of the detailed theoretical specification and database of ZAR-M. A careful interpretation of these results, aided by the use of ‘back-of-the-envelope’ analysis, is included. We conclude this chapter with some policy suggestions and general comments concerning labour and migration issues in South Africa.

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<sup>2</sup> In solving the model we used the Euler 32-step solution method. This multi-step approach is used to eliminate the linearisation error associated with the Johansen 1-step method. Other solution methods, such as the Gragg, did not produce noticeably different results. More details concerning the percentage-change approach and elimination of linearisation error is included in Appendix A.

## 5.2 Baseline Forecast

The baseline forecast simulation aims to produce a believable business-as-usual picture of the future evolution of the economy, excluding the impact of any policy under consideration. The impact of the policy may then be contrasted against this baseline scenario in a separate simulation. This allows the model to produce a more realistic estimate of the policy's impact and future evolution of the economy under the perturbed scenario. That is, if we wish to know what the difference in migrant flows will be in 2020 as a result of imposing certain policies, we must first establish what the flow of migrants would have looked like in 2020 without the imposition of such policies. In this section we describe the baseline forecast produced in the application of this study.

As noted in our discussion of the forecast closure in Chapter 4, we exogenise all macroeconomic variables for which we have sufficient and reliable forecast estimates. This allows us to apply exogenous projections to the ZAR-M model. Many of the model's naturally exogenous variables such as the average propensity to consume, preference twists and various shift variables must therefore be allowed to endogenously move to accommodate the exogenous forecasts. The baseline forecast simulation is applied from 2007–2020, incorporating available historical data up to 2009.<sup>3</sup> This 14-year simulation period is a short enough timeframe for results to remain significant and relevant for today's policymakers. It is also long enough to allow the model to adjust to the imposed policy shocks and produce a solution in which the economy has returned to a new 'steady-state' under the perturbed scenario.

Table 5A lists the economic variables in ZAR-M explicitly forecast in the baseline simulation. That is, these variables are all given non-zero exogenous year-to-year movements based on available historical data for 2007–2009 and forecast data for 2010 onwards. Other exogenous variables not explicitly forecast are simply assumed to remain unchanged over the

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<sup>3</sup> Given that the initial solution for ZAR-M in this study is based on 2006 data, we applied our baseline forecasts from 2007 onwards. At the time of writing, historical data up to 2009 were available for most macro variables. This information was incorporated to give a more accurate description of the economy in the baseline scenario following the Global Financial Crisis of 2008.

forecast period. The ability to set both employment and real wages exogenous in the model closure is facilitated by the introduction of a cost-neutral labour/capital preference twist variable in ZAR-M. In producing the baseline forecast, a wide variety of data sources were considered, including Altman (2005), Burger & Yu (2006), Dixon *et al* (2011), IMF (2010), Treasury (2010), Parsons *et al* (2007), SARB (2007; 2010), StatsSA (2008; 2009; 2010b; 2010c), UNDESA (2009a; 2010a; 2010b), UNDP (2009), and Ratha *et al* (2010). In addition, long-term trends estimated from available time-series data were also considered where appropriate.<sup>4</sup>

A list of the economic variables for which non-zero forecast values were exogenously specified in the baseline simulation is given below in Table 5A.

*TABLE 5A Exogenously Forecast Variables*

---

i)	aggregate employment (D)
ii)	real wages (RW)
iii)	household consumption (C)
iv)	investment expenditure (I)
v)	government expenditure (G)
vi)	exports (X)
vii)	imports (M)
viii)	consumer price index (CPI)
ix)	terms of trade (ToT)
x)	nominal exchange rate (ER)
xi)	number of households (HH)
xii)	new entrants to the labour market (NE)

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<sup>4</sup> Time-series data for most macro variables in ZAR-M are available from the South African Reserve Bank or Statistics South Africa website.

In Table 5B we report selected macroeconomic results from the baseline forecast simulation. Results for these variables are reported as cumulative percentage-changes from 2007 up to 2020 and represent deviations away from the base year values, or initial solution, given for 2006. The choice of variables displayed in Table 5B reflects our earlier exposition of the stylized model and forecast closure in Chapter 4. Where possible, it reports all the macro variables in ZAR-M equivalent to those shown in the stylized model. This should aid the reader in better understanding the results and underlying model closure for the baseline simulation conducted here. More details concerning the actual year-on-year forecast values for this study are included in Appendix C.

The results in Table 5B, for those variables listed in Table 5A, reflect the exogenous values imposed on them in the forecast simulation. With all components of GDP from the expenditure side set at their forecasted values, real GDP growth of 84.3 per cent is expected from 2007–2020. This is equivalent to an average annual growth rate of 4.5 per cent over the 14-year period. Investment expenditure is expected to continue growing at a faster rate than household consumption, with a slight increase in the average propensity to save also projected. The Global Financial Crisis of 2008 is the main reason for the relatively slow growth witnessed over the 2007–2010 forecast period, with household expenditure and trade most severely impacted. Increased stimulus from government, along with the finalising of FIFA World Cup related projects, are the main reasons that the ratio of public to private consumption ( $R_{GC}$ ) increased up to 2010. The expected recovery in household consumption allows this ratio to fall somewhat in later years. Given the respective forecasts imposed, very little change in the budget deficit to GDP ratio ( $R_{DEFGDP}$ ) is projected over the forecast period.

With employment ( $D$ ) exogenous and capital growth ( $K$ ) linked to investment expenditure ( $I$ ) via the capital accumulation mechanism, economy-wide technical change ( $A$ ) of 14.9 per cent is required to balance GDP from the supply side. Given the exogenous export demand ( $X$ ) and terms of trade ( $TofT$ ) settings, a cumulative shift in world demand for exports ( $F_X$ ) of 174.3 per cent over the forecast period is endogenously determined by the model.

The shift in  $F_X$  reflects the expected growth in world trade and demand for locally-produced exports, consistent with the exogenous trade forecasts imposed on the model. This result is equivalent to an annual average growth rate, or outward shift, of around 7.5 per cent in  $F_X$  over the 14-year period. At first glance, the 174.3 per cent shift of the export-demand curve may seem unusually large compared to some of the other results in Table 5B. Recall from Chapter 2 that the export-demand curve is downward sloping in the theoretical specification of ZAR-M. A large shift in  $F_X$  is therefore required for the model to reconcile the exogenously given increase in  $X$  of 69.3 per cent with the increase in the ToFT of 15.5 per cent. The result produced here also reflects the given export elasticities in ZAR-M of between 2 and 4 for all commodities.

The slightly lower levels of trade in the economy relative to GDP over the forecast period reflect the lingering impact of the Global Financial Crisis on imports ( $M$ ) and exports ( $X$ ). This is particularly evident over the 2007–10 period of the baseline. Globalisation and ‘love-of-variety’ from the modern-day consumer allows trade to recover in later years, with growth almost on par with GDP projected over the entire forecast period. As noted earlier, the ToFT is forecast to improve up to 2020. This is defined as an increase in export prices relative to import prices. A natural shift towards relatively cheaper imports in the local import/domestic sales mix should therefore occur. However, the exogenously imposed values for imports and exports in the baseline forecast do not suggest any significant change in the import/domestic sales mix relative to the given change in the ToFT. To calibrate the forecasted values for the trade balance with the ToFT, a cost-neutral import/domestic preference twist (TWS) is introduced. As may be expected, a cost-neutral twist in preferences by users holding back imports in the local/import domestic mix of sales is generated by the model. The negative value of –26.1 per cent generated for TWS over the baseline period reflects this shift in preferences away from the use of imports required by the model to reconcile the different exogenously given values of  $M$ ,  $X$  and ToFT with each other.

TABLE 5B Baseline Forecast for Selected Macroeconomic Variables (Cumulative Percentage Change)

<i>Macroeconomic Variable</i>	2007–2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
real gross domestic product (GDP)	15.1 %	20.5 %	26.2 %	32.3 %	38.7 %	45.4 %	52.4 %	59.8 %	67.6 %	75.7 %	84.3 %
private consumption (C)	12.5 %	17.5 %	22.8 %	28.4 %	34.1 %	40.2 %	46.5 %	53.1 %	60.0 %	67.2 %	74.7 %
average propensity to consume (APC)	-1.8 %	-2.2 %	-2.8 %	-3.5 %	-4.1 %	-4.8 %	-5.5 %	-6.1 %	-6.8 %	-7.5 %	-8.2 %
investment (I)	26.7 %	34.3 %	43.0 %	52.3 %	62.2 %	72.8 %	84.0 %	96.0 %	108.7 %	122.3 %	136.7 %
government expenditure (G)	19.3 %	24.4 %	29.4 %	34.5 %	39.9 %	45.5 %	51.3 %	57.4 %	63.7 %	70.2 %	77.0 %
ratio of public to private consumption (R_GC)	6.0 %	5.8 %	5.3 %	4.8 %	4.3 %	3.8 %	3.3 %	2.8 %	2.3 %	1.8 %	1.3 %
ratio of budget deficit to GDP (R_DEFGDP)	-0.01 %	-0.01 %	-0.02 %	-0.02 %	-0.03 %	-0.03 %	-0.04 %	-0.04 %	-0.05 %	-0.05 %	-0.06 %
exports (X)	4.3 %	9.3 %	14.6 %	20.4 %	26.4 %	32.7 %	39.3 %	46.3 %	53.6 %	61.3 %	69.3 %
export world demand shift (F_X)	29.9 %	39.8 %	50.7 %	62.4 %	75.1 %	88.7 %	103.3 %	119.1 %	136.2 %	154.5 %	174.3 %
imports (M)	9.2 %	14.7 %	20.4 %	26.4 %	32.8 %	39.4 %	46.4 %	53.7 %	61.4 %	69.4 %	77.9 %
import/domestic preference twist (TWS)	-14.9 %	-15.7 %	-16.8 %	-17.9 %	-19.1 %	-20.2 %	-21.4 %	-22.6 %	-23.8 %	-24.9 %	-26.1 %
overall technical change (A)	3.6 %	5.1 %	6.5 %	7.9 %	9.1 %	10.3 %	11.4 %	12.4 %	13.3 %	14.1 %	14.9 %
capital stock (K)	14.3 %	18.7 %	23.4 %	28.6 %	34.3 %	40.5 %	47.3 %	54.6 %	62.6 %	71.2 %	80.5 %
compensation of employees (CoE)	6.9 %	9.6 %	12.2 %	14.9 %	17.8 %	20.6 %	23.6 %	26.6 %	29.7 %	32.9 %	36.2 %
labour/capital preference twist (TWLK)	12.7 %	14.8 %	17.3 %	20.1 %	23.3 %	26.7 %	30.5 %	34.5 %	38.8 %	43.4 %	48.3 %



TABLE 5B *Baseline Forecast for Selected Macroeconomic Variables (Cumulative Percentage Change) (continued)*

<i>Macroeconomic Variable</i>	2007–2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
employment in local occupations (D)	6.6 %	9.3 %	12.0 %	14.8 %	17.7 %	20.7 %	23.7 %	26.8 %	29.9 %	33.2 %	36.5 %
labour supply to local occupations (L)	6.1 %	7.9 %	9.8 %	11.8 %	13.8 %	15.8 %	17.9 %	20.0 %	22.2 %	24.2 %	26.6 %
total labour force (LF)	6.1 %	7.6 %	9.2 %	10.7 %	12.4 %	14.0 %	15.7 %	17.4 %	19.1 %	20.8 %	22.6 %
change in unemployment rate (UR)	-2.0 %	-2.5 %	-3.0 %	-3.5 %	-4.0 %	-4.5 %	-4.9 %	-5.4 %	-5.8 %	-6.3 %	-6.7 %
average real wage in local occupations (RW)	1.6 %	2.1 %	2.6 %	3.1 %	3.7 %	4.2 %	4.7 %	5.2 %	5.7 %	6.3 %	6.8 %
nominal rental price of capital (Q)	40.5 %	52.1 %	65.1 %	78.9 %	93.5 %	108.7 %	124.6 %	141.2 %	158.5 %	176.6 %	195.3 %
nominal price of investment (PI)	29.0 %	36.4 %	44.3 %	52.7 %	61.6 %	71.1 %	81.1 %	91.6 %	102.9 %	114.7 %	127.3 %
consumer price index (CPI)	28.6 %	36.4 %	44.5 %	53.2 %	62.4 %	72.2 %	82.5 %	93.4 %	105.0 %	117.3 %	130.4 %
GDP price index (PY)	30.5 %	38.3 %	46.7 %	55.8 %	65.5 %	75.8 %	86.7 %	98.4 %	110.7 %	123.9 %	137.9 %
GNE price index (PGNE)	28.8 %	36.0 %	44.0 %	52.4 %	61.4 %	71.0 %	81.1 %	91.9 %	103.3 %	115.4 %	128.2 %
terms of trade (ToFT)	4.6 %	5.6 %	6.7 %	7.7 %	8.8 %	9.9 %	11.0 %	12.1 %	13.2 %	14.3 %	15.5 %
nominal exchange rate (ER)	-2.0 %	-2.5 %	-3.0 %	-3.5 %	-3.9 %	-4.4 %	-4.9 %	-5.4 %	-5.8 %	-6.3 %	-6.8 %
number of households (HH)	4.1 %	5.1 %	6.2 %	7.1 %	8.1 %	9.0 %	10.0 %	11.0 %	12.0 %	13.0 %	14.0 %
number of new entrants to the labour market (NE)	4.1 %	5.1 %	6.2 %	7.2 %	8.3 %	9.4 %	10.5 %	11.6 %	12.7 %	13.8 %	14.9 %

A similar approach may be used to explain the labour/capital preference twist (TWLK) in the baseline forecast. With strong forecasted growth in investment expenditure (I), capital growth (K) of 80.5 per cent is projected over the 14-year period. This occurs despite a relatively large increase in the real rental price of using capital (Q/CPI). Good aggregate employment (D) growth of 36.5 per cent is forecast, whilst real wages (RW) are forecast to increase by 6.8 per cent over the same period. With technical change (A) of 14.9 per cent projected, the increase in RW of only 6.8 percent may be viewed as somewhat low given the expected rise in the marginal product of labour associated with such technical improvements. Nonetheless, taking this information as given, the use of labour is then expected to become relatively cheaper than capital in the primary-factor composite of industries. In the baseline forecast however, employment (D) and investment (I) are exogenously forecast, with capital (K) pseudo exogenous via the capital accumulation mechanism in ZAR-M. The K/L ratio is therefore implicitly set via these forecasts. Contrary to our expectation of a fall in the K/L ratio based on their relative price and marginal product changes, our exogenous forecasts impose a rise in the K/L ratio.

There are several reasons why this result is plausible. Foremost is the continued strength of both labour unions and labour legislation expected in South Africa, which may act as a deterrent in employers' decisions to hire more labour. Industrial action instigated by unions is commonplace during periods of wage negotiation, with employers often forced to yield to the demands of workers. Moreover, the high unemployment rate in South Africa also suggests that current real wages are far above the market-clearing level. Even if the real wage rises slower than capital rental rates, this may not be enough incentive for employers to hire additional labour, especially with high expected rates of return on their capital investments.

To calibrate the forecasted increase in the K/L ratio with the projected increase in effective cost of using capital relative to labour, a cost-neutral labour/capital preference twist (TWLK) is introduced. As may be expected, a cost-neutral twist in preferences by users holding back labour in the primary-factor composite is generated by the model. The positive value of 48.3 per cent generated for TWLK over the baseline period reflects this strong shift in preferences

away from the use of labour required by the model to reconcile the seemingly incompatible exogenously given values of the K/L ratio and primary-factor prices with each other.

To better explain this result we must look at the theoretical specification of TWLK within ZAR-M. Input demand equations by industries are derived subject to a CES aggregation function with substitution elasticities ( $\sigma$ ) between primary factors set at 0.3. With  $\sigma \neq 1$  in the linearised input demand equations, this allows cost-neutral preference twists (TWLK) accommodating exogenous forecasts in the primary-factor market to be introduced and converted into technical or taste changes.<sup>5</sup> For readers familiar with GEMPACK, equations (E5.1) and (E5.2) show these linearised demand equations as they appear in the ZAR-M model code.

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$$cap = z - \sigma S_L (p_{cap} - p_{lab}) + S_L TWLK \quad (E5.1)$$

$$lab = z - \sigma S_K (p_{lab} - p_{cap}) - S_K TWLK \quad (E5.2)$$


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Following our notational convention, *cap* and *lab* represent the percentage change in industry demands for capital ( $K$ ) and labour ( $L$ ), respectively. We note that in the absence of any change in output ( $z$ ) and relative factor prices ( $p_{cap} - p_{lab}$ ), this representation gives  $[cap - lab = TWLK]$  and  $[S_K * cap + S_L * lab = 0]$ . Thus, if the K/L ratio in ZAR-M increases by 10 per cent beyond what is explained by relative factor price movements, then TWLK will equal 10. The twist is therefore equivalent to movements in  $A_K$  and  $A_L$  that satisfy equations (E5.3) and (E5.4).

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<sup>5</sup> This method of implementing cost-neutral preference twists eliminates a problem arising with these variables when set as exogenous in policy simulations. The same method is used to implement cost-neutral import/domestic preference twists. For a more technical explanation, see Dixon & Rimmer (2004).

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$$S_L TWLK = a_K - \sigma S_L (a_K - a_L) \quad (E5.3)$$

$$-S_K TWLK = a_L - \sigma S_K (a_L - a_K) \quad (E5.4)$$


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We can further show that by implementing the twist via the technical change variables, we are in effect assuming that  $\left[ a_K = \frac{S_L}{1-\sigma} * TWLK \right]$  and  $\left[ a_L = -\frac{S_K}{1-\sigma} * TWLK \right]$ . With  $\sigma$  below one, a positive TWLK value is equivalent to a cost-neutral capital-using technical change combined with a labour-saving technical change.

To provide additional insight into the composition of our K/L movements, we use the two ‘back-of-the-envelope’ equations shown in (E5.5) and (E5.6). Assuming competitive conditions, the marginal products of capital and labour would equal their respective factor payments. In our exposition of (E5.5) we recognise that the marginal product of capital ( $F_K$ ) is negatively related to the K/L ratio. In (E5.6) we recognise that the marginal product of labour ( $F_L$ ) is positively related to the K/L ratio. These equations are similar to (E4.12) and (E4.13) presented earlier in Chapter 4, but with technical change terms now included.

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$$F_K A \left[ \frac{\left( \frac{K}{A_K} \right)}{\left( \frac{L}{A_L} \right)} \right] \approx \frac{Q}{P_i} * \frac{P_i}{P_y} \quad (E5.5)$$

$$F_L A \left[ \frac{\left( \frac{K}{A_K} \right)}{\left( \frac{L}{A_L} \right)} \right] \approx \frac{W}{P_c} * \frac{P_c}{P_y} \quad (E5.6)$$


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Given the implementation of our labour/capital twist via the technical change variables, we are now able to use our ‘back-of-the-envelope’ equations to help interpret the results for TWLK shown in Table 5B. In (E5.5), a preference twist affecting capital will therefore be transmitted via the technical change variable ( $A_K$ ). In (E5.6), a preference twist affecting labour will similarly be transmitted via the technical change variable ( $A_L$ ). Our baseline forecast results show strong primary-factor technical change in favour of capital and against labour. This is reflected in the positive value of 48.3 per cent generated for TWLK. We may equate this result to an increase in  $A_K$  in (E5.5) which will decrease the marginal product of capital requiring a rise in the relative amount of capital used. For labour, this may be equated to a decrease in  $A_L$  in (E5.6) which will increase the marginal product of labour requiring a fall in the relative amount of labour used. In the absence of this cost-neutral preference twist, captured via the technical change variables, forecasts related to the K/L ratio and relative primary-factor payments as applied here could not have been accommodated simultaneously.

Taking into account net migration flows, growth in aggregate employment (D), or demand for labour, is expected to exceed growth in supply of labour (L) to local occupations groups in South Africa. As shown in Table 5B, forecasted employment growth is also expected to exceed total labour force (LF) growth.<sup>6</sup> The overall unemployment rate (UR) is projected to fall by 6.7 percentage points between 2007 and 2020 as a result. ZAR-M allows a large fraction of foreign-born migrants who would otherwise have become unemployed in South Africa to flow back home. In addition, ZAR-M theory specifies that potential illegal immigrants to South Africa only flow into the country if they actually manage to find a job. That is, foreign-illegals are not allowed to flow directly into unemployment activities if offering to local jobs from outside South Africa. These modelling assumptions contribute to the lower unemployment rate projected over the forecast period. Comparing labour market data from the start of our base year in 2006, to results produced at the end of our forecast

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<sup>6</sup> In our definition as applied to ZAR-M, the ‘total labour force’ also includes discouraged work-seekers not actively looking for a job. This is different from the ‘supply of labour to local occupations’ which only includes those workers actively supplying or offering their services to local work activities. Any person, employed or unemployed, who offers to become (voluntary unemployment) or remain (discouraged work-seeker) unemployed will therefore be excluded from the calculation of ‘supply of labour to local occupations’.

period in 2020, we find a drop in the overall unemployment rate from 37 per cent to just below 30 per cent. This number includes all short-term unemployed and discouraged work-seekers in the domestic labour market. An initial number of new entrants to the labour market in 2006 are specified for the base year period.<sup>7</sup> These projections are consistent with historical population and labour force growth trends. From here, the annual number of new entrants (NE) is exogenously forecast to grow by 1 per cent each year up to 2020.

The labour market specification in ZAR-M defines workers according to their labour force functions, birthplace, and legal status characteristics. As discussed in Chapters 2 and 3, initial settings in the base year data for 2006 play an important role in the natural evolution and momentum of the economy over the baseline forecast period. Table 5C presents initial stock values for 2006 with cumulative projections for employment, unemployment and labour offers up to 2020. Table 5D presents initial migrant flows for 2006 with annual projections on inflow and outflow of migrants from 2010 up to 2020. Readers should note the difference in cumulative stocks versus annual flows presented in the respective tables.

Both Tables 5C and 5D present disaggregated forecasts for the labour market according to the birthplace and legal status characteristics of workers. Base year values for 2006 are also included. The results shown here confirm some of the macro results presented earlier in Table 5B. As shown in Table 5C, aggregate employment in local occupations is forecast to rise to 18.16 million workers by the end of 2020. This is an increase of 4.86 million workers, or 36.5 per cent, compared to end of 2006 employment levels in the base year. Employment results show relatively even growth across all groups in line with projected aggregate employment growth. With relative pre-tax wages between different labour groups unchanged in the baseline, this result is to be expected.

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<sup>7</sup> This specification of new entrants to the labour market in ZAR-M distinguishes between the number of domestic-born new entrants, as well as the number of foreign-born new entrants viewed as potential legal or illegal immigrants.

With regard to unemployment, domestic-born workers fare best over the forecast period. This is mainly due to the difference in relative growth rates in labour offers between groups projected over the forecast period. Labour offers from domestic-born workers are set to grow by 3.93 million workers, or 23.2 per cent, between 2007 and 2020. In comparison, both foreign-born legal and illegal labour offers to local occupations grow in excess of 40 per cent. The jump in stock of domestic-born unemployed shown in 2010 compared to 2006 is largely due to the slowdown in employment growth witnessed in 2008-09. With GDP and employment growth set to recover from 2010 onwards, the number of domestic-born unemployed is projected to fall again over time. Between 2010 and 2020, the number of domestic-born workers in unemployment is projected to fall by around 500,000. Considering the expected growth in labour supply, this equates to a drop in the unemployment rate of domestic-born workers of close to 7 percentage points over the forecast period.

Another number from Table 5C worth highlighting is the total number of illegal immigrants in the domestic labour market. For the end of our base year in 2006, we model 2.130 million illegals in the local market of which 1.715 million are employed and 415,200 unemployed.<sup>8</sup> Foreign-illegals therefore represent close to 13 per cent of total number of employed workers in the base year. By the end of the forecast period in 2020, the total number of illegals is projected to have grown to over 2.860 million. These estimates may be viewed as conservative, especially with regard to the number of unemployed foreign-illegals, but should be assessed within the structure of the ZAR-M modelling environment. As noted before, we do not attempt to model those migrants who operate outside the given economic framework.

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<sup>8</sup> Subtracting 1 per cent of all end-of-year  $t$  numbers will give start-of-year  $t+1$  numbers. This 1 per cent reduction represents annual deaths or retirement from the labour force modelled between years in ZAR-M.

TABLE 5C Baseline Forecast for Selected Labour Market Variables (Cumulative Stocks)

<i>Labour Market Stock Variable</i>	2006 *	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
employment (local/domestic/legal)	11.45 million	12.21 mil	12.52 mil	12.83 mil	13.15 mil	13.48 mil	13.82 mil	14.16 mil	14.52 mil	14.88 mil	15.25 mil	15.63 mil
employment (local/foreign/legal)	138,300	147,300	151,100	154,900	158,900	162,900	167,000	171,200	175,600	180,000	184,500	189,000
employment (local/foreign/illegal)	1.715 million	1.829 mil	1.875 mil	1.922 mil	1.970 mil	2.019 mil	2.070 mil	2.121 mil	2.175 mil	2.229 mil	2.285 mil	2.343 mil
total employment in local occupations	13.30 million	14.19 mil	14.54 mil	14.90 mil	15.28 mil	15.66 mil	16.05 mil	16.45 mil	16.87 mil	17.29 mil	17.72 mil	18.16 mil
unemployment (local/domestic/legal)	7.414 million	7.731 mil	7.703 mil	7.672 mil	7.635 mil	7.594 mil	7.547 mil	7.495 mil	7.437 mil	7.374 mil	7.305 mil	7.231 mil
unemployment (local/foreign/legal)	2,600	6,900	7,000	7,200	7,400	7,600	7,800	8,000	8,200	8,400	8,600	8,900
unemployment (local/foreign/illegal)	415,200	422,800	427,000	432,800	440,000	448,400	457,900	468,300	479,500	491,400	504,100	517,500
total unemployment in local market	7.832 million	8.161 mil	8.138 mil	8.112 mil	8.083 mil	8.049 mil	8.012 mil	7.971 mil	7.925 mil	7.874 mil	7.818 mil	7.757 mil
labour offers (domestic/legal/local)	16.94 million	17.81 mil	18.08 mil	18.37 mil	18.67 mil	18.97 mil	19.27 mil	19.58 mil	19.90 mil	20.22 mil	20.54 mil	20.87 mil
labour offers (foreign/legal/local)	157,800	172,500	176,600	181,500	186,500	191,600	196,800	202,200	207,600	213,100	218,800	224,600
labour offers (foreign/illegal/local)	3.82 million	4.20 mil	4.31 mil	4.42 mil	4.53 mil	4.65 mil	4.76 mil	4.88 mil	5.00 mil	5.12 mil	5.25 mil	5.38 mil
total offers to local employment	20.92 million	22.19 mil	22.57 mil	22.97 mil	23.39 mil	23.81 mil	24.24 mil	24.67 mil	25.11 mil	25.56 mil	26.01 mil	26.48 mil

\* Base year data for ZAR-M. Employment and unemployment values for 2006 in Table 5D represent end-of-year stock data.

Labour offer values for 2006 in Table 5D represent the sum of offers made to all local work activities during 2006.



TABLE 5D Baseline Forecast for Selected Labour Market Variables (Annual Flows)

<i>Labour Market Flow Variable</i>	2006 *	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
inflow of illegal migrants	225,500	150,200	161,800	166,200	170,200	174,300	178,600	182,900	187,300	191,800	196,400	201,100
outflow of illegal migrants	77,000	89,100	89,600	90,300	91,400	92,600	93,900	95,400	97,000	98,700	100,500	102,300
net inflow of illegal migrants	148,500	61,100	72,200	75,900	78,800	81,700	84,700	87,500	90,300	93,100	95,900	98,800
inflow of domestic-born legal migrants	8,100	12,200	13,900	14,900	16,100	17,200	18,400	19,600	20,900	22,200	23,600	25,100
inflow of foreign-born legal migrants	16,300	19,900	20,900	21,300	21,800	22,300	22,800	23,300	23,800	24,300	24,800	25,300
total inflow of all legal migrants	24,400	32,100	34,800	36,200	37,900	39,500	41,200	42,900	44,700	46,500	48,400	50,400
outflow of domestic-born legal migrants	60,500	61,400	61,900	62,600	63,200	63,900	64,500	65,200	65,900	66,500	67,200	67,900
outflow of foreign-born legal migrants	12,200	15,300	15,400	15,700	16,100	16,400	16,800	17,100	17,500	17,800	18,200	18,600
total outflow of all legal migrants	72,700	76,700	77,300	78,300	79,300	80,300	81,300	82,300	83,400	84,300	85,400	86,500
net outflow of all legal migrants	48,300	44,600	42,500	42,100	41,400	40,800	40,100	39,400	38,700	37,800	37,000	36,100

\* Base year data for ZAR-M. Labour market values for 2006 in Table 5E represent migrant flows that occurred during 2006.

Subsequent values for 2010–2020 represent projected migrant flows for each of those years in the baseline forecast.

Some modelling assumptions<sup>9</sup>, along with the adverse effects of the Global Financial Crisis, also contribute to the somewhat peculiar foreign-born unemployment numbers found in 2010 relative to base 2006 stocks and flows. For example, the unemployment numbers for foreign-born legals jumps from 2,600 in the base year to 6,900 at the end of 2010. After this initial jump, unemployment then increases at a stable rate in line with projected labour supply growth. In contrast, the number of foreign-born illegal unemployed does not change much between 2006 and 2010. In this case, a large outflow of unemployed illegals returning home is offset by other illegals becoming unemployed due to the recession but choosing to remain in South Africa. As with the foreign-born legals, the number of unemployed illegals then rises steadily from 2010 onwards in line with projected labour supply growth.

Table 5D presents additional labour market forecasts. Selected flows for migrant workers in the base year are given with annual projections from 2010 onwards also included. Readers should note that in order for the annual net flow numbers in Table 5D to be consistent with the change in end-of-year stocks in Table 5C, they must allow for a 1 per cent reduction in the labour force between years. For example, at the end of 2010 we modelled a total of 2.252 million foreign-illegals in South Africa. This number is obtained by adding the 1.829 million employed and 422,800 unemployed foreign-illegals shown in Table 5C. At the end of 2010 we then reduce the stock of foreign-illegals by 1 per cent, or 22,520, to allow for deaths and retirements between years. With a net inflow of 72,200 foreign-illegals projected for 2011, this gives the total stock of 2,302 million foreign-illegals at the end of 2011.

A net inflow of 148,500 illegal immigrants is modelled for the base year in 2006. In addition, we model a net outflow of 48,300 skilled emigrants in 2006. In comparison to the flows in the base year, we notice a much smaller net inflow of illegal migrants in later years. This result can be attributed to the recession and spate of xenophobic attacks witnessed in South Africa during 2008–09 and is reflected in both the inflow and outflow numbers of

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<sup>9</sup> As discussed in Chapter 2 and 3, we impose a number of simplifying assumptions and restrictions on the flow of foreign-born migrants in ZAR-M. This may lead to some large changes in flows produced for the first year of the baseline forecast, relative to the base, as the model adjusts. The jump in number of unemployed foreign-legals shown between 2006 and 2010 is such a case, where voluntary offers to unemployment relative to the small initial stock of unemployed dominate any outflows from unemployment activities. These jumps are restricted to the first year of the forecast period, with no significant impact on the results produced in this study.

illegal migrants. Also contributing is our modelling assumptions that prevent foreign-illegals from flowing directly into unemployment activities if unsuccessfully offering to local jobs from outside South Africa. As strong economic growth and employment returns in later periods, so do the illegal immigrants. As shown in Table 5D, annual inflows of illegal immigrants are projected to increase again from 150,200 in 2010 to over 200,000 in 2020. Over the same period, outflows of illegal migrants increase slightly from 89,100 to 102,300. This yields a projected increase in net inflows of illegal migrants from 61,100 in 2010 to 98,800 per annum in 2020. The cumulative sum of these annual net inflows gives the increase in total number of illegals expected over the forecast period, allowing for deaths and retirements.

Annual flows of skilled migrants projected in the baseline forecast suggest a continued net outflow of workers over the forecast period.<sup>10</sup> However, as indicated in Table 5D, the net outflow of all skilled-legal migrants is expected to decline from 48,300 in the base year to 36,100 in 2020. Encouragingly, a sharp increase in domestic-born legal migrants returning home is projected. Apart from the strong growth forecast for the local economy, this result also indicates the slowdown in demand for foreign-migrant labour witnessed around the world in the aftermath of the Global Financial Crisis. Migrant workers who lost their jobs or who faced deteriorating work opportunities during this time were often found to return home. In addition, organisations such as the Homecoming Revolution has done good work in actively supporting the return of South African citizens working abroad to help build the local skill base.

Within the ZAR-M modelling framework, the success of these repatriation efforts rely heavily on sufficient employment growth and job vacancies generated in the local economy to entice those working abroad to return home. The annual outflow of domestic-born workers is projected to remain between 60,000 and 70,000 over the forecast period. The largest flow included here is the outflow of domestic-born new entrants to the labour market. This is consistent with reports suggesting that many new matriculants and graduates

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<sup>10</sup> All foreign-born workers with legal status in ZAR-M are modelled as potential skilled migrants.

go overseas for a period of time after completing their studies. Typically, this may be to do part-time jobs while travelling or to gain international experience in their field of interest to further their professional careers. Over the baseline period we also project a steady increase in inflows from foreign-born legal migrants, up from 16,300 in 2006 to over 25,000 in 2020. This reflects the continued improvement by South African companies and institutions in attracting skilled migrants. However, with a slight increase in the outflow of foreign-born legal migrants also projected, the net inflow related to these migrants show very little change over the forecast period.

This section presented the main macroeconomic and labour market projections generated over the baseline period by the ZAR-M model. It should be noted that many of the exogenous forecasts imposed on the model were based on data and reports prepared by outside sources. The baseline reflects relatively stable growth across all sectors of the economy, with most migration flows set to continue along trends established by initial settings in the base year data. Against the backdrop of this 'business-as-usual' picture of the economy, the model is now able to generate more a realistic estimate of any perturbed scenario in future years. In the following section we discuss the relevant policy simulations conducted in the application of this study. A complete analysis and discussion of results are included.

### 5.3 Policy Simulations

Policy simulations using the ZAR-M model aim to produce a detailed set of results indicating the impact of a proposed policy or perturbed scenario on the South African economy over a given period of time. The impact of the policy is contrasted against the ‘business-as-usual’ baseline scenario described in the previous section. This allows results to be calculated and reported as deviations away from the baseline. In the following section we give a quantitative description of the economic impact of a policy-induced reduction in illegal immigration to South Africa. The policy simulation conducted in this study takes a counterfactual approach. That is, it estimates the impact on the South African economy, relative to the baseline, had the proposed policy interventions been successfully implemented.

The policy simulation is conducted and analysed under two different sets of assumptions or scenarios. The *first* policy scenario reflects typical economic modelling assumptions in the wage adjustment process that include upward-sloping labour supply curves and equilibrium rates of unemployment. These assumptions also correspond to those implemented in the Dixon *et al* (2011) study for the United States. The *second* policy scenario attempts to capture a more realistic picture of prevailing labour market conditions in South Africa. This scenario accounts for the high unemployment rates among low skilled workers and the setting of a legal minimum wage. The actual policy shock, described hereafter, is the same in each scenario. The reason why two modelling scenarios are reported in this study is to clearly contrast our sets of results to those in the Dixon *et al* study. Whilst only the second policy scenario would have been required to answer the policy question posed in this study, including both scenarios also answers the question as to how and why the results generated in the second scenario are different from those in both the first scenario and the Dixon *et al* study. Results from *Scenario 1* will be reported and discussed in full detail. Results from *Scenario 2* will be reported after, with emphasis placed on significantly different outcomes resulting directly from the change in assumptions.

*The policy shocks*

The policy simulation conducted in this study considers the impact of a reduction in illegal immigration via a reduction in the preferences of foreign-born workers for moving illegally into South African employment activities. A change in such supply-side preferences of illegal migrants may be brought on in a number of ways. As noted before, we consider policies that will increase border security around South Africa or improve economic and political stability in neighbouring countries. This policy change aims to effectively increase the opportunity cost to illegal migrants of entering South Africa, thereby reducing their preference for moving to South Africa. The policy shock is introduced in ZAR-M as a 44 per cent reduction in the marginal utility to potential illegal immigrants from earning money in South Africa. That is, the cost to illegal migrants for moving to South Africa is effectively increased by 44 per cent. The policy shock is calibrated to produce a reduction in illegal workers in South Africa of around 30 per cent over the period 2007–2020. This is in line with suggested outcomes of successfully implementing the proposed policy of reducing illegal immigration.

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$$L_t(cat; act) = CAT_t(cat) * \left[ \frac{[PREF_t(cat; act) * ATRW_t(act)]^\eta}{\sum_q [PREF_t(cat; q) * ATRW_t(q)]^\eta} \right] \quad (E2.39)$$

*for all cat & act*

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In terms of equation (E2.39) described earlier in Chapter 2, the shocks in the policy simulation are a 44 per cent reduction in  $PREF_t(cat; act)$  for  $cat = (fgn, ill, zmn \text{ or } new)$  and  $act = (fgn, ill, o)$  where  $o$  is any local South African occupation. The categories  $zmn$  and  $new$  capture foreign-born workers plus new entrants in neighbouring countries that may be viewed as potential illegal immigrants to South Africa. The shocks are introduced as a 30 per cent reduction in 2008 and a 20 per cent reduction in 2009.

Understanding and interpreting the policy shock implemented via equation (E2.39) is made easier when written in its linearised percentage-change form.

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$$l_t(cat; act) = cat_t(cat) + \eta * [atrw_t(act) - atrw_t^{ave}(cat)] + \eta * [pref_t(cat; act) - pref_t^{ave}(cat)]$$

(E2.41)

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In the linearised equation (E2.41), also previously described in Chapter 2, the lowercase symbols  $l_t(cat; act)$ ,  $cat_t(cat)$ ,  $atrw_t(act)$  and  $pref_t(cat; act)$  are percentage changes in the variables denoted by their corresponding uppercase symbols in (E3.39). The symbols  $atrw_t^{ave}(cat)$  and  $pref_t^{ave}(cat)$  are weighted averages, with the weights reflecting the share of activity  $q$  in the offers from people in category  $cat$ . From here, interpretation of the policy shocks imposed on the  $pref_t(cat; act)$  variable and the role of the parameter  $\eta$  in the labour supply function becomes clear.<sup>11</sup> In the next section we present and interpret the policy simulation results under *Scenario 1*, followed by an analysis and comparison of results under *Scenario 2*. The policy shocks are identical in both scenarios.

### *Simulation results – Scenario 1*

Figure 5A shows the employment paths for illegal workers in the baseline forecast and policy simulation. In the baseline, employment of illegal migrants grows from 1.760 million in 2007 to 2.343 million in 2020. This represents an increase of 33.1 per cent between 2007 and 2020. The total number of illegal migrants in South Africa, including those in unemployment, grows from 2.169 million in 2007 to 2.861 million in 2020.<sup>12</sup> Employment

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<sup>11</sup> The parameter  $\eta$  has the value 1.5. This suggests that the number of people who wish to change jobs is quite sensitive to changes in relative wage rates between activities or exogenous changes to their preferences for earning money in a particular activity.

<sup>12</sup> Readers who view these estimates as conservative should bear in mind that we do not attempt to model those migrants who operate outside the given economic framework of ZAR-M. Estimates of illegal migrants in ZAR-M are at the conservative end of estimates purported in the popular media.

of legal residents is projected to grow at a similar rate in the baseline, increasing from 11.877 million in 2007 to 15.820 million in 2020. The share of illegals in aggregate employment is projected to remain constant at around 12.9 per cent up to 2020. This result is mainly due to our forecast assumptions which allow little change in relative wage rates between labour groups, and exogenously projects only the change in aggregate employment. Because illegal migrants are assumed to have low paid jobs, their share in the total wage bill is considerably less than their share in total employment. Over the forecast period, the wage bill share of illegal workers is projected to decline slightly from 3.94 per cent in 2007, to 3.84 per cent in 2020.

The most prominent impact of the policy shock is to reduce the number of illegal migrants employed in the South African labour market. In the policy simulation employment of illegal migrants falls to 1.609 million in 2020. This represents a drop of 734,000 illegal workers in South Africa compared to the baseline in 2020. The total number of illegals in South Africa, including those in unemployment, falls to 2.034 million in 2020. Under *Scenario 1*, the policy therefore has the effect of reducing illegal employment in South Africa by 31.3 per cent, and the total number of illegal migrants in South Africa by 28.9 per cent, over the forecast period.

Figure 5B shows that the policy of tighter border security combined with improved economic and political stability in neighbouring countries affects flows of illegal migrants in both directions. The shocks have a direct affect on inflows by reducing the number of unskilled people outside South Africa who want to move illegally into local employment activities. The shocks also have an indirect effect on outflows by lowering the number of illegal migrants in South Africa and thereby lowering the number who seek to go home. In terms of (E2.39) in the ZAR-M model, the shocks reduce the number of people in those categories in South Africa that offer to supply labour to low skilled foreign destinations ( $zmn$ ). That is, the shocks reduce the number of people in  $CAT_t(cat)$  where  $cat$  is a foreign-illegal category in South Africa.



FIGURE 5A *Employment of Illegal Workers (Scenario 1)*

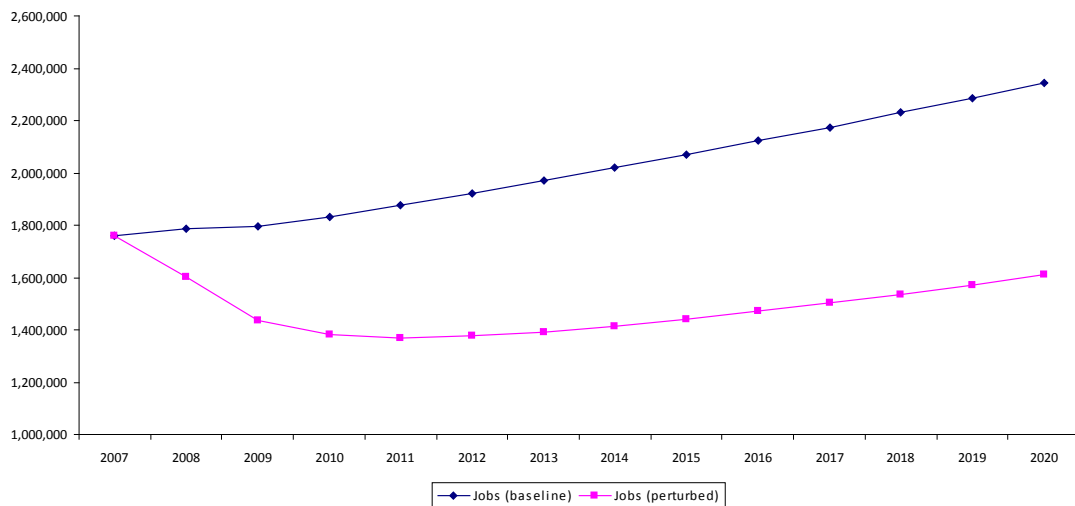
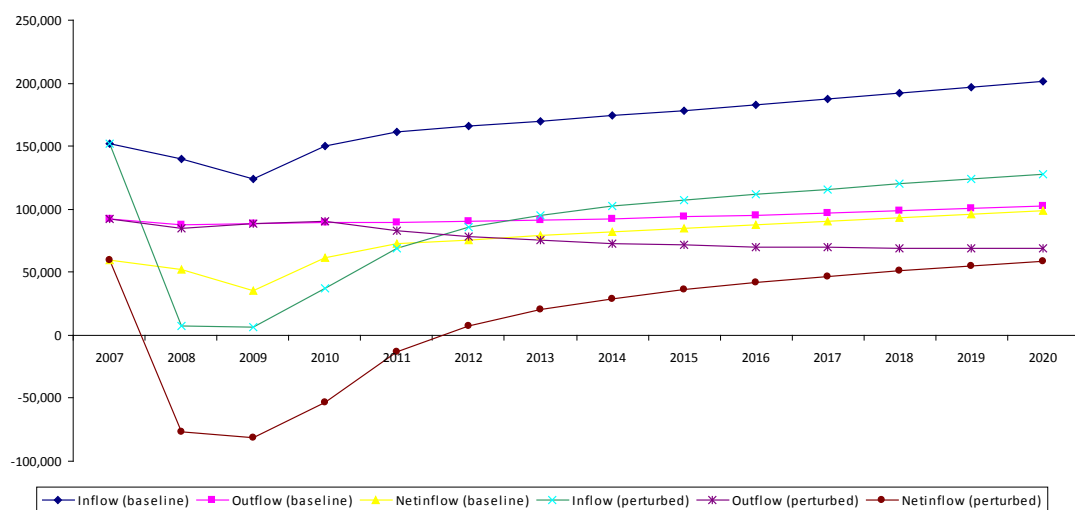


FIGURE 5B *Flows of Illegal Migrants (Scenario 1)*



Two features of Figure 5B warrant further comment. First, it implies that the net inflow of 60,000 foreign illegals to the local labour force in 2007 was generated by a gross inflow of about 152,000 and a gross outflow of about 92,000. Despite a fair degree of uncertainty, these estimates are consistent with strong inflows of illegal migrants reported over the past decade as well as illegal migrants making frequent trips home. As noted before, there are no reliable data on gross flows. Fortunately our results for the effects of reducing illegal employment in South Africa are not found to be sensitive to our assumptions concerning the initial level of gross flows.

The second notable feature of Figure 5B is the sharp decline in the early years of the policy run in the net and gross inflows of foreign illegals to local employment activities, followed by recovery in later years. It appears that increased border security combined with improved conditions in neighbouring countries would have a much greater effect on flows of illegal migrants in the short run than in the long run. To explain this result we start with equation (E2.39). This equation suggests that the initial impact of the policy should approximate a 58 per cent decline in labour offers from category ( $zmn$ ) to local employment activities.<sup>13</sup> However, the policy-induced decline in gross inflow shown in Figure 5B for 2009 is 94.7 per cent and the net inflow in the policy run is negative. Indeed, the model projects a negative net inflow for each year from 2008–2011. The decline in labour supply from low skilled workers in neighbouring countries ( $zmn$ ) causes an increase in wage rates for illegal workers, as dictated by (E2.52), and a decrease in local demand for illegal labour via the nested labour demand equations in ZAR-M described earlier in Chapter 2.

The growth rate in demand for illegal labour for the period 2008–2011 turns from positive in the baseline to negative in the policy run. Because the level of net inflow of foreign illegals depends on local growth in demand for their services, negative growth in this demand

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<sup>13</sup> In equation (E2.39)  $\eta$  has the value 1.5. Where  $cat$  is the foreign-unskilled category ( $zim$ ) and  $act$  is any local employment activity ( $occ$ ), a 44 per cent reduction in  $PREF_i(cat; act)$  generates a reduction in  $L_i(cat; act)$  of approximately 58 per cent  $\left( = 100 * \left[ (1 - 0.44)^{1.5} - 1 \right] \right)$ . We can ignore the change in the denominator on the RHS of (E2.39) because offers to South Africa are a relatively small part of total labour supply from category ( $zim$ ).

translates into negative net inflow requiring a dramatic reduction in gross inflow. Eventually, wages for illegal workers rise sufficiently to reconcile demand with the reduced supply at which point demand in the policy run recommences growth at approximately the same rate as in the baseline. This allows net and gross inflow of foreign illegals to partially recover. In 2020 net inflow in the policy run is 40.6 per cent less (58,600 versus 98,800) than in the baseline and gross inflow is 36.5 per cent less (127,600 versus 201,100).

Figures 5C and 5D show the macroeconomic impacts of the policy on both the supply and demand side of the economy under *Scenario 1*. In each case, impacts are expressed as percentage deviations from the baseline forecast. Figure 5C is concerned with the supply side of the economy. It shows that the policy causes a relatively large reduction in the number of employed workers. In 2020, the total number of jobs is 775,000 or 4.3 per cent lower in the policy run than in the baseline. This mainly reflects the reduction of 734,000 in the number of foreign-illegal jobs. Since the lost jobs are mainly for low-paid illegal workers, the reduction in wage bill weighted labour input in 2020 is less than 4.3 per cent. This measure of labour input is calculated as the number of jobs weighted by wage rates to reflect differences in the productivities of workers across skill or occupation groups. Measured this way, we might expect the percentage loss in labour input to be about 31.3 percent of 3.84 per cent (that is, the reduction in illegal employment times the illegal share in the baseline wage bill for 2020). However, this gives only a 1.2 per cent loss in wage bill weighted labour input compared to the actual projected loss of 2.9 per cent. The additional loss in labour input in *Scenario 1* is mainly due to the restriction of illegal employment causing the occupational mix or composition of employment to shift towards lower-paid occupations. A similar result was also found in Dixon *et al* (2011). Reasons for this projected shift in the occupational mix of employment and its welfare implications for legal residents are explained later.

The reduction in the capital stock of 2.2 per cent up to 2020 carries the same trend as the reduction in labour input. Indeed, the longer the simulation period is extended for, the closer these two results tend towards each other. As noted earlier, we assume no change in either long-run rates of return or technical change as a result of the policy, implying little change to the K/L ratio. With capital and labour inputs down by their respective amounts and no change in technology, GDP is projected to be 2.4 per cent lower over the simulation period under *Scenario 1* as a result of the policy. This is equivalent to a reduction in the average annual growth of GDP from 4.5 per cent in the baseline to around 4.3 per cent in the policy run. As a result of the cut in illegal immigration, the total size of the labour force in South Africa falls by 3.1 per cent, indicating a slight increase in overall GDP per capita. However, with the number of legal residents seeking job opportunities largely unaffected in the long run, this does not translate into a meaningful welfare gain for the legal population in South Africa.

Figure 5D is mainly concerned with the demand side of the economy. The long-run impacts of the policy on all aggregate expenditure variables are shown to be negative and ranged around that for GDP. We assume that the policy would have no effect on the private to public consumption ratio. Both these macro variables fall by 2.2 per cent at the end of the simulation period relative to the baseline. The drop is slightly less severe than that in GDP since the policy improves the South African terms of trade. This boosts GNP relative to GDP. To explain this result we start by considering the negative impact of the policy on the overall size of the economy, or GDP. With a smaller economy, the long-run deviation for exports is negative as shown in Figure 5D. With no shock to foreign-demand curves for South African exports in the policy run, the cut in export volumes is accompanied by an increase in their foreign-currency prices. On the import side we assume that changes in local demand have no effect on foreign-currency prices. An improvement in the terms of trade, that is, the price of exports relative to the price of imports, allows the local economy to increase its consumption (both public and private) relative to its GDP.

FIGURE 5C GDP, Capital and Labour (Scenario 1) (Percentage Deviation)

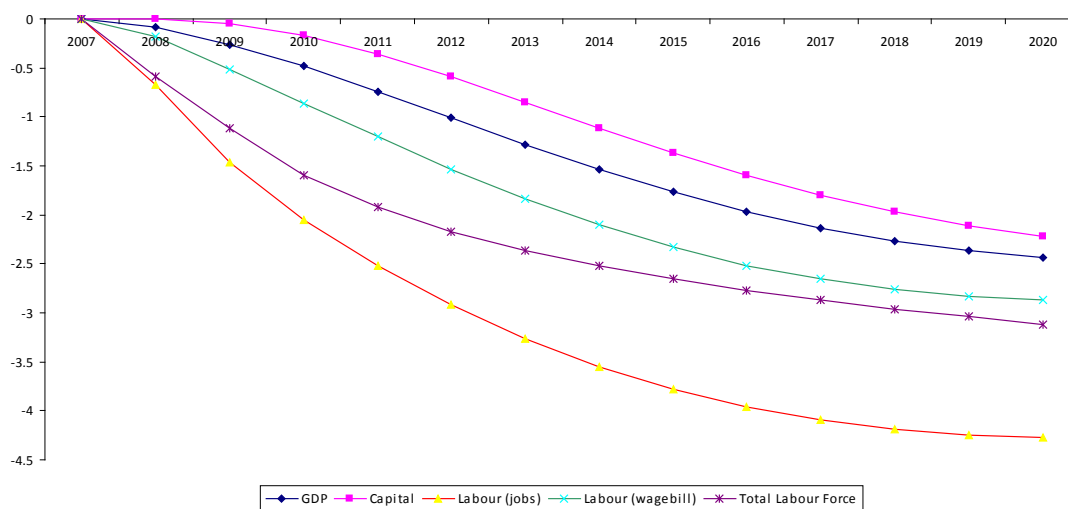
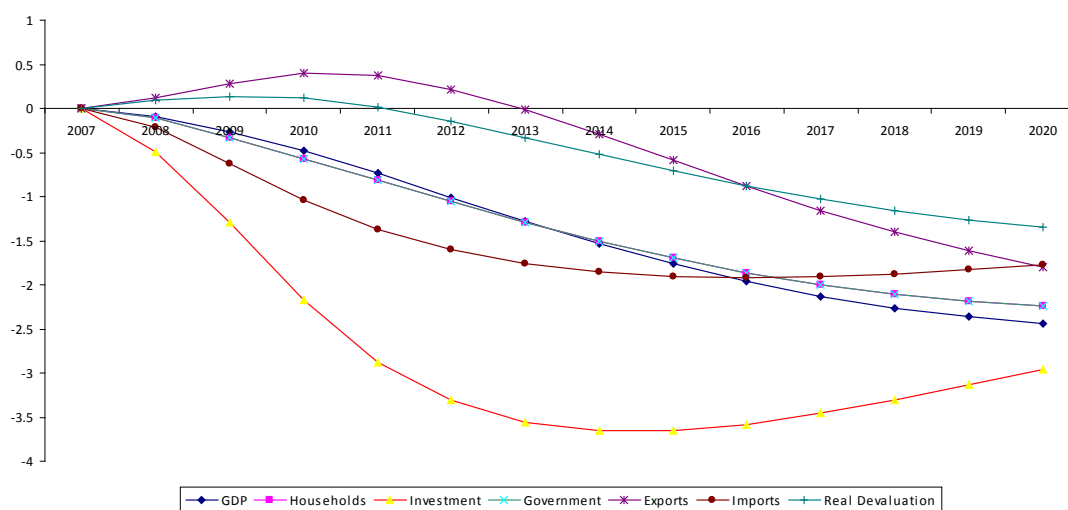


FIGURE 5D Expenditure Aggregates (Scenario 1) (Percentage Deviation)



As witnessed by the trends in Figure 5D, the eventual increase in consumption relative to GDP causes a deteriorating real trade balance ( $X-M$ ), supported by long-run real appreciation. Investment falls relative to GDP mainly because the capital stock is not fully adjusted after the policy shock and still falling relative to the baseline at the end of the simulation period in 2020. If we extend the simulation period we find that perturbed capital growth reaches a steady-state level relative to the baseline slightly below its 2020 level and that results for investment relative to GDP tend even closer over time.

The short-run results in Figure 5D are driven by the economy's adjustment towards a lower capital stock in the policy run relative to the baseline. In the short run, the policy causes a relatively sharp reduction in investment along with a real devaluation. This temporarily stimulates exports whilst limiting imports. As the adjustment in capital stock nears completion in 2020, investment recovers, causing the local currency to appreciate, exports to fall and imports to rise.

Figure 5E shows the results for output of selected industries relative to the baseline under *Scenario 1*. Analysis of these results finds that industry-level results can be explained largely by the macro results.<sup>14</sup> In the short run, investment-related industries such as construction are most severely affected. This reflects the adjustment of the economy to a lower capital stock which causes a sharp negative deviation in investment relative to GDP as previously shown in Figure 5D. Trade-exposed industries do comparatively better in the short run as a result of the real devaluation associated with the weakening of the investment to GDP ratio. This includes tourism-related industries such as hotels and transport services, and import-competing industries such as agriculture and textiles. Consumption-oriented industries such as health and social services also do comparatively better over time due to the increase in the ratio of domestic absorption ( $C+G$ ) to GDP.

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<sup>14</sup> A detailed sales destination matrix is included in Bohlmann (2010). This matrix shows the macro links of each industry in terms of sales for 2007 in the baseline. For example, it shows that the construction industry sells a majority of all commodities they produce to the investment sector, the gold industry exports virtually all of its output, the health and social services industry sells mainly to final households and that a significant proportion of commodities produced by the trade and transport services industries are sold as margins.

FIGURE 5E Selected Industry Outputs (Scenario 1) (Percentage Deviation)

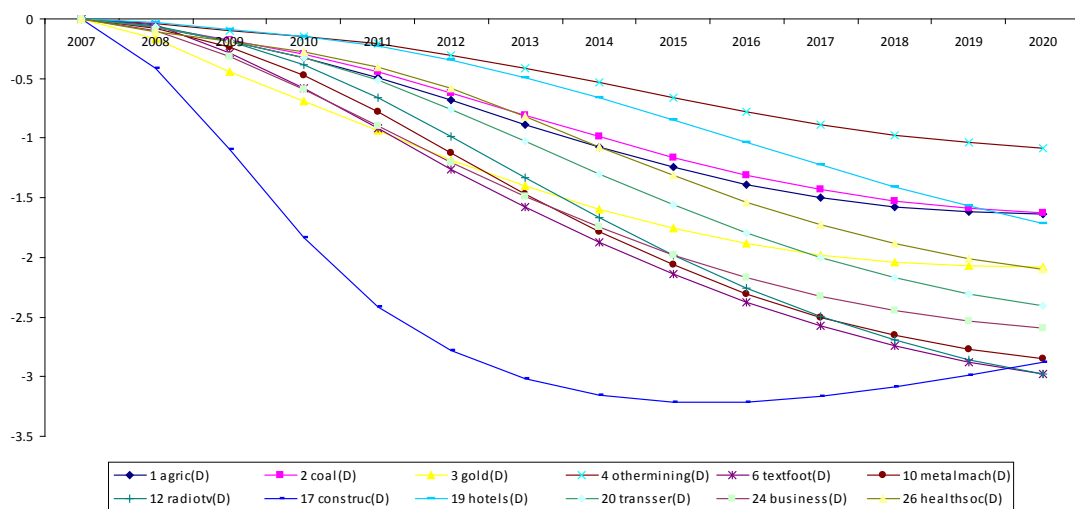
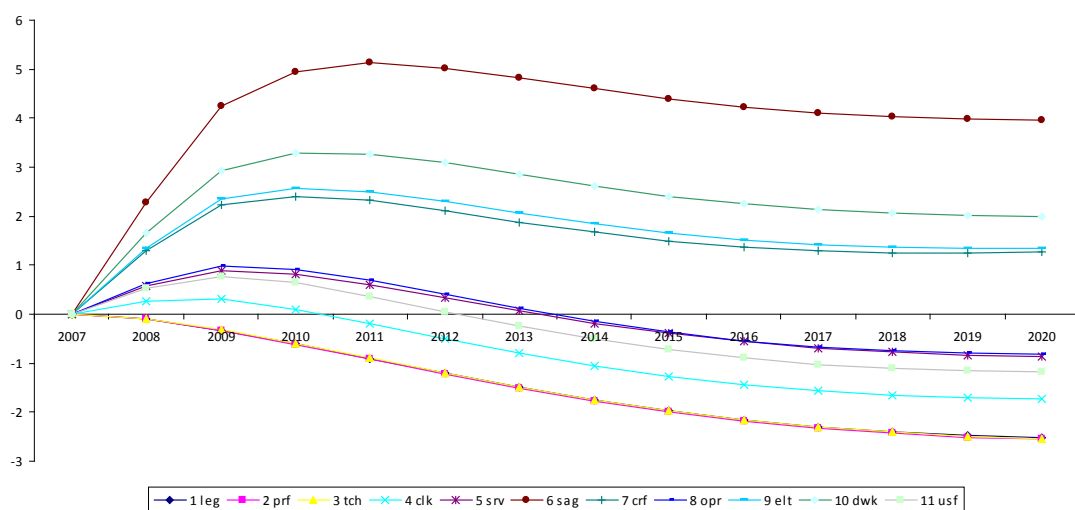


FIGURE 5F Employment of Legal Workers (Scenario 1) (Percentage Deviation)



In the long run, the simulation shows a real appreciation of the currency. Consequently, trade-exposed industries, including the manufacturing sector, start to deteriorate more rapidly in later years relative to the baseline. In the mining sector, coal and other mining do better than gold since all production by the gold industry is exported. Demand elasticities for exports are typically higher than for other final uses. Virtually all of the commodity gold is produced by the gold industry. Thus, gold faces a higher overall demand elasticity, despite having the same export elasticity as coal and other mining in its downward-sloping export demand curve. The increase in the foreign-currency export price of gold, driven by the appreciation of the local currency in the long run, leads to a decrease in demand for gold exports. Investment-related industries continue to show output deviations that are more negative than GDP as investment levels have not recovered sufficiently by the end of the simulation period. As the full set of industry-level results in Appendix C show, output deviations for most industries are quite close to that of GDP in the long run. Gaps between individual industries and that of GDP strongly reflect changes in the long-run expenditure composition of GDP, taking into consideration the relevant demand and supply elasticities for different industries. An implication of our explanation that industry results are mainly driven by their links to macro mechanisms is that the long-run effect of the policy on the industry outputs is not closely linked to the industry's use of illegal labour. An investigation comparing the illegal labour shares of industries to their long run output deviations suggests this to be true.<sup>15</sup> Although illegal labour shares play a role in the output deviations of industries, regression analysis suggests their overall impact seems to be trivial relative to the importance of the industry's macro link.

Some of the most interesting results in this particular simulation relate to the change in legal employment by occupation type shown in Figure 5F. The reduction in legal employment of legislators and managers (*lsm*), professionals (*prf*) and technical and associate professionals (*tch*) may at first seem surprising given the policy under consideration has no direct impact on legal or skilled employment. This result is due to the occupational-mix effect alluded to earlier. The policy shock directly reduces the labour supply of potential

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<sup>15</sup> The wage bill share of foreign-illegal workers in each industry for 2007 in the baseline is shown in the Appendix.



illegal immigrants to local South African occupations. The first round effect of the policy shock therefore is to reduce labour supply and consequently illegal employment in lesser skilled occupation groups. Real wages in these occupations rise in the short run as labour becomes scarcer. The second round effect of the policy shock relates to the increase in job vacancies at the lower-end of the market as a result of the policy shock. Combined with the real wage increase, lower skilled occupation groups are now expected to attract relatively more legal labour offers in future years.

The greater the share of illegals employed in any occupation, the larger these first and second round impacts will be in ZAR-M.<sup>16</sup> Local occupations such as ‘agriculture and fishery’ (*sag*), ‘craft and related trades’ (*crf*), ‘elementary’ (*elt*) and ‘domestic workers’ (*dwk*) employ a substantial share of illegal labour in the baseline. As a result, we find that new legal entrants to the labour market, or those in unemployment, who previously may have considered paying large sums in further training and education to find a skilled job, are now more likely to offer their services to one of these lower skilled jobs.<sup>17</sup>

Figure 5F shows a positive deviation in the number of legal jobs in some lower skilled occupation groups. As suggested, the largest positive deviations occur in those jobs that previously employed the highest share of foreign-illegal labour, that is, the *sag*, *crf*, *elt* and *dwk* occupation groups. These jobs therefore yielded the greatest number of vacancies and percentage increase in real wage to workers as a result of the policy. The policy-induced diversion of legal job offers towards lesser skilled occupations reduces the number of workers employed in the three well-paid skilled occupations (*lsm*, *prf*, *tch*) by an average of around 2.6 per cent over the simulation period. This explains the greater than expected loss in wage bill weighted labour input referred to earlier.

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<sup>16</sup> In ZAR-M we assume that there are no foreign-illegals employed in skilled jobs. That is, all workers in local skilled occupation groups (*lsm*, *prf*, *tch*) are considered to have legal status.

<sup>17</sup> The theoretical specification of ZAR-M prevents the occupational-mix effect from impacting directly on incumbent skilled workers as they are not allowed to offer from a position of employment to any of the lesser skilled occupations.

Readers unfamiliar with CGE modelling and the percentage-deviation approach should again note that this result is relative to the unperturbed baseline scenario. For example, in the baseline the number of technical and associate professional (*tch*) workers was forecast to rise from 1.260 million in 2007 to 1.673 million in 2020. In the policy run, employment of these workers is now expected to rise to only 1.630 million in 2020, a drop of 2.6 per cent relative to the baseline. Alternatively put, the policy has the effect of slowing growth in employment of skilled labour from 2.3 per cent per annum in the baseline to around 2.0 per cent in the policy run.

Figure 5G summarises the impact of the policy on aggregate employment and real wages. The long-run deviation in illegal employment of –31.3 per cent is equivalent to the loss of 734,000 illegal jobs reported in Figure 5A. Consistent with the sharp fall in their labour supply, real wages for illegal workers rise quickly to 13 per cent above the baseline before steadying. The total number of legal jobs falls by close to 0.3 per cent, or 41,000 workers, with the average real wage increasing by 2.2 per cent. The combined loss of legal and illegal jobs therefore adds up to the 775,000 shown earlier in Figure 5C.

*FIGURE 5G Employment and Real Wages (Scenario 1) (Percentage Deviation)*



The overall loss of legal jobs, most notably in skilled occupations, is mainly due to the occupational-mix effect. We trace this to a shift in the occupational composition of legal employment towards lesser skilled occupations in which there are higher equilibrium rates of unemployment. That is, the occupation-mix effect described before pushes legal workers towards occupations where relatively high rates of unemployment can be sustained with little wage pressure. This allows the deviation in aggregate employment of legal workers to be negative without producing an employment-increasing reduction in their average real wage rate.

As a corollary to the policy, it also has a negative impact on the overall welfare of legal residents measured via a combination of their private and public consumption expenditure. This measure is closely linked to the household disposable income of legal residents which falls by 2.2 per cent over the simulation period. Similar to many of the other results reported in this section, we find that this outcome closely mirrors that of the Dixon *et al* (2011) study on illegal immigration in the United States.

The welfare loss found under *Scenario 1* in this simulation using our ZAR-M model for South Africa can also be traced to a number of factors. These include i) the direct illegal labour effect, ii) legal employment effect, and iii) occupation-mix effect. The direct illegal labour effect occurs as a result of moving towards a smaller economy in the perturbed scenario. Assuming the wage rate of illegal migrants equal the value of their marginal product to employers, the direct loss to South African GDP can easily be approximated using a simple demand and supply diagram that reflects the shock to illegal labour supply in the policy run. The loss of welfare to legal residents can then be tied to the deadweight loss of producer surplus and the increased transfers from local employers to illegal migrants because of higher wage rates. Additional welfare loss linked to the direct illegal labour effect can be expected from reduced tax revenue on wages earned by illegals. The legal employment effect is tied to the loss of legal jobs as a result of the policy which imposes a direct welfare loss on legal residents.

Other contributing factors detailed in Dixon *et al* (2011) also play a small role in the net loss of welfare seen in this simulation under *Scenario 1*. These include iv) the capital effect, v) public expenditure effect, and vi) terms of trade price effect. The capital effect is tied to the reduction in capital stock as a result of the policy, and the share of local versus foreign-owned capital in South Africa. Taking into consideration the trade deficit and data on foreign-owned capital in South Africa (SARB, 2007; 2010), we expect the combined loss of capital income for legals and taxes collected from capital income to negatively affect the welfare of legal residents.

The public expenditure effect is tied to the reduced amount of spending required by government on the number of foreign-illegals in the country. In ZAR-M we assume that public expenditure per illegal migrant is half the public expenditure per legal resident. With the 28.9 per cent reduction in the number of illegal migrants, government spending reaching illegals is also reduced. This amounts to a small welfare gain for legal residents.

The terms of trade price effect also generates a minor welfare gain for legal residents. This effect is derived via the increase in the GDP price deflator relative to the consumer price index (CPI) and also the GNE price deflator as a result of the terms of trade improvement of 0.7 per cent. This effectively increases the consuming power of income earned by legal residents. Despite the welfare gains attributable to these last two effects, the net impact on overall welfare of legal residents as a result of the policy remains negative. Our analysis suggests that the largest negative contributions to consumption by legal residents under *Scenario 1* are the occupation-mix effect and the direct illegal labour effect.

It is worth pausing at this stage to review the modelling evidence presented to date under *Scenario 1*. Most results have ready explanations. However, the reduction in overall employment of legal workers may be interpreted as a counterintuitive outcome. In explaining this result, it is worth noting that this does not imply that large numbers of existing skilled workers would give up their jobs in high paid occupations and shift towards lower paid occupations. This effect mainly influences employment outcomes for future new

entrants and those in unemployment. For each occupation, two different factors influencing legal employment must be considered. The first is an increase in job opportunities or vacancies for legal workers in those lower skilled jobs previously held by illegal workers. The second is a general reduction in opportunities for all legal workers as a result of moving towards a smaller economy.

As seen in Figure 5F, the positive replacement effect for legals dominate in the low-paid occupations that employ large numbers of illegal migrants in the baseline. The negative effect of having a smaller economy dominates in the higher-paid occupations that employ very few illegal migrants. Higher equilibrium rates of unemployment in lower skilled occupations play an important role in this shift of occupational composition. The increase in vacancies in low paid, low skilled occupations relative to the high paid occupations allows these occupations to absorb an increased proportion of both new legal entrants and unemployed workers. In terms of equation (E2.56) described earlier in Chapter 2, the policy has the second round impact of increasing  $V_t(act)$  in the lower skilled occupations relative to high skilled occupations. This results in more non-incumbents finding employment in these occupations.

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$$H_t(cat; act) = V_t(act) * \left[ \frac{L_t(cat; act)}{\sum_{v \neq act} L_t(v; act)} \right] \quad \text{for } cat \neq act \quad (E2.56)$$


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It is often asserted that a reduction in illegal immigration would reduce unemployment rates of lower skilled local workers. This idea is counteracted by noticeable long-run evidence of the occupation-mix effect in ZAR-M under *Scenario 1*. As pointed out in Dixon *et al* (2011), under such labour market conditions, it may even be true that with cuts in illegal immigration, lower skilled workers find themselves under increased pressure from high skilled workers who can no longer find vacancies in high skilled occupations.

Thus, the rather controversial legal employment outcome is interpretable too – under *Scenario 1*. The question therefore becomes whether the labour market conditions and assumptions implied by *Scenario 1* are believable. A study of the local labour market quickly reveals the concept of equilibrium rates of unemployment for lower skilled South Africans to be inappropriate. Our alternative set of assumptions in *Scenario 2* addresses this issue.

### *Simulation results – Scenario 2*

The wage adjustment mechanism in ZAR-M plays a central role in the functioning of the labour market. Equation (E2.52) shows that an increase in labour demand relative to supply would place upward pressure on wages over time. With steady-state conditions in the base year and unemployment present in the model, (E2.52) implies the existence of equilibrium rates of unemployment. Workers who are unemployed in the base under *Scenario 1* are therefore seen as unwilling to lower their asking wage in order to obtain employment. That is, they continue to present upward-sloping supply curves despite not being able to find a job at the prevailing wage rate.

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$$\left( \frac{ATRW_t(b, s, o)}{ATRW_t^{base}(b, s, o)} \right) - \left( \frac{ATRW_{t-1}(b, s, o)}{ATRW_{t-1}^{base}(b, s, o)} \right) = \beta(s, o) \left( \frac{D_t(b, s, o)}{D_t^{base}(b, s, o)} - \frac{L_t(b, s, o)}{L_t^{base}(b, s, o)} \right)$$

(E2.52)

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The reality in South Africa is different though. Millions of workers are actively looking for jobs every week. Most are unsuccessful. With a minimum wage in place, unemployed workers are not legally allowed to reduce their wage to match their marginal productivities to employers.<sup>18</sup> Our alternative labour market assumptions in the wage adjustment process underlying *Scenario 2* account directly for these factors.

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<sup>18</sup> Not surprisingly, most evidence suggests that a legal minimum wage contribute to increased levels of unemployment, especially among low skilled workers (Neumark & Wascher, 2007).

To achieve our desired modelling behaviour in ZAR-M for *Scenario 2* we set the  $\beta(s,o)$  parameter in equation (E2.52) to a suitably small number for legal workers in low skilled occupations. As explained in Chapter 2, the  $\beta(s,o)$  parameter governs the speed of adjustment in the labour market for gaps between labour demand and supply. Under *Scenario 1*, this parameter value typically takes on a value of around 0.5 for most occupation groups. This suggests that any labour market shock is absorbed within a couple of years and equilibrium employment and wage conditions restored. Reducing the value of  $\beta(s,o)$  to a sufficiently tiny number for all  $s$  ‘legal’ and  $o$  the subset of ‘low skilled’ occupations alters this behaviour and allows us to create a more realistic modelling environment for the South African labour market.

By effectively ‘switching off’ the wage adjustment behaviour of low skilled legal workers in (E2.52), increases in labour demand for these workers can be met with matching shifts of the labour supply curve without inducing any wage pressure. Labour supply curves in the policy run’s wage adjustment process for low skilled occupations are therefore simulated as being close to perfectly elastic under *Scenario 2*. This is consistent with a situation in which an excess supply of labour exists at a given legal minimum wage level. A required condition in ZAR-M under the assumptions implied by *Scenario 2* is that the increase in demand for labour does not exceed the available excess supply. With such a large stock of unemployed workers in South Africa, this condition was easily satisfied in our simulation under *Scenario 2*.

Figures 5I to 5N show the impacts of the policy-induced cut to illegal immigration under *Scenario 2* assumptions. Since the policy shock has remained the same, it is not surprising that simulation results show a similar pattern to those generated under *Scenario 1*. That is, although there are differences in the levels of results between the two scenarios, reasons for divergences between macro aggregates or industry performance relative to GDP remain fundamentally the same. It is immediately evident though that the local economy is better off under the conditions implied in *Scenario 2*. The divergence between the two sets of results is naturally traced to the change in assumptions between the two scenarios.

The policy shock reduces the supply of low skilled illegal labour. This reduces demand for these workers as a result of increased wage rates over time. During the adjustment period, a large number of vacancies in the jobs previously done by illegal workers become available via equation (E2.56). With illegal labour now relatively more expensive, employers shift some of their demand for low skilled labour to legal workers. The substitution between legal and illegal workers is governed by the system of nested CES demand equations in (E2.45–E.2.47). It is at this stage where the two scenarios produce meaningfully different results. With  $\beta(s,o)$  in equation (E2.52) set at around 0.5 under *Scenario 1*, increased demand for low skilled legals generated an increase in their real wage via (E2.52). As a result, the increase in demand for low skilled workers in later years was choked off somewhat. Along with increased competition from workers who previously might have pursued a skilled job (the occupation-mix effect), employment gains for legal workers under *Scenario 1* was limited to only four occupation groups.<sup>19</sup>

As shown in Figure 5N, the increase in demand for legal workers produces almost no increase in their real wage under *Scenario 2*. This is because *Scenario 2* allows the excess supply of low skilled legals to accommodate the increase in their demand as a result of the policy shock. With virtually no wage pressure, Figure 5M shows that demand for legal workers now increases in virtually all low skilled occupations relative to the baseline. This result at the lower end of the market, also contributes to an improved outcome for skilled workers in *Scenario 2* via a reduction in the occupation-mix effect. Significantly, the policy now generates a healthy boost to overall legal employment. Whereas the number of legal jobs previously fell by around 0.3 per cent, the same policy shock now generates an increase of 2.0 per cent, or 313,000 legal jobs overall. This result drives the divergence in level of macro outcomes between *Scenario 1* and *Scenario 2*.

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<sup>19</sup> This result was illustrated by Figure 5F. As discussed in the policy results for *Scenario 1*, the occupations that employed the largest share of illegal workers, i.e. *sag*, *crf*, *elt* and *duk* also opened up the largest amount of vacancies for legals leading to increased employment in these low skilled jobs for legals in the policy run relative to the baseline.



FIGURE 5I *Flows of Illegal Migrants (Scenario 2)*

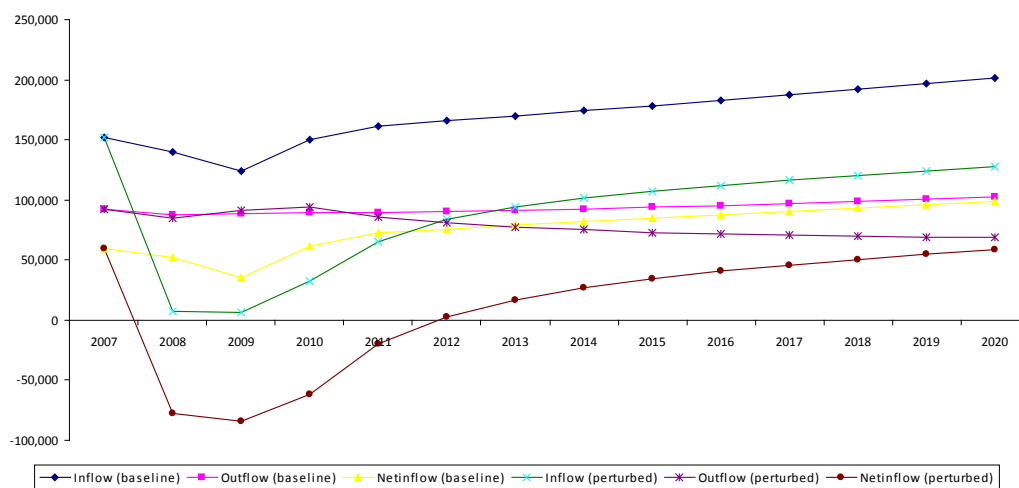


FIGURE 5J *GDP, Capital and Labour (Scenario 2) (Percentage Deviation)*

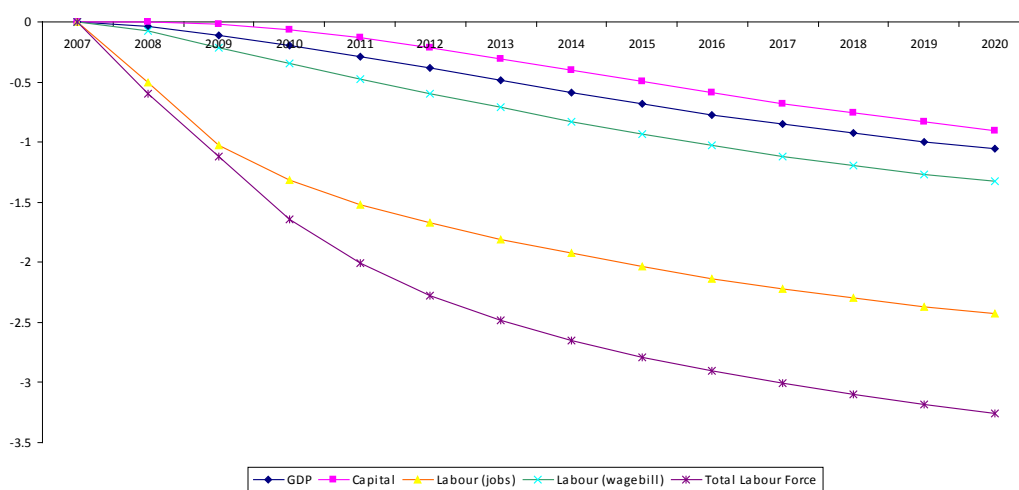


FIGURE 5K Expenditure Aggregates (Scenario 2) (Percentage Deviation)

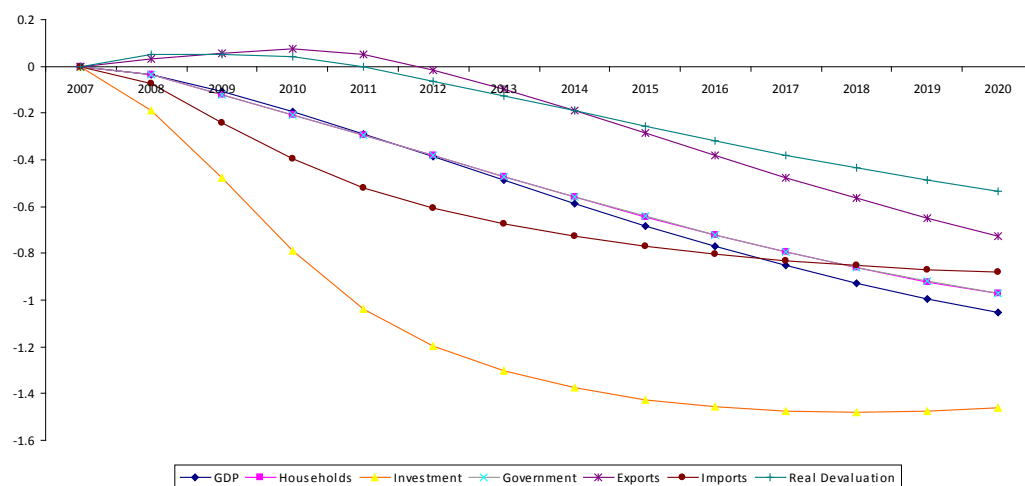


FIGURE 5L Selected Industry Outputs (Scenario 2) (Percentage Deviation)

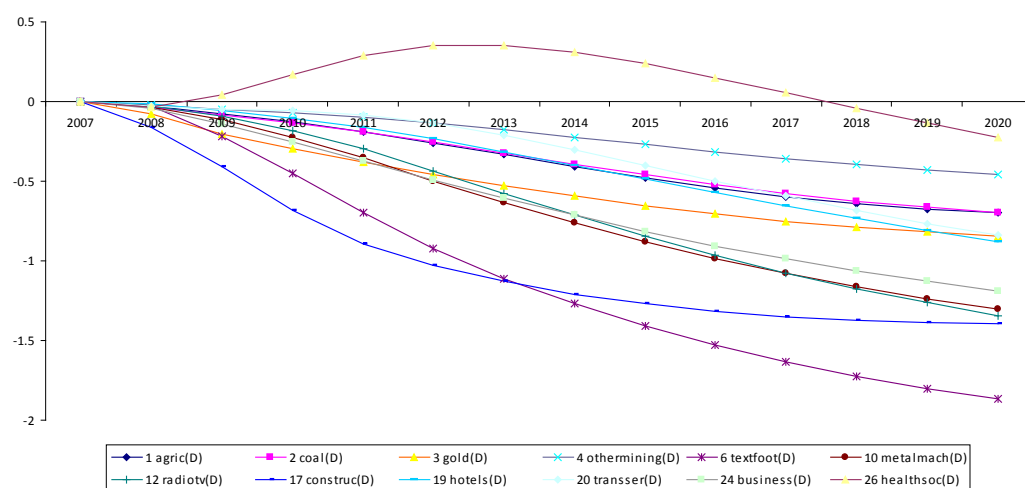


FIGURE 5M Employment of Legal Workers (Scenario 2) (Percentage Deviation)

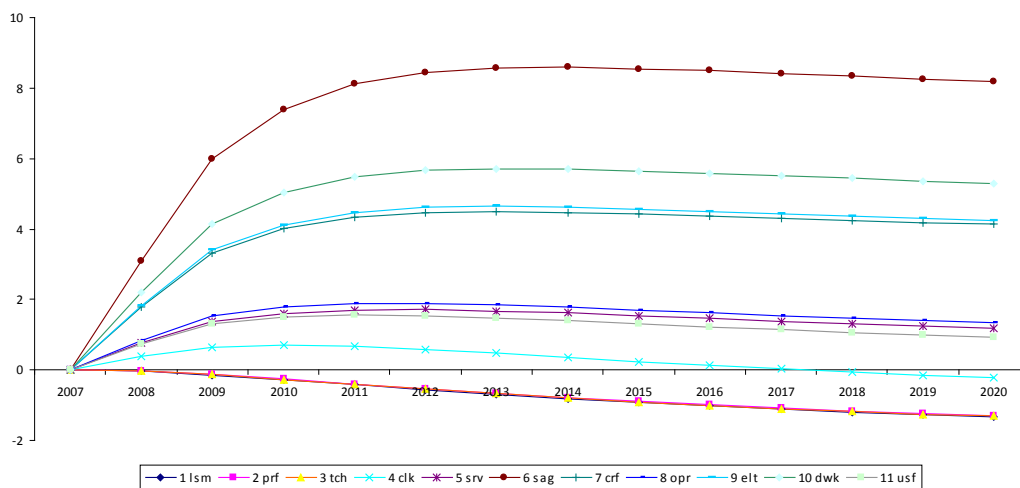
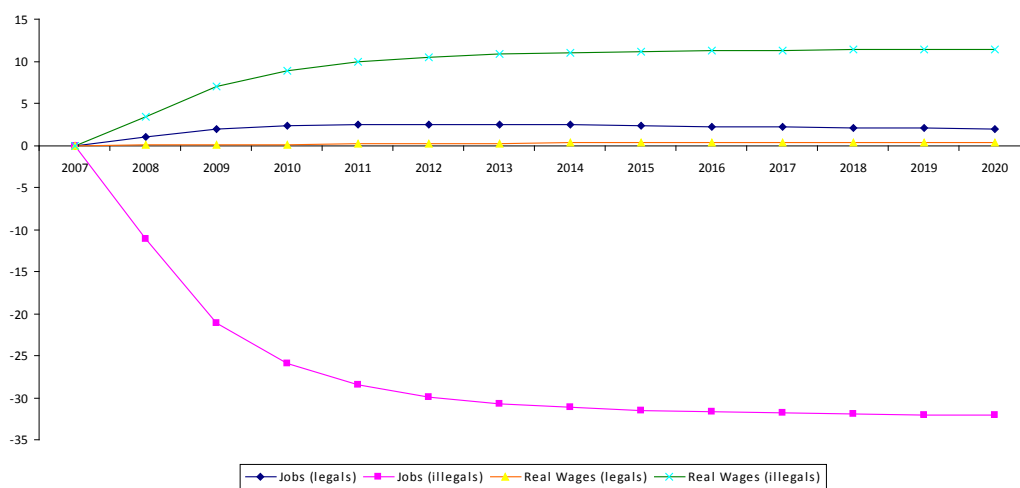


FIGURE 5N Employment and Real Wages (Scenario 2) (Percentage Deviation)



By comparing the scale of the results between *Scenario 1* and *Scenario 2* we can also gauge how important considering factors specific to the South African labour market is to our analysis. As expected, we do not observe much change in the shape and pattern of results over time, but the level of results do show some marked differences in a number of areas.

The drop in illegal employment numbers show little change between the two scenarios. This is because the shock directly affects the inflow, and thus employment, of illegal workers. Relative to the unperturbed baseline, there was a drop in illegal employment of 734,000 or 31.3 per cent under *Scenario 1* and 752,000 or 32.1 per cent under *Scenario 2*. Of greater interest to local policymakers should be the different outcomes with regard to employment of legal workers. Whereas the total number of legal jobs surprisingly fell by 41,000 or 0.3 per cent under *Scenario 1*, our results for *Scenario 2* show a healthy increase of 313,000 or 2 per cent in legal employment. On a more micro level, we find that all semi and lower-skilled employment groups now have a positive gain in legal employment under *Scenario 2*. This outcome is clearly illustrated by comparing Figure 5F from *Scenario 1* with Figure 5M from *Scenario 2*. The additional gain in lesser skilled legal jobs is as a direct consequence of allowing excess labour in the economy to reduce wage pressure when demand for workers in these occupation groups increase. The negative outcome in skilled occupation groups is also significantly reduced under *Scenario 2*. The lower wage in lesser-skilled jobs under *Scenario 2* relative to *Scenario 1* means fewer potentially skilled-legal workers will be lured into these jobs over time. That is, the occupation-mix effect is reduced. In total, 93,000 skilled jobs were lost and 52,000 semi and lower-skilled jobs gained under *Scenario 1*. Results under *Scenario 2* showed a loss of only 48,000 skilled jobs, whilst 361,000 more semi and lower-skilled jobs were taken up by legal workers. The small loss in skilled jobs relative to the baseline under *Scenario 2* can still mainly be ascribed to the occupation-mix effect, with the strong gain in lesser skilled legal jobs down to reduced wage pressure and therefore greater substitution with now more expensive illegal workers in the wake of the policy shock.

The legal employment outcome under *Scenario 2* is the driving force behind its improved GDP results. The 2.4 per cent drop in number of jobs translates into a loss of only 1.3 per cent on the wage bill. The smaller loss in terms of wage bill weighted labour input can once again be attributed to the relatively low pay in the type of jobs lost, that is, low skilled jobs performed by illegal workers. The smaller reduction in labour input also leads to a smaller reduction in capital under *Scenario 2*. Where capital previously fell by 2.2 per cent, it now falls by only 0.9 per cent relative to the baseline. As illustrated by Figure 5J, GDP declines by just over 1 per cent as a result of these labour and capital outcomes, a significant improvement over the 2.4 per cent drop in GDP seen under *Scenario 1*. A loss in GDP is virtually unavoidable given the nature of the policy shock, that is, to ‘remove’ a fairly significant number of workers from the economy. However, of greater concern here should be the impact of the policy on the welfare of legal workers in the economy.

We find that under *Scenario 2* the welfare loss to legal workers, measured via changes to household disposable income relative to the baseline, falls to only 0.7 per cent compared to the loss of 2.2 per cent under *Scenario 1*. Reasons for this loss in welfare remain the same as previously described for *Scenario 1*. The magnitude of the direct illegal labour effect is virtually unchanged between the two scenarios and negatively affects welfare. The legal employment effect is largely reversed under *Scenario 2* as the total number of legal jobs now rises, improving the welfare of legal residents. The occupation-mix effect continues to have a negative impact on welfare, shown via the loss of skilled legal jobs. However, this effect is greatly reduced under *Scenario 2*. The much smaller reduction in capital projected under *Scenario 2* also reduces the welfare loss associated with this effect. The public expenditure and terms of trade price effects also continue to show small welfare gains to legal workers. Apart from the changes in these effects, Figure 5J also highlights the improved outcome in terms of GDP per capita under *Scenario 2*.

It is clear from the set of simulation results presented for *Scenario 2* that a policy induced cut to illegal immigration may indeed pose benefits to the legal residents of South Africa. In particular, legal workers competing for jobs at the lower end of the market will benefit from

this policy. Comparing these results to those of *Scenario 1*, or indeed those of Dixon *et al* (2011) for the United States, it is evident that the assumptions we impose on the behaviour and functioning of the labour market is very important. Capturing specific features of the South African economy, in this case, high unemployment rates and a minimum wage at the lower end of the labour market has proven to be most worthwhile. Our simulation results under *Scenario 2* are readily explainable and in line with expectations, yet also continue to point to some unanticipated effects of the policy in the form of the occupation-mix effect.

### *Simulation results – Sensitivity analysis*

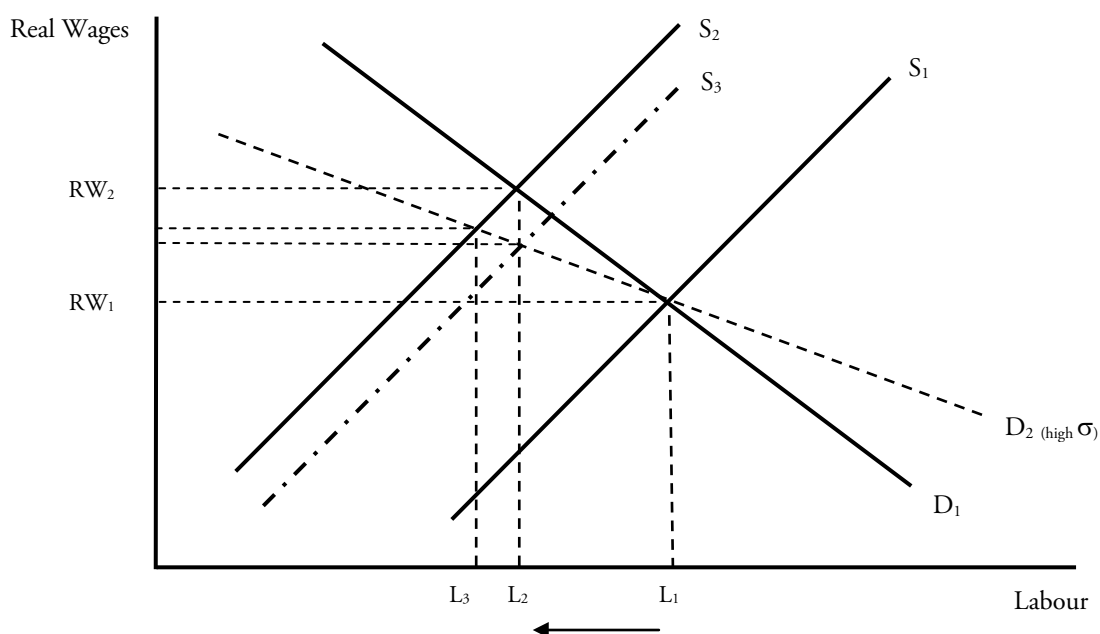
We noted earlier that a fair degree of uncertainty is contained within our choice of key demand and supply side parameters in the model's labour market equations. This is mainly due to a lack of reliable data, and subsequently, econometric estimates of these elasticities. ZAR-M features nested CES labour input demand equations. Sensitivity analysis concerning the choice of substitution elasticities ( $\sigma_s$  and  $\sigma_b$ ) in equation (E2.49) for labour demand by legal status ( $s$ ) and birthplace ( $b$ ) indicate both sets of results to be robust and not overly sensitive to realistic alternatives for these parameter values. Under *Scenario 2*, doubling the substitution elasticities in the labour input demand equations only elevated the gain in overall legal employment from 2.0 to 2.3 per cent in 2020. Apart from a slight increase in the benefit to legal residents, no discernable change to the pattern and structure of results were evident.

Figure 5P illustrates this sensitivity analysis in terms of changes to employment and real wages for illegal migrants as a result of the policy shock. An increase in the substitution elasticities or demand parameters flattens the demand curve ( $D_1$  to  $D_2$ ) for foreign-born illegal workers, effectively allowing domestic-born legal workers to become better substitutes. To maintain the desired reduction in illegal employment ( $L_1$  to  $L_2$ ), a smaller supply shift or policy shock would therefore be required ( $S_1$  to  $S_3$  as opposed to  $S_1$  to  $S_2$ ). Alternatively, if the magnitude of the supply curve shift was to be maintained ( $S_1$  to  $S_2$ ), a larger reduction in

illegal employment will be generated ( $L_1$  to  $L_3$ ), opening up even more vacancies for legal workers. Either scenario slightly improves the welfare of local residents via a reduced deadweight loss and reduced wages payable to illegal workers. It is also evident from Figure 5P that varying the key labour supply substitution elasticity in (E2.41), that is, the slope of the supply curve, would have very little impact on the simulation result if the policy shock was scaled to maintain a reduction in illegal employment of  $L_1$  to  $L_2$ .

A further concern observers may have is the baseline projections used in this simulation for illegal immigration. Critically though, a different baseline scenario in terms of illegal migrant flows would have very little impact in terms of our percentage change deviation outcomes. Different assumptions and projections in our baseline would naturally impact on the future levels position of the economy, but that is not our main concern in this type of analysis. Estimating the impact of the policy only, that is, the deviation it creates away from the unperturbed baseline is of greater interest when evaluating the merits of any policy.

FIGURE 5P Sensitivity Analysis



## 5.4 Concluding Remarks

In this chapter we described the simulations relevant to conducting our study of illegal immigration in South Africa and carefully interpreted the most important policy results produced by the ZAR-M model. We first constructed a plausible baseline forecast of the local economy in the absence of any proposed immigration policy changes. Using available macro forecasts and a number of assumptions about the future evolution of the economy, the baseline was used to contrast the impact of the policy-induced cut in illegal immigration to South Africa against.

The policy shock was designed to adversely affect the labour supply preferences of potential migrants for illegally moving to South Africa. The policy simulation was run under two different sets of assumptions. The labour market assumptions underlying *Scenario 1* was based on a similar study by Dixon *et al* (2011) for the United States. This incorporates the idea of equilibrium rates of unemployment. *Scenario 2* abandons the wage adjustment process in (E2.52) for lower-skilled legal workers and allows increases in their demand to be met without much wage pressure. This adjustment to the model recognises two distinctive features of the South African labour market: high unemployment among lower-skilled workers and a legal minimum wage for legal workers. The macro results in *Scenario 1* closely followed those in Dixon *et al* (2011). Reporting results from both scenarios highlighted the importance of including country-specific elements to the model as well as clearly explaining why the results in *Scenario 2* were different.

Simulation results showed that under *Scenario 1*, legal residents are worse off in terms of employment and welfare as a result of the cut in illegal immigration. As expected, the pattern of results closely mirrored those produced in Dixon *et al* (2011) under similar labour market assumptions. Alternative assumptions introduced in *Scenario 2* attempted to create a more realistic modelling environment for South Africa. Policy results under *Scenario 2* showed a much more positive impact on legal residents. Despite minor occupational-mix effects, overall employment for legal residents increased, with large gains in lower-skilled



occupations. There were also short-term gains in private consumption, and a long-term increase in public consumption, both indicative of increases in overall welfare of legal residents. Given various sensitivity analysis tests, we are confident that the results produced by the ZAR-M model are both plausible and robust.

The policy simulations conducted in this paper do not take any cost factors into consideration. The main reason for this is that there is a large degree of uncertainty regarding the cost of implementation. The exclusion of cost factors in these policy simulations does not invalidate their results by any means. We can expect the pattern and general trend of all results to remain virtually unchanged once all direct cost factors are accounted for. Naturally, there will be a small negative impact on the level of the economy as a result of ‘paying’ for the policy. For policies that focus solely on restricting inflows via supply side mechanisms, these implementation costs may be extensive. Combining such efforts in a cost-effective manner with policies that would reduce demand for illegal migrants may be more advantageous in terms of economic welfare to legal residents.<sup>20</sup> These alternative simulation scenarios will be investigated in future research.

Another important conclusion can be made based on the policy simulation results presented in this chapter. Whilst limiting the strong inflow of illegal immigrants will benefit the lower-skilled segment of the local labour market, this policy is by no means a silver bullet to fixing all of the labour market problems in South Africa. Despite the improvement in overall employment of legal residents, unemployment levels are still very high. Policymakers should therefore recognise the limits of the policy’s impact in isolation. High structural unemployment remains the source of many socio-economic problems in South Africa. The legacy of Apartheid should no doubt carry much of the blame for this situation. However, the current government must continue to strengthen their efforts in creating an environment conducive to job creation and training of those in the labour force that may be viewed as unemployable in today’s modern economy. Implementation of policies that promote economic and political stability on a macroeconomic scale must therefore become a priority.

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<sup>20</sup> Dixon *et al* (2011) suggests that implementing a demand-reducing tax instead of various supply-restricting actions would yield greater benefit to legal residents.

Within such a framework, the positive impact of the immigration policy simulated in this chapter will most likely be enhanced.

Finally, no attempt was made, nor do we seek to make any moral judgment in this study on distressed workers who seek employment opportunities illegally. Workers who have legitimate cause for asylum are also not considered here. This study merely aims to inform policy discussions on the economic consequences of reducing the inflow of illegal migrants to South Africa via supply-side policies. In this regard, we believe the simulation results presented and explained in this chapter to be insightful.





## CHAPTER 6

## CONCLUSION

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## 6.1 Overview of Study

This study evaluated the economic consequences of a policy-induced cut to employment of illegal immigrants in South Africa. The policy was simulated as a reduction in the preferences of foreign-born workers with illegal status for moving to and earning money in South Africa, effectively reducing the labour supply of illegal immigrants.

To conduct our analysis we used the ZAR-M economic model of South Africa. ZAR-M is a state-of-the-art MONASH-style dynamic computable general equilibrium (CGE) model, describing the South African economy in considerable detail. To facilitate the analysis of all relevant labour and migration flows, ZAR-M incorporated a labour market mechanism similar to that introduced in Dixon *et al* (2011). An overview of the theoretical structure of the ZAR-M model, including a full description of its capital and labour market theory, was included in Chapter 2. The key features of the model's database and different closure settings were described in Chapter 3 and 4, respectively. Chapter 5 contains all the modelling simulations and policy analysis.

In choosing between competing methodologies to conduct our analysis, criteria concerning the model's validity and reliability were considered. In test simulations the ZAR-M model was found to be functioning properly and generating results in a manner consistent with its intended theory. Following Dixon & Rimmer (2012), extensive model validation exercises were conducted to ensure the integrity of our calculations. These included i) checking the model code via homogeneity tests; ii) using GDP identities to check that both nominal and real GDP from the income side is equal to GDP from the expenditure side; and iii) using a 'back-of-the-envelope' model to elucidate relevant aspects of the full model and establish some intuition regarding macroeconomic outcomes. Along with our ability to produce sound qualitative and quantitative analysis for all simulations results, the model's performance in these tests gave us confidence in the validity of all the ZAR-M modelling outcomes.

## 6.2 Main Findings

Reducing the inflow and employment of low-skilled and illegal immigrants may have contrasting outcomes on the welfare of legal residents depending on the state of the labour market. Under *Scenario 1*, equilibrium rates of unemployment were assumed in the wage adjustment process. This feature is typically associated with economies at or near full employment. Policy simulation results showed a negative impact on the economy and welfare of legal residents when employment of illegal migrants were reduced via supply-restricting policies. The adverse change to the occupation or skill mix of legal workers was the leading cause of this welfare loss in the long-run. This evidence suggests that low-skilled immigrant workers, regardless of their legal status, make a positive contribution to the economy under tight labour market conditions.

The assumptions in *Scenario 1* may well be true in many countries, but in South Africa's case it crucially neglects the high rates of unemployment and legal minimum wage at the lower end of the labour market. With a vast reserve of lower-skilled legal labour, an increase in demand for these workers should be met with very little wage pressure. This behaviour is correctly reflected in the wage adjustment process for *Scenario 2*. Policy simulation results under *Scenario 2* showed a much smaller welfare loss to legal workers as a result of the occupation-mix effect. More previously unemployed legal workers were also able to find work in the jobs previously done by illegal workers. Employment and welfare gains from the positive replacement effect at the lower end of the market far outweighed losses from the negative occupation-mix effect under *Scenario 2*. This evidence suggests that the South African government is correct in limiting low-skilled immigration given current labour market conditions. Enforcing policies that further restrict the flow of illegal immigrants is also well advised. These findings do not intend to pass any moral judgment on the plight of distressed workers who seek employment opportunities illegally. Legitimate asylum seekers are also not considered here. Other findings and concluding remarks related to the modelling process in this study are contained in the final section of each chapter.

### 6.3 Future Research

The analytical framework produced in this study opens up a number of avenues for future research regarding labour and migration issues in South Africa. In addition, there are also various aspects to our current modelling effort which may be improved upon. First we identify some future research topics made possible by the development of the ZAR-M model.

Our study on immigration reform only considered a reduction in the supply of foreign-born illegal immigrants as a way of achieving a cut in their employment levels. One immediate alternative to this simulation is to measure the impact of a similar cut to their employment levels via demand-side policies. This would involve targeting employers of illegal workers and those facilitating their illegal movement into the country. By imposing taxes and fines on these companies or individuals, the increased cost and risk to such an activity will have a negative effect on the demand, and subsequent employment, of foreign-born illegal labour. As shown in Dixon *et al* (2011), the choice of policy may well have an effect on the welfare of the native population. Supply-restricting policies tend to raise the wage rate of foreign-illegal workers who remain in the country. In contrast, demand-restricting policies are expected to lower the wage rate of illegal workers, increasing the welfare of the legal population. The transfer of revenue collected from penalties on transgressing employers to the National Treasury should further boost local welfare.

Another future topic of research closely related to the work presented here, relates to the issue of legalisation versus restriction of illegal workers. Dixon & Rimmer (2009) compared the economic impact of these policy reform alternatives for the United States. In terms of the income and welfare of legal workers, they found strong evidence in favour of legalisation under an optimal visa tax. Given the large stock of illegal workers believed to be active in the South African labour market, this is a topic well worth investigating. South African authorities face a daunting task on how to best construct and implement required immigration reform policies. These future research topics have the potential to definitively



answer the question of how to best deal with the country's illegal immigration problem and what the expected economic outcomes might be. Only minor changes to the current ZAR-M model would be required to facilitate such analysis and work on these topics has already begun.

The study of skilled emigration and related policy issues are already possible within the current structure of the model. Skilled emigration is another research topic that should be of great interest to policymakers and social scientists. Migration data suggests that there continues to be a net outflow of skilled labour from South Africa. In the event where such emigration is permanent without any remittances flows, the impact on the local economy is most likely to be negative as it represents an outflow of human capital and productive labour. Detailed analysis on the potential economic impact of policies that attempt to either reduce the rate of skilled emigration or increase the inflow of skilled migrants and expatriates returning home would therefore be a worthwhile exercise.

Apart from policy analysis, future research may also focus on forecasting of the labour market and flow of migrants. Producing a detailed forecast of the South African labour market for the next decade and beyond is important in assisting government and industries prepare for any labour market pressures that may arise. For example, a detailed forecast may give industries early warning of looming skill shortages. These types of forecasts allow a range of appropriate policies to be developed in a timely fashion. This may include responses in key areas such as training, immigration and labour market flexibility. Inheriting all the versatility of a MONASH-style model, ZAR-M could easily be used to produce forecasts of the South African labour market. This would typically also involve using appropriate model closures and available data to construct an historical decomposition of the economy leading up to the forecast period. Estimates obtained from this analysis are then used to help produce a forecast. Including macroeconomic forecasts from specialists in their field and considering the expert opinion from industry insiders can easily be accommodated to improve the labour market forecasting performance of the model.

Improving our current effort on measuring the impact of a reduction in illegal immigration is another avenue for future research. Supply-restricting options may carry significant implementation costs. Our present analysis omits these costs, thereby understating the welfare loss, or overstating the welfare gain, to legal residents of programs to reduce illegal employment. Although we do not expect much difference to our analysis, including the various costing aspects of the policy may prove useful to some interested parties. As it is, we may overcome this omission by simply treating the expected cost of the policy as a fixed cost item to the local economy that would reduce local welfare by that amount.

One last improvement to our analysis would be through the inclusion of more reliable estimates with regard to certain key parameters and migration flows in the model database. Fortunately, our sensitivity analysis has indicated that our policy simulation results are robust and not overly sensitive to realistic alternatives for these values. This suggests that the time required to improve these estimates may well be better spent elsewhere.





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## APPENDIX A

### THE PERCENTAGE-CHANGE APPROACH

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## A.1 Introduction

In the first part of this appendix we describe the percentage-change approach in presenting CGE models in GEMPACK and then give an example of how the linearisation error introduced with this method is largely eliminated.

For a detailed CGE model, the number of equations ( $m$ ) and variables ( $n$ ) can be very large. Many of the equations in a CGE model may also be non-linear in its levels form. To avoid the computational problems involved in solving a large non-linear system, Johansen (1960) approximated his model by a system of linear equations in changes or percentage changes of the variables. This linear system was then solved by matrix manipulations, giving the approximate effects on the ( $m$ ) endogenous variables of changes in the ( $n-m$ ) exogenous variables. In GEMPACK we follow this Johansen-style percentage-change approach in the implementation and solving of the model.

Therefore, instead of writing the system of equations in the model as

$$Y = f(X_1, X_2) \quad (\text{EA.1})$$

where  $Y$  is output and  $X_1$  and  $X_2$  inputs, a Johansen-style model uses the linear percentage-change form

$$y - \varepsilon_1 x_1 - \varepsilon_2 x_2 = 0 \quad (\text{EA.2})$$

where  $\varepsilon_i$  is the elasticity of output with respect to inputs of factor  $i$ , and  $y$  and  $x_i$  are the percentage changes in  $Y$  and  $X_i$ . In matrix notation, a Johansen-style model can be represented by

$$Av = 0 \quad (\text{EA.3})$$

where  $A$  is a matrix of coefficients and  $v$  is the vector of percentage changes in the model's variables.

In this representation  $A$  is a  $(m \times n)$  matrix, with the number of variables  $(n)$  usually considerably more than the number of equations  $(m)$ . To close the model and compute a solution as shown in equation (EA.5) we therefore have to choose an appropriate combination of  $m$  endogenous and  $(n - m)$  exogenous variables. We can express the changes or percentage changes in our endogenous variables as linear functions of the changes or percentage changes in our exogenous and predetermined variables by rearranging equation (EA.3) in the form

$$A_1 y + A_2 x = 0 \quad (\text{EA.4})$$

where  $y$  is the vector of percentage changes in those variables chosen to be endogenous and  $x$  is the vector of percentage changes in the predetermined variables and those chosen to be exogenous.  $A_1$  and  $A_2$  are matrices formed by selecting appropriate columns of  $A$ . Therefore, to compute the effects of changes in any of the exogenous variables on the endogenous variables, we need to solve (EA.5) via matrix inversion.

$$y = -A_1^{-1} \cdot A_2 x \quad (\text{EA.5})$$

A system of equations in the form of (EA.4) is much easier to solve and interpret than when written in its underlying, and often complex, non-linear form. The choice of different parameter values, in particular, becomes much easier to interpret under the percentage-change form. We illustrate this point later in the appendix by deriving the percentage-change form of the Cobb-Douglas and CES function and contrasting the results to their respective levels form. However, because  $A$  is assumed constant, these approximate solutions produced by (EA.5) are only accurate for small changes in the vector of exogenous variables otherwise unacceptably large linearisation error may occur. Eliminating the linearisation error in solving large-scale CGE models has become a relatively easy task thanks to GEMPACK and modern-day computing power as users are now able to enjoy features such as multi-step solution and Richardson's extrapolation without much cost in terms of time. These issues will be discussed more thoroughly in the following section.

## A.2 Linearisation Error

In the following section we use a simple example to illustrate how the linearisation error introduced with the Johansen-approach is eliminated using techniques available to GEMPACK users. We do this by contrasting the true result produced by a levels-form equation to the approximate result of a linearised-form equation.

We start by using equation (EA.6).

$$X = Y.Z \quad (\text{EA.6})$$

From here we can write

$$(X + \Delta X) = (Y + \Delta Y).(Z + \Delta Z) \quad (\text{EA.7})$$

$$(X + \Delta X) = Y.Z + Y.\Delta Z + \Delta Y.Z + \Delta Y.\Delta Z \quad (\text{EA.8})$$

Since we know that  $X = Y.Z$  we can subtract  $X$  on both sides to find the expression for total change in  $X$  shown in (EA.9).

$$\Delta X = Y.\Delta Z + \Delta Y.Z + \Delta Y.\Delta Z \quad (\text{EA.9})$$

To find the percentage change in  $X$  we divide across by  $X$  and multiply with 100.

$$\frac{\Delta X}{X} * 100 = \frac{Y.\Delta Z}{Y.Z} * 100 + \frac{\Delta Y.Z}{Y.Z} * 100 + \frac{\Delta Y.\Delta Z}{X} * 100 \quad (\text{EA.10})$$

From here we follow the GEMPACK convention of writing the percentage-change form of a particular variable in lower-case symbols. For example, the percentage-change in the variable represented by the upper-case symbol  $X$  will be represented by the lower-case symbol  $x$

determined as  $x = \frac{dX}{X} * 100$  or alternatively  $dX = \frac{X.x}{100}$ . Equation (EA.10) can therefore

be written as (EA.11) following this notational convention.

$$x = \frac{Y.\Delta Z}{Y.Z} * 100 + \frac{\Delta Y.Z}{Y.Z} * 100 + \frac{\Delta Y.\Delta Z}{X} * 100 \quad (\text{EA.11})$$

Applying similar notation to express the percentage-change in  $Y$  and  $Z$  allows us to write the true percentage change in  $X$  as

$$x = y + z + \frac{\Delta Y.\Delta Z}{X} * 100 \quad (\text{EA.12})$$

This shows that the true percentage change in  $X$  is equal to the percentage change in  $Y$ , plus the percentage change in  $Z$ , plus a second-order term  $\frac{\Delta Y.\Delta Z}{X} * 100$ .

We can confirm this through a simple numerical example. Assume that in equation (EA.6)  $Y = 20$  and  $Z = 5$  to give us an initial solution for  $X$  of  $20 * 5 = 100$ . If we impose an exogenous increase of 20 per cent in the value of both  $Y$  and  $Z$ , the new value for  $X$  would be  $20(1.20) * 5(1.20) = 24 * 6 = 144$ .

Substituting these values into equation (EA.12) confirms that the true increase in  $X$  is 44 per cent.

$$y + z + \frac{\Delta Y.\Delta Z}{X} * 100 = x$$

$$20 + 20 + \left( \frac{4 * 1}{100} \right) * 100 = 44 \quad (\text{EA.13})$$

In GEMPACK, we use the total differential of an equation to find the approximate percentage change in the dependent variable as a result of a change in one of the independent variables. By applying this derivative-based method to the same equation and example as before, we are able to illustrate and contrast the approximate result produced by the linearised-form equation with the true result produced in (EA.13) via equation (EA.12). We again use equation (EA.6).

$$X = Y.Z$$

From here we can write the total differential of  $X$  for a given change in  $Y$  and  $Z$  as

$$dX = \frac{\partial X}{\partial Y} dY + \frac{\partial X}{\partial Z} dZ \quad (\text{EA.14})$$

$$dX = 1(Y^{1-1}.Z)dY + 1(Y.Z^{1-1})dZ \quad (\text{EA.15})$$

$$dX = Z.dY + Y.dZ \quad (\text{EA.16})$$

Remember that we can express the percentage-change of a variable as  $x = \frac{dX}{X} * 100$  or

alternatively  $dX = \frac{X.x}{100}$ . Applying this notational convention again allows us to find equation (EA.18).

$$\frac{x.X}{100} = \left( \frac{y.Y}{100} \right) Z + \left( \frac{z.Z}{100} \right) Y \quad (\text{EA.17})$$

$$x.X = y.YZ + z.ZY \quad (\text{EA.18})$$

Since we know that  $X = Y.Z$  we can divide both sides with  $X$  to find the expression for total approximate percentage change in  $X$  shown in equation (EA.19).

$$x = y + z \quad (\text{EA.19})$$

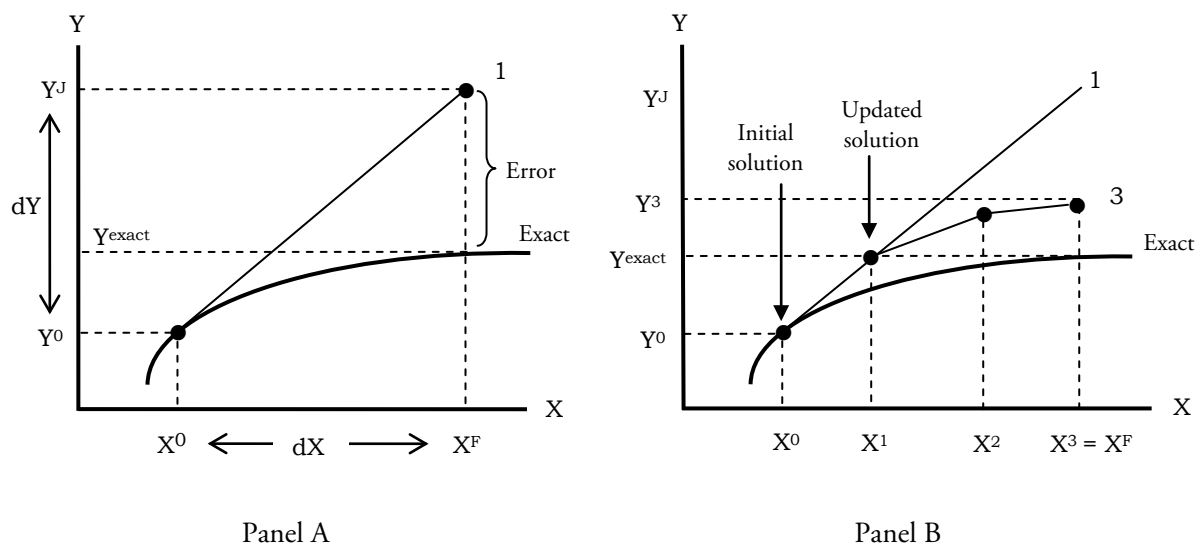


In comparing the true change in  $X$  from equation (EA.12) with the approximate change in  $X$  derived from equation (EA.19), we can see that the linearisation error when using the percentage-change form of an equation is equivalent to the second-order term  $\frac{\Delta Y \cdot \Delta Z}{X} * 100$  shown in equation (EA.12). This suggests that the larger the change in  $Y$  or  $Z$ , the greater the proportional error in  $x$ . Conversely, this also suggests that as changes to the independent variables, in this case  $Y$  and  $Z$ , tend closer to zero, the answer for  $x$  generated via equation (EA.19) will become a better approximation of the true answer generated through equation (EA.12). That is, as the term  $(\Delta Y \cdot \Delta Z)$  becomes smaller and smaller, the linearisation error would tend closer to zero.

This observation forms the basis to how the linearisation error is eliminated in GEMPACK. When breaking large changes in  $Y$  or  $Z$  into a number of steps, for each sub-change in  $Y$  or  $Z$ , we use the linear approximation to derive the consequent sub-change in  $X$ . Then, using the new values of  $Y$  and  $Z$ , we recomputed the coefficient matrices equivalent to  $A_1$  and  $A_2$  in equation (EA.5). Essentially, this updates the sales and cost shares imbedded in the  $A$  matrix. This process is repeated for each step. We can show that, given sensible restrictions on the derivatives of the model coefficients, we can obtain a solution as accurate as we like by dividing the process into sufficiently many steps.

This technique, known as the Euler method, is the simplest of several related techniques of numerical integration, that is, the process of using differential equations to move from one solution to another. To highlight the benefits of this multi-step solution technique, Figure A1 contrasts the accuracy of a 3-step Euler solution in Panel B with a 1-step Johansen solution in Panel A. The sub-intervals shown along the 3-step solution path in Panel B represent the updating of the coefficient matrices equivalent to  $A_1$  and  $A_2$  in equation (EA.5). GEMPACK offers users the choice of several such techniques. The accuracy of these multi-step solution techniques can further be improved by using Richardson's extrapolation.

FIGURE A1 Single-Step Johansen versus Multi-Step Euler Solution



Source: Adapted from Horridge (2000)

### A.3 The Cobb-Douglas Function

In the next part of this appendix we use the standard constrained optimisation problem facing consumers to give a complete exposition on how to derive the percentage-change equation of a simple Cobb-Douglas function.

Features of the Cobb-Douglas function include i) homogeneous of degree  $(\alpha + \beta)$  and in the special case of  $(\alpha + \beta) = 1$  it is linearly homogeneous, ii) its isoquants are negatively sloped and strictly convex for positive values of inputs  $X_1$  and  $X_2$ , iii) since the function is homogenous, it is also homothetic, i.e. consumer's preferences or budget shares depend only on the ratio of prices of goods, not income, and iv) like Leontief, Cobb-Douglas functions are easy to parameterize.

The function's linear homogeneity, that is  $(\alpha + \beta) = 1$ , is easily seen from the fact that, using a standard Cobb-Douglas production function given as  $Y = f(K, L) = A(K^\alpha \cdot L^\beta)$ , and changing inputs  $K$  and  $L$  to  $zK$  and  $zL$  respectively, the output will be changed to  $A(zK^\alpha \cdot zL^\beta) = z^{(\alpha + \beta)} \cdot A(K^\alpha \cdot L^\beta) = zY$ . Note that the function is not linear however. The generalized Cobb-Douglas production function is characterized by a constant, unitary elasticity of substitution ( $\sigma = 1$ ). The derivation of this result in no way relies upon the assumption that  $(\alpha + \beta) = 1$ . Thus, the elasticity of substitution of the production function  $Y = A(K^\alpha \cdot L^\beta)$  will be unitary even if  $(\alpha + \beta) \neq 1$ . The elasticity of substitution can be interpreted, under competitive conditions, as the sensitivity of the K/L ratio for a given change in the real wage to rental of capital ratio.

In the following optimisation problem, choose amounts of goods  $X_1$  and  $X_2$  to consume that will maximise utility  $U = U(X_1, X_2) = X_1^\alpha \cdot X_2^\beta$ , with  $\alpha$  and  $\beta$  positive fractions that sum to 1 indicating the share of income allocated to  $X_1$  and  $X_2$ , respectively, subject to the budget constraint  $X_1 \cdot P_1 + X_2 \cdot P_2 = M$ .

First set up the Lagrange function.

$$L = X_1^\alpha \cdot X_2^\beta + \lambda M - \lambda X_1 \cdot P_1 - \lambda X_2 \cdot P_2 \quad (\text{EA.20})$$

Taking the partial derivatives of  $L$  with respect to  $X_1$ ,  $X_2$  and  $\lambda$  in (EA.20) yields the following first-order conditions in (EA.21–EA.23).

$$\frac{\partial L}{\partial X_1} = \alpha \cdot X_1^{\alpha-1} \cdot X_2^\beta - \lambda P_1 = 0 \quad (\text{EA.21})$$

$$\frac{\partial L}{\partial X_2} = \beta \cdot X_1^\alpha \cdot X_2^{\beta-1} - \lambda P_2 = 0 \quad (\text{EA.22})$$

$$\frac{\partial L}{\partial \lambda} = M - X_1 \cdot P_1 - X_2 \cdot P_2 = 0 \quad (\text{EA.23})$$

Divide (EA.21) with (EA.22) to eliminate  $\lambda$ , simplify and write in terms of  $X_1 \cdot P_1$  to find the expression in (EA.25).

$$\frac{\lambda P_1}{\lambda P_2} = \frac{\alpha \cdot X_1^{\alpha-1} \cdot X_2^\beta}{\beta \cdot X_1^\alpha \cdot X_2^{\beta-1}} \quad (\text{EA.24})$$

$$X_1 \cdot P_1 = \frac{\alpha}{\beta} \cdot X_2 \cdot P_2 \quad (\text{EA.25})$$

Now substitute (EA.25) into (EA.23) and solve for  $X_2$ .

$$M = \left( \frac{\alpha}{\beta} \cdot X_2 \cdot P_2 \right) + X_2 \cdot P_2 \quad (\text{EA.26})$$

Remember that  $(\alpha + \beta) = 1$  which allows us find (EA.27).

$$M = (X_2 \cdot P_2) \left( \frac{1}{\beta} \right) \quad (\text{EA.27})$$

From here, the demand equation for good  $X_2$  can be written as shown in (EA.28).

$$X_2 = \beta \cdot \frac{M}{P_2} \quad (\text{EA.28})$$

In the case of  $(\alpha + \beta) \neq 1$  the demand equation for good  $X_2$  can be written as

$$X_2 = \frac{\beta}{\alpha + \beta} \cdot \frac{M}{P_2} \quad (\text{EA.29})$$

Given that  $(\alpha + \beta) = 1$  we can similarly show that the demand equation for good  $X_1$  can be written as

$$X_1 = \alpha \cdot \frac{M}{P_1} \text{ since } (1 - \beta) = \alpha \quad (\text{EA.30})$$

By writing the demand equations shown in (EA.28) and (EA.30) in terms of  $\alpha$  and  $\beta$  we can easily interpret these parameters as the budget shares, or share of expenditure on good  $X_1$  and  $X_2$ , respectively, by agents in their total income  $M$ . This is shown by (EA.31).

$$\beta = \frac{X_2 \cdot P_2}{M} \text{ and } \alpha = \frac{X_1 \cdot P_1}{M} \quad (\text{EA.31})$$

To write the demand equation for a Cobb-Douglas function in percentage-change form as shown in (EA.36) requires us to take the total differential of the general demand function derived from equations (EA.28) and (EA.30), which can be written as

$$X_i = \beta_i \cdot \frac{M}{P_i} \text{ with } \sum_i \beta_i = 1$$

From this general levels-form demand equation we can write  $X_i = \beta_i.M.P_i^{-1}$ .

The total differential of this expression is

$$dX_i = (\beta_i.P_i^{-1})dM + [-1(\beta_i.M.P_i^{-2})]dP_i \quad (\text{EA.32})$$

which can be simplified and written as

$$dX_i = (\beta_i.P_i^{-1})dM - (\beta_i.M.P_i^{-2})dP_i \quad (\text{EA.33})$$

Remember that the percentage change of a variable can be written as  $x = \frac{dX}{X} * 100$  or

alternatively  $dX = X.x * 0.01$ .

$$X_i.x_i * 0.01 = (\beta_i.P_i^{-1})M.m * 0.01 - (\beta_i.M.P_i^{-2})P_i.p_i * 0.01 \quad (\text{EA.34})$$

Since we know that  $X_i = \beta_i.M.P_i^{-1}$  equation (EA.34) can be simplified to

$$X_i.x_i * 0.01 = X_i.m * 0.01 - X_i.p_i * 0.01 \quad (\text{EA.35})$$

From here, the percentage-change form of the demand equation for a Cobb-Douglas function can be written as (EA.36) by dividing both sides with  $X_i * 0.01$ .

$$x_i = m - p_i \quad (\text{EA.36})$$

From the percentage-change form in (EA.36) it is easy to show that the Cobb-Douglas function exhibits unitary income elasticities. Income elasticities of demand can be defined as

$\frac{dX_i}{dM} \cdot \frac{M}{X_i}$  or alternatively  $\frac{x_i}{m}$ , and holding prices constant ( $p_i = 0$ ) it must therefore be

that  $x_i = m$ . The income elasticity defined as above must also be equal to one for all  $x_i$  given  $m \neq 0$ . It is also clear from (EA.36) that the Cobb-Douglas function exhibits own-price elasticities equal to one and cross-price elasticities equal to zero as a change in  $p_k$  will have no effect on  $x_i$ .

#### A.4 The CES Function

In the next part of this appendix we use the standard constrained optimisation problem facing producers to give a complete exposition on how to derive the percentage-change equation of a Constant Elasticity of Substitution (CES) function.

Features of the CES function include i) it is homogeneous of degree 1 and therefore exhibits constant returns to scale, ii) its isoquants are negatively sloped and strictly convex for positive values of  $X_1$  and  $X_2$ , and iii) since the function is homogenous, it is also homothetic, i.e. consumer's preferences or budget shares depend only on the ratio of prices of goods, not income, and iv) it relaxes the imposed substitution elasticities in the Leontief ( $\sigma = 0$ ) and Cobb-Douglas ( $\sigma = 1$ ) functions.

In the following optimisation problem, choose inputs  $X_i$  (with  $i = 1 \dots n$ ) to minimise the cost  $\sum_i P_i X_i$  of producing given output  $Z$  subject to the CES production function.

$$Z = \beta \left( \sum_i \delta_i X_i^{-\rho} \right)^{-1/\rho}$$

In this equation,  $X_i$  represents types of inputs, and  $\beta$ ,  $\delta$  and  $\rho$  are three parameters, with  $\beta$  the efficiency or technology parameter,  $\delta_i$  the distribution parameter indicating the relative factor shares in the product, and  $\rho$  the substitution parameter which determines the value of the (constant) elasticity of substitution defined as  $\sigma = \frac{1}{1 + \rho}$ .

First set up the Lagrange function.

$$L = \sum_i P_i X_i + \lambda \left[ Z - \beta \left( \sum_i \delta_i X_i^{-\rho} \right)^{-1/\rho} \right] \quad (\text{EA.37})$$

Taking the partial derivatives of  $L$  with respect to  $X_i$ ,  $X_k$  and  $\lambda$  yields the following first-order conditions in (EA.38–EA.40).

$$\frac{\partial L}{\partial X_i} = P_i - \lambda \beta \cdot \left(-\frac{1}{\rho}\right) \left(\sum_i \delta_i X_i^{-\rho}\right)^{-(1/\rho)-1} \cdot (-\rho \delta_i X_i^{-\rho-1}) = 0 \quad (\text{EA.38})$$

$$\frac{\partial L}{\partial X_k} = P_k - \lambda \beta \cdot \left(-\frac{1}{\rho}\right) \left(\sum_i \delta_i X_i^{-\rho}\right)^{-(1/\rho)-1} \cdot (-\rho \delta_k X_k^{-\rho-1}) = 0 \quad (\text{EA.39})$$

$$\frac{\partial L}{\partial \lambda} = Z - \beta \left(\sum_i \delta_i X_i^{-\rho}\right)^{-1/\rho} = 0 \quad (\text{EA.40})$$

Divide (EA.39) with (EA.38) to eliminate  $\lambda$ , simplify and write in terms of  $X_i^{-\rho}$  to find the expression in (EA.44).

$$\frac{P_k}{P_i} = \frac{\lambda \beta \cdot \left(-\frac{1}{\rho}\right) \left(\sum_i \delta_i X_i^{-\rho}\right)^{-(1/\rho)-1} \cdot (-\rho \delta_k X_k^{-\rho-1})}{\lambda \beta \cdot \left(-\frac{1}{\rho}\right) \left(\sum_i \delta_i X_i^{-\rho}\right)^{-(1/\rho)-1} \cdot (-\rho \delta_i X_i^{-\rho-1})} \quad (\text{EA.41})$$

$$\left(\frac{P_k}{P_i}\right)^{\frac{1}{1+\rho}} = \left(\frac{\delta_k}{\delta_i}\right)^{\frac{1}{1+\rho}} \left(\frac{X_i}{X_k}\right) \quad (\text{EA.42})$$

Remember that  $\left(\frac{\delta_k}{\delta_i}\right)^{\frac{1}{1+\rho}}$  can be written as  $\left(\frac{\delta_k^{\frac{1}{1+\rho}}}{\delta_i^{\frac{1}{1+\rho}}}\right)$  which allows us to find (EA.44).

$$\left(\frac{\delta_i \cdot P_k}{\delta_k \cdot P_i}\right)^{\frac{1}{1+\rho}} \cdot X_k = X_i \quad (\text{EA.43})$$

$$X_i^{-\rho} = \left(\frac{\delta_i \cdot P_k}{\delta_k \cdot P_i}\right)^{-\frac{\rho}{(1+\rho)}} \cdot X_k^{-\rho} \quad (\text{EA.44})$$



Now substitute (EA.44) into (EA.40) and solve for  $X_k$ .

$$Z = \beta \left[ \sum_i \delta_i \left\{ \left( \frac{\delta_i \cdot P_k}{\delta_k \cdot P_i} \right)^{\frac{\rho}{(1+\rho)}} \cdot X_k^{-\rho} \right\} \right]^{-1/\rho} \quad (\text{EA.45})$$

Remember that  $X_k^{-\rho \cdot \left(\frac{1}{\rho}\right)} = X_k^1$  which allows us to find (EA.46).

$$Z = X_k \beta \left[ \sum_i \delta_i \left\{ \left( \frac{\delta_k \cdot P_i}{\delta_i \cdot P_k} \right)^{\frac{\rho}{(1+\rho)}} \right\} \right]^{-1/\rho} \quad (\text{EA.46})$$

From here, we can find the input demand equation by writing (EA.46) in terms of  $X_k$ .

$$X_k = Z \cdot \beta \left[ \sum_i \delta_i \left\{ \left( \frac{\delta_k \cdot P_i}{\delta_i \cdot P_k} \right)^{\frac{\rho}{(1+\rho)}} \right\} \right]^{1/\rho} \quad (\text{EA.47})$$

From (EA.47) we can use some algebraic manipulation to find equation (EA.49).

$$X_k = Z \cdot \beta \cdot \left( \frac{\delta_k}{P_k} \right)^{\frac{1}{(1+\rho)}} \left[ \sum_i \delta_i \left( \frac{P_i}{\delta_i} \right)^{\frac{\rho}{(1+\rho)}} \right]^{1/\rho} \quad (\text{EA.48})$$

$$X_k = Z \cdot \beta \cdot \left( \frac{\delta_k}{P_k} \right)^{\frac{1}{(1+\rho)}} \left[ \sum_i \delta_i^{1-\frac{\rho}{(1+\rho)}} \cdot P_i^{\frac{\rho}{(1+\rho)}} \right]^{1/\rho} \quad (\text{EA.49})$$

Since  $1 - \frac{\rho}{(1+\rho)}$  can be written as  $\frac{1}{(1+\rho)}$ , this allows us to find equation (EA.50).

$$X_k = Z \cdot \beta \cdot \delta_k^{\frac{1}{(1+\rho)}} \cdot P_k^{-\frac{1}{(1+\rho)}} \left[ \sum_i \delta_i^{\frac{1}{(1+\rho)}} \cdot P_i^{\frac{\rho}{(1+\rho)}} \right]^{1/\rho} \quad (\text{EA.50})$$

Some more algebraic manipulation of (EA.50) yields the input demand equation for  $X_k$  as shown in equation (EA.51). The form shown in (EA.51) represents the non-linear levels-form of a CES equation.

$$X_k = Z \cdot \beta \cdot \delta_k^{\frac{1}{1+\rho}} \left[ \frac{P_k}{P_{ave}} \right]^{-\frac{1}{(1+\rho)}} \quad (\text{EA.51})$$

$$\text{with } P_{ave} = \left( \sum_i \delta_i^{\frac{1}{1+\rho}} \cdot P_i^{\frac{\rho}{1+\rho}} \right)^{\frac{1+\rho}{\rho}} \quad \text{or alternatively} \quad \frac{1}{P_{ave}} = \left( \sum_i \delta_i^{\frac{1}{1+\rho}} \cdot P_i^{\frac{\rho}{1+\rho}} \right)^{-\frac{1+\rho}{\rho}}$$

$$\text{since } \left[ \left( \sum_i \delta_i^{\frac{1}{1+\rho}} \cdot P_i^{\frac{\rho}{1+\rho}} \right)^{-\frac{1+\rho}{\rho}} \right]^{-\frac{1}{(1+\rho)}} \quad \text{must be equal to} \quad \left[ \sum_i \delta_i^{\frac{1}{1+\rho}} \cdot P_i^{\frac{\rho}{1+\rho}} \right]^{1/\rho}$$

To find the exponent used for  $P_{ave}$  in the expression above we had to find an exponent which multiplied with  $-\frac{1}{(1+\rho)}$  would yield  $\frac{1}{\rho}$  from equation (EA.50).

The demand equation derived from the CES optimisation problem given earlier can now be converted to percentage-change form as shown in (EA.54). This requires us to take the total differential of the demand function derived above in (EA.51). To find (EA.52) we continue to use the convention of writing percentage-change variables in lower case, that is,

$$x = \frac{dX}{X} * 100 \quad \text{or alternatively} \quad dX = \frac{X \cdot x}{100}.$$

$$X_k \cdot x_k * 0.01 = \left( \beta \cdot \delta_k^{\frac{1}{1+\rho}} \left[ \frac{P_k}{P_{ave}} \right]^{-\frac{1}{(1+\rho)}} \right) Z \cdot z * 0.01 + \left( Z \cdot \beta \cdot \delta_k^{\frac{1}{1+\rho}} \left( -\frac{1}{1+\rho} \right) \left[ \frac{P_k}{P_{ave}} \right]^{-\frac{1}{(1+\rho)}-1} \right) d \frac{P_k}{P_{ave}} \quad (\text{EA.52})$$

Using the quotient rule, we can write the  $d\left(\frac{P_k}{P_{ave}}\right)$  component in equation (EA.52) as

$$\left(\frac{P_{ave} \cdot P_k' - P_k \cdot P_{ave}'}{P_{ave}^2}\right) \text{ which simplifies to } \left[\left(\frac{P_k \cdot 0.01}{P_{ave}}\right)(p_k - p_{ave})\right].$$

Given that we also know  $X_k = Z \cdot \beta \cdot \delta_k^{\frac{1}{1+\rho}} \left[\frac{P_k}{P_{ave}}\right]^{\frac{1}{1+\rho}}$ , equation (EA.52) can therefore be simplified and written as (EA.53).

$$X_k \cdot x_k \cdot 0.01 = X_k \cdot z \cdot 0.01 + \left(X_k \cdot \left[\frac{P_{ave}}{P_k}\right]\right) \left(\frac{P_k \cdot 0.01}{P_{ave}}\right) (p_k - p_{ave}) \quad (\text{EA.53})$$

From here, the percentage-change form of the demand equation for a CES function can be written as (EA.54). The economic meaning of  $S_i$  can be interpreted as the cost share of input  $X_i$  in producing output  $Z$ .

$$x_k = z - \sigma(p_k - p_{ave}) \quad (\text{EA.54})$$

$$\text{with } p_{ave} = \sum_i S_i p_i \text{ and where } \sigma = \frac{1}{1+\rho} \text{ and } S_i = \frac{\delta_i^{\frac{1}{1+\rho}} \cdot P_i^{\frac{\rho}{1+\rho}}}{\sum_k \delta_k^{\frac{1}{1+\rho}} \cdot P_k^{\frac{\rho}{1+\rho}}}$$

By writing  $\sigma$  in terms of  $-\rho$  the standard CES function  $Z = \beta \left(\sum_i \delta_i X_i^{-\rho}\right)^{-1/\rho}$  expressed earlier could also be written as

$$Z = \beta \left(\sum_i \delta_i X_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{-\sigma}{1-\sigma}} \text{ or alternatively } Z = \beta \left(\sum_i \delta_i X_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad (\text{EA.55})$$

### A.5 The Klein-Rubin Function

In the final part of this appendix we briefly describe the Klein-Rubin utility function used in the determination of household demand in ZAR-M.

The Klein–Rubin utility function is used in the top nest of household demand in the ZAR-M model. The Klein–Rubin function (also known as the Stone–Geary or Linear Expenditure System) is non-homothetic which means that rising income will cause the preferences of consumers (budget shares) to change, even with price ratios fixed. That is, the marginal rate of substitution changes as income increases.

Total demand or consumption of household goods consists of a combination of subsistence goods and luxury goods, which can be written as  $X_i = X_i^{sub} + X_i^{lux}$ . From here we are able to derive an algebraic equation of Klein-Rubin household demand by solving the optimisation problem of choosing amounts of  $X_i^{sub}$  and  $X_i^{lux}$  (with  $i = 1 \dots n$ ) that will maximise utility  $U = (X_1 \dots X_i) = \sum_i S_i^{lux} * \ln(X_i - X_i^{sub})$  subject to the consumer's budget constraint  $\sum_i X_i \cdot P_i = M$ .

Using a simple algebraic exposition, we are able to derive an expression for household demand equivalent to that used in ZAR-M. We start again by defining total household demand as the sum of subsistence and luxury consumption.

$$X_i = X_i^{sub} + X_i^{lux} \quad (\text{EA.56})$$

Assuming that all income is spent, we can express luxury spending as

$$Y^{lux} = Y - \sum_i P_i \cdot X_i^{sub} \quad (\text{EA.57})$$

If we further assume that luxury spending on good  $i$  is a fixed proportion of total supernumerary income, we can show that

$$P_i \cdot X_i^{lux} = S_i^{lux} \cdot Y^{lux} \quad (\text{EA.58})$$

which can be written as

$$X_i^{lux} = \left[ \frac{S_i^{lux}}{P_i} \right] * Y^{lux} \quad (\text{EA.59})$$

From here we can substitute (EA.57) into (EA.59) to find equation (EA.60)

$$X_i^{lux} = \left[ \frac{S_i^{lux}}{P_i} \right] * \left( Y - \sum_i P_i \cdot X_i^{sub} \right) \quad (\text{EA.60})$$

which can then be substituted into (EA.56) to find equation (EA.61).

$$X_i = X_i^{sub} + \left[ \frac{S_i^{lux}}{P_i} \right] * \left( Y - \sum_i P_i \cdot X_i^{sub} \right) \quad (\text{EA.61})$$

Equation (EA.61) shows the general levels-form of a Klein-Rubin household demand equation, with expenditure on each good  $i$  a linear function of prices and income.

A complete exposition on deriving the percentage-change form of the Klein-Rubin household demand equation is contained in Dixon *et al* (1982:96-103).



# APPENDIX B

## MODEL CLOSURES

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## B.1 Exogenous Variables in the Decomposition Closure for ZAR-M

<i>Variable</i>	<i>Size / Dimension</i>	<i>Description</i>
a0	COM*IND	technical change, output augmenting
a1	IND	technical change, input augmenting, production
a2	IND	technical change, input augmenting, investment
a1Ind	IND	technical change, land saving
a1oct	IND	technical change, other cost tickets augmenting
a3com	COM	combined change in household tastes and preferences
a5mar	COM*SRC*MAR	technical change, margins for government use
ac	COM	technical change, commodity use
del_ror	IND	expected rates of return
del_r_tot	1	allows for equal changes in expected rates of return
del_unity	1	homotopy variable, normally shocked from zero to one
d_f_eeqror	1	shifter, capital growth & rates of return equation
d_f_fd_t	1	shifter, foreign debt (NFL), start of year
d_f_psd_t	1	shifter, public sector debt, start of year
d_f_p3tot_l	1	shifter, lagged CPI, usually CPI in year t-1
d_rint	1	real interest rate
emp_jobs	1	aggregate employment, jobs weighted
f0tax	IND	shifter, production taxes
f0tax_i	1	shifter, general production tax
f0tax_s	COM	shifter, general sales taxes
f1oct	IND	shifter, price of other cost tickets
f1tax_csi	1	shifter, tax on intermediate usage
f2tax_csi	1	shifter, tax on investment
f3tax_cs	1	shifter, tax on household consumption
f4gen	1	shifter, export demand curve
f4p	COM	shifter, price or vertical shift on export demand curves
f4q	COM	shifter, quantity or horizontal shift on export demand curves
f4tax	COM	shifter, tax on exports
f4tax_c	1	shifter, tax on exports
f5	COM	shifter, government demand
f5tot	1	shifter, ratio of government to household demand
fx6	COM*SRC	shifter, inventories
fa1c	COM	shifter, technical change in production
fa1ci	COM*IND	shifter, technical change in production
fa1mar	COM*SRC*MAR*IND	shifter, technical change in margins, intermediate
fa1marc	COM	shifter, technical change in margins, intermediate
fa2c	COM	shifter, technical change in investment
fa2ci	COM*IND	shifter, technical change in investment
fa2mar	COM*SRC*MAR*IND	shifter, technical change in margins, investment
fa2marc	COM	shifter, technical change in margins, investment
fa3mar	COM*SRC*MAR	shifter, technical change in margins, consumption
fa3marc	COM	shifter, technical change in margins, consumption
fa4mar	COM*MAR	shifter, technical change in margins, exports
fa4marc	COM	shifter, technical change in margins, exports
ftwistlk	IND	shifter, labour/capital preference twist



ftwist_src	COM	shifter, import/domestic preference twist
f_a1cap	IND	shifter, capital saving technical change
f_a1lab_o	IND	shifter, labour saving technical change
f_a1csi	COM*SRC*IND	shifter, technical change in production
f_a2csi	COM*SRC*IND	shifter, technical change in capital creation
f_a3cs	COM*SRC	shifter, change in household tastes and preferences
f_a1prim	IND	shifter, primary factor technical change
ff_a1prim	1	shifter, primary factor technical change
f_t0imp	COM	shifter, power of tariff
ff_t0imp	1	shifter, power of tariff
f_pf0cif	COM	shifter, foreign currency import prices
f_pf0cif_c	1	shifter, foreign currency import prices
f_emp_o	1	shifter, employment forecast
f_rwage_o	1	shifter, real wage forecast
f_tax_r	1	ratio of capital to labour tax rates
phi	1	exchange rate, mid year, \$Foreign/ZAR
pop	1	population
q	1	number of households
r_inv_cap	IND	shifter, investment/capital ratio
r_inv_cap_u	1	shifter, investment/capital ratio
tax_l_r	1	tax rate on labour income
twist_c	1	shifter, import/domestic preference twist
twist_i	1	shifter, labour/capital preference twist
x1lnd	IND	land use by industry

<i>Variable</i>	<i>Size / Dimension</i>	<i>Description</i>
a28	1	technical change, 'zmn' industry
a29	1	technical change, 'auk' industry
b_pref	OCCPM*BP*LS*NONNEW	policy variable to alter labour supply preferences
d_f_rw_ptd_A	UNEMP*BP*LS	shifter, real returns to unemployment
d_f_rw_ptd_C1	BP*LS	shifter, real post-tax wages in 'zmn'
d_f_rw_ptd_C2	BP*LS	shifter, real post-tax wages in 'auk'
f1lab_bs	BP*LS	shifter, nominal wages
f1lab_oibs	OCC*BP*LS	shifter, nominal wages
ff_bca3	OCC*BP*LS	shifter, forecast/policy transfer for real wages
ff_bca4	OCC*BP*LS	shifter, forecast/policy transfer for employment
f_jobldr	OCC*BP*LS	shifter, forecast/policy transfer for labour supply
f_lfcoffer_oi	OCCPM*BP*LS	shifter, labour offers from categories
ff_tax_l_r	1	shifter, overall rate of tax on labour income
f_tax_l_rd	BP*LS	shifter, tax rate on labour income
f_x1labose	BP*LS	used to shock numbers of new entrants
p0ind28	1	price of 'zmn' output
p0ind29	1	price of 'auk' output
rem_rate	1	remittance rate for foreign illegals
delunity	1	homotopy variable, normally shocked from zero to one

## B.2 Swap Statements Applied to Generate the Forecast Closure

*Previously exogenous variables*

*Newly exogenous variables*

Step 1 – Moving from a one-period long-run closure to a year-on-year short-run closure  
(Dixon & Rimmer, 2002: 262-263)

del_ror .....	d_f_cap_t
r_inv_cap .....	d_f_eeqror_j

Step 2 – Incorporating available macroeconomic forecast data  
(Dixon & Rimmer, 2002: 264-265)

apc_gnp .....	x3tot
d_f_eeqror .....	x2tot_i
f5tot .....	x5tot
f4gen .....	x4tot
twist_c .....	x0cif_c
phi .....	p3tot
ff_a1prim .....	phi
f_pf0cif_c .....	p0toft

Step 3 – Real wage rate and miscellaneous variables  
(Dixon & Rimmer, 2002:267-268)

twist_i .....	real_wage_c
a1 .....	del_f_a1
a2 .....	del_f_a2

Step 4 – Optional swaps

emp_jobs .....	f1lab_io
f_tax_r .....	tax_k_r
f1lab_bs .....	x1labh_bs

### B.3 Description of Newly Exogenous Variables in the Forecast Closure

#### *Newly exogenous variables*

#### *Description*

d_f_cap_t .....	shifter, year-to-year capital growth
d_f_eeqror_j .....	shifter, capital growth & rates of return equation
x3tot .....	aggregate real household consumption
x2tot_i .....	aggregate real investment expenditure
x5tot .....	aggregate real government expenditure
x4tot .....	aggregate export volume index
x0cif_c .....	aggregate import volume index
p3tot .....	consumer price index
p0toft .....	terms of trade
real_wage_c .....	real consumer wage
del_f_a1 .....	shifter, cost neutralisation, production
del_f_a2 .....	shifter, cost neutralisation, investment

## B.4 Swap Statements Applied to Generate the Policy Closure

*Previously exogenous variables*

*Newly exogenous variables*

Step 1 – Endogenise macro variables again to evaluate impacts of the policy  
(Dixon & Rimmer, 2002: 268-274)

x3tot .....	apc_gnp
x2tot_i .....	d_f_eeqror
x5tot .....	f5tot
x4tot .....	f4gen
x0cif_c .....	twist_c
phi .....	ff_a1prim
p0toft .....	f_pf0cif_c
real_wage_c .....	twist_i

Step 2 – Labour migration variables

f1lab_oibs .....	del_f_wage_ptd
emp_jobs .....	f1lab_io
ff_bca4 .....	x1lab_obso
f_joblrd .....	emp_jobslrd
f_emp_o .....	emp_jobs_o
f_rwage_o .....	real_wage_c_o

Step 3 – Other variables

q .....	f_q
f3tax_cs .....	d_gov_def
f_tax_r .....	tax_k_r
del_f_a1 .....	a1
del_f_a2 .....	a2
fa1ci .....	a1_s
fa2ci .....	a2_s
f_a1csi .....	a1csi
f_a2csi .....	a2csi
f_a3cs .....	a3cs
f_aicap .....	a1cap
f_a1lab_o .....	a1lab_o

## B.5 Description of Newly Exogenous Variables in the Policy Closure

### *Newly exogenous variables*

### *Description*

del_f_wage_ptd .....	shifter, post-tax wage rate in work activity o,b,s
f1lab_io .....	shifter, overall wage rate
x1lab_obso .....	employment in work activity o,b,s, forecast
emp_jobslrdo .....	labour supply to work activity o,b,s, forecast
emp_jobs_o .....	aggregate employment, jobs weighted, forecast
real_wage_c_o .....	real consumer wage, forecast
f_q .....	shifter, number of households
d_gov_def .....	change in public sector deficit
tax_k_r .....	tax rate on capital rental income
a1 .....	technical change, input augmenting, production
a2 .....	technical change, input augmenting, investment
a1_s .....	technical change, input saving, production
a2_s .....	technical change, input saving, investment
a1csi .....	technical change, input saving, production
a2csi .....	technical change, input saving, investment
a3cs .....	commodity augmenting change in household tastes
a1cap .....	technical change, capital saving
a1lab_o .....	technical change, labour saving



# APPENDIX C

## SIMULATION RESULTS

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TABLE AC1 Baseline Forecast (Year-on-Year Percentage Change) (Macro Variables)

<i>Macros</i>	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
x0cif_c	9.00	2.50	-6.00	4.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
x0gdpexp	4.92	4.15	1.30	3.94	4.70	4.78	4.81	4.82	4.83	4.84	4.85	4.86	4.87	4.87
x0gne	5.91	3.94	1.64	4.00	4.76	4.81	4.82	4.83	4.84	4.84	4.85	4.86	4.87	4.87
x0imp_c	9.00	2.50	-6.00	4.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
x1cap_i	2.83	3.35	3.77	3.68	3.77	3.97	4.21	4.43	4.64	4.82	5.00	5.15	5.30	5.43
x2tot_i	9.00	8.00	2.50	5.00	6.00	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50	6.50
x3tot	5.50	2.50	0.50	3.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50
x4tot	6.00	3.00	-8.00	3.80	4.80	4.90	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00
x5tot	4.50	4.50	4.50	4.50	4.30	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
p0cif_c	7.61	7.42	2.79	5.03	5.19	5.32	5.29	5.27	5.27	5.28	5.28	5.29	5.30	5.31
p0gdpexp	7.03	7.33	7.34	5.85	5.94	6.12	6.18	6.21	6.22	6.23	6.23	6.24	6.24	6.25
p0gne	7.05	7.18	6.32	5.55	5.64	5.83	5.88	5.91	5.92	5.93	5.94	5.94	5.95	5.95
p0imp_c	7.61	7.42	2.79	5.03	5.19	5.32	5.29	5.27	5.27	5.28	5.28	5.29	5.30	5.31
p0realdev	0.55	0.08	-4.25	-0.78	-0.71	-0.76	-0.84	-0.88	-0.89	-0.90	-0.90	-0.89	-0.89	-0.89
p0toft	0.00	0.50	3.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
p1cap_i	11.30	10.70	6.54	7.04	8.27	8.54	8.37	8.13	7.87	7.62	7.40	7.18	6.98	6.79
p2tot_i	8.17	7.63	5.32	5.21	5.60	5.91	5.83	5.84	5.84	5.84	5.85	5.85	5.85	5.85
p3tot	7.00	7.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
p4tot	7.61	7.95	5.87	6.07	6.24	6.37	6.34	6.32	6.32	6.33	6.33	6.34	6.35	6.36
p5tot	5.92	7.23	8.54	4.48	4.56	5.17	5.52	5.67	5.74	5.79	5.83	5.87	5.90	5.93
phi	-0.50	-1.50	0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50
emp_jobs	2.50	1.50	0.50	2.00	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
employ_io	2.36	1.51	0.81	2.09	2.44	2.42	2.44	2.45	2.45	2.45	2.45	2.45	2.45	2.45
real_wage_c	0.50	0.50	0.10	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
jobsupply_dl	0.78	1.61	1.43	1.23	1.49	1.61	1.61	1.61	1.61	1.61	1.61	1.60	1.60	1.60
jobsupply_fi	3.51	2.29	2.04	1.93	2.41	2.56	2.56	2.54	2.52	2.51	2.49	2.48	2.46	2.45
jobsupply_fl	3.42	2.74	1.71	1.10	2.42	2.79	2.74	2.73	2.71	2.70	2.68	2.67	2.65	2.64



TABLE AC2 Baseline Forecast (Year-on-Year Percentage Change) (Industry Output)

<i>Industry</i>	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
1 agric	3.39	3.21	-0.01	2.98	4.00	4.14	4.12	4.04	3.95	3.86	3.76	3.66	3.55	3.43
2 coal	3.41	2.92	-1.04	2.44	3.58	3.82	3.90	3.89	3.85	3.80	3.74	3.68	3.61	3.54
3 gold	4.11	2.91	-2.67	1.80	2.98	3.24	3.35	3.36	3.34	3.28	3.21	3.13	3.05	2.96
4 othermining	3.35	2.75	-0.61	1.97	2.82	2.98	3.04	3.03	2.99	2.95	2.89	2.82	2.75	2.68
5 foodbev	4.06	2.90	-0.42	3.41	4.12	3.96	3.91	3.89	3.90	3.92	3.94	3.97	4.00	4.04
6 textfoot	3.96	2.99	-0.01	3.98	4.08	3.87	3.81	3.79	3.81	3.84	3.89	3.94	3.99	4.04
7 woodpaper	3.99	3.88	1.60	4.22	4.51	4.49	4.50	4.50	4.52	4.54	4.56	4.59	4.62	4.65
8 petrochem	4.03	3.85	0.89	4.27	4.91	4.94	4.99	5.00	5.03	5.06	5.09	5.13	5.16	5.20
9 glassnonmet	3.90	3.95	1.34	4.46	4.93	4.97	5.04	5.06	5.08	5.11	5.14	5.17	5.21	5.24
10 metalmach	4.38	4.57	1.13	4.47	5.17	5.29	5.41	5.45	5.48	5.51	5.54	5.57	5.60	5.64
11 electrical	5.01	5.57	2.54	4.96	5.40	5.49	5.52	5.52	5.53	5.55	5.57	5.59	5.62	5.64
12 radiotv	4.52	4.68	1.19	4.83	5.71	5.87	6.02	6.09	6.14	6.19	6.23	6.27	6.31	6.35
13 transequip	4.93	5.26	1.86	5.15	5.71	5.79	5.84	5.83	5.84	5.85	5.87	5.89	5.91	5.93
14 othermanuf	4.02	3.92	0.39	3.20	4.49	4.87	5.18	5.38	5.54	5.65	5.74	5.81	5.86	5.90
15 electricity	4.45	3.60	0.39	3.78	4.55	4.61	4.63	4.64	4.67	4.70	4.73	4.77	4.80	4.84
16 water	4.34	3.35	0.38	3.66	4.22	4.28	4.29	4.31	4.35	4.38	4.41	4.45	4.48	4.51
17 construc	7.03	6.09	1.86	5.29	5.86	6.08	6.10	6.11	6.13	6.15	6.16	6.17	6.18	6.19
18 trade	4.31	3.77	1.27	4.22	4.64	4.67	4.71	4.74	4.77	4.80	4.84	4.87	4.91	4.94
19 hotels	5.20	4.03	0.19	3.34	4.12	4.23	4.33	4.39	4.46	4.53	4.60	4.68	4.76	4.83
20 transser	5.59	4.48	0.54	4.28	5.37	5.54	5.60	5.60	5.59	5.59	5.58	5.58	5.58	5.58
21 comm	6.57	5.03	0.95	3.87	5.42	5.72	5.73	5.72	5.70	5.69	5.68	5.68	5.68	5.68
22 financial	6.30	4.47	0.47	4.20	5.38	5.50	5.50	5.48	5.47	5.47	5.46	5.46	5.46	5.47
23 estate	6.86	5.16	1.12	2.90	4.50	4.95	4.98	4.98	4.98	5.00	5.02	5.05	5.07	5.10
24 business	5.24	4.60	1.42	4.33	4.98	5.06	5.10	5.11	5.12	5.14	5.15	5.17	5.19	5.21
25 gengov	4.51	4.45	4.16	4.45	4.33	4.06	4.06	4.07	4.07	4.07	4.07	4.08	4.08	4.08
26 healthsoc	7.01	4.98	0.34	4.68	6.34	6.69	6.76	6.72	6.65	6.57	6.49	6.41	6.34	6.27
27 otherser	5.41	3.57	-0.33	3.80	4.74	4.74	4.71	4.67	4.66	4.65	4.64	4.63	4.63	4.62

TABLE AC3 Policy Simulation – Scenario 1 (Cumulative Percentage Deviation from Baseline) (Macro Variables)

<i>Macros</i>	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
x0cif_c	0	-0.21	-0.62	-1.04	-1.37	-1.60	-1.76	-1.86	-1.91	-1.93	-1.91	-1.88	-1.83	-1.78
x0gdpexp	0	-0.09	-0.26	-0.48	-0.74	-1.01	-1.28	-1.54	-1.77	-1.97	-2.13	-2.27	-2.37	-2.44
x0gne	0	-0.18	-0.51	-0.88	-1.22	-1.51	-1.76	-1.96	-2.12	-2.25	-2.33	-2.39	-2.42	-2.43
x0imp_c	0	-0.21	-0.62	-1.04	-1.37	-1.60	-1.76	-1.86	-1.91	-1.93	-1.91	-1.88	-1.83	-1.78
x1cap_i	0	0.00	-0.05	-0.17	-0.35	-0.59	-0.85	-1.12	-1.37	-1.60	-1.80	-1.98	-2.12	-2.22
x2tot_i	0	-0.49	-1.29	-2.17	-2.88	-3.32	-3.56	-3.66	-3.67	-3.59	-3.47	-3.31	-3.14	-2.97
x3tot	0	-0.11	-0.33	-0.57	-0.81	-1.05	-1.29	-1.51	-1.70	-1.86	-2.00	-2.11	-2.18	-2.24
x3tot_ill	0	-7.18	-13.85	-16.94	-18.60	-19.51	-20.08	-20.49	-20.83	-21.12	-21.39	-21.62	-21.83	-22.00
x3tot_leg	0	0.01	-0.09	-0.28	-0.51	-0.75	-0.98	-1.21	-1.40	-1.57	-1.71	-1.82	-1.91	-1.97
x4tot	0	0.11	0.28	0.40	0.37	0.22	-0.02	-0.29	-0.59	-0.88	-1.16	-1.41	-1.62	-1.80
x5tot	0	-0.11	-0.33	-0.57	-0.81	-1.05	-1.29	-1.51	-1.70	-1.86	-2.00	-2.11	-2.18	-2.24
x5tot_ill	0	-5.64	-10.77	-15.54	-18.82	-21.22	-23.08	-24.56	-25.76	-26.75	-27.56	-28.22	-28.75	-29.18
x5tot_leg	0	0.24	0.32	0.37	0.33	0.23	0.11	-0.02	-0.13	-0.22	-0.29	-0.34	-0.36	-0.37
p0cif_c	0	0.08	0.11	0.11	0.03	-0.11	-0.27	-0.43	-0.60	-0.76	-0.91	-1.03	-1.13	-1.21
p0gdpexp	0	0.00	-0.02	-0.01	0.01	0.04	0.07	0.09	0.10	0.11	0.12	0.13	0.13	0.13
p0gne	0	0.01	0.01	0.03	0.05	0.06	0.06	0.05	0.03	0.01	-0.01	-0.03	-0.04	-0.06
p0imp_c	0	0.08	0.11	0.11	0.03	-0.11	-0.27	-0.43	-0.60	-0.76	-0.91	-1.03	-1.13	-1.21
p0realdev	0	0.09	0.14	0.12	0.01	-0.15	-0.33	-0.52	-0.71	-0.88	-1.03	-1.16	-1.26	-1.35
p0toft	0	-0.04	-0.10	-0.14	-0.13	-0.07	0.02	0.12	0.23	0.34	0.44	0.54	0.61	0.68
p1cap_i	0	-0.30	-0.78	-1.15	-1.39	-1.52	-1.57	-1.55	-1.48	-1.38	-1.26	-1.13	-0.99	-0.86
p2tot_i	0	-0.03	-0.13	-0.16	-0.15	-0.16	-0.19	-0.24	-0.28	-0.32	-0.36	-0.39	-0.42	-0.44
p3tot	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
p4tot	0	0.05	0.02	-0.03	-0.10	-0.18	-0.25	-0.31	-0.37	-0.42	-0.47	-0.50	-0.52	-0.54
p5tot	0	0.08	0.19	0.31	0.43	0.51	0.55	0.54	0.52	0.48	0.42	0.36	0.31	0.25
phi	0	-0.08	-0.11	-0.11	-0.03	0.11	0.27	0.44	0.61	0.77	0.92	1.04	1.15	1.23
emp_jobs	0	-0.67	-1.47	-2.05	-2.52	-2.92	-3.26	-3.55	-3.78	-3.96	-4.10	-4.19	-4.25	-4.28
employ_io	0	-0.18	-0.51	-0.86	-1.20	-1.53	-1.84	-2.10	-2.33	-2.52	-2.66	-2.76	-2.83	-2.87
real_wage_c	0	0.31	0.76	1.16	1.48	1.72	1.88	1.96	1.97	1.94	1.88	1.80	1.70	1.60
jobsupply_dl	0	0.01	0.16	0.28	0.30	0.28	0.24	0.19	0.15	0.11	0.08	0.06	0.04	0.03
jobsupply_fi	0	-17.80	-29.06	-31.43	-33.05	-33.85	-34.37	-34.74	-35.03	-35.27	-35.48	-35.65	-35.80	-35.92
jobsupply_fl	0	0.00	0.04	0.18	0.37	0.52	0.61	0.63	0.59	0.51	0.40	0.27	0.14	0.01

TABLE AC4 Policy Simulation – Scenario 1 (Cumulative Percentage Deviation from Baseline) (Industry Output)

<i>Industry</i>	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
1 agric	0	-0.07	-0.19	-0.32	-0.49	-0.68	-0.88	-1.08	-1.25	-1.39	-1.50	-1.58	-1.62	-1.63
2 coal	0	-0.07	-0.18	-0.30	-0.44	-0.62	-0.80	-0.99	-1.16	-1.31	-1.43	-1.53	-1.59	-1.63
3 gold	0	-0.17	-0.44	-0.69	-0.94	-1.18	-1.40	-1.59	-1.76	-1.89	-1.98	-2.04	-2.07	-2.08
4 othermining	0	-0.04	-0.10	-0.15	-0.21	-0.30	-0.41	-0.54	-0.66	-0.78	-0.89	-0.97	-1.04	-1.09
5 foodbev	0	-0.06	-0.25	-0.51	-0.80	-1.10	-1.38	-1.63	-1.85	-2.05	-2.21	-2.35	-2.45	-2.53
6 textfoot	0	-0.05	-0.28	-0.58	-0.92	-1.26	-1.58	-1.88	-2.14	-2.38	-2.58	-2.75	-2.88	-2.98
7 woodpaper	0	-0.08	-0.30	-0.58	-0.90	-1.22	-1.53	-1.81	-2.06	-2.27	-2.45	-2.59	-2.70	-2.77
8 petrochem	0	-0.04	-0.15	-0.32	-0.54	-0.82	-1.11	-1.39	-1.66	-1.90	-2.11	-2.29	-2.43	-2.54
9 glassnonmet	0	-0.06	-0.17	-0.35	-0.59	-0.88	-1.19	-1.48	-1.76	-2.01	-2.23	-2.41	-2.55	-2.66
10 metalmach	0	-0.07	-0.23	-0.47	-0.78	-1.13	-1.47	-1.79	-2.07	-2.31	-2.50	-2.66	-2.77	-2.85
11 electrical	0	-0.12	-0.38	-0.72	-1.11	-1.49	-1.82	-2.11	-2.35	-2.54	-2.69	-2.79	-2.87	-2.91
12 radiotv	0	-0.06	-0.18	-0.38	-0.66	-0.99	-1.34	-1.67	-1.98	-2.26	-2.50	-2.70	-2.86	-2.98
13 transequip	0	-0.07	-0.24	-0.47	-0.79	-1.13	-1.48	-1.79	-2.06	-2.30	-2.49	-2.63	-2.74	-2.82
14 othermanuf	0	-0.03	-0.09	-0.16	-0.26	-0.40	-0.58	-0.78	-1.00	-1.22	-1.42	-1.61	-1.78	-1.92
15 electricity	0	-0.07	-0.24	-0.47	-0.73	-1.01	-1.28	-1.55	-1.78	-1.99	-2.17	-2.31	-2.43	-2.51
16 water	0	-0.06	-0.26	-0.54	-0.82	-1.11	-1.37	-1.62	-1.84	-2.02	-2.18	-2.31	-2.41	-2.48
17 construc	0	-0.41	-1.10	-1.84	-2.42	-2.79	-3.03	-3.16	-3.22	-3.22	-3.17	-3.09	-2.99	-2.88
18 trade	0	-0.08	-0.27	-0.52	-0.81	-1.11	-1.40	-1.67	-1.91	-2.13	-2.30	-2.45	-2.56	-2.64
19 hotels	0	-0.03	-0.09	-0.15	-0.23	-0.34	-0.49	-0.66	-0.85	-1.04	-1.23	-1.41	-1.57	-1.72
20 transser	0	-0.07	-0.18	-0.33	-0.52	-0.75	-1.02	-1.30	-1.56	-1.80	-2.01	-2.17	-2.31	-2.41
21 comm	0	-0.05	-0.15	-0.27	-0.46	-0.71	-0.99	-1.27	-1.53	-1.76	-1.96	-2.13	-2.26	-2.35
22 financial	0	-0.07	-0.20	-0.37	-0.60	-0.86	-1.13	-1.40	-1.65	-1.87	-2.05	-2.20	-2.32	-2.40
23 estate	0	-0.01	-0.11	-0.28	-0.54	-0.84	-1.14	-1.42	-1.66	-1.88	-2.05	-2.19	-2.30	-2.37
24 business	0	-0.10	-0.32	-0.59	-0.89	-1.20	-1.49	-1.75	-1.98	-2.17	-2.33	-2.45	-2.54	-2.60
25 gengov	0	-0.11	-0.33	-0.57	-0.81	-1.06	-1.30	-1.52	-1.72	-1.89	-2.02	-2.13	-2.21	-2.27
26 healthsoc	0	-0.11	-0.19	-0.28	-0.40	-0.58	-0.82	-1.07	-1.32	-1.54	-1.73	-1.89	-2.01	-2.10
27 otherser	0	-0.17	-0.54	-0.93	-1.34	-1.71	-2.05	-2.34	-2.59	-2.79	-2.94	-3.05	-3.13	-3.17

TABLE AC5 Policy Simulation – Scenario 2 (Cumulative Percentage Deviation from Baseline) (Macro Variables)

<i>Macros</i>	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
x0cif_c	0	-0.07	-0.24	-0.40	-0.52	-0.61	-0.67	-0.73	-0.77	-0.80	-0.83	-0.85	-0.87	-0.88
x0gdpexp	0	-0.03	-0.11	-0.19	-0.29	-0.39	-0.49	-0.59	-0.68	-0.77	-0.85	-0.93	-0.99	-1.05
x0gne	0	-0.07	-0.19	-0.32	-0.45	-0.55	-0.64	-0.73	-0.81	-0.89	-0.95	-1.01	-1.05	-1.09
x0imp_c	0	-0.07	-0.24	-0.40	-0.52	-0.61	-0.67	-0.73	-0.77	-0.80	-0.83	-0.85	-0.87	-0.88
x1cap_i	0	0.00	-0.02	-0.06	-0.13	-0.21	-0.31	-0.40	-0.50	-0.59	-0.68	-0.76	-0.83	-0.90
x2tot_i	0	-0.19	-0.48	-0.79	-1.04	-1.20	-1.30	-1.38	-1.43	-1.46	-1.47	-1.48	-1.47	-1.46
x3tot	0	-0.04	-0.12	-0.21	-0.30	-0.38	-0.47	-0.56	-0.64	-0.72	-0.79	-0.86	-0.92	-0.97
x3tot_ill	0	-8.00	-15.50	-19.15	-21.18	-22.29	-22.92	-23.31	-23.55	-23.72	-23.84	-23.92	-23.97	-24.01
x3tot_leg	0	0.11	0.15	0.12	0.06	-0.02	-0.11	-0.20	-0.29	-0.37	-0.45	-0.53	-0.59	-0.65
x4tot	0	0.03	0.05	0.08	0.05	-0.01	-0.10	-0.19	-0.29	-0.38	-0.48	-0.57	-0.65	-0.73
x5tot	0	-0.04	-0.12	-0.21	-0.30	-0.38	-0.47	-0.56	-0.64	-0.72	-0.79	-0.86	-0.92	-0.97
x5tot_ill	0	-5.58	-10.72	-15.72	-19.15	-21.60	-23.46	-24.90	-26.05	-26.98	-27.73	-28.34	-28.84	-29.25
x5tot_leg	0	0.32	0.54	0.76	0.89	0.96	1.00	1.01	1.02	1.01	1.01	1.00	0.99	0.99
p0cif_c	0	0.05	0.04	0.03	-0.01	-0.06	-0.11	-0.16	-0.22	-0.28	-0.33	-0.39	-0.43	-0.48
p0gdpexp	0	0.00	-0.01	-0.02	-0.01	0.01	0.02	0.03	0.03	0.04	0.05	0.05	0.05	0.05
p0gne	0	0.00	-0.01	-0.01	0.00	0.01	0.01	0.01	0.00	0.00	-0.01	-0.01	-0.02	-0.02
p0imp_c	0	0.05	0.04	0.03	-0.01	-0.06	-0.11	-0.16	-0.22	-0.28	-0.33	-0.39	-0.43	-0.48
p0realdev	0	0.05	0.05	0.04	0.00	-0.06	-0.13	-0.19	-0.25	-0.32	-0.38	-0.44	-0.49	-0.53
p0toft	0	-0.01	-0.02	-0.02	-0.01	0.01	0.04	0.07	0.11	0.14	0.18	0.21	0.24	0.27
p1cap_i	0	-0.09	-0.28	-0.41	-0.49	-0.53	-0.56	-0.57	-0.58	-0.57	-0.56	-0.54	-0.52	-0.50
p2tot_i	0	0.01	-0.03	-0.05	-0.05	-0.05	-0.06	-0.07	-0.09	-0.11	-0.13	-0.15	-0.16	-0.18
p3tot	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
p4tot	0	0.04	0.02	0.00	-0.02	-0.05	-0.07	-0.09	-0.11	-0.13	-0.15	-0.17	-0.19	-0.21
p5tot	0	0.00	-0.01	0.02	0.05	0.08	0.10	0.11	0.12	0.13	0.13	0.13	0.12	0.12
phi	0	-0.05	-0.04	-0.03	0.01	0.06	0.11	0.16	0.22	0.28	0.33	0.39	0.44	0.48
emp_jobs	0	-0.50	-1.02	-1.32	-1.52	-1.67	-1.81	-1.93	-2.04	-2.13	-2.22	-2.30	-2.37	-2.43
employ_io	0	-0.07	-0.21	-0.35	-0.47	-0.59	-0.71	-0.83	-0.93	-1.03	-1.12	-1.20	-1.27	-1.33
real_wage_c	0	0.13	0.29	0.43	0.54	0.63	0.71	0.77	0.81	0.85	0.87	0.89	0.91	0.92
jobsupply_dl	0	0.00	0.20	0.38	0.47	0.51	0.52	0.53	0.52	0.51	0.50	0.49	0.49	0.48
jobsupply_fi	0	-17.92	-29.44	-32.15	-34.09	-35.09	-35.73	-36.15	-36.45	-36.67	-36.83	-36.95	-37.04	-37.09

TABLE AC6 Policy Simulation – Scenario 2 (Cumulative Percentage Deviation from Baseline) (Industry Output)

<i>Industry</i>	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
1 agric	0	-0.03	-0.08	-0.13	-0.19	-0.26	-0.33	-0.41	-0.48	-0.54	-0.60	-0.64	-0.68	-0.70
2 coal	0	-0.03	-0.09	-0.13	-0.19	-0.25	-0.32	-0.39	-0.46	-0.52	-0.58	-0.62	-0.66	-0.69
3 gold	0	-0.08	-0.21	-0.30	-0.38	-0.46	-0.53	-0.59	-0.65	-0.71	-0.75	-0.79	-0.82	-0.84
4 othermining	0	-0.02	-0.05	-0.07	-0.10	-0.13	-0.18	-0.22	-0.27	-0.32	-0.36	-0.40	-0.43	-0.45
5 foodbev	0	-0.03	-0.15	-0.32	-0.51	-0.68	-0.83	-0.96	-1.07	-1.17	-1.26	-1.34	-1.40	-1.46
6 textfoot	0	-0.03	-0.22	-0.45	-0.70	-0.92	-1.11	-1.27	-1.41	-1.53	-1.63	-1.72	-1.80	-1.87
7 woodpaper	0	-0.03	-0.17	-0.33	-0.50	-0.65	-0.79	-0.92	-1.03	-1.13	-1.22	-1.31	-1.38	-1.44
8 petrochem	0	-0.02	-0.08	-0.17	-0.28	-0.41	-0.53	-0.64	-0.75	-0.86	-0.95	-1.04	-1.11	-1.18
9 glassnonmet	0	-0.02	-0.08	-0.16	-0.26	-0.37	-0.49	-0.60	-0.72	-0.82	-0.92	-1.01	-1.09	-1.16
10 metalmach	0	-0.03	-0.12	-0.22	-0.35	-0.50	-0.63	-0.76	-0.88	-0.98	-1.08	-1.16	-1.24	-1.30
11 electrical	0	-0.05	-0.17	-0.30	-0.46	-0.61	-0.74	-0.86	-0.97	-1.06	-1.15	-1.22	-1.29	-1.34
12 radiotv	0	-0.03	-0.09	-0.18	-0.30	-0.43	-0.58	-0.71	-0.84	-0.96	-1.08	-1.18	-1.26	-1.34
13 transequip	0	-0.03	-0.11	-0.20	-0.32	-0.45	-0.59	-0.71	-0.83	-0.93	-1.03	-1.11	-1.19	-1.25
14 othermanuf	0	-0.01	-0.04	-0.05	-0.05	-0.05	-0.07	-0.11	-0.16	-0.22	-0.29	-0.36	-0.43	-0.50
15 electricity	0	-0.03	-0.11	-0.22	-0.34	-0.46	-0.58	-0.69	-0.80	-0.89	-0.98	-1.06	-1.13	-1.19
16 water	0	-0.02	-0.14	-0.30	-0.45	-0.59	-0.71	-0.82	-0.92	-1.01	-1.09	-1.17	-1.23	-1.28
17 construc	0	-0.16	-0.41	-0.68	-0.89	-1.03	-1.13	-1.21	-1.27	-1.32	-1.35	-1.37	-1.39	-1.40
18 trade	0	-0.03	-0.13	-0.26	-0.40	-0.53	-0.66	-0.78	-0.88	-0.98	-1.07	-1.15	-1.21	-1.27
19 hotels	0	-0.02	-0.06	-0.10	-0.16	-0.24	-0.32	-0.40	-0.49	-0.57	-0.66	-0.74	-0.81	-0.88
20 transser	0	-0.03	-0.05	-0.06	-0.08	-0.13	-0.21	-0.30	-0.40	-0.50	-0.59	-0.68	-0.77	-0.84
21 comm	0	-0.02	-0.03	-0.02	-0.04	-0.09	-0.16	-0.25	-0.35	-0.45	-0.55	-0.63	-0.72	-0.79
22 financial	0	-0.02	-0.05	-0.08	-0.12	-0.19	-0.28	-0.37	-0.47	-0.57	-0.66	-0.75	-0.82	-0.89
23 estate	0	0.00	-0.04	-0.09	-0.18	-0.28	-0.39	-0.49	-0.59	-0.68	-0.77	-0.85	-0.92	-0.98
24 business	0	-0.04	-0.14	-0.25	-0.38	-0.49	-0.61	-0.71	-0.81	-0.91	-0.99	-1.06	-1.13	-1.19
25 gengov	0	-0.03	-0.12	-0.22	-0.31	-0.40	-0.49	-0.58	-0.66	-0.74	-0.82	-0.89	-0.95	-1.00
26 healthsoc	0	-0.03	0.04	0.17	0.29	0.35	0.35	0.31	0.24	0.15	0.06	-0.04	-0.14	-0.23
27 otherser	0	-0.06	-0.25	-0.43	-0.61	-0.77	-0.91	-1.04	-1.16	-1.26	-1.36	-1.44	-1.51	-1.58